KYBERNETIKA — VOLUME *54* (2018), NUMBER 1, PAGES 155–174

ADAPTIVE HIGH GAIN OBSERVER EXTENSION AND ITS APPLICATION TO BIOPROCESS MONITORING

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The adaptive version of the high gain observer for the strictly triangular systems subjected to constant unknown disturbances is proposed here. The adaptive feature is necessary due to the fact that the unknown disturbance enters in a way that cannot be suppressed by the high gain technique. The developed observers are then applied to a culture of microorganism in a bioreactor, namely, to the model of the continuous culture of *Spirulina maxima*. It is a common practice that just the biomass (or substrate) concentration is directly measured as the output of the process for monitoring and control purposes. This paper thereby shows both by theoretical analysis and numerical simulation that the adaptive high-gain observers offer a realistic option of online software sensors for substrate estimation.

Keywords: adaptive observers, nonlinear systems, bioprocess

Classification: 93C95, 90C46

1. INTRODUCTION

The deterministic observer theory goes back to the well-known Luenberger observer for the linear system and their extensions to the case of more complex dynamics, see e. g. [21, 27] and references within there. Observability theory for nonlinear systems was introduced in the seminal paper [18]. Along with this line of research considering a more general classes of systems than the linear one, the problem of observers construction for the systems with unknown parameters was introduced as well. More specifically, during the early seventies the adaptive observer for linear systems with a parameter adaptation algorithm was presented in [15]. For the multi-input multi-output (MIMO) linear system, assuming detectability, as well as persistent excitation (PE) of some observer internal signal, one may basically conclude that both the state and the unknown parameter can be asymptotically estimated. Furthermore, the adaptive observers with exponential convergence were proposed in [23]. This concept can be extended to systems having a possibly nonlinear input and output injection term at the right-hand side of the state equation, which is perhaps the most general case where PE can be checked through some clear test. Some earlier results on nonlinear observers during the eighties go back to [3], continued in the nineties by [9, 28, 29], all these efforts were nicely and comprehensively

DOI: 10.14736/kyb-2018-1-0155

presented in the well-know monograph [27]. Relatively recent development is related with the series of mutually related works [5, 25, 36] commented in detail later on. As an alternative to the adaptive approach, various robust techniques were considered, most importantly, the sliding mode technique, see e.g [4, 10, 13], or the high gain technique, see e.g [14],[21]. The recent observers research field is a very broad one, including observers design for classes of implicit and descriptor systems [1, 17], time delayed systems [19, 34] and fractional order systems [35]. Observer concept finds also its application in other system related tasks, like fault detection and fault tolerant control [20, 22], to mention just a few. The above overview is by no mean complete as the current paper aims anyway to concentrate to a certain more narrow segment of the overall extensive and broad adaptive observers research subarea.

The aim of the current paper is twofold. First, to provide combination of the high-gain approach with the dynamic adaptive observers that are the generalization of those in [25, 36]. Secondly, to use these results, to tackle an important practical problem of the observation of the unknown state of the waste water treatment facility with the simultaneous estimation of the unknown constant component of the dilution factor of the substrate inlet.

The class of systems where adaptive observers are to be designed is the so-called strictly triangular one [27] and [11], as far as the dependence of the right-hand side on the state and the input is concerned. Nevertheless, the unknown constant additive input disturbance is assumed to be present as well, i. e. this constant disturbance enters, in general, every row of the system right-hand side. Therefore, one can not simply regard the unknown input component as an artificial additional state component to be estimated as the resulting structure would be no more strictly triangular. Furthermore, the constant unknown disturbance is multiplied by the vector field having the strictly triangular dependence on the state. Therefore, one can not use the static-like observers from [27], as they require that such a field is a constant vector multiplied by scalar nonlinearity depending on output only, not speaking about the additional strong assumption enabling to use the strict positive realness property. Neither can one use directly the results [25, 36] as they require that vector field to be output dependent only. Moreover, these results require some a priori stability assumptions that might be difficult to achieve. In our approach, combining the high-gain design with dynamical adaptation allows to handle more general vector field that multiplies uncertain parameter. In [12], the high-gain observer for a class of uniformly observable single-output nonlinear system is presented where the compulsory use of the high-gain parameter makes necessary to analyze the case when it goes to infinity, therefore leading to look for a palliative to the PE condition as stated in [11]. It is important to note that due to our new constructive proof of the basic high-gain technique one can constructively design gains with reasonable values. In [24], on the basis of the Extended Kalman Filter, the adaptive observer was considered. Nevertheless, the current paper proposes a different array based on the adaptive extension of the high-gain technique capable to deal with the robust observer problem. Summarizing, the present work deals with the design of nonlinear observers for strictly triangular systems in presence of the input disturbances. The adaptation feature then allows to determine the unknown constant parameter which extends the work reported in [31].

An important motivation to investigate the class of strictly triangular systems is the necessity to observe the model of the waste water treatment facility. Nowadays, biological and biotechnological processes have gained a prominent place within the basic and applied research due to its high impact in the industrial processes. In turn, *Spirulina maxima* is used as a dietary supplement for humans and under stress condition it is capable to produce an expensive dietary pigment for the food industry, moreover, it is employed for the waste water treatment [2, 6, 8, 33]. In such a process, the reliable control and monitoring requires the online measurements of biomass and nutrients, which is a complex issue as it involves nonlinear dynamics, high parameter sensitivity, external disturbances, ambient noise, etc.

The rest of the paper is organized as follows. The preliminary definitions and results on nonlinear observability and observers are collected in the next section including the derivation of the classical high-gain exponentially stable observer and the alternative proof of its convergence compared to [14, 21]. Section 3 presents the main contribution of the paper being two versions of the dynamic adaptive high-gain observer. The waste water system transformation from the standard Monod bioprocess description to a more convenient one together with a useful forward invariant property is presented in Section 4. Finally, the specific bioprocess observer design and its numerical simulations are provided in Section 5. Final section contains conclusions and outlooks for the possible future research.

2. DEFINITIONS AND PRELIMINARY RESULTS

Consider the following single-input single-output nonlinear system

$$\dot{x} = A_0 x + f(x) + g(x)u, \quad y = h(x), \tag{1}$$

where $A_0, f(x), g(x), h(x)$ are such that (1) has the following form

$$\dot{x} = \begin{bmatrix} x_2 \\ x_3 \\ \vdots \\ x_n \\ 0 \end{bmatrix} + \begin{bmatrix} f_1(x_1) \\ f_2(x_1, x_2) \\ \vdots \\ f_{n-1}(x_1, \dots, x_{n-1}) \\ f_n(x_1, \dots, x_n) \end{bmatrix} + \begin{bmatrix} g_1(x_1) \\ g_2(x_1, x_2) \\ \vdots \\ g_{n-1}(x_1, \dots, x_{n-1}) \\ g_n(x_1, \dots, x_n) \end{bmatrix} u, \tag{2}$$
$$y = x_1.$$

In other words, $A_0, f(x), g(x), h(x)$ are given as

$$A_0 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \vdots & & \ddots & \\ & & & 1 \\ 0 & \cdots & & 0 \end{bmatrix}, \quad f(x) = \begin{bmatrix} f_1(x_1) \\ f_2(x_1, x_2) \\ \vdots \\ f_{n-1}(x_1, \dots, x_{n-1}) \\ f_n(x_1, \dots, x_n) \end{bmatrix}, \tag{3}$$

$$g(x) = \begin{bmatrix} g_1(x_1) \\ g_2(x_1, x_2) \\ \vdots \\ g_{n-1}(x_1, \dots, x_{n-1}) \\ g_n(x_1, \dots, x_n) \end{bmatrix}, \quad h(x) = x_1, \tag{4}$$

where $f(x)$, $g(x)$ and $h(x)$ are smooth vector fields and function, respectively. Further, assume that $f(x)$, $g(x)$ are globally uniformly Lipschitz on a given subset Ω of R^n and that Ω is forward invariant with respect to (1,2) for any admissible input. In the sequel, (1,2) will be referred to as the so-called **strictly triangular system (STS)**.

Definition 2.1. Asymptotic observer of the system (1) is the following system

$$\begin{aligned} \dot{z}(t) &= A_0 z(t) + f(z) + g(z)u(t) + \alpha(z, u, y) \\ z(t) &= \beta(z, u, y), \quad z \in R^n, \end{aligned} \tag{5}$$

provided that $\forall z_0, x_0 \in R^n$ and every bounded input $u(t)$ it holds:

i) $z_0 = x_0 \implies x(t, x_0) = z(t, z_0)$ for all $t \geq t_0$,

ii) $\lim_{t\to\infty} \|z(t, z_0) - x(t, x_0)\| = 0$.

Here, $x(t, x_0)$ stands for the solution of (1) with $x(t_0, x_0) = x_0$ while $z(t, z_0)$ is the solution of (5) with $z(t_0, z_0) = z_0$. The corresponding observer is called as the global one if the condition *ii)* holds for any z_0, x_0 and it is called as the exponential one if the convergence in *ii)* is the exponential one. Where no confusion arises, we will write in the sequel just $x(t), z(t)$. The component z_1 will be occasionally referred to as the output of the observer (5).

For STS (1), the observer in the sense of Definition 2.1) is provided by the following

Theorem 2.2. Assume that all right hand side vector fields in (1) are globally uniformly Lipschitz on $\Omega \subset R^n$ being forward invariant with respect to (1) for any bounded input $u(t), t \geq t_0$. Then for any real gains $l_1, l_2, \dots, l_n$, such that the matrix

$$A_l = \begin{bmatrix} l_1 & 1 & 0 & \cdots & 0 \\ l_2 & 0 & 1 & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 \\ l_{n-1} & 0 & \cdots & 0 & 1 \\ l_n & 0 & \cdots & 0 & 0 \end{bmatrix} \tag{6}$$

is the Hurwitz one, there exists a real number $r > 0$ such that the following system

$$\dot{z} = \begin{bmatrix} z_2 \\ z_3 \\ \vdots \\ z_n \\ 0 \end{bmatrix} + \begin{bmatrix} f_1(x_1) \\ f_2(x_1, z_2) \\ \vdots \\ f_{n-1}(x_1, z_2, \dots, z_{n-1}) \\ f_n(x_1, z_2, \dots, z_n) \end{bmatrix} \tag{7}$$

$$+\begin{bmatrix} g_1(x_1) \\ g_2(x_1, z_2) \\ \vdots \\ g_{n-1}(x_1, z_2, \dots, z_{n-1}) \\ g_n(x_1, z_2, \dots, z_n) \end{bmatrix} u + \begin{bmatrix} rl_1 \\ r^2 l_2 \\ \vdots \\ r^{n-1} l_{n-1} \\ r^n l_n \end{bmatrix} (z_1 - x_1) \tag{8}$$

is the exponential observer of (1), provided the system trajectory starts in the interior of Ω while the initial observer estimate starts sufficiently close to it. If $\Omega = R^n$, then the corresponding observer is the global exponential one.

The results similar to the above theorem were already presented in the literature before, see e. g.[14]. The novelty of the current paper is the alternative proof, presented later on, which allows to obtain a reasonable value of the "high-gain parameter" $r > 0$, including a constructive way how to determine it. Note also, that the observer presented by Theorem 2.2 has the usual form of the copy of the system to be observed[1] and the injection of the output error evaluated as $e_1 = z_1 - x_1$.

P r o o f. First, let us obtain the error dynamics for the estimation error $e = z - x$, namely, subtracting (1) from (8) gives:

$$\dot{e} = \begin{bmatrix} e_2 \\ e_3 \\ \vdots \\ e_n \\ 0 \end{bmatrix} + \begin{bmatrix} rl_1 \\ r^2 l_2 \\ \vdots \\ r^{n-1} l_{n-1} \\ r^n l_n \end{bmatrix} e_1 + \begin{bmatrix} 0 \\ k_2(x_1, z_2, u) - k_2(x_1, x_2, u) \\ \vdots \\ k_{n-1}(x_1, z_2, \dots, z_{n-1}, u) - k_{n-1}(x_1, \dots, x_{n-1}, u) \\ k_n(x_1, z_2, \dots, z_n, u) - k_n(x_1, \dots, x_{n-1}, x_n, u) \end{bmatrix}, \tag{9}$$

where $k_i(\cdot, u) = f_i(\cdot) + g_i(\cdot)u$, $i = 2, \dots, n$. Equation (9) can be rewritten as follows

$$\dot{e} = \begin{bmatrix} rl_1 & 1 & 0 & \cdots & 0 \\ r^2 l_2 & 0 & 1 & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 \\ r^{n-1} l_{n-1} & 0 & \cdots & 0 & 1 \\ r^n l_n & 0 & \cdots & 0 & 0 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ \vdots \\ e_n \end{bmatrix} + U(t) \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ \vdots \\ e_n \end{bmatrix}. \tag{10}$$

The matrix $U(t)$ has by $e_j = z_j - x_j,\ j = 1, \dots, n$, the following form

$$U(t) = \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 \\ 0 & u_{22} & 0 & \ddots & \vdots \\ \vdots & \vdots & \vdots & \ddots & 0 \\ 0 & u_{n-1,2} & \cdots & u_{n-1,n-1} & 0 \\ 0 & u_{n2} & \cdots & u_{n,n-1} & u_{nn} \end{bmatrix}, \tag{11}$$

$$u_{i,j}(t) := \frac{k_i(x_1, \dots, x_{j-1}, z_j, \dots, z_i, u) - k_i(x_1, \dots, x_j, z_{j+1}, \dots, z_i, u)}{z_j - x_j}, \tag{12}$$

[1]The minor difference is that in the system copy on the observer (8) right hand side the component z_1 is replaced by the directly measured output of the system to be observed x_1 which is realizable as an additional output injection and it may obviously only help to improve the observer convergence.

where $i = 2, \dots, n$, $j = 2, \dots, i$ and $z_k, \dots, z_l$, should be replaced by the empty space if $k > l$. Furthermore, let $\|\cdot\|$ stands for some suitable matrix norm compatible with the Euclidean vector norm. By the uniform Lipschitz property assumption there obviously exists a fixed suitable constant $M > 0$ such that

$$\|U(t)\| \leq M, \quad \forall t \geq t_0, \tag{13}$$

provided both trajectory to be observed and the estimate stay inside the set where the uniform Lipschitz property holds. In case of the global Lipschitz on R^n property such a provision is obviously valid, while in case of Lipschitz property on $\Omega \subset R^n$ that provision obviously holds if the error dynamics is exponentially stable and the initial error is sufficiently small. Summarizing, both claims to be proved are indeed proved if the above error dynamics is shown to be stable for some $r > 0$.

To proceed with, recall that the gains $l_1, \dots, l_n$ were selected in such a way that A_l given by (6) is Hurwitz, therefore there exists matrix $S = S^\top > 0$ such that:

$$SA_l + A_l^\top S = -I \tag{14}$$

and for any $r > 0$ introduce the matrix

$$S(r) := \Theta^{-1}(r) S \Theta^{-1}(r), \quad \Theta(r) = diag(1, r, \dots, r^{n-1}). \tag{15}$$

Notice, that for all $r > 0$ the matrix $S(r)$ is also positively definite and symmetric as $\Theta(r)$ is nonsingular and diagonal matrix. Next, introduce yet another notation

$$A(r) = \begin{bmatrix} rl_1 & 1 & 0 & \cdots & 0 \\ r^2 l_2 & 0 & 1 & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 \\ r^{n-1} l_{n-1} & 0 & \cdots & 0 & 1 \\ r^n l_n & 0 & \cdots & 0 & 0 \end{bmatrix}, \tag{16}$$

in particular, $A(0) = A_0$, $A(1) = A_l$, $\Theta(1) = I_n$, $S(1) = S$ and for all $r \geq 1$

$$\Theta^{-1}(r) A(r) \Theta(r) = r A_l, \quad \|S(r)\| \leq \|S\|, \quad \left\|\Theta^{-1}(r) x\right\| \leq \|x\|,$$

which can be checked by straightforward computations. Note, that $S(r)$ is, indeed, symmetric as S is symmetric and $\left[\Theta^{-1}(r)\right]^\top = \Theta^{-1}(r)$ by the diagonality of $\Theta(r)$. Moreover, $S(r)$ is positive definite since $\forall e \neq 0$ it holds

$$e^\top S(r) e = e^\top \Theta^{-1}(r) S \Theta^{-1}(r) e = \left[\Theta^{-1}(r) e\right]^\top S \left[\Theta^{-1}(r) e\right] > 0,$$

where the last inequality is the consequence of the positive definiteness of S. Moreover, by the lower triangularity of $U(t)$, by the definition of $\Theta(r)$ in (15) and by (13) it holds

$$\|\Theta^{-1}(r) U(t) \Theta(r)\| \leq \|U(t)\| \leq M, \quad \forall t \geq t_0. \tag{17}$$

Next, consider the parameterized Lyapunov function candidate

$$V_r(e) = e^\top S(r) e$$

having the full time derivative along the trajectories of the error dynamics (9,10,11)

$$\frac{dV_r(e)}{dt} = e^\top \left[S(r)A(r) + A(r)^\top S(r)\right] e(t) + e^\top \tilde{A}_r e(t),$$

where $\tilde{A}_r(t) = S(r)U(t) + U(t)^\top S(r)$. Straightforward computations using (15) give

$$\begin{aligned} \tfrac{dV_r(e)}{dt} &= e^\top \left[\Theta^{-1}(r)S\Theta^{-1}(r)A(r) + A(r)^\top \Theta^{-1}(r)S\Theta^{-1}(r)\right] e(t) + e^\top \tilde{A}_r e(t), \\ \tfrac{dV_r(e)}{dt} &= e_\Theta^\top \left[S\Theta^{-1}(r)A(r)\Theta(r) + \left(\Theta^{-1}(r)A(r)\Theta(r)\right)^\top S\right] e_\Theta + e^\top \tilde{A}_r e(t) = \\ & re_\Theta^\top [SA_l + A_l^\top S] e_\Theta + e_\Theta^\top [S\Theta^{-1}(r)U(t)\Theta(r) + (\Theta^{-1}(r)U(t)\Theta(r))^\top S] e_\Theta, \end{aligned} \tag{18}$$

where $e_\Theta = \Theta^{-1}(r)e$ and the last equality is again by (15) and by (14). Denote

$$\overline{A}_r = S\Theta^{-1}(r)U(t)\Theta(r) + (\Theta^{-1}(r)U(t)\Theta(r))^\top S. \tag{19}$$

Summarizing,

$$\frac{dV_r(e)}{dt} = -re_\Theta^\top e_\Theta + e_\Theta^\top \overline{A}_r e_\Theta = -r\left\|e_\Theta\right\|^2 + e_\Theta^\top \overline{A}_r e_\Theta. \tag{20}$$

Furthermore, by (17) and by (19) it holds that

$$\left\|\overline{A}_r\right\| \leq 2M\left\|S\right\|, \quad \forall t \in R, \forall r \geq 1,$$

and therefore

$$\frac{dV_r(e(t))}{dt} \leq -r\left\|e_\Theta\right\|^2 + 2M\left\|S\right\|\left\|e_\Theta\right\|^2 = -(r - 2M\left\|S\right\|)\left\|e_\Theta\right\|^2.$$

This gives by $e_\Theta = \Theta^{-1}(r)e$ that

$$\frac{dV_r(e(t))}{dt} \leq -(r - 2M\left\|S\right\|)\|\Theta^{-1}(r)\|\|e\|^2.$$

Finally, selecting any $r > 2M\left\|S\right\|$ gives

$$\tfrac{dV_r(e(t))}{dt} \leq -K\left\|e\right\|^2, \quad K := (r - 2M\left\|S\right\|)\|\Theta^{-1}(r)\| > 0, \tag{21}$$

so that the above introduced parameterized Lyapunov function candidate $V_r(e)$ becomes for the selected $r > 2M\left\|S\right\|$ the Lyapunov function proving the exponential stability of the error dynamics (9). □

Remark 2.3. The above proof shows that r can be, indeed, searched in a constructive way, moreover, it is possible to attempt to optimize the value of r via the selection of the matrix S and the gains $l_1, \ldots, l_n$, mutually related by (14). Indeed, the adequate LMI formulation of this problem would be straightforward. In such a vein, the sufficiency of condition $r > 2M\left\|S\right\|$ proves, in particular, that the observer design based on the LMI optimization approach [30, 37] has a feasible solution.

3. MAIN RESULTS

In general, the observer state estimation may be lost when system is subjected to external unknown disturbances acting on the system unless these disturbances are properly managed. Such a task and its solvability depends crucially on the class of these disturbances. Here, the constant disturbances without known range are to be considered. Namely, consider the following perturbed system given by

$$\begin{aligned} \dot{x} &= A_0 x + f(x) + g(x)u + p(x)\delta, \\ y &= x_1 = cx, \\ c &= [1, 0, \ldots, 0], \end{aligned} \tag{22}$$

where $f(x), g(x)$ are as in (1) while $p(x)$ has the similar strictly triangular structure as f, g do have there. Furthermore, $x \in R^n$ is the state vector, while y, u, δ are the scalar output, input and constant parameter perturbation, respectively. In the sequel, we assume the same Lipschitzean properties as in Theorem 2.2.

First, for the sake of completeness let us repeat the well-known definition of the persistent excitation and its consequences, see e. g. [27].

Definition 3.1. A bounded and piecewise continuous function $\varphi(t) \in R^n$ is called as persistently exciting (PE) if there exist $\alpha, \beta \in R$, $0 < \alpha \leq \beta$ and $T > 0$ such that $\forall t \geq 0$ it holds

$$\alpha \leq \int_t^{t+T} \varphi^\top(t)\varphi(t)\,\mathrm{d}\tau \leq \beta.$$

Lemma 3.2. Let $\varphi(t) \in R^n$ be persistently exciting, then the system

$$\dot{\eta} = -\varphi^\top(t)\varphi(t) \cdot \eta, \quad \eta \in R,$$

is globally exponentially stable.

Now we are ready to state the main result of the paper being a combination of the high gain design with dynamic adaptation similar to [25, 36]. High gain technique enables to treat more general state space dependence structure as in these previous results. First, let us state the following

Theorem 3.3. Consider system (22) with unknown constant $\delta \in R$. Let f, g, p be smooth and denote, respectively, by $F(x), G(x), P(x)$ the Jacobians of $f(x), g(x), p(x)$ at x and let $P(x)$ is globally bounded. Consider a state trajectory $x(t)$ of (22) generated by a finite input $u(t)$ such that $F(x(t)), G(x(t))$ are uniformly bounded $\forall t \geq t_0$. Then, for any n-tuple of gains $l_1, l_2, \ldots, l_n$ such that the matrix A_l defined in (6) is Hurwitz and $\forall k > 0$ there exists $r \geq 1$ and $\delta_0 > 0$ such that for all $\phi_0 \in (\delta - \delta_0, \delta + \delta_0)$ the following system

$$\begin{aligned} \dot{z} &= A_0 z + f(z) + g(z)u + [l_1 r, \ldots, l_n r^n]^\top (z_1 - x_1) + p(z)\phi + \gamma(t)\dot{\phi}, \\ \dot{\phi} &= -k\gamma^\top(t)c^\top ce, \quad \phi(0) = \phi_0, \\ \dot{\gamma}(t) &= [A(r) + F(z) + G(z)u + P(z)\phi]\gamma(t) + p(z), \end{aligned} \tag{23}$$

is the local (with respect to initial observation error) adaptive exponential observer of the considered above trajectory $x(t), t \geq t_0$, of (22) provided that the signal $\gamma^\top(t)c^\top$ is the persistently exciting for all $r \geq 1$.

P r o o f. First, notice that the observer error $e(\cdot) = z(\cdot) - x(\cdot)$ has the following dynamics

$$\begin{aligned}\dot{e} &= A(r)e + [f(x+e) + g(x+e)u - f(x) - g(x)u] \\ &+ p(x+e)\phi - p(x)\delta + \gamma(t)\dot{\phi},\end{aligned} \tag{24}$$

where $A(r)$ is given by (16). Further, (24) can be obviously rearranged as follows

$$\begin{aligned}\dot{e} &= A(r)e + [f(x+e) + g(x+e)u - f(x) - g(x)u \\ &+ p(x+e)\delta - p(x)\delta] + p(x+e)(\phi - \delta) + \gamma(t)\dot{\phi}.\end{aligned} \tag{25}$$

In the sequel, denote the error of the unknown parameter estimate as

$$\epsilon := \phi - \delta. \tag{26}$$

As the unknown parameter is a constant with respect to time, it clearly holds that

$$\dot{\epsilon} = \dot{\phi}. \tag{27}$$

Furthermore, introduce the so-called combined error variable $\bar{e}$ as follows

$$\bar{e} = e - \gamma(t)\epsilon. \tag{28}$$

Using the system equations, the observer equations and those for adaptation one has

$$\begin{aligned}\dot{\bar{e}} &= \dot{e} - \dot{\gamma}(t)\epsilon - \gamma(t)\dot{\epsilon} = A(r)e \\ &+ [f(x+e) + g(x+e)u - f(x) - g(x)u + p(x+e)\delta - p(x)\delta] \\ &+ p(x+e)(\phi - \delta) + \gamma(t)\dot{\phi} - \dot{\gamma}(t)\epsilon - \gamma(t)\dot{\epsilon},\end{aligned} \tag{29}$$

which gives by (27) that

$$\begin{aligned}\dot{\bar{e}} &= A(r)e + p(x+e)(\phi - \delta) - \dot{\gamma}(t)\epsilon \\ &+ [f(x+e) + g(x+e)u + p(x+e)\delta - f(x) - g(x)u - p(x)\delta],\end{aligned} \tag{30}$$

and, hence, by the third row of (23) and by (26)

$$\begin{aligned}\dot{\bar{e}} &= A(r)e + p(x+e)\epsilon \\ &+ [f(x+e) + g(x+e)u + p(x+e)\delta - f(x) - g(x)u - p(x)\delta] \\ &- \big[[A(r) + F(x+e) + G(x+e)u + P(x+e)\phi]\,\gamma(t) + p(x+e)\big]\epsilon.\end{aligned} \tag{31}$$

Canceling some terms and using (26)

$$\begin{aligned}\dot{\bar{e}} &= A(r)e - A(r)\gamma\epsilon \\ &+ [f(x+e) + g(x+e)u + p(x+e)\delta - f(x) - g(x)u - p(x)\delta] \\ &- [F(x+e) + G(x+e)u + P(x+e)\delta]\gamma(t)\epsilon + P(x+e)\gamma(t)\epsilon^2.\end{aligned} \tag{32}$$

As a consequence, by (28) and by Taylor first order expansion of the first three terms in the middle row of (32) with respect to a small $e - \bar{e} = \gamma\epsilon$ one has after the cancellation of the terms with Jacobians F, G, P the following equation

$$\begin{aligned}\dot{\bar{e}} &= A(r)\bar{e} + [f(x+\bar{e}) + g(x+\bar{e})u + p(x+\bar{e})\delta - f(x) - g(x)u - p(x)\delta] \\ &+ P(x+e)\gamma(t)\epsilon^2 + o(e - \bar{e}).\end{aligned} \tag{33}$$

Here, $o(e-\bar{e}) = o(\gamma\epsilon)$ stands for the higher order terms and using Taylor expansion again one gets the following expression for the combined error time derivative

$$\dot{\bar{e}} = A(r)\bar{e} + (F(x) + G(x)u + P(x)\delta)\bar{e} + P(x+e)\gamma(t)\epsilon^2 + o(\bar{e}) + o(e-\bar{e}). \tag{34}$$

Summarizing, one has by $e - \bar{e} = \gamma\epsilon$ that $o(e-\bar{e}) = o(\gamma\epsilon) = o(\epsilon)$ and by the theorem assumption on global boundedness of $P(\cdot)$ that it holds

$$\dot{\bar{e}} = A(r)\bar{e} + (F(x) + G(x)u + P(x)\delta))\bar{e} + o([\bar{e}, \epsilon]^\top). \tag{35}$$

By the theorem assumptions on boundedness of $F(\cdot), G(\cdot)$ along the trajectory to be observed, which, in turn, is generated by the bounded input $u(\cdot)$, there obviously exists a positive constant W such that:

$$\|(F(x) + G(x)u + P(x)\delta)\| < W. \tag{36}$$

Recalling the strict triangular structure of the system and choosing r sufficiently large in a analogous way as during the proof of Theorem 2.2, namely, as

$$r > 2W\,\|S\| \tag{37}$$

one has that $\bar{e}(t) \to 0$ as $t \to \infty$, exponentially for (35) when $o([\epsilon, \bar{e}]^\top)$ is neglected. In turn, the middle adaptation equation for $\epsilon(\cdot)$ given by (23) can be rewritten as

$$\dot{\epsilon} = -k\gamma^\top(t)c^\top c\gamma(t)\epsilon - k\gamma^\top(t)c^\top c\bar{e} \tag{38}$$

where the part

$$\dot{\epsilon} = -k\gamma^\top(t)c^\top c\gamma(t)\epsilon$$

is exponentially stable due to PE assumption and Lemma 3.2. Therefore, recalling $\bar{e}(t) \to 0$ as $t \to \infty$, one has by straightforward arguments that linear approximation of (35,38) is exponentially stable and therefore (35,38) is locally exponentially stable. Then, by definition of $\bar{e}$ also e and ϵ go exponentially to zero as t goes to infinity. □

Remark 3.4. It is generally well-known that the PE property is quite difficult to check in a constructive way. Moreover, the PE property is required by the formulation of Theorem 3.3 to be valid for all $r \geq 1$. Nevertheless, the following interesting and useful observation can be made based on the proof that has been just concluded. Indeed, note that the "high gain" parameter r can be fixed as any value $r > 2W\|S\|$, *i.e.* r does not need to go to infinity and PE property needs to be valid for limited value of parameter r only. This aspect represents a clear advantage with respect to other approaches, like [11] or [12].

Yet, the above PE assumptions is quite hard to be checked theoretically for a given general system. Indeed, (23) defining γ is a linear equation with time dependent coefficients and those time dependent coefficients depend on z and y obtained during the particular observer course. So the PE property actually should be checked for each possible output and observer trajectory. On the other hand, it usually holds in practice and can be easily checked during the simulations for particular trajectories. Another drawback is the local character with respect to ϵ, so that initial error of unknown parameter estimate should be reasonable small. Despite these facts, Theorem 3.3 seems to be the only option how to treat the above more complex state space structure of the system containing an unknown parameter.

In case that the unknown parameter perturbing vector field $p(\cdot)$ depends on the output only, one can obtain even better version of the above theorem. Namely, consider the system (22) as follows:

$$\begin{aligned} \dot{x} &= A_0 x + f(x) + g(x)u + p(y)\delta, \\ y &= x_1 = cx, \\ c &= [1, 0, \ldots, 0], \end{aligned} \tag{39}$$

where the same triangular-like assumptions on the right-hand vector fields as before are made. Then one has the following

Theorem 3.5. Consider system (39) with unknown constant $\delta \in R$. Assume that p is continuous, f, g are smooth and denote, respectively, by $F(z), G(z)$ the Jacobians of $f(x), g(x)$ at x. Furthermore, let $F(x(t)), G(x(t)), p(x(t))$ are uniformly bounded $\forall t \geq t_0$ where $x(t)$ is a given state trajectory of (22) to be observed and generated by a bounded input $u(t)$. Then, for any n-tuple of gains $l_1, l_2, \ldots, l_n$ such that the matrix A_l defined in (6) is Hurwitz and some $\forall k > 0$ there exists $r > 1$ and $\delta_0 > 0$ such that for all $\phi_0 \in (\delta - \delta_0, \delta + \delta_0)$ the following system

$$\begin{aligned} \dot{z} &= A_0 z + f(z) + g(z)u + [l_1 r, \ldots, l_n r^n]^\top (z_1 - e_1) + p(y)\phi + \gamma(t)\dot{\phi}, \\ \dot{\phi} &= -k\gamma^\top(t) c^\top c e, \quad \phi(0) = \phi_0, \\ \dot{\gamma}(t) &= [A(r) + F(z) + G(z)u]\gamma(t) + p(y), \end{aligned} \tag{40}$$

is the adaptive exponential observer of the above given trajectory $x(t), t \geq t_0$, of (39) provided that the signal $\gamma^\top(t)c^\top$ is persistently exciting for all $r \geq 1$.

Notice, that Theorem 3.5 is not a particular case of Theorem 3.3 as it is actually a bit stronger result making more advantage of the simpler system structure.

P r o o f. Nevertheless, the proof of Theorem 3.5 follows very similar arguments as the one of Theorem 3.3. Moreover, it is greatly simplified by the fact that all terms related to $p(\cdot)$ and $P(\cdot)$ are not present during the combined error dynamics analysis. In other words, one can repeat the steps (24-31) of the proof of Theorem 3.3 and then to realize that all terms related to $p(\cdot)$ are eliminated in (31) as both $p(x+e)$ and $p(x)$ are replaced everywhere by $p(y)$. Moreover, $P(\cdot)$ is also not present in this proof adapted version of (31) due to (40). So, one can easily finish the rest of the proof in the same way as for Theorem 3.3 just erasing $p(\cdot)$ and $P(\cdot)$ everywhere. The latter enables to make the same conclusions as in Theorem 3.3, but with the weaker assumptions of Theorem 3.5. □

Similar observations as in the case of Theorem 3.3 regarding the PE property are valid again. Indeed, the PE property is difficult to check practically, it and can be just verified in the course of simulation for particular trajectories. Let us mention that the case of systems of the form $\dot{x} = Ax + bu + f(y, u, t) + p\delta, \quad y = cx$, where p is a constant vector, represents perhaps the most general case where PE can be checked through some clear test. The analysis has been extended to the so-called state affine systems assuming the existence of some nominal exponentially stable observer and the PE property, this last is no more easy to test, see [5, 25, 36].

Finally, note that the above vector field $p(y)$ multiplied by that unknown parameter δ may be dependent on time and the input u, in addition to the dependence on the output y, and all the above treatment would remain the same.

4. WASTE WATER SYSTEM TRANSFORMATION

In this section, the interest is focused on showing both the useful invariance property and the waste water system transformation from the standard Monod bioprocess description to the more convenient one. It is supposed that the constant dilution factor of the substrate input is not known precisely. Nevertheless, this constant unknown dilution factor may be conveniently complemented by some additive time-varying dilution factor of the substrate input, the latter may be then naturally considered as the active control input. In such a way, the constant unknown perturbation enters the system to be controlled and observed through the same channel as the known and controlled input.

More specifically, a waste water system consists of a bioreactor where chemical and biochemical reactions between alive organisms and substances occur. The process can be anaerobic or aerobic and a great number of biotechnological process are formed by such systems. One way of modeling a biological non structured process is by mass balance where the rule for the rate of growth plays a fundamental role, [26]. Among several possible formulations, the so-called Monod formula is one of the most popular ones due to its simplicity and consistency with the saturation of growth due to substrate availability.

To be more precise, let us denote the concentration the biomass x and the one of the substrate s. Then the mass balance in a continuous culture is typically described by the following model:

$$\begin{aligned} \dot{x} &= x\mu(s) - x(u + \delta) \\ \dot{s} &= -a_3^{-1} x\mu(s) + (a_4 - s)(u + \delta), \end{aligned} \tag{41}$$

where the Monod rate of growth is given by

$$\mu(s) = \frac{a_1 s}{a_2 + s}. \tag{42}$$

The parameter a_1 is the maximal rate of growth, a_2 represents the Monod saturation constant, a_3 is the yield coefficient and finally a_4 is the input substrate concentration. The control input is the rate of the dilution $u(t)$ feeding the bioreactor and the input channel is biased by the constant unknown dilution factor $\delta > 0$.

Note, that system (41) is not in the strictly triangular form (1). Nevertheless, such a form can be achieved after suitable coordinate change. Namely, introduce the smooth state transformation introducing new variables $x_{1,2}$ as follows

$$x_1 = \ln(x), \qquad x_2 = \mu(s). \tag{43}$$

Notice, that the new variable $x_1 \in R$ is defined for all $x > 0$, which is not restrictive due to the biological interpretation of x. New variable x_2 ranges in $[0, a_1]$ and it is defined for $s > 0$ as well. In other words, the relation (43) defines a local diffeomorphism. These new coordinates give the following transformed state equations

$$\begin{aligned}\dot{x}_1 &= x_2 - u - \delta,\\ \dot{x}_2 &= (a_2 + \tfrac{a_2 x_2}{a_1 - x_2})^{-2}\big[-x_2 \exp(x_1) a_1 a_2 a_3^{-1} + (a_4 - \tfrac{a_2 x_2}{a_1 - x_2}) a_1 a_2 (u + \delta)\big],\end{aligned} \tag{44}$$

that can be further simplified as follows

$$\begin{aligned}\dot{x}_1 &= x_2 - u - \delta,\\ \dot{x}_2 &= -a_1^{-1} a_2^{-1} a_3^{-1} \exp(x_1) x_2 (a_1 - x_2)^2\\ &\quad + a_1^{-1} a_2^{-1} (a_4 a_1 - (a_2 + a_4) x_2)(a_1 - x_2)(u + \delta).\end{aligned} \tag{45}$$

Note, that the above simpler form avoids possible singularities. Its right hand side is not globally Lipschitz on the whole R^2, but for any bounded input and perturbation trajectories stay bounded, cf. Proposition 4.1 bellow, so that system is globally Lipschitz on its forward invariant set where it evolves. Therefore, one can use the high-gain argument explained in the theoretical part of this paper.

The following proposition demonstrates the biological consistency of the model and specify the forward invariant subset where system always evolves. Note, that for practical reasons the substrate concentration in the tank should always be after some time less than a_4 as the latter is the input substrate concentration.

Proposition 4.1. Consider the system

$$\begin{aligned}\dot{x} &= x\mu(s) - x(u + \delta),\\ \dot{s} &= -a_3^{-1} x\mu(s) + (a_4 - s)(u + \delta),\\ \mu(s) &= \tfrac{a_1 s}{a_2 + a_1 s}.\end{aligned} \tag{46}$$

The following properties hold:

(i) The strip inside the positive orthant defined as $\mathcal{O}^+ := \{x > 0,\ a_4 > s > 0\}$ is forward invariant with respect to the above system for every integrable input signal $u(t)$ and constant perturbation δ.

(ii) Every solution of the above system starting in $\mathcal{O}^+ := \{x > 0,\ a_4 > s > 0\}$ is bounded for every integrable input signal $u(t)$ and perturbation δ.

Proof. To prove (i) it is sufficient to check the right hand side on the boundary of $\mathcal{O}^+$. Note, that $x = 0$ is invariant due to the first equation. Therefore, every trajectory starting at $x(0) > 0$ should stay in the open right half-plane. Moreover, on the set $s = 0$ one has $\dot{s} \geq 0$ while on $s = a_4$ it holds $\dot{s} \leq 0$.

To prove (ii), introduce new coordinates

$$\tilde{x} = x + a_3 s - a_3 a_4, \qquad \tilde{s} = s. \tag{47}$$

Then, equations are transformed into the following form

$$\begin{aligned}\dot{\tilde{x}} &= -(u+\delta)\tilde{x},\\ \dot{\tilde{s}} &= -a_3^{-1}(\tilde{x} - a_3\tilde{s} + a_3a_4)\mu(\tilde{s}) + (a_4 - \tilde{s})(u+\delta).\end{aligned} \tag{48}$$

Now, consider function $V = \frac{1}{2}\tilde{x}^2$, then $\dot{V} = -(u+\delta)\tilde{x}^2$. This means that any trajectory starting at the point $\tilde{x}_0, \tilde{s}_0$ at $t = 0$ stays for $t \geq 0$ inside the set where $|\tilde{x}| \leq \tilde{x}_0$ for every integrable input and perturbation signals. The latter set is in the original coordinates the strip formed by two lines parallel with the line $x + sa_3 = 0$. Note that intersection of any such strip and positive orthant is obviously bounded for any $a_3 > 0$, so that boundedness of the trajectory follows by (i). □

5. OBSERVER DESIGN AND NUMERICAL SIMULATION

This section is devoted to the high gain adaptive observer design described in Theorem 3.3 for the particular system, namely, for the waste water treatment model described in the previous section. Note that in this case the perturbation channel coincides with the input one, therefore we may set in the sequel $p(x) = g(x)$. We aim to perform the adaptive observer design in the new coordinates description given by (24-25). In other words, let us consider the system written in the following compact form

$$\begin{aligned}\dot{x}_1 &= x_2 + g_1(x)(u+\delta),\\ \dot{x}_2 &= f_2(x) + g_2(x)(u+\delta),\end{aligned} \tag{49}$$

where

$$\begin{aligned}&f_1(x) = 0,\ f_2(x) = -a_1^{-1}a_2^{-1}a_3^{-1}\exp(x_1)x_2(a_1 - x_2)^2,\\ &g_1(x) = -1,\ g_2(x) = a_1^{-1}a_2^{-1}(a_4a_1 - (a_2+a_4)x_2)(a_1 - x_2).\end{aligned} \tag{50}$$

In turn, according to (40) the observer dynamics is given by

$$\begin{aligned}\dot{z}_1 &= z_2 + g_1(z)(u+\phi) + rl_1(z_1 - x_1) + \gamma_1(t)\dot{\phi},\\ \dot{z}_2 &= f_2(z) + g_2(z)(u+\phi) + r^2l_2(z_1 - x_1) + \gamma_2(t)\dot{\phi},\end{aligned} \tag{51}$$

where the observed parameter dynamics obeys the expression

$$\dot{\phi} = -\kappa\gamma_1(t)(z - x), \quad \kappa > 0. \tag{52}$$

Furthermore, the adaptive gains are expressed as

$$\begin{aligned}\dot{\gamma}_1 &= rl_1\gamma_1 + \gamma_2 + p_1(z),\\ \dot{\gamma}_2 &= r^2l_2\gamma_1 + F_{21}(z)\gamma_1 + F_{22}(z)\gamma_2 + G_{22}(z)(u+\phi)\gamma_2 + p_2(z),\end{aligned} \tag{53}$$

where

$$\begin{aligned}&p_1(z) = g_1(z) = -1, \quad p_2(z) = g_2(z) = a_1^{-1}a_2^{-1}(a_4a_1 - (a_2+a_4)z_2)(a_1 - z_2),\\ &F_{21}(z) = -a_1^{-1}a_2^{-1}a_3^{-1}\exp(z_1)z_2(a_1 - z_2)^2,\\ &F_{22}(z) = -a_1^{-1}a_2^{-1}a_3^{-1}\exp(z_1)(a_1 - z_2)(a_1 - 3z_2),\\ &G_{22}(z) = -2a_1^{-1}a_2^{-1}\left[a_4a_1 - (a_2+a_4)z_2\right] - 1.\end{aligned} \tag{54}$$

The parameters of the nominal model were adjusted from the experimental data of a batch culture of *Spirulina maxima* which is a blue green microalgae able to remove some pollutants from the domestic and industrial activity like phosphates and nitrates, see [6]. To be precise, nominal parameters, initial conditions, input (perturbed) dilution rate u, initial state and the observer gains are:

1) Nominal parameters:

a_1	a_2	a_3	a_4
0.027	25	3.45	205

2) Initial conditions of the system and observer, observer gains:

$x_1(0) = z_1(0)$	$x_2(0)$	$z_2(0)$	r	l_1	l_2	k
6.4	0.019	0.021	1	-1	-0.25	5

3) Input and perturbations:

	$t(h) : 0 - 500$	$t : 500 - 1000$	$t : 1000 - 1500$
δ	0	0.003	-0.007

In what follows we shall consider the observation problem when the input $u(t)$ is a non-constant time function, given below. Indeed, when a constant input is applied, the observation of both the state an perturbation δ is a trivial matter as can be computed in a direct way from the equilibrium. The observed state z_2 issued from both the standard high gain observer and the adaptive version are presented in Figure 1. Notice that, in the absence of perturbation, for $t \in [0, 500\text{h.}]$, both observers are such that $z_2(t)$ approaches asymptotically to $x_2(t)$; indeed the convergence of the high gain observer is faster than the adaptive one, certainly adaptation takes longer due to dynamics of both $\phi(\cdot)$ and $\gamma(\cdot)$ (see, (52), (53)). However, when constant perturbation δ is present at time intervals $t \in [500h, 1000]$ and $t \in [1000\text{h}, 1500\text{h}]$, the sole high gain observer is not able to give a good estimate of $x_2(t)$, after a short transient a constant error would persist. It can be further reduced by adjusting the high gain term, but it can not converge to zero again not speaking about the well-known price for the gains too high. On the contrary, the adaptive version of the high gain observers developed in the theoretical part of this paper is able to deliver a good asymptotic estimate of $x_2(t)$ despite the presence of perturbation δ. Actually, in Figure 2 is shown the observed "perturbed" state ϕ according to eq. (52). Notice that, the external disturbance δ is asymptotically found, indeed at the time when it is applied the estimation error goes to zero after a short adaptation time. The speed of convergence can be modified with the gain k of eq. (52); the actual unknown parameter points out the important issue of the unknown parameter influence rejection and, actually, the unknown parameter identification.

As mentioned before, the PE condition can be verified only through the numerical simulation. In Figure 3, one can see the demonstration of the PE property of function $c\gamma(t)$ which is positive, so that Lemma 3.2 is applicable as the PE introduced by Definition 3.1 is valid.

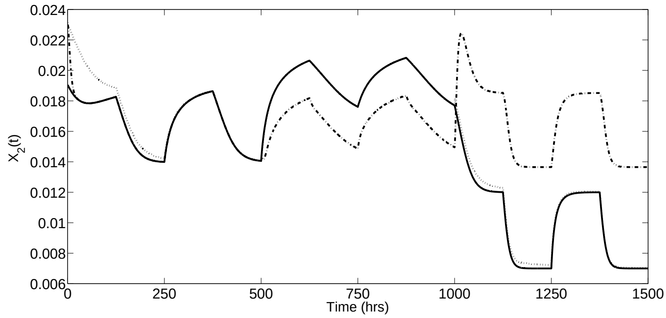

Fig. 1. Observed state $z_2(t)$: Perturbed system (continuous bold line), standard high gain observer (dashed line) and adaptive observer (soft dashed line).

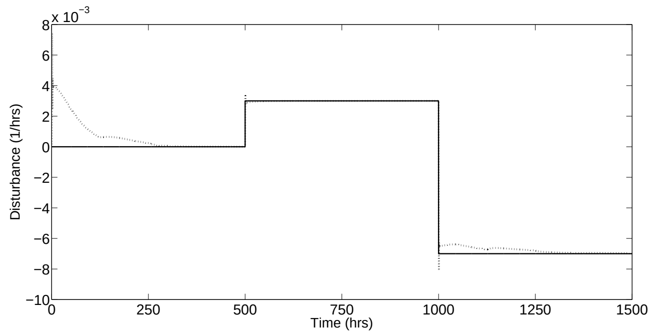

Fig. 2. Input disturbance δ: unknown disturbance (continuous bold line), estimated disturbance (dashed line) issued form high gain adaptive observer.

Finally, in order to give a complete view of the simulation study, the measured output $y = x_1$ is given in Figure 4. The input function is reported in Figure 5, where a relatively fast frequency of change is proposed to verify the performance of the observer given in this work.

6. CONCLUSION

In this paper, the high-gain observers are extended to a new type of the structure incorporating the adaptive high-gain correction terms. A new proof of asymptotic stability of the estimation error is given for the so-called strict triangular systems (STS), which turned out to be instrumental for the adaptive version of the high-gain observers, allowing to handle the unknown external disturbances later on. An interesting feature is the lower bound of the so-called high-gain parameter which can be further optimized through

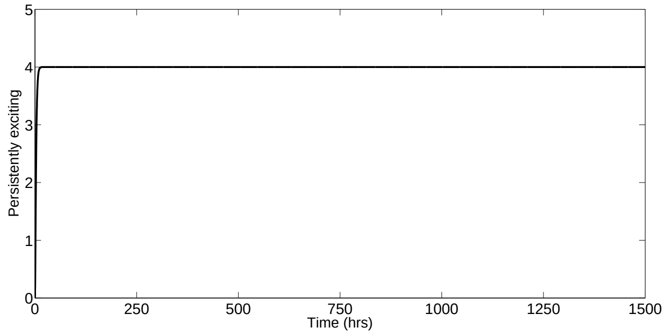

Fig. 3. Persistent excitation property of function $c\gamma(t)$.

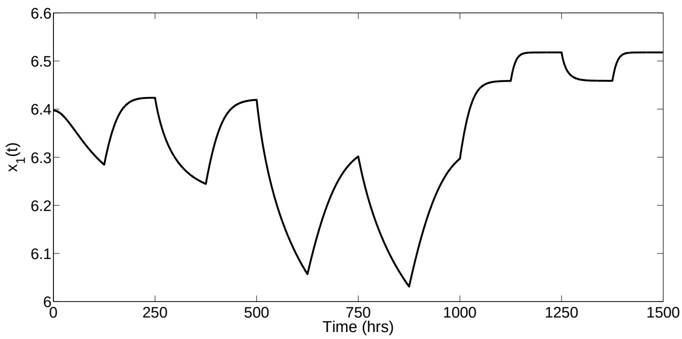

Fig. 4. Measured output in presence of external disturbance δ.

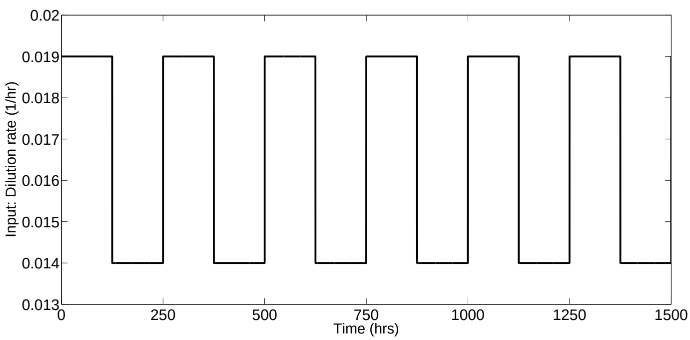

Fig. 5. Input reference function u, without disturbance δ.

the LMI approach and it also simplifies the analysis of the effect of such parameter on the unavoidable PE property. In general, the usual high-gain nonlinear observers for strictly triangular systems are not robust against constant input perturbations except the special case when both the input and the perturbation has the maximal relative degree, leading to constant substrate estimation errors in the bioreactor, for instance. In this vein, it has been shown that the addition of the adaptive action, together with the high-gain concept, allows the joint estimation of the state and the perturbation. Disturbance estimation is an interesting issue on its own and may provide the efficient solution of the problem of disturbance rejection by the adaptive observer techniques. This last mentioned topic is actually a subject of our current research work.

ACKNOWLEDGEMENTS

This research was supported by the joined CONACyT and The Czech Academy of Sciences project "Advanced Nonlinear Methods for Modelling, Control and Detection in Biosystems".

(Received May 25, 2017)

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