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REGULAR ARTICLE

Prospect Theory in the Heterogeneous Agent Model

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Abstract Using the Heterogeneous Agent Model framework, we incorporate an extension based on Prospect Theory into a popular agent-based asset pricing model. This extension covers the phenomenon of loss aversion manifested in risk aversion and asymmetric treatment of gains and losses. Using Monte Carlo methods, we investigate behavior and statistical properties of the extended model and assess how our extension is manifested in different strategies. We show that, on the one hand, the Prospect Theory extension keeps the essential underlying mechanics of the model intact, but on the other hand it considerably changes the model dynamics. Stability of the model is increased and fundamentalists may be able to survive in the market more easily. When only the fundamentalists are loss-averse, other strategies profit more.

Keywords Heterogeneous Agent Model · Prospect Theory · Behavioral finance · Stylized facts

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1 Introduction

This paper introduces features of loss aversion and gain–loss asymmetry into the popular Brock and Hommes (1998) asset pricing model. Our work is based on findings of the iconic Prospect Theory (PT) of Kahneman and Tversky (1979), which describes the way people choose between probabilistic alternatives involving risk and is inherently a critique of other, more normative decision-making economic theories. Back in 1979, Kahneman and Tversky found that the actual behavior of human beings might be very different to what major economic theories had assumed, namely in relation to risk and attitude towards losses. According to PT, instead of behaving fully rationally and using perfect cognitive calculations, people make decisions with respect to gains and losses rather than the final outcome. Losses also have a greater emotional impact than an equivalent amount of gains; they hurt more than equal gains please. The extension that we develop in this work is aimed at accounting for these empirically observed irrationalities. Over the years, PT has become one of the most influential theories, merging psychology with economics. As Belsky and Gilovich (2010, p. 52) aptly remark, “If Richard Thaler’s concept of mental accounting is one of two pillars upon which the whole of behavioral economics rests, then Prospect Theory is the other”. Kahneman and Tversky (1979) paper is the most cited work to ever appear in Econometrica (Chang et al. 2011, p. 30).

In contemporary economic theory, there is little doubt that economic agents are heterogeneous to some extent. In the late 1980s and early 1990s, empirical micro studies reported heterogeneity as an empirically significant phenomenon. Frankel and Froot (1990) attribute the reason for the divergence of the US dollar interest rate from macroeconomic fundamentals at the beginning of the 1980s to the existence of speculative traders; Hansen and Heckman (1996, p. 101) indicate a “considerable interest in Heterogeneous Agent Models in the real business cycle literature research”; and Brock and Hommes (1997, 1998) theoretically prove that it may be individually ‘rational’ for agents not to follow rational expectations and to behave, instead, according to simple predictors. Evans and Honkapohja (2001) explain that agents lack the required sophistication to rationally form expectations; Mankiw et al. (2004) draw attention to statistically significant disagreement in survey data on inflation expectations even among professional economists; Branch (2004) summarizes studies documenting “failure of the rational expectations hypothesis to account for survey data on inflationary expectations”; and Vissing-Jorgensen (2004) conducts an analysis of qualitative telephone survey data on US stock markets from 1998 to 2002, in which the author concludes that there is significant disagreement among the investors regarding expected profits. As an important experimental contribution to the hypothesis of heterogeneity of market participants, Hommes (2011) provides ‘evidence from the lab’ of the presence of heterogeneous expectations in an experimental financial market.

The primary objective of this paper is thus to extend the original Brock and Hommes (1998) model with features of PT and, at the same time, keep the intrinsic mechanics of the model intact in order to preserve its simple, stylized nature. The original Brock and Hommes (1998) Adaptive Belief System (ABS) is a financial market application of the evolutionary selection system proposed by Brock and Hommes (1997), in which agents switch among different forecasting strategies according to past relative profitability of these strategies. The ABS is a discounted value asset pricing model extended to heterogeneous beliefs, in which the agents have the possibility to invest in either a risk-free or a risky asset. Our analysis consists of using Monte Carlo methods to investigate the behavior and statistical properties of the extended versions of the model and assess the economic relevance of results.

One of the most important stimuli to induce the development of Agent-based Models (ABMs) in economics was certainly an erosion of trust in the Efficient Market Hypothesis (EMH)—the EMH asserts, in Eugene Fama's words, that "…security prices at any time 'fully reflect' all available information…" (Fama 1970, p. 383)—and in the Rational Expectations Theory in the late 1970s and early 1980s. This was largely due to increased focus on the study of several stylized empirical facts—according to Cont (2001, p. 224), "The seemingly random variations of asset prices do share some quite non-trivial statistical properties. Such properties, common across a wide range of instruments, markets and time periods are called stylized empirical facts". The most essential difference between natural sciences and economics is arguably the fact that decisions of economic agents are determined by their expectations of the future and contingent on them; hence, the study of how these beliefs are formed plays a vital part of any economic theory.

Several scholars have published papers which confront the EMH with empirical data mainly from the perspective of non-normal returns,[1] systematic deviations of asset prices from their fundamental value, and excessive stock price volatility; it has proved impossible to attribute these phenomena to the EMH or to explain them within the rational expectations framework. Offering an insightful survey on the volatility issue at that time, West (1988) summarizes and interprets the literature related to this field. The author finds that neither rational bubbles nor any standard models for expected returns adequately explain stock price volatility and emphasizes the necessity to introduce alternative models to offer a better explanation of the apparent contradiction between the EMH, the Rational Expectations Theory, and empirical findings.

This paper is structured as follows: immediately after the present Introduction, Sect. 2 summarizes the main features of Prospect Theory and Sect. 3 describes the mathematical structure and underlying mechanics of the original Brock and Hommes (1998) model; Sect. 4 develops the behavioral extension based on Prospect Theory, while Sect. 5 describes the numerical simulations using Monte Carlo methods; Sect. 6 highlights the main results of the simulations and Sect. 7 concludes the paper.

2 Prospect Theory

Proposed in the seminal paper of Kahneman and Tversky (1979), PT is a critique of then-mainstream expected utility theory. Using convincing evidence obtained from

[1] According to Ehrentreich (2007, p. 56), at the time when the foundations of the EMH were laid, logarithmic asset returns were assumed to be distributed normally and the prices therefore followed the log-normal distribution.

questionnaires, the authors illustrate several issues with the concept of expected utility and its applicability to real-life human decision-making. The most critical objection is the incapacity of the expected utility theory to explain certain ‘irrational’[2] choices made by people. As a result, Kahneman and Tversky (1979)—and later Tversky and Kahneman (1992)—propose a brand new descriptive[3] theory which takes all such ‘irrational’ choices into account and explains them rigorously, using the so-called weighting and value functions. Three major features of PT are the following:

1. *Existence of a reference point* PT suggests that people make decisions in relation to gains and losses with respect to a certain reference point, rather than in terms of final wealth.
2. *Differences in treatment of gains and losses* While most people are risk-seeking towards losses, the same people are risk-averse towards gains. Moreover, most are generally loss-averse which explains why the value function is steeper for losses than for gains.
3. *Distorted understanding of probability* According to PT, the average person underestimates large probabilities and overestimates small probabilities. Given the proposed specification and shape of the weighting function, weighting is not linear in probability.

2.1 Value and weighting functions

According to PT, any selection process consists of two parts, editing and evaluation. In the former, an individual conducts a preliminary analysis of the available prospects in order to facilitate the selection, and in the latter, the individual evaluates the edited prospects, assigns a value to each of them, and makes the final decision. The reader can find details about the editing phase in Kahneman and Tversky (1979, pp. 274–275); we present here the most essential properties of the evaluation phase.

The overall value V of an edited prospect is formulated in terms of the weighting function π and the value function v. π expresses probabilities of the prospect’s respective outcomes, while v assigns a specific value to each of these outcomes. Denoting by $(x, p; y, q)$ a prospect which pays x, y, or 0 with probability p, q, and $1 - p - q$, respectively, the basic equation which assigns value to a regular prospect[4] is given as follows:

$$V(x, p; y, q) = \pi(p) \cdot v(x) + \pi(q) \cdot v(y), \tag{1}$$

where it is assumed that $v(0) = 0$, $\pi(0) = 0$, and $\pi(1) = 1$. It is important to note that the weighting function is not a probability measure and typical properties of probability need not be valid for it, and that the value function is defined with respect to a reference point, which is usually given as $x = 0$—that is, the point at which a

[2] The ‘irrationality’ is meant within the expected utility theory.

[3] PT is descriptive in the sense that it tries to capture the real-world decision-making whereas the expected utility theory is de facto normative—it models how people are supposed to decide.

[4] Regular prospect is a prospect such that either $p + q < 1$, $x \geqslant 0 \geqslant y$, or $x \leqslant 0 \leqslant y$. Evaluation of prospects which are not regular follows a different rule—details are provided in Kahneman and Tversky (1979, p. 276).

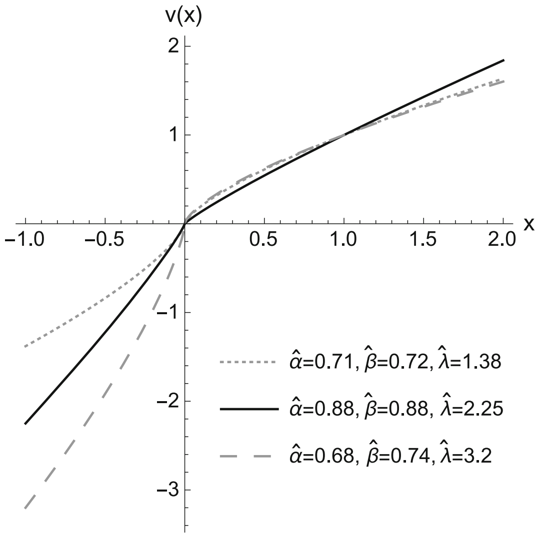

Fig. 1 Estimates of the value function $v(x)$ using results of Harrison and Rutström (2009) in dotted gray, Tversky and Kahneman (1992) in solid black, and Tu (2005) in dashed gray

gain changes to a loss and vice versa. In our extension of the model, we do not directly consider the effect of the weighting function and, hence, do not describe its properties in detail here.

The value function v satisfies the following properties: it is increasing $\forall x$, i.e., $v'(x) > 0$ always holds, convex below the reference point, i.e., $v''(x) > 0$ for $x < 0$, and concave above it, i.e., $v''(x) < 0$ for $x > 0$. Additionally, it is usually thought to be steeper for losses than for gains. A number of scholars have estimated the shape of the value function, most often using a piecewise power function proposed by Tversky and Kahneman (1992). This function is of the following form:

$$v(x) = \begin{cases} x^{\alpha}, & x \geqslant 0; \\ -\lambda \cdot (-x)^{\beta}, & x < 0; \end{cases} \tag{2}$$

where the parameters α and β determine the curvature of the function for gains and losses, respectively, relative to the reference point of $x = 0$, and λ is a parameter that measures the degree of loss aversion.

Estimating Eq. 2, Tversky and Kahneman (1992) report $\hat{\alpha} = 0.88$, $\hat{\beta} = 0.88$, and $\hat{\lambda} = 2.25$, Harrison and Rutström (2009) $\hat{\alpha} = 0.71$, $\hat{\beta} = 0.72$, and $\hat{\lambda} = 1.38$, and, e.g., Tu (2005) $\hat{\alpha} = 0.68$, $\hat{\beta} = 0.74$, and $\hat{\lambda} = 3.2$. All these versions are plotted in Fig. 1.

2.2 Relevance for financial markets

Since the time of the formulation of PT, several studies have confirmed its relevance for financial markets. One of the most cited applications of PT is an aid in explanation of the so-called disposition effect. The term was first coined by Shefrin and Statman (1985) and refers to a tendency to "…sell winners too early and ride losers too long"

(Shefrin and Statman 1985, p. 778), which essentially means that traders tend to hold value-losing assets too long and not hold value-gaining assets long enough. Using the PT value function, the authors explain the disposition effect for an investor who owns a losing stock as a gamble between selling the stock now and thereby realizing a loss, or holding the stock for an additional period given, say, a 50–50 chance between loosing further value or breaking even. As the investor finds himself in the 'negative domain' with respect to the reference point given here as the break-even point (that is, $x \leqslant 0$), the choice between the two options is associated with the convex part of the value function. This implies that the investor selects the second option and thus 'rides the loser too long'.

Li and Yang (2013) also attempt to explain the disposition effect using findings of PT. The authors build a general equilibrium model and, besides the disposition effect, also focus on trading volume and asset prices. The results suggest that loss aversion implied by PT tends to predict a reversed disposition effect and price reversal for stocks with non-skewed dividends. Yao and Li (2013), on the other hand, investigate trading patterns in the market with Prospect-Theoretical investors, who base their choices on the value and weighting functions and related features of PT. The authors find that the three main features of PT can be regarded as behavioral causes for the negative-feedback trading. The authors subsequently construct a market with traders with PT preferences on the one hand, and traders who maximize the Constant Relative Risk Aversion (CRRA) utility function on the other hand, and discover that the individual PT preferences might cause contrarian noise trading.

Some other research efforts related to the study of PT traits in financial markets are made by Grüne and Semmler (2008) who try to attribute some of the most frequently observed asset price characteristics—yet not explained by 'standard' preferences—to the loss aversion feature of traders. Giorgi and Legg (2012) use the weighting function and show that dynamic models of portfolio choice might be consistently and meaningfully extended by probability weighting. Zhang and Semmler (2009) further investigate properties of the model proposed by Barberis et al. (2001) using time series data; they conclude that models with PT features are able to better explain some financial 'puzzles', such as the equity premium puzzle.[5] Finally, for instance, Giorgi et al. (2010) explore aspects of Cumulative Prospect Theory—a modification of the original PT developed by Tversky and Kahneman (1992)—and find that financial markets' equilibria need not exist under the assumptions of PT.

3 Heterogeneous Agent Modeling framework

Our modeling framework follows the Brock and Hommes (1998) Heterogeneous Agent Model (HAM) approach, slightly reformulated in Hommes (2006). We consider a risk-free asset that pays a fixed rate of return r and is perfectly elastically supplied and a risky asset that pays an uncertain dividend. Denoting by p_t and y_t

[5] The equity premium puzzle is a phenomenon that the average return on equity is far greater than return on a risk-free asset. Such a characteristic has been observed in many markets. The term was first coined by Mehra and Prescott (1985).

the ex-dividend price of the risky asset and its random dividend process, respectively, and z_t the amount of the risky asset an agent purchases at time t, each agent's wealth dynamics takes the following form:

$$W_{t+1} = R \cdot W_t + z_t \cdot (p_{t+1} + y_{t+1} - R \cdot p_t), \tag{3}$$

where R is the gross risk-free return rate equal to $1 + r$. There are H forecasting strategies or, equivalently, H distinct classes of agents. Let $E_{h,t}$ and $V_{h,t}$, respectively, denote the belief of an agent who uses forecasting strategy h about conditional mean and conditional variance of wealth, $1 \leqslant h \leqslant H$. It is assumed that all agents maximize the same Constant Absolute Risk Aversion (CARA) utility function of wealth in the form $F(W) = -\exp(-a \cdot W)$, where a is a risk aversion parameter. Given the mean–variance maximization, the optimal demand $z^*_{h,t}$ for the risky asset of agents of type h then solves the following maximization problem:

$$\max_{z_{h,t}} \left\{ E_{h,t}(W_{t+1}) - \frac{a}{2} \cdot V_{h,t}(W_{t+1}) \right\}. \tag{4}$$

The demand $z^*_{h,t}$ is then

$$z^*_{h,t} = \frac{E_{h,t}(p_{t+1} + y_{t+1} - R \cdot p_t)}{a \cdot V_{h,t}(p_{t+1} + y_{t+1} - R \cdot p_t)}, \tag{5}$$

which, assuming that $V_{h,t} \equiv \sigma^2 \; \forall h, t$, simplifies to

$$z^*_{h,t} = \frac{E_{h,t}(p_{t+1} + y_{t+1} - R \cdot p_t)}{a \cdot \sigma^2}. \tag{6}$$

Denoting by z^s the supply of outside risky shares per trader and $n_{h,t}$ the proportion of agents using forecasting strategy h, the demand–supply equilibrium is

$$\sum_{h=1}^{H} n_{h,t} \cdot \frac{E_{h,t}(p_{t+1} + y_{t+1} - R \cdot p_t)}{a \cdot \sigma^2} = z^s, \tag{7}$$

where, again, H is the total number of forecasting strategies. In the case of zero supply of outside shares, i.e., $z^s = 0$, Eq. 7 becomes

$$R \cdot p_t = \sum_{h=1}^{H} n_{h,t} \cdot E_{h,t}(p_{t+1} + y_{t+1}). \tag{8}$$

Now, if all traders were identical and their expectations homogeneous, we would obtain a simplified version of Eq. 8 called the arbitrage market equilibrium of the form

$$R \cdot p_t = E_{h,t}(p_{t+1} + y_{t+1}). \tag{9}$$

Equation 9 asserts that the price of the risky asset in this period is equal to the expected sum of next period's price and dividend, discounted by the gross risk-free interest rate. In this case of homogeneous expectations, provided that the transversality condition

$$\lim_{t\to\infty} \frac{E_t\left(p_{t+k}\right)}{(1+r)^k} = 0 \tag{10}$$

holds,[6] the fundamental price of the risky asset is given as

$$p_t^* = \sum_{k=1}^{\infty} \frac{E_t\left(y_{t+k}\right)}{(1+r)^k}. \tag{11}$$

The price p_t^* is the equilibrium price of the risky asset in a perfectly efficient market with fully rational traders and, as can be seen directly from Eq. 11, it depends on the expectation of the stochastic dividend process y_t, $E_t\left(y_t\right)$. Assuming the dividend process y_t is independent, identically distributed with mean $\bar{y}$, the fundamental price p_t^* becomes constant $\forall t$ and is given by

$$p^* = \sum_{k=1}^{\infty} \frac{\bar{y}}{(1+r)^k} = \frac{\bar{y}}{r}. \tag{12}$$

The deviation from the fundamental price is defined as follows:

$$x_t = p_t - p_t^*. \tag{13}$$

There are two additional assumptions made by Brock and Hommes (1998):

1. Expectations about future dividends y_{t+1} are the same for all agents, regardless of the specific forecasting strategy they use, and are equal to the true conditional expectation. In other words, $E_{h,t}\left(y_{t+1}\right) = E_t\left(y_{t+1}\right)\ \forall h, t$.
2. Agents believe that the stock price might deviate from the fundamental price p_t^* by a certain function f_h, which depends on previous deviations from the fundamental price $x_{t-1}, \ldots, x_{t-K}$. This assumption can be stated as

$$E_{h,t}\left(p_{t+1}\right) = E_t\left(p_{t+1}^*\right) + f_h\left(x_{t-1}, \ldots, x_{t-K}\right)\ \forall h, t. \tag{14}$$

It is now important to note two crucial facts: first, Assumption 1 above implies that all agents have homogeneous expectations about future dividends, that is, the heterogeneity of the model lies in Assumption 2. Secondly, the asset price in period $t+1$, p_{t+1}, is predicted using the price realized in period $t-1$—not in period t—as the agents are yet unaware of the price p_t when they make their predictions. This fact directly follows from Eq. 7.

[6] Hommes (2013, p. 162) remarks that Eq. 9 is also satisfied by the so-called rational bubble solution of the form $p_t = p_t^* + (1+r)^t \cdot \left(p_0 - p_0^*\right)$. However, this solution does not satisfy the transversality (or 'no-bubbles') condition.

Next, Brock and Hommes (1998) define realized excess return as $\mathcal{R}_{t+1} = p_{t+1} + y_{t+1} - R \cdot p_t$. The realized excess return over period t to period $t+1$ can be expressed in deviations from the fundamental value as follows:

$$\begin{aligned}\mathcal{R}_{t+1} &= p_{t+1} + y_{t+1} - R \cdot p_t = x_{t+1} + p^*_{t+1} + y_{t+1} - R \cdot x_t - R \cdot p^*_t \\ &= x_{t+1} - R \cdot x_t + \underbrace{p^*_{t+1} + y_{t+1} - E_t\left(p^*_{t+1} + y_{t+1}\right)}_{\delta_{t+1}} \\ &\quad + \underbrace{E_t\left(p^*_{t+1} + y_{t+1}\right) - R \cdot p^*_t}_{=0} \\ &= x_{t+1} - R \cdot x_t + \delta_{t+1}, \end{aligned} \tag{15}$$

where the latter underbrace holds because Eq. 12 is satisfied. The term δ_{t+1} is a Martingale Difference Sequence with respect to an information set $\mathcal{F}_t$, that is, we have $E\left(\delta_{t+1}|\mathcal{F}_t\right) = 0\ \forall t$.

3.1 Fitness measure

The fitness measure of strategy h, $U_{h,t}$, depends on the stochastic dividend process of the risky asset and is defined as

$$U_{h,t} = \mathcal{R}_{t+1} \cdot z^*_{h,t} = (x_{t+1} - R \cdot x_t + \delta_{t+1}) \cdot z^*_{h,t}. \tag{16}$$

Two cases will now be distinguished:

1. The case of $\delta_{t+1} = 0$ corresponds to deterministic nonlinear pricing dynamics with constant dividend $\bar{y}$. According to Hommes (2006, p. 1168) who uses a slightly modified understanding of the time notation,[7] Eq. 16, written in deviations, reduces to
$$U_{h,t} = (x_t - R \cdot x_{t-1}) \cdot \frac{f_{h,t-1} - R \cdot x_{t-1}}{a \cdot \sigma^2}, \tag{17}$$
where $f_{h,t-1}$ is the forecasting function of type h.
2. The case in which the dividend is given by a stochastic process $y_t = \bar{y} + \epsilon_t$, where ϵ_t is an independent, identically distributed random variable with uniform distribution. Under these circumstances, $\delta_{t+1} = \epsilon_{t+1}$.

[7] The notation difference consists of 'shifting' time subscripts of realized excess return by one period—for this reason, Eq. 16 is reduced to Eq. 17 only after this shift.

3.2 Market proportions

Equation 8 can be reformulated in terms of deviations from the fundamental price by a substitution using Eq. 14 as

$$R \cdot x_t = \sum_{h=1}^{H} n_{h,t} \cdot E_{h,t}(x_{t+1}) \equiv \sum_{h=1}^{H} n_{h,t} \cdot f_h(x_{t-1}, \ldots, x_{t-K}), \tag{18}$$

where $n_{h,t}$ denotes the proportion of agents using the forecasting function h for their predictions. These proportions are modeled using the multinomial logit model as

$$n_{h,t} = \frac{\exp\left(\beta \cdot U_{h,t-1}\right)}{Z_{t-1}}, \tag{19}$$

where $Z_{t-1} \equiv \sum_{h=1}^{H} \exp\left(\beta \cdot U_{h,t-1}\right)$ is a normalization factor such that the proportions $n_{h,t}$ add up to 1, and β, $\beta \geqslant 0$, is a parameter called the intensity of choice, which measures the agents' sensitivity to selection of the best-performing forecasting strategy. Two extreme cases will be distinguished—if $\beta = \infty$, all agents unerringly choose the best strategy, while if $\beta = 0$, the proportions $n_{h,t}$ remain constant in time and fixed to $1/H$. The former case corresponds to the situation in which there is no noise and thus all agents select the optimal strategy, while the latter implies the existence of noise with infinite variance and thus the inability of agents to switch between strategies at all.

The functions $f_{h,t}$ are crucial for the formation of agents' expectations. Brock and Hommes (1998) propose simple forecasting rules of the form

$$f_{h,t} = g_h \cdot x_{t-1} + b_h. \tag{20}$$

The term g_h is a trend parameter which determines the trend following (or possibly reverting) strength of a particular strategy, and the term b_h is a bias parameter. For $g_h = b_h = 0$, the function $f_{h,t}$ is reduced to $f_{h,t} \equiv 0$ and corresponds to the fundamentalist belief of no price deviations from the fundamental value. Additionally, if $g_h \neq 0$, then such a trader type is called a chartist. This class of traders can be further divided into four categories: a chartist is called a pure trend chaser if $0 < g_h \leqslant R$, a strong trend chaser if $g_h > R$, a contrarian if $-R \leqslant g_h < 0$, and a strong contrarian if $g_h < -R$. Finally, the term b_h determines the nature (if any) of each agent class' bias—if $b_h < 0$ the bias is downward, while if $b_h > 0$ the bias is upward.

4 Prospect Theory extension

Despite the indisputable relevance of findings by PT for the study of human decision-making, no PT extensions of the Brock and Hommes (1998) HAM framework apparently exist. The possible reason for the absence of such ABM designs is relatively self-evident: the HAM developed by Brock and Hommes (1998) is populated with agents with CARA utility function and demand for the risky asset is derived by maximization of expected utility.

As the origins of PT are based on the critique of the expected utility theory, and subsequent development of a diametrically different approach to decisions under risk, the very basic component of the ABS—the CARA utility function—seems to be incompatible with PT. Yet, although the authors do not use the original Brock and Hommes (1998) model, Shimokawa et al. (2007) propose a relatively straightforward method to implement a number of PT features into ABMs in which the agents have CARA preferences.

There is literature that deals with the notion of PT within the field of ABMs, allowing the agents' strategies to determine the parameters of their utility functions. Cao et al. (2010) develop a framework that combines heterogeneous expectations and attitudes toward risk to test the impact of endogenous changes of agents' behavior preferences on market efficiency. They find that markets that show risk preference are more volatile, exhibit stronger volatility clustering, and are distant from efficiency. Chiarella et al. (2009) construct a model in which agents use a combination of the fundamental value of the asset and a set of chartist rules to form expectations about stock returns. The agents in this model differ in the degree of risk aversion. Tedeschi et al. (2012) introduce an order-driven market model based on the model developed by Chiarella et al. (2009). The agents in this extended model imitate each other and the authors study how a 'guru' can rise, influence the market, and subsequently fall. Castro and Parsons (2014) create an agent model based on an extension of PT called the Smooth Prospect Theory, which deals with several possible outcomes per prospect and allows for continuous probability distributions. The authors compare the price defined by a set of heterogeneous trading agents to a price obtained from a real stock exchange.

4.1 Loss aversion inclusion

The core of the model remains identical; however, extending the original Brock and Hommes (1998) model, we introduce the features of PT into the model as follows: PT traders maximize utility function of the form

$$F_l\left(W\right) = -\exp\left(-a \cdot B_{h,t} \cdot W\right), \tag{21}$$

where we denote the loss aversion parameter by $B_{h,t}$. Generally, this loss aversion parameter may differ for each agent's class and time period, hence the two subscripts. Furthermore, the subscript l distinguishes the utility functions of these PT traders from those of 'standard' traders specified in the original model; we refer to the PT traders as *loss-averse* traders since this trait is the main feature of PT that is possible to incorporate into the model using the utility function defined in Eq. 21. Other symbols in Eq. 21 have their usual meanings as given in Sect. 3. We assume that agents' wealth follows the form given in Eq. 3.

The crucial aspect of the utility function given in Eq. 21 is the loss aversion parameter $B_{h,t}$ and its specification. Partially following the proposition of Shimokawa et al. (2007, p. 211), we define the loss aversion parameter at time t and strategy h as follows:

$$B_{h,t} = \begin{cases} c_g, & U_{h,t} > 0; \\ c_l, & U_{h,t} \leqslant 0; \end{cases} \tag{22}$$

where c_g and c_l are gain and loss parameters, respectively, and $0 < c_g < c_l$. That is, the loss aversion parameter of strategy h is set to c_g if the fitness measure of that strategy is positive in the next period and to c_l if it is negative or equal to zero. This specification means that, if traders expect to incur a gain, they become loss-averse and their loss aversion parameter is set to c_g; while if they expect a loss, their loss aversion parameter is set to c_l. This allows us to mimic the value function component of Eq. 1; in this model we do not focus on the weighting function component. Note that the agents do not explicitly evaluate multiple prospects: they only set their risk aversion parameter to either c_g or c_l if they expect a gain or loss, respectively. It is important to emphasize that each agent maximizes either the original utility function $F(W) = -\exp(-a \cdot W)$ or the 'augmented' utility function F_l with the loss aversion parameter given in Eq. 21. However, any agent's forecast of the price of the risky asset in the next period is governed by Eq. 14, whether the agent is loss-averse or not.

Optimal demand $z^*_{l,t}$ of the loss-averse traders for the risky asset then solves the well-known maximization problem

$$\max_{z_{l,t}} \left\{ E_{h,t}(W_{t+1}) - \frac{a \cdot B_{h,t}}{2} \cdot V_{h,t}(W_{t+1}) \right\}, \tag{23}$$

where $V_{h,t}(W_{t+1})$ is the (loss-averse) traders' belief about next period conditional variance of wealth, and is thus given by

$$z^*_{l,t} = \frac{E_{h,t}(p_{t+1} + y_{t+1} - R \cdot p_t)}{a \cdot B_{h,t} \cdot \sigma^2}. \tag{24}$$

The fundamentals of the model remain the same: there are H distinct trading strategies or classes of agents, and each agent class maximizes a CARA utility function. L classes of agents, $0 \leqslant L \leqslant H$, are endowed with the above-specified PT feature—optimal demand of agents of these L classes for the risky asset is given by Eq. 24—while the agents of the $H - L$ remaining classes are 'standard' in that they do not exhibit the PT behavior. The general specification of the optimal demand for the risky asset, $z^*_{h,t}$, $1 \leqslant h \leqslant H$, is thus the same and given by Eq. 6 where, if hth class of agents has the PT feature (i.e., for $h \leqslant L$, $1 \leqslant h \leqslant H$), we use $z^*_{l,t}$ given by Eq. 24 instead of $z^*_{h,t}$.

The definition of the parameter $B_{h,t}$ given in Eq. 22 enables us to mimic the first two of the three major features of PT listed in the beginning of Sect. 2, that is, the loss aversion and biased treatment of gains and losses, and the relationship of decisions under risk to a reference point, by using an 'imitation' of the value function. In this application, however, we omit the third major feature of PT, the probability weighting and the weighting function, to keep the model within the bounds of the stylized, simple framework proposed by Brock and Hommes (1998). Furthermore, the curvature of the value function is neither studied within nor incorporated into the model as it is well approximated by a linear function (see Fig. 1).

4.2 Reference point

The reference point is given by the value of the fitness measure in the next period. The choice of specific numerical values for the gain and loss parameters c_g and c_l is relatively free. The inequality $0 < c_g < c_l$ is the only condition that must always hold in order to properly capture the loss aversion feature.

Note that Eq. 24 introduces a form of circularity into the framework: to be able to calculate the value of $z^*_{l,t}$, the optimal demand of the loss-averse agents for the risky asset, the agents must know the value of the loss aversion parameter $B_{h,t}$. According to Eq. 22, however, this parameter depends on the expected value of the fitness measure in the next period, which in turn depends on the value of $z^*_{l,t}$. To circumvent this issue, agents with the PT feature using strategy h first compute the fitness measure $U_{h,t}$ according to Eq. 17 as if they did not have the PT feature, then, based on Eq. 22, they set their loss aversion parameter to c_g if they expect the fitness measure to be positive or to c_l if they expect it to be negative or equal to zero, and finally they calculate the value of $z^*_{l,t}$. This value then expresses the final fitness measure $\tilde{U}_{h,t}$ of the hth strategy at time t. In other words, the calculation of the fitness measure $\tilde{U}_{h,t}$ values for the strategies that have the PT feature is a two-step process, during which the agents first assess the overall profitability of their strategy by computing the fitness measure of the analogous strategy without the PT feature, and then they scale this value by the $B_{h,t}$ parameter's value.

To summarize, the ABS extended with the PT loss aversion becomes

$$\begin{aligned} R \cdot x_t &= \sum_{h=1}^{H} n_{h,t} \cdot f_{h,t} + \varepsilon_t, \\ n_{h,t} &= \frac{\exp\left(\beta \cdot U_{h,t-1}\right)}{\sum_{h=1}^{H} \exp\left(\beta \cdot U_{h,t-1}\right)}, \\ U_{h,t-1} &= \begin{cases} (x_{t-1} - R \cdot x_{t-2}) \cdot \frac{f_{h,t-2} - R \cdot x_{t-2}}{a \cdot \sigma^2}, & h > L; \\ \tilde{U}_{h,t-1}, & h \leqslant L; \end{cases} \end{aligned} \tag{25}$$

where the first L of the H agent classes are endowed with the PT feature. As explained above, the term $\tilde{U}_{h,t-1}$ is defined as

$$\tilde{U}_{h,t-1} = \frac{1}{B_{h,t-1}} (x_{t-1} - R \cdot x_{t-2}) \frac{f_{h,t-2} - R \cdot x_{t-2}}{a \cdot \sigma^2} \tag{26}$$

and represents the fitness measure of the strategies with the PT feature calculated in two steps. $f_{h,t}$ is the forecasting function of strategy h at time t and ε_t is a noise term which represents natural uncertainty about the performance of economic fundamentals. It replaces the term $\delta_t = \epsilon_t$ defined in Sect. 3. The system of Eq. 25 is, in essence, a generalization of the original ABS: for $L = 0$, one obtains the 'benchmark' case used for the PT extension impact evaluation in Sect. 5.

5 Monte Carlo analysis

5.1 Model setup

The inevitable downside of the ABS is the leeway in choice of the parameters of the model, especially of β, the forecasting functions $f_{h,t}$, and the distribution of the noise term ε_t. We follow a number of previous studies, e.g., Barunik et al. (2009), Vacha et al. (2012), Kukacka and Barunik (2013, 2017), and adopt the following settings:

1. We use two different sets of forecasting functions $f_{h,t}$:
   (a) First, we follow the approach suggested by Brock and Hommes (1998) and apply the simple forecasting functions of the form $f_{h,t} = g_h \cdot x_{t-1} + b_h$. In this case, the random trend and bias parameters g_h and b_h are drawn from the normal distributions with means of zero and variances of 0.16 and 0.09, respectively, unless we state otherwise. If we ex ante indicate presence of fundamentalists in the model, the fundamentalist strategy is the first of the H strategies; the algorithm sets both of the parameters g_1 and b_1 to 0, and the term $n_{1,t}$ corresponds to the proportion of fundamentalists in the market.
   (b) Second, we use a set of fixed forecasting rules inspired by the strategies proposed by Anufriev and Hommes (2012). Using the fixed forecasting rules eliminates a source of noise in the model caused by the random trend and bias parameters g_h and b_h when the rules are not fixed.
2. The noise terms for each time period, ε_t, are drawn from the uniform distribution $U(-0.05, 0.05)$. Kukacka and Barunik (2013) investigate behavior of the model with the noise term drawn from several different uniform distributions and conclude that such behavior is largely similar for all of them.
3. Other parameters are set as follows: the gross risk-free rate of return R, $R = 1+r$, to 1.0001 and the term $\frac{1}{a \cdot \sigma^2}$ to 1. Note that a and σ^2 are mere scaling factors of the fitness measure U that do not influence the dynamics of the model.

Each simulation consists of 11 runs, each of which is characterized by a different intensity of choice parameter β that takes on values from 5 to 505 in increments of 50. There are 1000 repeat cycles in each run. If we use the simple, non-fixed forecasting rules, then the parameters g_h and b_h are randomly drawn from the aforementioned distributions in each cycle to guarantee robust simulation results. Finally, there are 500 ticks in each cycle representing trading days.

5.2 Criteria for evaluation

Cont (2001) lists the following phenomena as the most frequent financial time series stylized facts: absence of autocorrelations, heavy or fat tails, volatility clustering, intermittency, gain–loss asymmetry, and several others. We focus on the first three stylized facts as the original Brock and Hommes (1998) model has been found capable of explaining them soundly (Chen et al. 2012).

1. *Absence of autocorrelations.* Autocorrelations of asset returns are insignificant at most times and for most time scales, except for very small time scales of approximately 20 min, in which micro structures may have an effect on the autocorrelations (Cont 2001).
2. *Fat tails* Probability distributions of many asset returns have large skewness or kurtosis values relative to the normal distribution. Additionally, the distributions exhibit power law or Pareto-like tails, with a tail index of $2 \leqslant \alpha \leqslant 5$ (Cont 2001), i.e., the (upper) tail $P(X > x) = \bar{F}(x) = x^{-\alpha} \cdot G(x)$, where $G(x)$ is a slowly varying function (Haas and Pigorsch 2009).
3. *Volatility clustering.* Absolute or squared returns of an asset are characterized by a significant, slowly decaying autocorrelation function, that is, $\text{corr}(|r_t|, |r_{t+\tau}|) > 0$ or $\text{corr}\left(r_t^2, r_{t+\tau}^2\right) > 0$, where the time span τ ranges from minutes to weeks or months (Cont 2007).

5.3 Simple forecasting rules with random trend and bias

In this subsection we summarize the results of the analysis when the simple forecasting functions of the form $f_{h,t} = g_h \cdot x_{t-1} + b_h$ are used.

5.3.1 Benchmark simulation

We run a benchmark simulation of the original model specified by the system of Eq. 25 without the PT feature, that is, we set $L = 0$. Number of total strategies in the model is four ($H = 4$) and fundamentalists are present in the model as the first strategy. In each repeat cycle, the first 5 % of realizations of x_t are discarded to allow the model to stabilize.

Table 1 shows selected descriptive statistics of the x_t time series obtained from the benchmark simulation. Clearly, the distributions of the deviations from the fundamental price are statistically different from the normal distribution, as indicated by small p values of the Jarque–Berra (J–B) test for all values of β. As β increases, the distributions exhibit decreasing kurtosis; for $\beta = 255$, the kurtosis is closest to that of the normal distribution. With the exception of $\beta = 5$, the sample variance of x_t tends to decrease as β increases. This is also true for maxima and minima, in absolute value.

Figure 2 compares, on a log–log scale, the complementary Cumulative Distribution Functions (CDFs) $\bar{F}_{|x_t|}(y)$, $\bar{F}_{|x_t|}(y) = P(|x_t| > y)$ for the 150 largest absolute deviations $|x_t|$ for one repeat cycle corresponding to four randomly selected illustrative sample time series generated with different β values, along with the OLS fits, for the benchmark case and the case with the PT feature employed (see Sect. 5.3.2). One might notice the similarity of the tails for $\beta = 5$ and $\beta = 505$, as well as various patterns of the tail departures for the two other values of β.

Although not rigorous, the slopes of the regression lines are, in absolute values, estimates of the respective tail indices. For the benchmark case (black circles), these are equal to 7.43 for $\beta = 5$ ($R^2 = 0.83$), 8.3 for $\beta = 105$ ($R^2 = 0.94$), 5.44 for

Table 1 Benchmark simulation summary statistics and p values of J–B test for normality of distribution of x_t in 11 runs with different β values

β	Mean	Var.	Skew.	Kurt.	Min	Max	Med.	J–B
5	−0.001	0.021	− 0.714	10.241	− 2.135	0.782	0.002	0.000
55	0.004	0.118	0.130	5.696	− 1.732	1.984	0.000	0.000
105	0.002	0.116	0.011	4.153	− 1.349	1.377	0.002	0.000
155	0.002	0.104	0.004	3.458	− 1.169	1.136	0.002	0.000
205	0.001	0.096	0.008	3.158	− 1.014	1.050	0.000	0.000
255	− 0.002	0.086	0.017	3.042	− 0.918	0.920	− 0.002	0.000
305	0.003	0.079	0.079	2.952	− 0.837	0.879	− 0.001	0.000
355	− 0.001	0.069	0.055	2.831	− 0.817	0.856	− 0.002	0.000
405	− 0.003	0.065	0.023	2.807	− 0.817	0.760	− 0.003	0.000
455	− 0.002	0.060	0.031	2.780	− 0.794	0.775	− 0.002	0.000
505	− 0.007	0.057	− 0.002	2.757	− 0.793	0.775	− 0.004	0.000

Simple random forecasting rules are used. There are fundamentalist and three other strategies present in the model

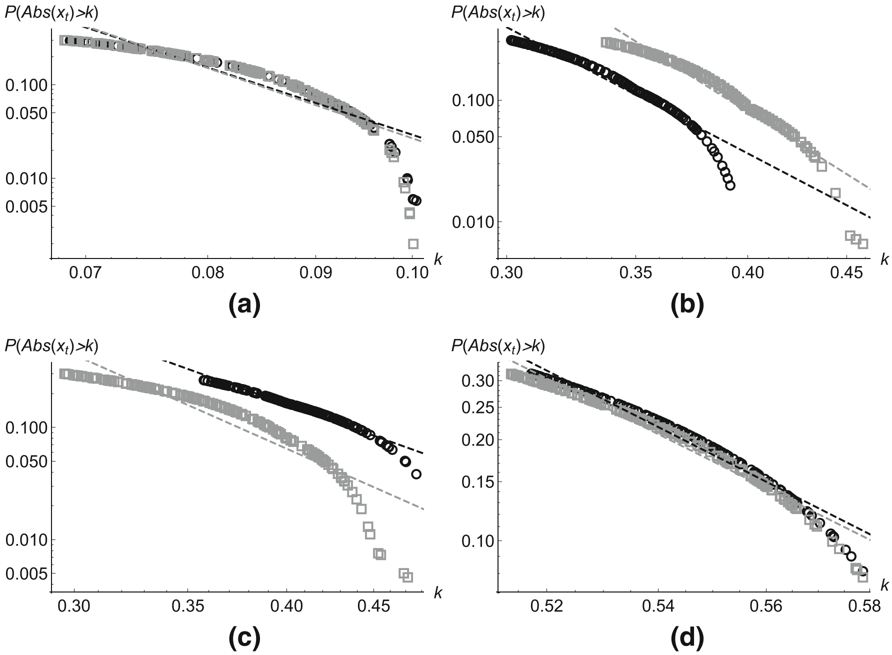

Fig. 2 Plots of the tails of sample x_t time series' empirical distributions and OLS fits. Black circles depict the benchmark case without the PT feature, gray squares depict the case with the PT feature employed. **a** $\beta = 5$. **b** $\beta = 105$. **c** $\beta = 305$. **d** $\beta = 505$

$\beta = 305$ ($R^2 = 0.96$), and 10.26 for $\beta = 505$ ($R^2 = 0.97$). Having only an informative character, the plots in Fig. 2 nonetheless show possible existence of a power law in tails of the sample distribution of $|x_t|$. It is important to emphasize, however, that the power law apparently does not hold universally for the whole tail. Most extreme observations—for which the imaginary curvature is relatively significant and the realizations clearly do not follow the linear pattern estimated for the complete collection of the 150 observations—might exhibit a tail index different from the remaining observations; the 'break point' seems to lie in the interval $\bar{F}_{|x_t|}(y) \in [0.1, 0.05]$.

5.3.2 Employment of Prospect Theory

The simulation with the PT traders is run with the same random seed as the benchmark simulation in Sect. 5.3.1, meaning that, for each repeat cycle, the same randomly generated parameters are used. Any differences between the benchmark and the PT simulations can therefore be solely attributed to the PT feature.

The gain and loss parameters c_g and c_l are set to 4/7 and 10/7, respectively, to properly account for the gain–loss asymmetry. The setting of c_g and c_l is crucial because the parameter a (see Eq. 26) determines both the slope of the utility function as well as the degree of risk aversion. Thus, PT traders might also potentially differ in their risk aversion, which might be on average $\frac{c_g+c_l}{2}$ times higher than those of non-PT traders for an arbitrary choice of c_g and c_l. The particular numerical values for c_g and c_l are therefore chosen to primarily satisfy the three following principles. First, the findings that "…losses hurt more than equal gains please; typically two to two-and-a-half times more" (van Kersbergen and Vis 2014, p. 163) or that "…the disutility of giving something up is twice as great as the utility of acquiring it," (Benartzi and Thaler 1993). Second, this setting is well justified by Fig. 1, which shows estimates of the PT value function. Finally, the fact that the risk aversion of the PT traders under $c_g = 4/7$ and $c_l = 10/7$ is on average the same as that of the non-PT traders—equal to 1. This allows us to compare these results to those produced by the original model in the benchmark simulations and to interpret the observed effects as arising purely from the differential treatment of gains and losses. Initially, all strategies exhibit the PT feature, i.e., $L = 4$, and we compare these results to the benchmark simulation with only non-PT traders.

Table 2 summarizes descriptive statistics along with p values of the J–B and Kruskal–Wallis (K–W) tests of the x_t time series. Using the K–W method, we test whether the x_t time series obtained from the PT simulation and those from the benchmark simulation originate from the same distribution (see Table 1). Addition of the PT feature clearly causes—potentially except for the cases of $\beta = 5$ and $\beta = 355$—significant differences of these distributions with respect to those of the benchmark simulation. Also notice the smaller variance of the time series with respect to the benchmark case, as well as smaller extreme values for most values of β. The sample means and variance are statistically different from those obtained from the benchmark run for all values of β.

For the PT case, Fig. 2 shows (gray squares), on a log–log scale, the complementary Cumulative Distribution Functions (CDFs) $\bar{F}_{|x_t|}(y)$ for the 150 largest absolute deviations $|x_t|$ corresponding to four randomly selected illustrative sample time series generated with different β values, along with the OLS fits. Estimates of the respective tail indices (i.e., the opposites of the estimated slope coefficients) are equal to 7.76 for

Table 2 PT simulation summary statistics of x_t and p values of J–B and K–W tests in 11 runs with different β values

β	Mean	Var.	Skew.	Kurt.	Min	Max	J–B	K–W
5	− 0.002	0.029	− 0.371	10.046	− 1.633	1.544	0.000	0.156
55	0.002	0.109	− 0.120	4.276	− 1.724	1.455	0.000	0.000
105	− 0.001	0.104	− 0.004	3.492	− 1.272	1.367	0.000	0.001
155	− 0.002	0.091	0.047	3.247	− 1.123	1.133	0.000	0.000
205	0.005	0.084	0.008	3.154	− 1.004	1.013	0.000	0.000
255	− 0.001	0.071	− 0.061	3.176	− 0.961	0.927	0.000	0.000
305	− 0.009	0.065	− 0.073	3.036	− 0.830	0.816	0.000	0.000
355	− 0.002	0.056	− 0.019	3.079	− 0.775	0.902	0.000	0.065
405	− 0.001	0.055	− 0.016	2.962	− 0.734	0.768	0.000	0.000
455	− 0.001	0.049	− 0.110	2.954	− 0.682	0.684	0.000	0.000
505	− 0.006	0.046	− 0.014	2.996	− 0.663	0.670	0.000	0.004

There are fundamentalist and three other strategies in the model, i.e., $H = 4$, and all strategies have the PT feature

$\beta = 5$ ($R^2 = 0.79$), 9.97 for $\beta = 105$ ($R^2 = 0.91$), 6.7 for $\beta = 305$ ($R^2 = 0.85$), and 10.13 for $\beta = 505$ ($R^2 = 0.96$). The OLS fits provide roughly the same R^2 compared to the benchmark case, although the most extreme observations do, again, exhibit considerable curvature and departure from any power law, mainly in the regions where $\bar{F}_{|x_t|}(y) < 0.05$.

5.3.2.1 PT vs. non-PT traders We will now relax the assumption that all trading strategies are endowed with the PT feature and examine the behavior of the model by running additional simulations in which some of the trading strategies exhibit loss aversion and gain–loss asymmetry and some do not. Table 3 summarizes simulations with $L = 1$, $L = 2$, $L = 3$. The fundamentalist strategy is present in the model as the first one, i.e., $L = 1$ corresponds to a situation in the market in which there are PT fundamentalists and three other random strategies. The K–W test compares, in this case, the distributions obtained from the simulations based on the PT feature with those obtained from a simulation without it.[8] To maintain mutual comparability, the same parameters g_h, b_h, and ε_t are used for each value of $L \neq 0$ and for $L = 0$.

Figure 3 examines, for $\beta = 255$, the cases in which $L = 1$ and $L = 4$, i.e., the situation in which only the fundamentalist strategy has the PT feature ($L = 1$) versus the one in which all strategies have the PT feature ($L = 4$). These situations are compared to the benchmark case of $L = 0$, where no strategies have that feature. Estimated densities of the x_t time series are plotted on the left-hand side of the figure while the right-hand side of the figure shows estimated densities of the $n_{1,t}$ time series, i.e., of the proportion of traders using the fundamentalist strategy. The densities of x_t

[8] We run another ‘benchmark’ simulation of the model without the proposed extensions, that is, for the K–W test, we use a different benchmark than that examined in Sect. 5.3.1.

Table 3 p values of the K–W test for different L and β

β	$L = 1$	$L = 2$	$L = 3$
5	0.952	0.000	0.000
55	0.950	0.000	0.012
105	0.762	0.230	0.174
155	0.367	0.000	0.000
205	0.391	0.000	0.000
255	0.232	0.000	0.000
305	0.673	0.000	0.000
355	0.579	0.000	0.000
405	0.433	0.802	0.000
455	0.584	0.000	0.000
505	0.983	0.000	0.000

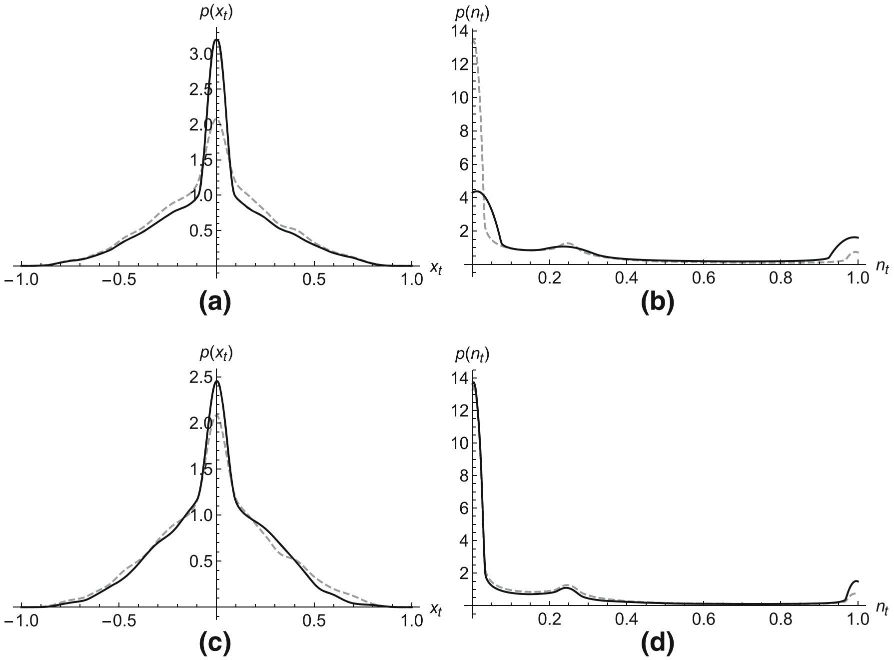

Fig. 3 Behavior of the model for different L (solid black line) versus the benchmark case of $L = 0$ (dashed gray line); $\beta = 255$. **a** PDFs of x_t, $L \in \{0, 1\}$. **b** PDFs of $n_{1,t}$, $L \in \{0, 1\}$. **c** PDFs of x_t, $L \in \{0, 4\}$. **d** PDFs of $n_{1,t}$, $L \in \{0, 4\}$

are to a large extent similar; however, the K–W test rejects the null hypothesis that both the sample from the benchmark simulation and the one from the PT simulation come from the same distribution for the case of $L = 4$ (p value < 0.000). We fail to reject the null hypothesis for the case of $L = 1$, as per Table 3. Yet, PT fundamentalists are driven out of the market *less* strongly for $L = 1$ than they are for $L = 4$. The expected value of $n_{1,t}$ is equal to 0.23 for $L = 1$ and 0.14 for $L = 4$. This finding

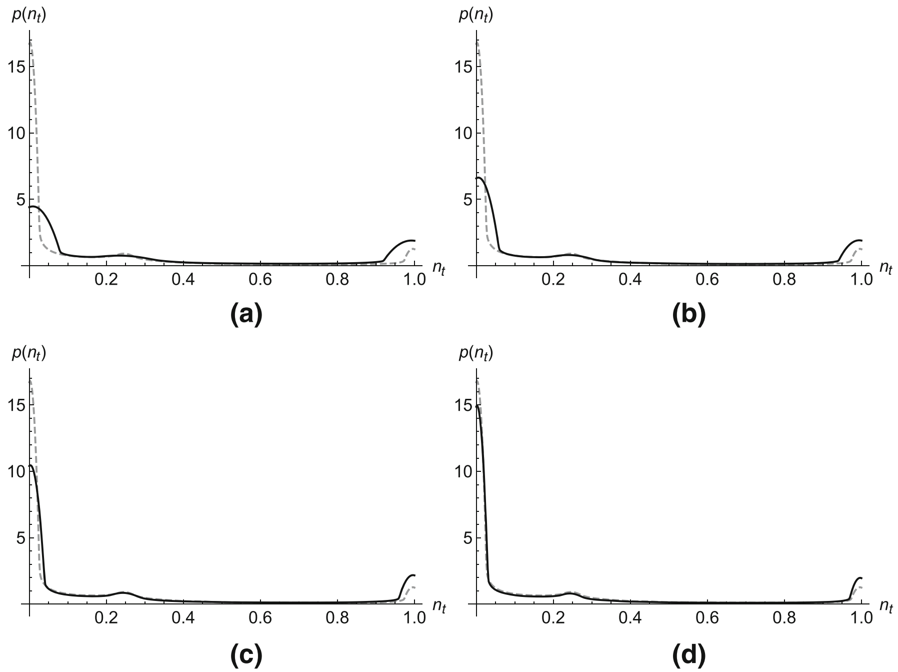

Fig. 4 Occurrence of loss-averse fundamentalists (solid black line) is less likely with increasing L. The distributions are compared to those in the benchmark case (dashed gray line) where no strategies have the PT feature ($L = 0$); $\beta = 455$. **a** PDFs of $n_{1,t}$, $L \in \{0, 1\}$. **b** PDFs of $n_{1,t}$, $L \in \{0, 2\}$. **c** PDFs of $n_{1,t}$, $L \in \{0, 3\}$. **d** PDFs of $n_{1,t}$, $L \in \{0, 4\}$

can be confirmed visually by comparing the peakedness of respective distributions in Fig. 3 around the point 0. This is an interesting result—the PT feature seems to be a heavier burden for fundamentalists if they have to face other loss-averse strategies than if they have a single strategy that is loss-averse.

This result holds for all values of β and is most pronounced for $L = H$. However, the pattern is clear for $L < H$, too: generally, the occurrence of loss-averse fundamentalists is less likely as L increases, as evident from Fig. 4. The expected value of $n_{1,t}$ decreases but it is still greater than in the benchmark case even for $L = 4$. That is, the PT feature makes the fundamentalists better off relative to the benchmark case in a situation when other strategies do not have this feature—the expected value of $n_{1,t}$ is equal to 0.13 in the benchmark case.

Generally, the distributions of x_t tend to differ more as L increases—the PT feature stabilizes the market and rules out the proportions of extreme price deviations which are present in the benchmark case. Figure 5 shows estimated densities of $n_{1,t}$ and $n_{4,t}$ for $L = 3$, i.e., proportions of PT fundamentalists and non-PT chartists in a model in which one chartist trading strategy does not have the PT feature, for different values of β. Notice that, for the smaller value β, the non-PT chartist strategy is more popular—the expected value of $n_{4,t}$ is 0.18 for $\beta = 55$ and 0.13 for $\beta = 455$. Again, fundamentalists are more likely to survive in the market than they are when they face

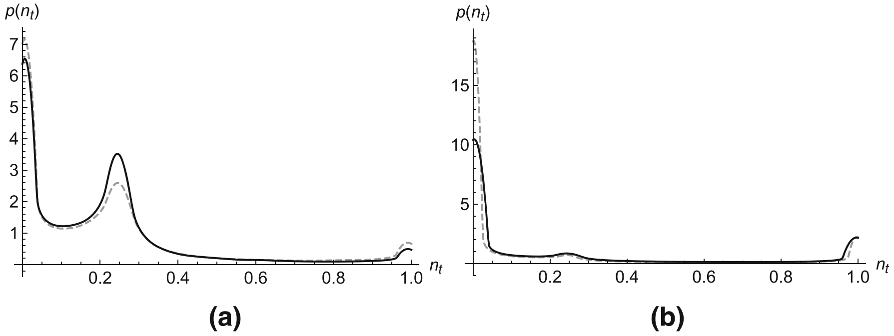

Fig. 5 Estimated densities of $n_{1,t}$ (solid black line) and $n_{4,t}$ (dashed gray line) for $L = 3$. **a** $\beta = 55$. **b** $\beta = 455$

only PT traders—for $\beta = 455$ this effect is revealed in the expected value of $n_{1,t}$ of 0.14 for $L = 4$ [solid black line in panel (d) of Fig. 4] and of 0.17 for $L = 3$ [solid black line in panel (b) of Fig. 5]. The more frequent occurrence of fundamentalists can thus be attributed to the presence of non-PT chartists.

5.4 Fixed forecasting rules

In this subsection we summarize the results of the analysis when the forecasting functions $f_{h,t}$ are fixed. Specifically, we employ the following five strategies:

1. *Fundamentalists (FND)* $f_{t,FND} = 0$,
2. *Weak trend-following rule (WTR)* $f_{t,WTR} = x_{t-1} + 0.4 \cdot (x_{t-1} - x_{t-2})$,
3. *Strong trend-following rule (STR)* $f_{t,STR} = x_{t-1} + 1.3 \cdot (x_{t-1} - x_{t-2})$,
4. *Adaptive heuristic (ADA)* $f_{t,ADA} = 0.65 \cdot x_{t-1} + 0.35 \cdot f_{t-1,ADA}$, and
5. *Anchoring and adjustment rule with learning factor (LAA)* $f_{t,LAA} = 0.5 \cdot \left(\frac{\sum_{i=1}^{t-1} x_i}{t-1} - x_{t-1} \right) + x_{t-1} - x_{t-2}$.

As before, we first run a benchmark simulation without the PT feature, that is, we set $L = 0$. Then, for each value of β, we run five additional simulations where we increment L by 1.

Table 4 shows the key descriptive statistics of the x_t time series. Notice that the values of the sample mean and sample variance, as well as the p values of the J–B test, are the same across different values of β with the precision of three decimal places. The elimination of the random bias parameter significantly stabilizes the behavior of the model. With increasing values of β, sample skewness decreases while sample kurtosis increases, as does the difference between the minimum and maximum values of x_t. The latter two results are in contrast with the findings from the benchmark simulation with random strategies (see Table 1). There, the sample kurtosis and the difference between the minimum and maximum values of the deviations decreases.

Table 5 shows, for $L = 1, \ldots, 5$ and selected values of β, the summary statistics of the x_t time series and p values of the K–W test where the respective distributions

Table 4 Benchmark simulation summary statistics and p values of the J–B test for normality of distribution of x_t in 11 runs with different β values when the five fixed forecasting rules are used

β	Mean	Var.	Skew.	Kurt.	Min	Max	Med.	JB
5	– 0.001	0.002	0.008	2.545	– 0.134	0.136	– 0.001	0.000
55	– 0.001	0.002	0.006	2.531	– 0.133	0.138	– 0.001	0.000
105	– 0.001	0.002	0.004	2.539	– 0.133	0.140	– 0.001	0.000
155	– 0.001	0.002	0.002	2.561	– 0.140	0.142	– 0.001	0.000
205	– 0.001	0.002	0.001	2.592	– 0.149	0.143	– 0.001	0.000
255	– 0.001	0.002	– 0.003	2.631	– 0.158	0.144	– 0.001	0.000
305	– 0.001	0.002	– 0.006	2.673	– 0.164	0.148	– 0.001	0.000
355	– 0.001	0.002	– 0.008	2.716	– 0.168	0.153	– 0.001	0.000
405	– 0.001	0.002	– 0.011	2.761	– 0.172	0.157	– 0.001	0.000
455	– 0.001	0.002	– 0.012	2.805	– 0.176	0.161	– 0.001	0.000
505	– 0.001	0.002	– 0.014	2.845	– 0.181	0.164	0.000	0.000

Table 5 PT simulation summary statistics of x_t and p values of K–W tests for selected values of β and $L = 1, \ldots, 5$

β	L	Mean	Var.	Skew.	Kurt.	Min	Max	K–W
305	1	– 0.001	0.002	– 0.015	2.723	– 0.168	0.149	0.980
305	2	– 0.001	0.002	– 0.010	2.737	– 0.170	0.153	0.835
305	3	– 0.001	0.002	– 0.013	2.979	– 0.196	0.238	0.745
305	4	– 0.001	0.002	– 0.014	2.975	– 0.196	0.241	0.636
305	5	0.000	0.002	– 0.019	2.808	– 0.185	0.168	0.397
405	1	– 0.001	0.002	– 0.023	2.840	– 0.178	0.167	0.622
405	2	– 0.001	0.002	– 0.016	2.836	– 0.180	0.163	0.734
405	3	0.000	0.002	– 0.010	3.158	– 0.198	0.244	0.816
405	4	– 0.001	0.002	– 0.015	3.135	– 0.197	0.245	0.627
405	5	0.000	0.002	– 0.023	2.930	– 0.192	0.173	0.342
505	1	– 0.001	0.002	– 0.029	2.957	– 0.184	0.180	0.314
505	2	– 0.001	0.002	– 0.019	2.922	– 0.186	0.167	0.620
505	3	0.000	0.002	– 0.005	3.296	– 0.206	0.245	0.958
505	4	– 0.001	0.002	– 0.013	3.253	– 0.198	0.245	0.618
505	5	0.000	0.002	– 0.024	3.029	– 0.196	0.183	0.380

are compared to those in the benchmark simulation where $L = 0$. Notice that the distributions are not statistically different, regardless of the value of $L = 0$. This fact, too, is in contrast with the results of the simulations with random strategies. The sample kurtosis tends to increase as β and L increase, although this relationship is not monotonic. The differences between the maximum and minimum values are markedly higher than in the benchmark case; note that this result is most pronounced for $L = 3$ and $L = 4$, that is, when the fundamentalists, weak trend followers, and strong trend

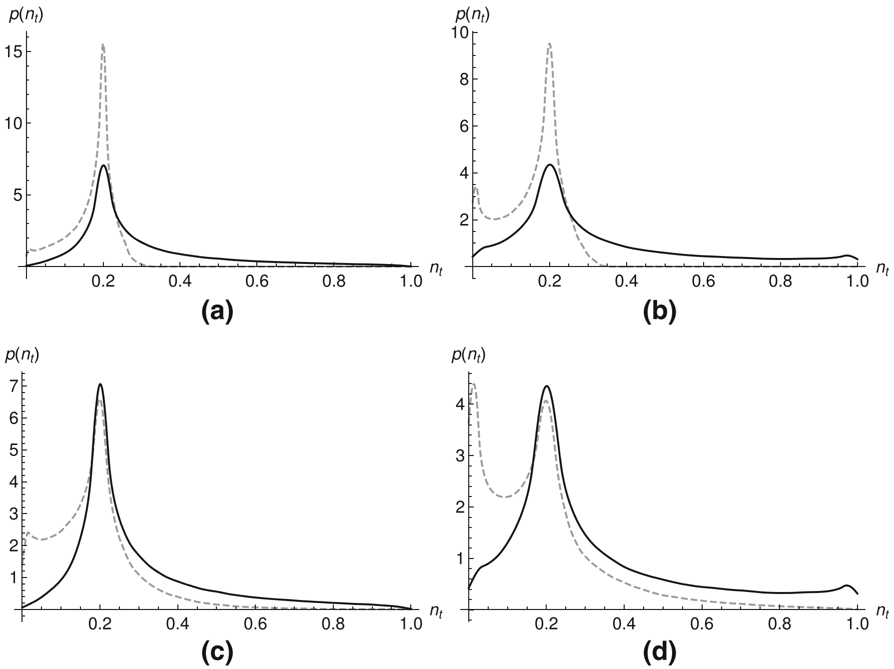

Fig. 6 Occurrences of both types of trend followers increase when the PT feature is introduced for fundamentalists. $L = 1$ (solid black line) and $L = 0$ (dashed gray line). **a** PDFs of $n_{WTR,t}$ for $\beta = 305$. **b** PDFs of $n_{WTR,t}$ for $\beta = 505$. **c** PDFs of $n_{STR,t}$ for $\beta = 305$. **d** PDFs of $n_{STR,t}$ for $\beta = 505$

followers all have the PT feature ($L = 3$) and when also the adaptive heuristic strategy has that feature ($L = 4$). When all strategies have the feature ($L = 5$), this difference is diminished. These findings also hold for the values of β that are not reported in the Table. The sample means are statistically different from those obtained in the benchmark run for all values of β; the same holds for the sample variances, except for the case of $\beta = 405$, $L = 5$.

5.4.1 *PT versus non-PT traders*

In this subsection we present a comparison between the occurrence of strategies with the PT feature and without it. Figure 6 shows the estimated densities of the $n_{WTR,t}$ and $n_{STR,t}$ time series, i.e., the proportions of weak and strong trend-followers in the market for different values of β. We compare the occurrence of these two types of trend followers in the market where no strategy has the PT feature (the benchmark case of $L = 0$) to their occurrence in the market where the fundamentalist strategy has the feature ($L = 1$). Notice the fact that the introduction of the PT feature into the fundamentalist strategy dramatically increases the trend followers' chances of survival in the market. The increase in the expected value of the distributions for $\beta = 305$ is equal to 0.126 for the weak trend-followers and to 0.112 for the strong trend-followers; for $\beta = 505$ these increments are, respectively, equal to 0.171 and 0.139.

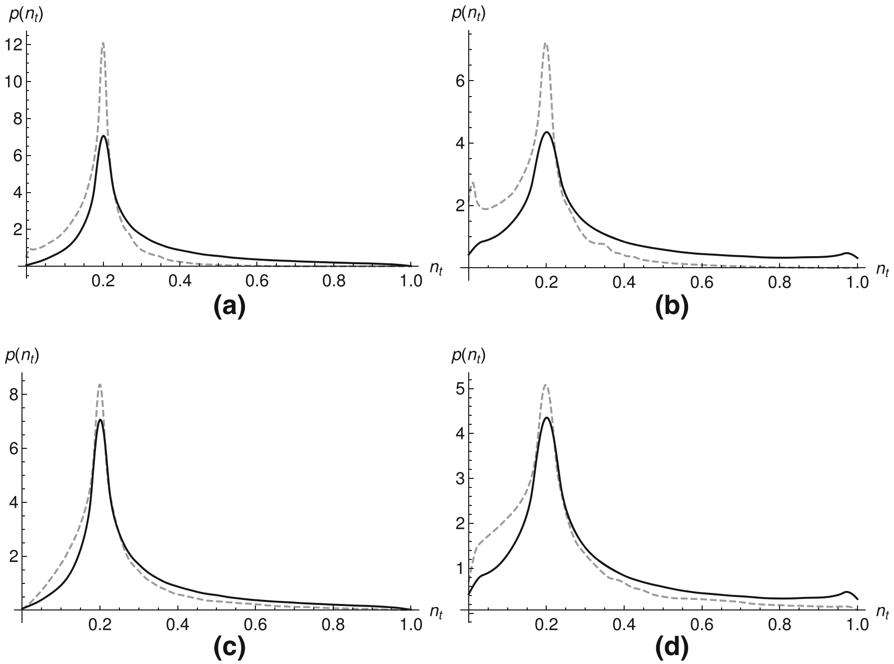

Fig. 7 Occurrences of the ADA and LAA strategies increase when the PT feature is introduced for fundamentalists. $L = 1$ (solid black line) and $L = 0$ (dashed gray line). **a** PDFs of $n_{ADA,t}$ for $\beta = 305$. **b** PDFs of $n_{ADA,t}$ for $\beta = 505$. **c** PDFs of $n_{LAA,t}$ for $\beta = 305$. **d** PDFs of $n_{LAA,t}$ for $\beta = 505$

Figure 7 shows the estimated densities of the $n_{ADA,t}$ and $n_{LAA,t}$ time series for different values of the parameter β. As before, we compare the presence of the two strategies in the market where no strategy has the PT feature to that in the market in which the fundamentalist strategy has the PT feature. The results are similar to those for the trend-following strategies. The introduction of the PT feature for the fundamentalist strategy increases the occurrence of the ADA and LAA strategies in the market. However, this improvement is not as significant for the LAA strategy as it is for the trend-following strategies or the ADA strategy. The increase in the expected value of the distribution for $\beta = 305$ is equal to 0.104 for ADA but only to 0.054 for LAA; for $\beta = 505$ these increments are, respectively, equal to 0.141 and 0.073. All these results are statistically significant.

Figure 8 shows estimated densities of the fundamentalist proportions, $n_{FND,T}$, for varying L. The differences between the expected values of the densities with the PT feature and without it are equal to 0.099 for $L = 1$, 0.082 for $L = 3$, 0.054 for $L = 4$, and -0.008 for $L = 5$. Notice that, as L increases, the advantage of having the PT feature is reduced, and it becomes a disadvantage for $L = 5$.

6 Results

Introduction of the PT feature into the model considerably changes its behavior. Nonetheless, some of the key characteristics remain the same as the underlying math-

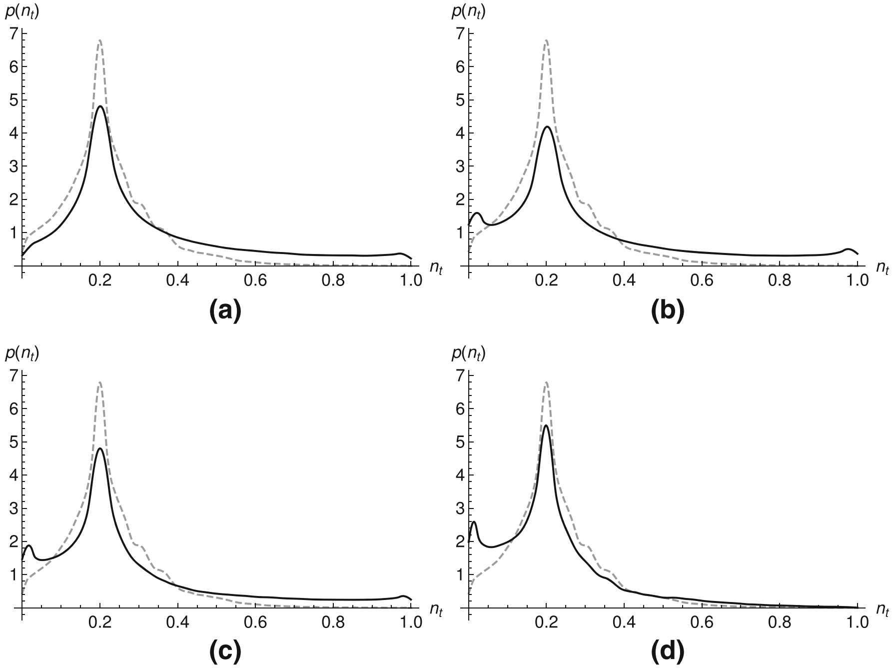

Fig. 8 PDFs of $n_{FND,t}$ for $\beta = 455$ and varying L. The benchmark case of $L = 0$ is depicted in dashed gray line. **a** $L = 1$. **b** $L = 3$. **c** $L = 4$. **d** $L = 5$

ematical structure of model is intact—the generated time series of the deviations from the fundamental price of the asset, x_t, exhibit decreased variance as the intensity of choice parameter β increases, extreme price deviations are less ‘extreme’ for larger β, and the deviations are still far from being normally distributed. However, the differences are considerable and non-negligible as indicated by the very low p values of the K–W tests. The main conclusions arising from the PT extension can be summarized as follows:

1. *Stability* When random strategies are used, the model is more stable when the strategies have the PT feature. Summarized in Table 2, the sample variance of the x_t time series is generally lower than in the benchmark case. Recall that the same random seed is used for both versions of the model. The difference in stability can therefore be attributed to the PT extension completely. When the fixed strategies are used, the addition of the PT feature does not change the variance of the x_t time series. Yet, the differences between the maximum and minimum values of x_t are markedly higher with the PT feature.
2. *Loss aversion matters* The number of strategies endowed with the PT feature, L, significantly affects the behavior of the model. Table 3 shows, for the case of random strategies, that if only the fundamentalist strategy is loss-averse (i.e., $L = 1$), the empirical distributions of x_t are statistically different at a reasonable significance level from those obtained from the model with $L = 0$ only for higher values of β. On the other hand, for $L > 1$ these distributions are statistically

(and also usually visually) different from those of the benchmark, $L = 0$ case. This finding does not hold for the case of fixed strategies, as seen in Table 5, where the p values of the K–W test are high. However, the PT feature significantly affects the probability of occurrence of the strategies. When we introduce it for the fundamentalist strategy, the chances of the four remaining strategies to survive in the market considerably increase. The LAA strategy (Anchoring and adjustment rule with learning factor) seems to be the least sensitive in this respect.

3. *Fundamentalists may survive more easily* In the original model, fundamentalists are, with increasing β, less likely to survive in the market than they are for lower values of β. In the case of random strategies, we find that the PT feature increases the chances of fundamentalists to survive in the market relative to the benchmark case. This effect is, interestingly, most pronounced for $L = 1$, rather than for higher values of L. The loss aversion feature increases the probabilities of occurrence of the fundamentalist strategy and the strength of this effect increases as L decreases (but remains greater than 0). This result partly holds when the strategies are fixed. This result does not hold only for $L = 5$, in which case the fundamentalists are worse-off.

7 Conclusions

Using a general idea proposed by Shimokawa et al. (2007), we extend the popular Brock and Hommes (1998) agent-based asset pricing model and include the most important features of the PT into the framework, namely the loss aversion with reference point dependence and distorted treatment of gains and losses. The main contribution of our work is the finding that the original model can be consistently and meaningfully extended with the most relevant features of PT, while its intrinsic 'stylized' structure may remain intact. Using Monte Carlo simulations and random strategies, we find that the distributions of the main variable are statistically different from those obtained from the original version of the model and that the stability of the model is increased as the proportion of extreme price deviations is ruled out. Furthermore, the occurrence of fundamentalists is more extreme and the PT feature increases the chances of fundamental traders to survive in the market compared to the benchmark simulation. We also investigate the behavior of the model with fixed strategies. In this case the distributions of the main variable are not statistically different from those occurring in the benchmark case; however, the PT feature does affect the likelihood of occurrence of the strategies.

As the Brock and Hommes (1998) model is inherently characterized by 'many degrees of freedom' and the extensions bring even more options in this regard, future research might focus on exploration of other possible combinations of the parameters. Additionally, the extended model could be estimated using real-world empirical data to reveal the natural values of some parameters, e.g., those expressing the degree of loss aversion present in the markets. Another field that could be explored with respect to the extended version of the model is a more thorough analysis of the volatility structure of the x_t time series.

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