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Predictive and Anisotropic Control Design for Robot Motion under Stochastic Disturbances

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Abstract— **The paper deals with the design and comparison of model-based predictive control and anisotropic control formulated for the motion control of industrial robots-manipulators. Stochastic disturbances, usually occurring and entering a control process, are taken into account in the design to attenuate their undesirable influences. The explanation refers a specific online control parameter tuning for predictive control and introduces a single-pass offline optimization for anisotropic control. The aim is to point out features of the proposed advanced approaches in transition situations.**

I. INTRODUCTION

The model predictive control (MPC) represents a well-known control strategy [1]. Its design pursues to minimize the expected cost of a relevant objective function that combines dominant powerful feedforward and complementary feedback. The feedforward is employed to optimize control actions for future reference values. The feedback serves for suppression of inaccuracies in the feedforward and for managing of bounded stochastic influences [2], [3]. Nevertheless, the broader use increases demands on MPC stability and robustness. The robustness property of MPC respecting uncertain model parameters and imprecisely known external stochastic disturbances is not provided in full for real time control of complex robotic systems [4].

The controllers operate usually under stochastic conditions including reference and system signals. The measured signals mostly include random errors. Furthermore, the real parameters of a controlled system can differ from the parameters of its model used for control design. A consideration of the conditions in the design may lead to a more efficient and safe actuation with a potential increase of the motion accuracy of industrial robots-manipulators.

There exist various approaches for a disturbance attenuation in the control theory. The typical $\mathcal{H}_2$ and $\mathcal{H}_\infty$ design for linear time invariant systems uses the evaluation of the $\mathcal{H}_2$ and $\mathcal{H}_\infty$ norms of matrix-valued transfer functions [5]. However, the $\mathcal{H}_2$ and $\mathcal{H}_\infty$ work only if assumptions on the disturbances are met well as well as a model of controlled system is accurate enough or else they may lead to low control quality or to undue conservatism [6].

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Such problems can be solved by a specific control design based on an anisotropic control theory. Anisotropic control is relatively novel approach [7], in which the statistic uncertainty of disturbance is measured in terms of a *relative entropy rate* using the mean anisotropy functional. The disturbance attenuation capabilities of the controlled system are quantified by a specific anisotropic norm [7], a stochastic counterpart of the $\mathcal{H}_\infty$ norm. The $\mathcal{H}_2$ and $\mathcal{H}_\infty$ norms are the limiting cases of the anisotropic norm. Minimization of such a norm (specifically, the norm of a closed-loop system as a performance criterion) leads to the controller, which is less conservative than the $\mathcal{H}_\infty$ and more efficient for attenuating the disturbances than the $\mathcal{H}_2$ (LQG).

As distinct from some other well-known approaches for hybridizing $\mathcal{H}_2$ and $\mathcal{H}_\infty$ control schemes, including minimum entropy controls, risk sensitive controls, and mixed $\mathcal{H}_2/\mathcal{H}_\infty$ controls, the anisotropy-based approach explicitly incorporates different representations of the stochastic disturbance distribution into a single performance index [8]. The solution leads to a unique optimal controller computed from the solution of cross-coupled algebraic Riccati equations. Recently, the suboptimal (but close to optimal) anisotropic design was focused on the solutions by linear matrix inequalities (LMI) and convex optimization [9], [10].

The paper aims at unified introduction of the predictive control and novel anisotropic control in relation to the robot motion under stochastic disturbances. The solution is introduced both for predictive control as a specific tuning problem and for anisotropic control as an integral part of the optimization.

In MPC, the criterion is an optimal value minimizing a quadratic cost function. It is very close to evaluation of $\mathcal{H}_2$ norm, which, in case of LQG control, can be expressed identically. This affinity substantiates to investigate motion control task with a relatively novel anisotropic control lying between $\mathcal{H}_2$ and $\mathcal{H}_\infty$ design. The investigated approaches (predictive and anisotropic control) are explained on a one generalized state-space model used for description of various multi-input multi-output robotic systems.

The paper is organized as follows. In the section II, there are definitions of used notation and control task specification. The sections III and IV introduce unified predictive and anisotropic motion control design. The section V deals with a description of used model of robotic structure [11] and shows time histories of simulation tests implemented on this structure. Finally, the section VI summarizes features and potentialities of proposed ways for practical use in the motion control.

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II. PRELIMINARIES

The notation of the paper arises from stochastic and robust control theory [5], [12]. This section makes its brief overview.

A. Generalized Model of the Controlled System

A linear discrete time-invariant (LDTI) generalized state-space model is considered. The model is defined with the following sequences: X for n_x-dimensional internal states x_k, W for n_w-dimensional noise inputs w_k, R for m_r-dimensional reference inputs r_k, U for m_u-dimensional control inputs u_k, Z for m_z-dimensional controlled outputs z_k, and Y for m_y-dimensional outputs y_k purely selected from states x_k. All signals are discrete-time sequences related to each other by a state-space equation (1), controlled output equation (2) and measured output equation (3) forming together LDTI generalized state-space model of the controlled system:

$$\begin{aligned} x_{k+1} &= A\; x_k + B_{xw}\, w_k + B\; u_k & (1)\\ z_k &= C_{zx}\, x_k + D_{zr}\; r_k + D_{zu}\, u_k & (2)\\ y_k &= C\; x_k & (3) \end{aligned}$$

where A is the state-space matrix, B_{xw} is the noise matrix, B is the input matrix; C_{zx} is the output-weight matrix, D_{zr} is the reference-weight matrix, D_{zu} is the control weight matrix; and C is the output matrix. The model corresponds to lower *linear fractional transformation* (LFT) used for synthesis problems [5]. Eqs. (1) and (3) follow mainly from nominal deterministic model of the controlled system from mathematical physical analysis. On the other hand, controlled output equation (2) represents tunable weighted terms of controlled system input and state or output, which determine the balance between input energy and demanded control accuracy.

As an available prior information, the disturbance sequence $W = (w_k)_{-\infty<k<+\infty}$ is assumed to be a stationary sequence of random vectors w_k with zero mean $Ew_k = 0$, unknown covariance matrix $Ew_k w_k^T = \Sigma_W \succ 0$, ($E$ denotes the expectation) and with Gaussian probability density function

$$p(w_k) := (2\pi)^{-m_w/2}(\det \Sigma_W)^{-1/2} \exp\left(-\frac{1}{2}\|w_k\|^2_{\Sigma_W^{-1}}\right) \quad (4)$$

where $\|w_k\|_{\Sigma_W^{-1}} = \sqrt{w_k^T \Sigma_W^{-1} w_k}$.

B. Control Law and Transfer Function of the Closed-Loop

A control task of the robot motion can be specified such that a given robot should perform the desired user motion trajectories represented by reference signals. The tracking of the reference signals should be provided by a controller taking into account feedback from the system, reference signals and the available mathematical model in a real (stochastic) environment. The described task is in Fig. 1, which shows the general block diagram of closed-loop system, which correspond to the model (1)-(3). A suitable controller can be expressed generally as follows

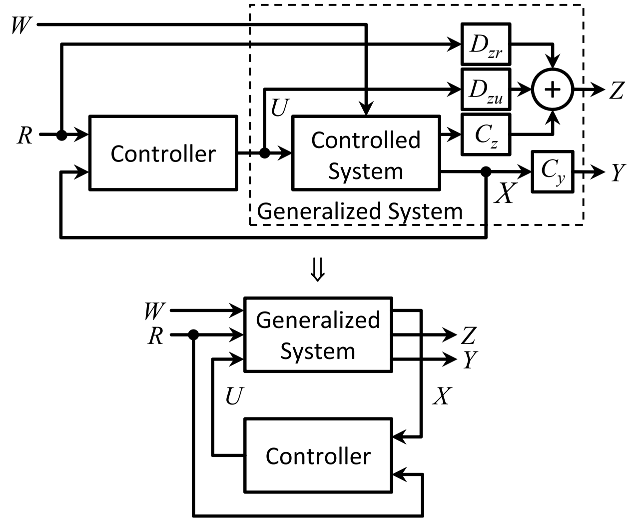

Fig. 1. Closed-loop system and appropriate diagram of lower LFT.

$$u_k = K_x\, x_k + K_r\, r_k \quad (5)$$

To describe the whole closed-loop system (Fig. 1), let us consider the following matrix transfer function

$$T_{zw}(z) = \mathcal{C}(zI - \mathcal{A})^{-1}\mathcal{B} + \mathcal{D} \quad (6)$$

that will be used in further explanation. It represents the closed-loop system from the external disturbance input W to the controlled output Z. Involved matrices in (6) are defined by the following way

$$\left[\begin{array}{c|c} \mathcal{A} & \mathcal{B} \\ \hline \mathcal{C} & \mathcal{D} \end{array}\right] = \left[\begin{array}{c|c} \widetilde{A} + \widetilde{B}K & \widetilde{B}_{xw} \\ \hline \widetilde{C}_{zx} + D_{zu}K & 0 \end{array}\right] \quad (7)$$

Individual submatrices arise from the generalized state-space model description (1)-(3) and parameter definitions (13) in the context of the motion control. They are defined as follows

$$\widetilde{A} := \begin{bmatrix} A & 0 \\ 0 & I_{m_r} \end{bmatrix}, \quad \widetilde{B}_{xw} := \begin{bmatrix} B_{xw} \\ 0 \end{bmatrix}, \quad \widetilde{B} := \begin{bmatrix} B \\ 0 \end{bmatrix},$$
$$\widetilde{C}_{zx} := \begin{bmatrix} C_{zx} & D_{zr} \end{bmatrix}, \quad \widetilde{C} := \begin{bmatrix} C & 0 \end{bmatrix} \quad (8)$$

A gain $K := \begin{bmatrix} K_x & K_r \end{bmatrix}$ represents the searched joint control gain corresponding to the assumptive control law (5).

III. MODEL PREDICTIVE CONTROL

MPC represents a multi-step control strategy, which allows the online optimization of control actions with respect to future reference signals and varying nonlinear robot dynamics. This is achieved by the optimization within a finite time-horizon towards future time instants. Specifically, in the each instant, MPC minimizes a quadratic cost function involving updated specific predictions of future system outputs. The predictions express future outputs in relation to searched control actions.

A. Equations of Predictions and Cost Function

The predictions are based on the model of the system (1) and (3). The cost function and predictions are expressed as follows [1]:

$$\begin{aligned} J_k &= E\sum_{j=1}^{N_p} \hat{z}_{k+j}^T \; \hat{z}_{k+j} \\ &= E\Big(\|Q_Y(\hat{Y}_{k+1} - R_{k+1})\|_2^2 + \|Q_U U_k\|_2^2\Big) \quad (9) \\ \hat{z}_{k+j} &= C_{zx}\,\hat{x}_{k+j} + D_{zr}\,r_{k+j} + D_{zu}\,u_{k+j-1} \quad (10) \\ \hat{Y}_{k+1} &= F_{p,k}\,x_k + G_{p,k}\,U_k \quad (11) \end{aligned}$$

where $\hat{z}_{k+j}$ is an expected controlled output adapted to predictive control [2]; vectors $\hat{Y}$, R and U represent sequences of predictions (future expected system outputs), references and control actions (searched system inputs) within a given horizon of prediction N_p: $\hat{Y}_{k+1} = \left[\hat{y}_{k+1}, \cdots, \hat{y}_{k+N_p}\right]^T$, $R_{k+1} = \left[r_{k+1}, \cdots, r_{k+N_p}\right]^T$, $U_k = \left[u_k, \cdots, u_{k+N_p-1}\right]^T$; and Q_Y and Q_U are square-roots of weighting matrices:

$$Q_Y = \begin{bmatrix} Q_y & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & Q_y \end{bmatrix},\ Q_U = \begin{bmatrix} Q_u & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & Q_u \end{bmatrix};$$

consist of output and input penalization matrices, selected usually as: $Q_y = q_y I_{m_y}$ and $Q_u = q_u I_{m_u}$; and matrices $F_{p,k}$ and $G_{p,k}$ are defined as follows:

$$F_{p,k} = \begin{bmatrix} CA_k \\ \vdots \\ CA_k^{N_p} \end{bmatrix},\ G_{p,k} = \begin{bmatrix} CB_k & \cdots & 0 \\ \vdots & \ddots & \vdots \\ CA_k^{N_p-1}B_k & \cdots & CB_k \end{bmatrix} \quad (12)$$

The cost function (9) corresponds to the following parameters of controlled output equation (2):

$$C_{zx} = \begin{bmatrix} Q_y\,C \\ 0 \end{bmatrix},\ D_{zr} = \begin{bmatrix} -Q_y \\ 0 \end{bmatrix},\ D_{zu} = \begin{bmatrix} 0 \\ Q_u \end{bmatrix} \quad (13)$$

B. Minimization Procedure

The minimization of the cost function (9) can be provided beside usual procedures [2] in one-shot as a least squares problem solution of algebraic system of equations [13], [14]:

$$\begin{bmatrix} Q_Y & 0 \\ 0 & Q_U \end{bmatrix}\begin{bmatrix} \hat{Y}_{k+1} - R_{k+1} \\ U_k \end{bmatrix} = 0$$

where $\hat{Y}_{k+1} = F_{p,k}\,x_k + G_{p,k}\,U_k$

$$\Rightarrow \begin{bmatrix} Q_Y & 0 \\ 0 & Q_U \end{bmatrix}\begin{bmatrix} G_{p,k} & R_{k+1} - F_{p,k}\,x_k \\ I & 0 \end{bmatrix}\begin{bmatrix} U_k \\ -I \end{bmatrix} = 0 \quad (14)$$

The usual result of the minimization is a sequence of control actions U_k, where only first term u_k of the sequence U_k is really applied to the controlled system.

However, for the comparison with $\mathcal{H}_2$ and other proposed control methods, a usual procedure of the cost minimization is used [15]:

$$\begin{aligned} u_k = MU_k &= M(G_{p,k}^T\,Q_Y^T\,Q_Y\,G_{p,k} + Q_U^T\,Q_U)^{-1} \\ &\quad \times G_{p,k}^T\,Q_Y^T\,Q_Y\,(R_{k+1} - F_{p,k}\,x_k) \quad (15) \end{aligned}$$

where a rectangular matrix M is defined as follows

$$M = \left[\, I_{m_u},\ 0_{m_u},\ \cdots,\ 0_{m_u} \,\right] \quad (16)$$

Thus, the matrix M selects only the appropriate control actions corresponding to the time instant k. The expression (15) can be decomposed and expressed by comparable control law as in case of $\mathcal{H}_2$ control as indicated:

$$u_k \;=\; MK_{X,k}\,x_k + MK_{R,k}\,R_{k+1} \quad (17)$$

where matrix gains K_R and K_X are given as follows:

$$K_{R,k} = (G_{p,k}^T\,Q_Y^T\,Q_Y\,G_{p,k} + Q_U^T\,Q_U)^{-1} G_{p,k}^T\,Q_Y^T\,Q_Y \quad (18)$$

$$K_{X,k} = -K_R\,F_{p,k} \quad (19)$$

If the selection $r_{k+j} = r_k$ for $j := 1, 2, \cdots, Np$ is considered, i.e. the future reference values are constant, unknown or equal to current reference value in the time instant k, then the control law is equivalent to the assumed law (5) with varying K_x and K_r given as follows

$$K_x = MK_{X,k},\ K_r = MK_{R,k}\,\left[\, I_{m_r}, I_{m_r}, \cdots, I_{m_r} \,\right]^T \quad (20)$$

The right expression in (20) represents only appropriate sums of elements of $MK_{R,k}$ with respect to the constant reference.

C. Tuning with Respect to Stochastic Influences

Predictive control usually runs under constant control parameters. However, sudden stochastic disturbances can generate sharp control actions. It is caused by discrepancy of used model and controlled system. Such discrepancy can partially be solved by specific tuning of the control parameters Q_y and Q_u. Such tuning can be realized by correspondence of parameters to so called precision or covariance matrices [16] with reasonable lower $\underline{Q_*}$ and upper $\overline{Q_*}$ element bounds

$$\underline{Q_y} \le Q_y|_{Q_y \propto\, C_y^{-1}} \le \overline{Q_y},\ \ \underline{Q_u} \le Q_u|_{Q_u \propto\, C_u^{-1}} \le \overline{Q_u} \quad (21)$$

Note that elements $Q_y < \underline{Q_y} \bigwedge Q_u > \overline{Q_u}$ would lead to excessive control attenuation whereas $Q_y > \overline{Q_y} \bigwedge Q_u < \underline{Q_u}$ to excessive control amplification causing system instability. Due to proportional dependency of control parameters, which was denoted in (21) by symbol $\propto$, it is sufficient to tune only one parameter e.g. let Q_u constant and tune Q_y only according to model precision evolution [17], [18]:

$$Q_{y_k} \propto C_{y_k}^{-1} = (E\{(y_k - \hat{y}_k)\,(y_k - \hat{y}_k)^T\})^{-1} \quad (22)$$

This solution is reasonable, but it is suitable as a temporal solution only. At continuing substantial stochastic influences, it can cause controller insensitivity or inadequate small control actions. More efficient solution in this point is offered by anisotropic control introduced in the next section.

IV. ANISOTROPIC CONTROL

This section deals with a novel formulation of anisotropic control theory for motion control tasks. Here, the input stochastic disturbance W entering the controlled system (Fig. 1) is characterized in terms of the mean anisotropy as a magnitude of the statistical uncertainty of the signal. The robust performance of the closed-loop control system with respect to statistically uncertain input is characterized by its anisotropic norm, which is an anisotropy-constrained stochastic version of the induced norm of the system. A solution of the tracking problem via anisotropic control is introduced as well. The anisotropic control can perform a standard reference tracking with powerful attenuation of stochastic influences of the input disturbances including a robust property for the model parameter imperfections.

A. Mean Anisotropy of Disturbance Inputs

To characterize the statistical uncertainty of the external disturbance W, the concept of the mean anisotropy is used [7]. Let $\mathbb{L}_2^m$ denote the class of square integrable $\mathbb{R}^m$-valued random vectors distributed absolutely continuously with respect to the m-dimensional Lebesgue measure. The external disturbance W is assumed to be a stationary sequence of vectors $w_k \in \mathbb{L}_2^m$ interpreted as a discrete-time random vector signal. Assemble the elements of W associated with a time interval $[s,t]$ into the column random vector $W_{s:t} := [w_s^T, \cdots, w_t^T]^T$. It is assumed that $W_{0:N}$ is distributed absolutely continuously for every $N \geq 0$. It should be noted that not only the one-point covariance matrix $Ew_k w_k^T$ is unknown, but in fact, the covariance matrix $E(W_{0,N}W_{0,N}^T)$ is supposed to be unknown.

The mean anisotropy of the sequence W is defined as the anisotropy production rate per time step [19]:

$$\overline{\mathcal{A}}(W) := \lim_{N\to+\infty} \frac{\mathcal{A}(W_{0:N})}{N}$$

where the anisotropy $\mathcal{A}(W_{0:N})$ is defined as the minimal value of relative entropy $\mathcal{D}(f_{W_{0:N}} \| f_{m(N+1),\lambda})$ with respect to the Gaussian distributions $f_{m(N+1),\lambda}$ in $\mathbb{R}^{m(N+1)}$ with zero mean and scalar covariance matrices $\lambda I_{m(N+1)}$:

$$\begin{aligned}\mathcal{A}(W_{0:N}) &:= \min_{\lambda>0} \mathcal{D}(f_{W_{0:N}} \| f_{m(N+1),\lambda}) \\ &= \frac{N+1}{2}\ln\left(\frac{2\pi \mathrm{e}}{m(N+1)}E(|W_{0:N}|^2)\right) - h(W_{0:N}) \end{aligned} \tag{23}$$

having a minimum at $\lambda = E(|W_{0:N}|^2)/(m(N+1))$, where $h(W_{0:N})$ denotes the differential entropy of $W_{0:N}$ (see e.g. [20]). The anisotropy functional (23) is an entropy theoretic measure of deviation of the unknown actual noise distribution from the family of Gaussian white noise laws.

Furthermore, the disturbance W is supposed to have the bounded mean anisotropy, i.e. $\overline{\mathcal{A}}(W) \leq a$. Thus, the input mean anisotropy level a represents a measure of the statistical uncertainty of the model.

B. Anisotropic Norm of System

The robust performance of the closed-loop system is characterized by its anisotropic norm [7]. Let us denote the set of the input signals with bounded mean anisotropy as follows

$$\mathcal{W}_a := \{W \in \ell_{\mathcal{P}}^m \colon \overline{\mathcal{A}}(W) \leq a\},$$

where

$$\ell_{\mathcal{P}}^m := \{W = (w_k)_{-\infty<k<+\infty} \colon w_k \in \mathbb{L}_2^m \text{ and } \|W\|_{\mathcal{P}} < +\infty\}$$

is the space of weakly stationary square-integrable sequences and the power-norm of W is generally defined as

$$\|W\|_{\mathcal{P}} := \left(\lim_{N\to\infty} \frac{1}{2N+1} \sum_{k=-N}^{N} E|w_k|^2\right)^{1/2}$$

Since the second moments $Ew_j w_k^T$ of the weakly stationary sequence depend only on the time difference j–k and $E|w_k|^2$ does not depend on k, then the sequence W is:

$$\|W\|_{\mathcal{P}} = \sqrt{E|w_k|^2} = \sqrt{E|w_0|^2}$$

with an arbitrary k. The anisotropic norm of the closed-loop system $T_{zw}(z)$ is defined as

$$|\!|\!| T_{zw} |\!|\!|_a := \sup_{W\in\mathcal{W}_a} \frac{\|Z\|_{\mathcal{P}}}{\|W\|_{\mathcal{P}}} \tag{24}$$

The anisotropic norm (24) is a nondecreasing continuous function of the mean anisotropy level a, which satisfies

$$\begin{aligned}\frac{1}{\sqrt{m}}\|T_{zw}\|_2 = |\!|\!| T_{zw} |\!|\!|_0 \ &\leq \ |\!|\!| T_{zw} |\!|\!|_a \\ |\!|\!| T_{zw} |\!|\!|_a \ &\leq \ \lim_{a\to+\infty} |\!|\!| T_{zw} |\!|\!|_a = \|T_{zw}\|_\infty\end{aligned}$$

These relations show that the scaled $\mathcal{H}_2$ and $\mathcal{H}_\infty$ norms are the limiting cases of the anisotropic norm as $a \to 0, +\infty$, respectively [7]. An important property of the anisotropic norm is that it coincides with the scaled $\mathcal{H}_2$ norm of the system for $a = 0$ and converges to the $\mathcal{H}_\infty$ norm as $a \to \infty$. Therefore, $|\!|\!| \cdot |\!|\!|_a$ is an anisotropy-constrained stochastic version of the induced norm of the system which occupies a unifying intermediate position between the $\mathcal{H}_2$ and $\mathcal{H}_\infty$ norms as control performance criteria [6].

C. Anisotropic Suboptimal Controller Synthesis

Now let us proceed to the synthesis problem statement: given LDTI state-space model (1) - (3), a mean anisotropy level $a \geq 0$ of the external disturbance W, and some designed threshold value $\gamma > 0$, find a time-invariant state-feedback controller in the form (5), which internally stabilizes the closed-loop system $T_{zw}(z)$ with the state-space realization (6) and ensures that its anisotropic norm does not exceed a threshold γ, i.e. the following inequality holds true:

$$|\!|\!| T_{zw} |\!|\!|_a < \gamma \tag{25}$$

The solution of this problem can be expressed as a system of convex inequalities. The inequality (25) holds true if there exist $\eta \in \left(\gamma^2, \gamma^2(1-\mathrm{e}^{-2a/m_w})\right)$ and some real $(n_x \times n_x)$-matrix $\Phi = \Phi^T \succ 0$ that satisfy following inequalities [9]

$$-\left(\det(\eta I_{m_w} - \mathcal{B}^T\Phi\mathcal{B})\right)^{1/m_w} < -\left(\eta - \gamma^2\right)\mathrm{e}^{2a/m_w} \tag{26}$$

$$\begin{bmatrix} \mathcal{A}^T\Phi\mathcal{A} - \Phi + \mathcal{C}^T\mathcal{C} & \mathcal{A}^T\Phi\mathcal{B} \\ \mathcal{B}^T\Phi\mathcal{A} & \mathcal{B}^T\Phi\mathcal{B} - \eta I_{m_w} \end{bmatrix} \prec 0 \tag{27}$$

To obtain the solution, let a suitable slack $(m_w \times m_w)$-matrix variable $\Psi = \Psi^{\mathrm{T}} \succ 0$ be considered so that

$$\eta - \left(\mathrm{e}^{-2a} \det \Psi\right)^{1/m_w} < \gamma^2 \tag{28}$$

$$\Psi \prec \eta I_{m_w} - \mathcal{B}^T\Phi\mathcal{B} \tag{29}$$

The inequality (28) is the first of the convex inequality system. Now, let us take into account the inequality (29). It is equivalent to

$$\Psi - \eta I_{m_w} - \mathcal{B}^T(-\Pi^{-1})\mathcal{B} \prec 0, \tag{30}$$

where $\Pi := \Phi^{-1}$. Applying Schur's Lemma [21] to this inequality with respect to (7), the second convex inequality can be obtained

$$\begin{bmatrix} \Psi - \eta I_{m_w} & \widetilde{B}_{xw}^T & 0 \\ \widetilde{B}_{xw} & -\Pi & 0 \\ 0 & 0 & -I_{p_z} \end{bmatrix} \prec 0 \tag{31}$$

By double application of Schur's Lemma [21] to the inequality (27) with further multiplication from both sides by $\mathrm{blockdiag}\,(\Pi, I_{m_w}, I_{n_x}, I_{p_z}) \succ 0$ and introduction of the linearizing change of variable $\Lambda := K\Pi$, the last convex inequality can be expressed as follows

$$\begin{bmatrix} -\Pi & 0 & \Pi\widetilde{A}^T + \Lambda^T\widetilde{B}^T & \Pi\widetilde{C}_{zx}^T + \Lambda^T D_{zu}^T \\ 0 & -\eta I_{m_w} & \widetilde{B}_{xw}^T & 0 \\ \widetilde{A}\Pi + \widetilde{B}\Lambda & \widetilde{B}_{xw} & -\Pi & 0 \\ \widetilde{C}_{zx}\Pi + D_{zu}\Lambda & 0 & 0 & -I_{p_z} \end{bmatrix} \prec 0 \tag{32}$$

Then, for suitably selected $a \geq 0$, $\gamma > 0$, the desired state-feedback controller exists if the system of inequalities above is feasible with respect to the scalar variable η, real $(m_w \times m_w)$-matrix Ψ, real $((n_x + p_r) \times (n_x + p_r))$-matrix Π

$$\eta > \gamma^2, \quad \Psi \succ 0, \quad \Pi \succ 0 \tag{33}$$

and real $(m_u \times (n_x + p_r))$-matrix Λ.

Thus, the solution of the system of convex inequalities (28), and (31) - (33) gives the unknown variables Ψ, Λ and Π and the searched state-feedback controller gain matrix is determined by

$$K = \begin{bmatrix} K_x & K_r \end{bmatrix} = \Lambda\Pi^{-1}. \tag{34}$$

Note that the inequalities (28), (31) - (33) are not only convex in Ψ and affine with respect to Π and Λ, but also linear in γ^2. Obviously, minimizing γ^2 under these convex inequalities, γ is minimized under the same constraints. So, the conditions (28), (31) - (33) allow to compute the minimal γ via solving the convex optimization problem

$$\begin{array}{c} \text{minimize} \quad \gamma^2 \\ \text{over} \quad \Psi, \Pi, \Lambda, \eta, \gamma^2 \quad \text{satisfying} \quad (28), (31)\text{-}(33). \end{array} \tag{35}$$

If the convex problem (35) is solvable, the state-feedback controller gain matrix is given by (34). The anisotropic controller for minimal γ^2 is called γ-optimal. The problem (35) can be efficiently solved offline e.g. by toolbox YALMIP [22]. The explanation in this section completes realizable implementation of the anisotropic control for the motion control under stochastic disturbances, compatible with the predictive control algorithms.

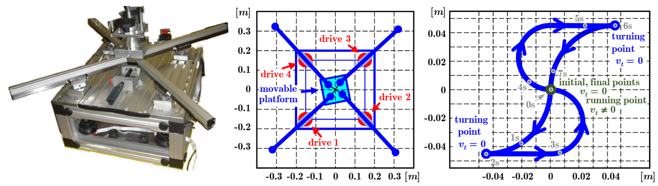

Fig. 2. Functional model of the robot 'Moving Slide'; wireframe representation; and used testing 'S'-shape trajectory with time marks.

V. Simulation Examples

This section demonstrates the proposed predictive and anisotropic control applied to robot motion specifically to a robotic system 'Moving Slide' (Fig. 2). It represents a planar parallel robot-manipulator [11] intended for top milling operations. It has four control inputs u (torques of drives) and three outputs y (positions x_c, y_c and angular rotation ψ_c of a robot platform). The control should ensure the motion of the platform along reference trajectory.

A. The Nominal Model of Robotic System

The robot model follows from Lagrange equations, which lead to the system of nonlinear differential equations (ideal mathematical-physical model)

$$\ddot{y}(t) = f(y(t), \dot{y}(t)) + g(y(t))u(t) \tag{36}$$

The system (36) for MPC is linearized using specific decomposition technique [23], keeping equalities $A(x(t))\,x(t) = [\dot{y}^T(t), f(y(t), \dot{y}(t))^T]^T$, $B(t) = [0, g(y(t))^T]^T$ and leading to the usual continuous state-space model, and discretized to the following discrete model

$$x_{k+1} = A_k\, x_k \; + \; B_k\, u_k \tag{37}$$

$$y_k = C \; x_k \tag{38}$$

The elements of state matrix A_k and input matrix B_k depend on current system state $x_k = [y_k, \dot{y}_k]^T$ that includes system output y and its time derivative $\dot{y}$, i.e. $x_k = x(t)|_{t=k\,T_s}$: $A(x_k) \rightarrow A_k$ and $\{A(x_k), B(x_k)\} \rightarrow B_k$.

MPC, within the finite horizon, applies state-dependent time-varying state-space model to each time step of online optimization. Anisotropy-based approach is primarily developed as single-pass offline robust control design considering one representative model averaged along the trajectory (Fig. 2, right) with the parameter uncertainty, i.e. for the sampling $T_s = 0.01s$ and given robot, the averaged state-space matrices were the following:

$$A = \begin{bmatrix} 1 & 0 & 0 & 0.01 & 1.8789\cdot10^{-9} & -6.3286\cdot10^{-13} \\ 0 & 1 & 0 & -5.8784\cdot10^{-10} & 0.01 & -4.0357\cdot10^{-12} \\ 0 & 0 & 1 & 1.9878\cdot10^{-6} & -1.7937\cdot10^{-5} & 0.01 \\ 0 & 0 & 0 & 1 & 3.8207\cdot10^{-7} & -1.2168\cdot10^{-10} \\ 0 & 0 & 0 & -2.2561\cdot10^{-7} & 1 & -8.0817\cdot10^{-10} \\ 0 & 0 & 0 & 0.00039757 & -0.0035874 & 1 \end{bmatrix} \tag{39}$$

$$B = \begin{bmatrix} -5.507\cdot10^{-5} & -5.4653\cdot10^{-5} & 5.5052\cdot10^{-5} & 5.4642\cdot10^{-5} \\ 5.3431\cdot10^{-5} & -5.3128\cdot10^{-5} & -5.346\cdot10^{-5} & 5.3159\cdot10^{-5} \\ -0.0017779 & -0.0017399 & -0.0017781 & -0.0017403 \\ -0.011014 & -0.010931 & 0.01101 & 0.010929 \\ 0.010686 & -0.010626 & -0.010692 & 0.010632 \\ -0.35558 & -0.34797 & -0.35561 & -0.34807 \end{bmatrix} \tag{40}$$

$$C = I_{m_y \times n_x}, \qquad B_{xw} = I_{n_x} \tag{41}$$

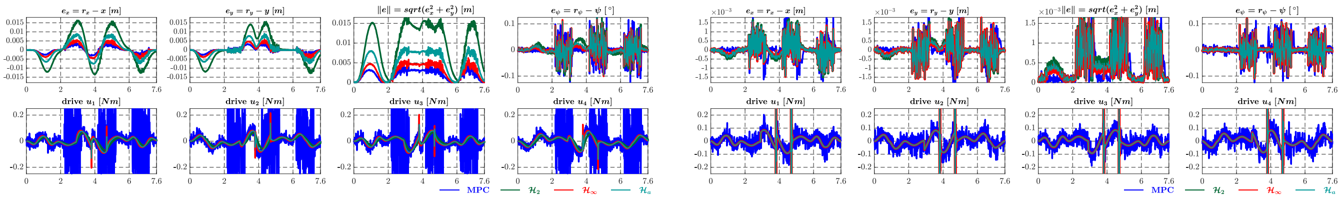

Fig. 3. Time histories [s] of errors of system outputs and control actions for the identical settings (left) and for the individual ('the best') settings (right).

B. Evaluation of the Examples

Obtained control actions were applied to the initial nonlinear model (36), used as a simulation model substituting the real robot (Fig. 2, left). The parameters of examples were selected with respect to the identical fixed and 'best' individual settings:

- the identical controller settings for the comparison: MPC, $\mathcal{H}_2$, $\mathcal{H}_\infty$, $\mathcal{H}_a$: $N_p = 10,\ Q_y = 5 \cdot 10^{-2}\ I_{m_y}$, $Q_u = 1 \cdot 10^{-4}\ I_{m_u}$, with $r_{k+j} = r_{k+1},\ j := 1, \cdots, N_p$
- the 'best' individual settings of the controllers leading to 'best' trajectory tracking (smaller control errors): MPC : $N_p = 10,\ Q_y$ tuned (III-C), $Q_u = 1 \cdot 10^{-4}\ I_{m_u}$ $\mathcal{H}_2$, $\mathcal{H}_\infty$, $\mathcal{H}_a$: $Q_y = 7 \cdot 10^{-1}\ I_{m_y},\quad Q_u = 5 \cdot 10^{-5}\ I_{m_u}$

The mean anisotropy level a was 0.25 in both cases to show difference among anisotropic, $\mathcal{H}_2$, and $\mathcal{H}_\infty$ strategies.

The examples were realized with noise disturbance variance $\mathrm{Var}(w_{k_i}) = (5 \cdot 10^{-5})^2,\ i := 1, \cdots, n_{x=6}$ with tenfold amplifications in time intervals $\langle 2.2, 3.2\rangle$, $\langle 4.2, 5.2\rangle$, $\langle 6.2, 7.2\rangle$ $[s]$. The noise simulates stochastic influences in the measurement of the robot state. Fig. 3 (left) for identical parameters shows the difference among $\mathcal{H}_2$, $\mathcal{H}_\infty$ and anisotropic control. It serves to markedly demonstrate intermediate anisotropic control behavior. Fig. 3 (right) is for individual the most suitable ('the best') settings that lead to the minimal control error of the trajectory tracking.

It is proved that MPC tracks the reference trajectory well especially if the trajectory shape is substantially changed e.g. in the transitions between abscissa and arc segments or two arc segments, where the kinematic parameters of the reference trajectory are changed rapidly.

However, at the disturbance increase, MPC (Fig. 3, left) tries to compensate that increase by the increase of control actions with their oscillation. In case of anisotropic, $\mathcal{H}_2$ and $\mathcal{H}_\infty$ control, the situation is different. Their tracking is smooth, but due to their static and single-step character, they are not able to manage changes of reference trajectory as MPC. In Fig. 3 (right), the MPC was under online tuning of weighting parameter Q_y according to idea described in subsection III-C. At increasing of uncertainty caused by increase of the noise, the weight Q_y is decreased and vice versa. This idea can suppress sharp changes in control actions but at the cost of accuracy control.

During execution, MPC run online with $T_s = 0.01s$ whereas anisotropic, $\mathcal{H}_2$ and $\mathcal{H}_\infty$ control used fixed single-pass offline pre-computed gains. The fixed gains were optimized by YALMIP with no more than 30 iterations per control for various param. setting (on average 22 iterations).

VI. Conclusion

The paper introduces a novel anisotropic control approach, as specific convex optimization problem, intended for motion control of industrial robotic systems in analogy to known MPC. The explanation focuses on the attenuation of stochastic influences. In this regard, specific online tuning of MPC and detailed synthesis of stochastic anisotropic control were shown. Advantage of the anisotropic control is in the continuous tuning between $\mathcal{H}_2$ and $\mathcal{H}_\infty$ as its limiting cases. It enables user to select adequate level of the control conservatism relative to required control accuracy.

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