

# Problem of competing risks with covariates: Application to an unemployment study

Petr Volf<sup>1</sup>

**Abstract.** This study deals with the methods of statistical analysis in the situation of competing risks in the presence of regression. First, the problem of identification of marginal and joint distributions of competing random variables is recalled. The main objective is then to demonstrate that the parameters and, in particular, the correlation of competing variables, may depend on covariates. The approach is applied to solution of a real example with unemployment data. The model uses the Gauss copula and Cox's regression model.

**Keywords:** statistical survival analysis, competing risks, copula, unemployment study, Cox's regression model.

**JEL classification:** C41, J64

**AMS classification:** 62N02, 62P25

## 1 Introduction and Motivation

The problem of competing risks arises when two (or more) mutually dependent random times to certain events (e.g. a failure of a device which can be caused by two different reasons) are followed and just the first (least) of them is registered. The phenomenon occurs frequently in areas of reliability, biostatistics and medical studies, as well as in demography studies, labor statistics, insurance, and in econometrics generally. The interest in the problem dates back to 70-ties of the last century. It is well known that, in general, without additional assumptions (e.g. on a parametric form of distributions) the model is not identifiable. It means that for each level of dependence we may obtain different estimate of joint probability distribution of both random times, and we are not able to choose among them. Further, Heckman and Honoré (1989) have shown (rather than proved, as their argumentation is more verbal than precisely mathematical) that when an additional information on covariates is available, identification is possible. Namely, they dealt with competing times with Cox's and AFT regression models. Their result is in the background of the unemployment duration study by Han and Hausman (1990), with two competing events of unemployment termination. However, they quite neglected the possibility that the parameter characterizing mutual dependence of risks (e.g. the correlation) may also depend on the covariate. When this is the case, the problem of non-identifiability arises anew. Later on the case of competing risks with covariates was solved by many other authors, in a more precise way, already with the aid of a copula describing the dependence, see e.g. Lee (2006), Berg et al. (2007). However, the problem of possible dependence of copula parameter on the covariates was not discussed. That is why the present paper opens this problem. On another unemployment data, which are commonly available on the Web (as the original data used in Han and Hausman are not available to me), the model of joint distribution is formulated, with the use of Gauss copula. Then, it is shown that the correlation, which characterizes the dependence of two random times, depends on the age of employee, here taken as a covariate. In fact, the result of Heckman and Honoré and of others can be used for at least approximate estimation of partially constant correlation in a moving window scheme. In such a way its dependence on the covariate can be demonstrated.

The outline of the paper is the following: The next section introduces the scheme of competing risks, presents the method of analysis of competing events incidence, and points to the problem of possible non-identifiability of their marginal probability distributions. We shall mention also certain identifiability results in the framework of regression models. Then the notion of copula is recalled and used in competing risks model formulation. In Section 3 the Gauss copula is introduced and the procedure of simultaneous maximum likelihood estimation of marginal distributions and correlation is described. Finally, in Section 4, the Gauss copula and Cox's regression model are jointly applied to a real example. We use the data taken from Kadane and Woodworth (2004) recording the employment and its termination in certain company. There are two competing events, dismissal or voluntary leaving the job, it is expected that their risks are dependent mutually and also on the age of employees, which is taken as a covariate. The objective is to show that the model parameters and, in particular, the correlation of both risks, depend on this covariate. The solution consists in a randomized search for the maximum likelihood estimate combined with the Metropolis MCMC algorithm in the Bayes framework.

---

<sup>1</sup>Department of Stochastic Informatics, ÚTIA AV ČR, Pod vodárenskou věží 4, 182 08 Praha 8, Czech Republic, volf@utia.cas.cz

## 2 Competing risks scheme

Let us recall briefly the competing risks scheme: There are  $K$  (possibly dependent) random variables, times  $T_j, j = 1, \dots, K$ , running simultaneously. Observation is terminated at minimum of them. Sometimes there is another random or deterministic variable  $C$  of right censoring assumed to be independent of all  $T_j$ . Standardly,  $C$  is the time of observation termination without expected event occurrence; in that case  $C < T_j$  for all  $j$ . We assume that all variables  $T_j$  are of continuous type. Let  $\bar{F}_K(t_1, \dots, t_K) = P(T_1 > t_1, \dots, T_K > t_K)$  be the joint survival function of  $\{T_j\}$ . However, instead the 'net' survivals  $T_j$  we observe just 'crude' data (sometimes called also 'the identified minimum')  $Z = \min(T_1, \dots, T_K, C)$  and the indicator  $\delta = j$  if  $Z = T_j$ ,  $\delta = 0$  if  $Z = C$ . Such data lead us to direct estimation of the distribution of  $Z^* = \min(T_1, \dots, T_K)$ , for instance its survival function  $S(t) = P(Z^* > t) = \bar{F}_K(t, \dots, t)$ . Further, we can estimate so called **incidence densities**

$$f_j^*(t) = dP(Z^* = t, \delta = j) = -\frac{\partial \bar{F}_K(t_1, \dots, t_K)}{\partial t_j} \Big|_{(t_1 = \dots = t_K = t)},$$

and also their integrals, cumulative incidence functions

$$F_j^*(t) = \int_0^t f_j^*(s) ds = P(Z^* \leq t, \delta = j).$$

Notice that  $\lim_{t \rightarrow \infty} F_j^*(t) = P(\delta = j) < 1$  if  $t \rightarrow \infty$ ,  $S(t) = 1 - \sum_{j=1}^K F_j^*(t)$ . Further, so called cause-specific hazard functions for events  $j = 1, 2, \dots, K$  are estimable by:

$$h_j^*(t) = \lim_{d \rightarrow 0} \frac{P(t \leq Z^* < t + d, \delta = j | Z^* \geq t)}{d}.$$

Overall hazard rate for  $Z^* = \min(T_1, \dots, T_K)$  is then:

$$h^*(t) = \lim_{d \rightarrow 0} \frac{P(t \leq Z^* < t + d | Z^* \geq t)}{d} = \sum_{j=1}^K h_j^*(t),$$

by integration the cumulated hazard rates  $H_j^*(t)$ ,  $H^*(t)$  are obtained. Consequently,  $S(t) = P(Z^* > t) = \exp(-H^*(t))$ . Then  $f_j^*(t) = h_j^*(t) \cdot S(t)$  and the cumulative incidence functions can be also written as

$$F_j^*(t) = P(Z^* \leq t, \delta = j) = \int_0^t S(s) \cdot h_j^*(s) ds.$$

As both components, i.e.  $S$  and  $h_j^*$ , are estimable consistently by standard survival analysis methods, it follows that there also exist consistent estimates of  $F_j^*$ , see for instance Lin (1997).

### 2.1 Non-identifiability problem

As it has already been said, in general, from data  $(Z_i, \delta_i)$ ,  $i = 1, \dots, N$  it is not possible to identify neither marginal nor joint distribution of  $\{T_j\}$ . A. Tsiatis (1975) has shown that for arbitrary joint model we can find a model with independent components having the same incidences, i.e. we cannot distinguish among the models. Namely, this 'independent' model is given by cause-specific hazard functions  $h_j^*(t)$ . It follows that it is necessary to make certain functional assumptions about the form of both marginal and joint distribution in order to identify them. Several such cases are specified for instance in Basu and Ghosh (1978). More recent results on identifiability can be found for example in Schwarz et al (2013) dealing with non-parametric setting, or in Escarela and Carriere (2003) considering Frank copula and parametric models.

Many authors have studied the role of additional information gathered from covariates in the framework of a regression model for examined random times. There are numerous results showing conditions for full identifiability of such a regression model, starting from already mentioned Heckman and Honoré (1989). Lee (2006) investigated more general transformation models of regression. Berg et al. (2007) have studied two competing transition rates from unemployment state. They have used a discrete-time multiplicative regression model with latent heterogeneities. A common identifiability assumptions consists in sufficiently rich structure of covariates. However, all these studies rely on the assumption that the dependence structure (in the next section given by a copula parameter) does not change with covariates. And this rather strong assumption is the target of our examination.

### 2.2 Copulas in models for competing risks

Let us reduce the model to just 2 competing events, random variables  $S, T$  (eventually with a censoring variable  $C$ ). The data are then given as realizations of  $N$  i.i.d. random variables  $Z_i = \min(S_i, T_i, C_i)$ ,  $\delta_i = 1, 2, 0$ ,  $i =$

$1, 2, \dots, N$ . The notion of copula offers a way how to model multivariate distributions, here the joint distribution function  $F_2(s, t)$  of  $S, T$ :

$$F_2(s, t) = \mathbf{C}(F_S(s), F_T(t), \theta), \quad (1)$$

$F_S, F_T$  are marginal distribution functions of  $S, T$ ,  $\mathbf{C}(u, v, \theta)$  is a copula, i.e. a two-dimensional distribution function on  $[0, 1]^2$ , with uniformly on  $[0, 1]$  distributed marginals  $U, V$ ,  $\theta$  is a copula parameter. The parameter is connected uniquely with correlation of  $U, V$ , hence also with correlation of  $S, T$ . It is seen that the use of copula allows to model the dependence structure separately from the analysis of marginal distributions. Hence, the identifiability of the copula (and its parameter) and marginals can be considered as two separate steps.

Zheng and Klein (1995) have proved that when the copula is known, the marginal distributions are estimable consistently (and then the joint distribution, too, from (1)), even in non-parametric (so that quite general) setting. However, in general, also value of  $\theta$  is needed, because (again due to Tsiatis, 1975) without fully determined copula we are not able to distinguish between the 'true' model and corresponding independent one. On the other hand, Zheng and Klein (1995) also argued that the selection of copula type is not crucial. The problem of proper copula choice is analyzed in a set of papers, let us mention here Kaishev et al (2007) comparing performance of several copula types. A common agreement is that the knowledge (or a good estimate) of parameter  $\theta$  is much more important for correct model of joint distribution.

As a consequence, because the knowledge of copula type is still an unrealistic supposition, we can try to use certain sufficiently flexible class of copulas, as approximation, and concentrate to reliable estimation of its parameter. There exist a large number of different copula functions, among them for instance a set of Archimedean copulas. Let us concentrate here to one rather universal and flexible copula type, namely to Gauss copula.

### 3 Use of Gauss copula

Let  $X, Y$  be standard normal random variables  $\sim N(0, 1)$  tied with (Pearson) correlation  $\rho = \rho(X, Y)$ . Let us denote by  $\varphi(x)$ ,  $\Phi(x)$  the univariate standard normal density and distribution function, further by  $\phi_2(x, y)$  distribution function and by  $\varphi_2(x, y)$  density function of two-dimensional Gauss distribution with both expectations equal to zero and covariance matrix  $\Sigma = [1, \rho; \rho, 1]$ . If we define  $U = \Phi(X)$ ,  $V = \Phi(Y)$ , we obtain that the couple  $(U, V)$  has a 2-dimensional copula distribution on  $(0, 1)^2$  with distribution function

$$\mathbf{C}(u, v) = \phi_2(\Phi^{-1}(u), \Phi^{-1}(v)). \quad (2)$$

Naturally,  $\rho(U, V) \neq \rho(X, Y)$  (though they are rather close, as a rule), while Spearman's correlations coincide, namely  $\rho_{\text{SP}}(X, Y) = \rho_{\text{SP}}(U, V) = \rho(U, V)$ . As our aim is to model the dependence of competing variables  $S, T$ , let us assume that their joint distribution function is given by Gauss copula (2),

$$F_2(s, t) = \phi_2(\Phi^{-1}(F_S(s)), \Phi^{-1}(F_T(t))), \quad (3)$$

and  $S = F_S^{-1}(\Phi(X))$ ,  $T = F_T^{-1}(\Phi(Y))$ . Again  $\rho_{\text{SP}}(S, T) = \rho_{\text{SP}}(U, V)$ , and "initial"  $\rho = \rho(X, Y)$  is the only parameter describing the dependence of  $S$  and  $T$ . It, naturally, differs from  $\rho(S, T)$ , however, all values  $\rho(S, T)$  can be achieved by convenient choice of  $\rho(X, Y)$ . Let us remark here that the real dependence among  $S, T$  can be much more complicated, nevertheless the use of Gauss copula offers here certain rather simple and sufficiently flexible (as regards the correlation) set of distributions.

#### 3.1 Estimation in Gauss copula model

When parameter  $\rho$  is known, copula (2) is fully defined and from Zheng, Klein (1995) it follows that the distribution of  $(S, T)$  can be estimated, even non-parametrically. On the other hand, without knowledge of  $\rho$  nonparametric model is not identifiable and in the parametric setting explicit proofs of identifiability are available for just certain types of marginal distributions specified for instance already in Basu and Ghosh (1978). We shall deal with a richer model including the covariates, and, similarly like Han and Hausman (1990), with the Cox's model of dependence on them. Let us first sketch the estimation procedure based on the maximum likelihood method. The data are  $(Z_i, \delta_i)$ ,  $i = 1, \dots, N$ , the likelihood function then has the form

$$L = \prod_{i=1}^N \left\{ -\frac{\partial}{\partial s} \bar{F}_2(s, t) \right\}^{I[\delta_i=1]} \cdot \left\{ -\frac{\partial}{\partial t} \bar{F}_2(s, t) \right\}^{I[\delta_i=2]} \cdot \bar{F}_2(s, t)^{I[\delta_i=0]},$$

evaluated at  $s = t = Z_i$ , with  $\bar{F}_2(s, t) = P(S > s, T > t) = 1 - F_S(s) - F_T(t) + F_2(s, t)$ . From (3) it follows that  $F_2(s, t) = \phi_2(x, y)$  with  $x = \Phi^{-1}(F_S(s))$ ,  $y = \Phi^{-1}(F_T(t))$ . For the first term we obtain that

$$\frac{\partial}{\partial s} \bar{F}_2(s, t) = -f_S(s) + \frac{\partial}{\partial s} \phi_2(\Phi^{-1}(F_S(s)), \Phi^{-1}(F_T(t))) =$$

$$= -f_S(s) + \frac{\partial}{\partial x} \phi_2(x, y) \cdot \frac{d\phi^{-1}(F_S(s))}{ds} = -f_S(s) + \phi_1(y; \rho x, 1 - \rho^2) \cdot \varphi(x) \cdot \frac{f_S(s)}{\varphi(x)},$$

where  $\phi_1(y; \mu, \sigma^2)$  denotes the distribution function of normal distribution  $N(\mu, \sigma^2)$ , evaluated at  $y$ . The second term of the likelihood can be processed in the same way, hence the likelihood can be expressed in the following manner:

$$L = \prod_{i=1}^N \left\{ f_S(Z_i) [1 - \phi_1(Y_i; \rho X_i, 1 - \rho^2)] \right\}^{I[\delta_i=1]} \cdot \left\{ f_T(Z_i) [1 - \phi_1(X_i; \rho Y_i, 1 - \rho^2)] \right\}^{I[\delta_i=2]} \cdot \left\{ 1 - F_S(Z_i) - F_T(Z_i) + \phi_2(X_i, Y_i) \right\}^{I[\delta_i=0]}, \quad (4)$$

again with  $X_i = \phi^{-1}(F_S(Z_i))$ ,  $Y_i = \phi^{-1}(F_T(Z_i))$ . Parameter  $\rho$  is hidden in  $\phi_1$  and in  $\phi_2$ . Distributions of  $S$  and  $T$  are present both explicitly and also implicitly, in transformed  $X_i$ ,  $Y_i$ . It is seen that the problem of maximization may be a difficult optimization task and has to be solved by a convenient numerical procedure. In the following real data example we search for the MLE of parameters. As it can be rather computationally involving, the search is performed with the aid of the MCMC method, in the Bayes approach framework starting from conveniently chosen uniform priors (c.f. Gamerman, 1997). Such method then allows to obtain estimated Bayes credibility intervals for model parameters.

## 4 Real data example

The data are taken from the Statlib database: <http://lib.stat.cmu.edu/datasets/caseK.txt>, the "Case K" data, appearing also in Kadane and Woodworth (2004). They did not use the idea of competing risks, the aim of their study was to explore whether older employees were or not "discriminated" having higher rate of dismissal. In fact, the same question is, though just implicitly, behind our analysis, the positive dependence of both risks revealed in the present paper can be interpreted also in such a way that under the risk of dismissal some people prefer to leave the job voluntarily, the change it in time. And the aim is to show that younger employees have higher tendency to do it. This phenomenon should be taken into account when comparing younger and older persons.

The data contain the records on all persons employed by a firm during the period of observation, from 1.1.1900 to 31.1.1995, namely their dates of birth, dates when persons were hired by the company and when they have left it, either voluntarily or were forced to leave (dismissed). There were together 412 people, from them 96 were fired, 108 left voluntarily, the rest, 208 employees, were still with the company at the end of data collection period. Hence, we deal with two competing risks of the end of work, and we assume that the risks are dependent. The time considered is the calendar time, in days, from 1 to 1857, the end of study is also the fixed time of censoring, namely  $C = 1857$  is the upper bound for each personal record (it is so called type I censoring by fixed value). It is expected that the development of the company can be the reason for changing rates of leaving it. There are also people joining the company during the followed period, thus changing the "risk set" of the study.

The age (in years) of employees at the moment of leave or censoring was taken as a covariate, because it was expected that the age can influence the decision and is changing the relation between the rate of compulsory and voluntary leaving. The age varied from 20 to 70 years, its median was 39. Figure 1 shows the times of leaving, distinguishing both ways. It is seen that the period of higher intensity of dismissals (possibly as a consequence of certain problems of the company) started at about day 800 and lasted almost another 800 days, and was more remarkable for the strata with higher age, in fact supporting the conjecture of Kadane and Woodworth. In order to prove the dependence of correlation of random times of both competing events on age, the sample was divided into two subsamples with  $\text{age} < 40$  and  $\text{age} \geq 40$  years, each subsample was analyzed separately. Number of employees in each group and observed incidence is given in Table 1 below.

### 4.1 Competing risks with Cox's regression model

The competing risks model together with the Gauss copula was sketched in preceding section. The influence of the covariate (age) was incorporated via the Cox's regression model. Hence, each of both random times was described by the hazard rate

$$h_j(t; x) = h_{0j}(t) \cdot \exp(\beta_j \cdot x),$$

where  $j = 1, 2$  for related risks of dismissal and voluntary leave, respectively,  $h_{0j}$  are baseline hazard rates and  $\beta_j$  are regression parameters. Covariate  $x$  is the age of persons (in years) at moment of job termination or of censoring,  $t$  is the time of study in days. Further, to make computation easier, we assume that the baseline distributions correspond to Weibull ones. The baseline hazard rates and their cumulated (integrated) versions have then the form

$$h_{0j}(t) = \frac{b_j \cdot t^{b_j-1}}{a_j^{b_j}}, \quad H_{0j}(t) = \left( \frac{t}{a_j} \right)^{b_j},$$

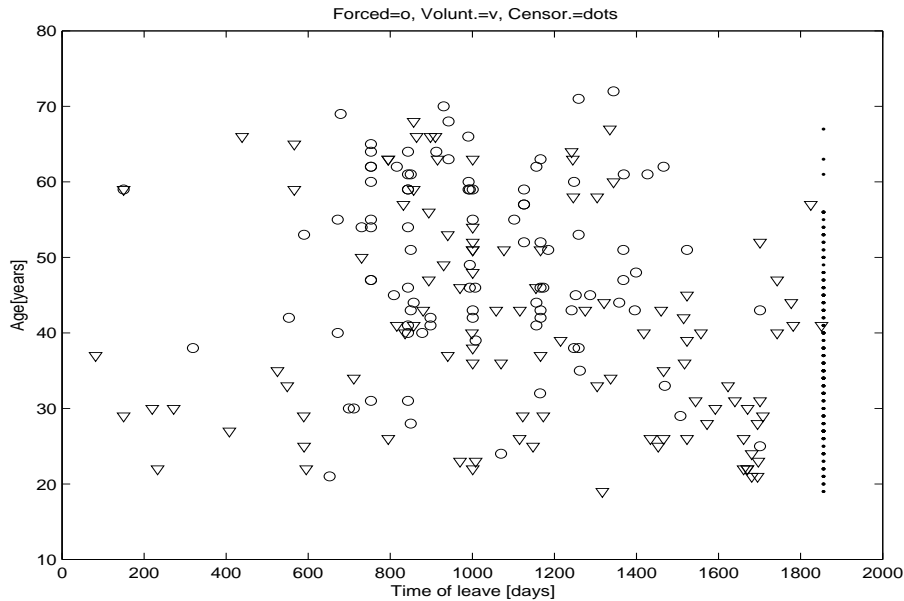


Figure 1 Graphical representation of data: circles=dismissals, triangles=voluntary leavings, dots=censored records.

where  $a_j, b_j, j = 1, 2$  are Weibull scale and shape parameters. When the Cox's regression term is added, the distribution of both random times for given covariate value  $x$  is still Weibull, with the same shape parameters  $b_j$  and scale parameters depending on covariate, i.e.

$$a_j(x) = a_j \exp\left(-\frac{\beta_j}{b_j} x\right).$$

The complete model then contains 7 unknown parameters,  $a_j, b_j, \beta_j$  for  $j = 1, 2$  and parameter  $\rho$  controlling the dependence of both competing risks via the Gauss copula.

## 4.2 Results

The model is fully parametrized, parameters were estimated by the MLE method. Already from (4) it is seen that the computation may be difficult, therefore the maximum of log-likelihood was found approximately with the aid of a random search. The results are collected in Table 1. In order to get also certain insight into the credibility of parameters values we have employed the Metropolis algorithm in the Bayes framework. Such a procedure yields the representation of the posterior distribution of parameters, hence also credibility intervals for them. Prior distributions of parameters were chosen uniform in reasonable intervals, the first rough estimate was obtained under the assumption of no dependence (i.e.  $\rho = 0$ ). Then, naturally, the value of parameter  $\rho$  was alternated, too. Each computation used 10000 iterations of the algorithm, credibility intervals were obtained from the last half of them. It is seen that estimated parameters differ in both parts of data, i.e. they depend on the covariate - age of employees. It concerns also parameter  $\rho$ . The interpretation of higher positive correlation in the group of younger employees could be that they are more flexible and in the case of symptoms of approaching negative changes in the company they are more prone to search for a new employment. Further, it is seen that also Cox's model parameters  $\beta$  characterizing the dependence of hazard rates on age differ for both risks. For instance, from the first column of Table 1 we can deduce that the risk of dismissal increases significantly with age (parameter  $\beta_1$ ). In fact, it could be understood as an indicator of existing discrimination of older employees in the sense of the study of Kadane and Woodworth (2004).

## 5 Conclusion

We have studied the problem of competing risks with regression, with the focus on assessing mutual dependence of competing random variables. The joint distribution was expressed with the aid of Gauss copula, while the Cox's regression model described the covariate influence. The model was utilized in an example with real unemployment data. Statistical analysis revealed positive correlation between times to both competing events, and also the dependence of correlation on the covariate. This was, in fact, the main purpose of the study. It has to be said that the Weibull model, used in order to simplify computations, is far from optimal. Further, the experience with present and similar computational procedures indicates that the log-likelihood function is, as a rule, rather flat,

	Data:	all data	with age < 40		with age $\geq$ 40	
Par.	$n_{1,2,0}$ :	96, 108, 208	16, 53, 141		80, 55, 67	
$a_1$	13723	(13150,15608)	4667	(3731,5920)	12165	(10418,14602)
$b_1$	1.975	(1.863,2.053)	1.444	(1.243,1.672)	2.070	(1.900,2.254)
$a_2$	3186	(3152,3420)	2817	(2582,3072)	8924	(6021,11395)
$b_2$	1.904	(1.776,1.910)	1.443	(1.300,1.617)	2.337	(2.117,2.597)
$\beta_1$	0.0713	(0.0652,0.0722)	0.0063	(0.0027,0.0120)	0.0717	(0.0616,0.0789)
$\beta_2$	0.0090	(0.0085,0.0100)	0.0002	(-0.0001,0.0003)	0.0600	(0.0507,0.0761)
$\rho$	0.221	(0.025,0.263)	0.778	(0.688,0.944)	0.216	(-0.073,0.580)

Table 1 Results of MCMC: Estimated parameters (modes of posterior distribution) and 90% credibility intervals. Scale parameters  $a_1, a_2$  of Weibull baseline distributions are related to time in days;  $n_1, n_2, n_0$  are numbers of people dismissed, leaving voluntarily, censored.

the convergence of computations to its maximum is slow, resulting confidence intervals are then quite wide. This phenomenon does not depend on the copula choice, it is a consequence of complicated model structure.

**Acknowledgement:** The research was supported by the grant No. 18-02739S of the Grant Agency of the Czech Republic.

## References

- [1] Basu, A.P. and Ghosh, J.K.: Identifiability of the Multinormal and Other Distributions under Competing Risks Model, *Journal of Multivariate Analysis* **8** (1978), 413–429.
- [2] Van den Berg, G.J., van Lomwel, A.G.C., and van Ours, J.C.: Nonparametric estimation of a dependent competing risks model for unemployment durations, *Empirical Economics* **34** (2008), 477–491
- [3] Escarela, G. and Carriere, J.F.: Fitting competing risks with an assumed copula, *Statistical Methods in Medical Research* **12** (2003), 333–349
- [4] Gamerman, D.: *Markov Chain Monte Carlo*. Chapman and Hall, New York, 1997.
- [5] Han, A. and Hausman J.A.: Flexible parametric estimation of duration and competing risk models, *J. of Applied Econometrics* **5** (1990), 1–28.
- [6] Heckman, J.J. and Honoré, B.E.: The identifiability of the competing risks model, *Biometrika* **76** (1989), 325–330.
- [7] Kaishev, V.K., Dimitrova, D.S., and Haberman, S.: Modelling the joint distribution of competing risks survival times using copula functions, *Insurance: Mathematics and Economics* **41**(2007), 339–361
- [8] Kadane, J.B. and Woodworth, G.G.: Hierarchical Models for Employment Decisions, *Journal of Business and Economic Statistics* **22** (2004), 182–193
- [9] Lee, S.: Identification of a competing risks model with unknown transformations of latent failure times, *Biometrika* **93** (2006), 996–1002.
- [10] Lin, D.Y.: Non-parametric inference for cumulative incidence functions in competing risks studies, *Statistics in Medicine* **16** (1997), 901–910.
- [11] Schwarz, M., Jongbloed, G., and Van Keilegom, I.: On the identifiability of copulas in bivariate competing risks models, *Canadian Journal of Statistics* **41** (2013), 291–303.
- [12] Tsiatis, A.: A nonidentifiability aspects of the problem of competing risks, *Proc. Nat. Acad. Sci. USA* **72** (1975), 20–22.
- [13] Zheng, M. and Klein, J.P.: Estimates of marginal survival for dependent competing risks based on an assumed copula, *Biometrika* **82** (1995), 127–138.

36<sup>th</sup> International Conference

# Mathematical Methods in Economics

September 12<sup>th</sup> – 14<sup>th</sup>, 2018, Jindřichův Hradec, Czech Republic



## Conference Proceedings

**Published by:**

MatfyzPress,

Publishing House of the Faculty of Mathematics and Physics Charles University

Sokolovská 83, 186 75 Praha 8, Czech Republic

as the 565. publication.

Printed by Reprintředisko MFF UK.

The text hasn't passed the review or lecturer control of the publishing company MatfyzPress.

The publication has been issued for the purposes of the MME 2018 conference.

The publishing house Matfyzpress is not responsible for the quality and content of the text.

Printed in Prague — September 2018

**Organized by:**

Faculty of Management, University of Economics, Prague

**Under auspices of:**

Czech Society for Operations Research

Czech Econometric Society

**Credits:**

Editors: Lucie Váchová, Václav Kratochvíl

L<sup>A</sup>T<sub>E</sub>Xeditor: Václav Kratochvíl

Cover design: Jiří Přibíl

using L<sup>A</sup>T<sub>E</sub>X's 'confproc' package, version 0.8 by V. Verfaillie

© L. Váchová, V. Kratochvíl (Eds.), 2018

© MatfyzPress, Publishing House of the Faculty of Mathematics and Physics  
Charles University, 2018

**ISBN: 978-80-7378-371-6 (printed version)**

**978-80-7378-372-3 (electronic version)**