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COMMENTS AND CORRECTIONS

Comments on "Stability, l_2-Gain, and Robust H_∞ Control for Switched Systems via *N*-Step-Ahead Lyapunov Function Approach"

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ABSTRACT This paper presents a momentous aspect that should be considered in the stability analysis and synthesis of switched systems by investigating an N-step ahead Lyapunov function (LF) approach. By considering these traits, the proposed incorrect theorems in the aforementioned paper are revised to new ones. An illustrative numerical example manifests the effectiveness of our proposed approach. Unlike the assertion in the aforementioned paper when the monotonic requirement of LF is relaxed, the switching is not necessarily gentler. Also, our results indicate that our approach guarantees lower average dwell time.

INDEX TERMS Switched system, average dwell time, N-step ahead Lyapunov function, robust control.

I. INTRODUCTION

Switched systems as a class of hybrid systems consist of a family of subsystems and a rule orchestrating the switching among them. Switched systems provide an ever-increasing application in control systems, flight control and power systems [1]. Given the diverse application of switched systems, many promising results are reported on their stability and stabilization problem. The stability analysis for switched positive linear systems with ADT switching in discrete and continuous-time is tackled in [2], the stability analysis for slowly switched systems utilizing a multiple discontinuous LF approach is developed in [3], the switched system stability and stabilization with mode-dependent ADT switching in discrete and continuous-time is studied in [4], the stabilization problem for a class of switched nonlinear systems using ADT switching comprising unstable subsystems is presented in [5], the problem of output feedback H_∞ control for a nonlinear fuzzy system with dynamic parameter adjustment is studied in [6] and the trajectory tracking control of nonaffine stochastic nonlinear switching systems using robust fuzzy adaptive control under arbitrary switching is addressed in [7].

The aforementioned paper [8] investigates an N-step ahead LF in the stability analysis and synthesis of a discrete-time switched system. Such an approach allows a non-monotonic behavior of LF both at the switching instants and during the running time of each subsystem but guarantees LF convergence to zero in the limit. To this end, average dwell time switching logic has been utilized. Although the use of N-step ahead LF for stability analysis and synthesis of switching systems is interesting, not considering some points ends in inefficiency and improperness of theorems presented in [8].

In this note, the main drawback of the mathematical procedure to derive the stability theorem in [8] based on N-step ahead LF is highlighted. Then, the corrected form of the stability criterion and the minimal ADT constraint are proposed in section II. This section is carried on by formulating the revisited problem of robust H_∞ control. A numerical example is given to demonstrate the superiority and effectiveness of the proposed criteria in section III. Finally, some conclusions are drawn in Section IV.

II. MISTAKES WITH THEOREMS

The relaxed GUAS stability criterion and the minimal ADT constraint via a class of N-step ahead LFs are first formulated in section 3 of [8] for the discrete-time switched system (1) without considering uncertainty and disturbance.

$$
\begin{aligned}
X_{k+1} &= (A_\delta(k) + \Delta A_\delta(k))x_k + (B_\delta(k) + \Delta B_\delta(k))u_k \\
&\quad + B_{\delta w}(k)w_k \\
Z_k &= C_\delta(k)x_k + D_\delta(k)u_k
\end{aligned} \tag{1}
$$

First, the original form of Theorem 1 [8] is given.

Theorem 1 [8]: Given constants $0 < \alpha < 1$ and $\mu > 1$, the switched system (1) without uncertainties and disturbance is GUAS, if there exist a set of symmetric matrices $Q_{i,f}$ ($f \in \mathbb{N}_{1,N-1}$) and $P_i > 0$ such that for $N = 1$,

$$A_i^T P_i A_i + (\alpha - 1)P_i \leq 0$$

for $N = 2$

$$A_i^T P_i A_i - Q_{i,1} \leq 0$$
$$A_i^T Q_{i,1} A_i + (\alpha - 1)P_i \leq 0$$

for $N \geq 3$

$$A_i^T P_i A_i - Q_{i,N-1} \leq 0$$
$$A_i^T Q_{i,N-f+1} A_i - Q_{i,N-f} \leq 0, \quad f \in \mathbb{N}_{2,N-1}$$
$$A_i^T Q_{i,1} A_i + (\alpha - 1)P_i \leq 0$$

for all $i, j \in \Omega$

$$P_i \leq \mu P_j$$

and the ADT of the switching signal $\sigma(k)$ satisfies

$$\tau_a > \tau_a^* = -N \ln \mu \big/ \ln(1 - \alpha)$$

then the underlying system is GUAS.

In order to prove this theorem in [8], the mathematical procedure is divided into two separate but related parts which are briefly expressed. The derived linear matrix inequalities (LMIs) based on N-step ahead LF guarantee the convergence of LF to zero with a predefined decay rate for each subsystem. Also, the ADT constraint guarantees that although the LF may increase at the switching instants, its average should have a lower value when a new switching instant occurs.

In order to achieve the ADT constraint in Xie *et al.* [8] assume that the switching behavior is much slower than the sampling time step. Then according to this assumption, the following inequality is concluded.

$$V_{\sigma(k)}(x_k) \leq (1 - \alpha)^{\frac{k-k_l}{N}} V_{\sigma(k)}\left(x_{k_l}\right) \tag{2}$$

This inequality is not correctly deduced and is only reliable when $N = 1$. This assumption together with the LMIs derived based on N-step ahead LF do not guarantee that the LF is decreasing between two time steps in each subsystem. Fig.1 shows that although the LF is decreasing in two steps, the value of LF in k^{th} time step is much more than the value of LF in k_l^{th} time step.

In addition to the aforementioned main drawback, focusing on conservativeness reduction by fixing α and μ ends in performance degradation as the switching must be gentle as N increases. It is clear that in this point of view, when α is fixed, the decay rate of LF for different values of N is different. In other words, if $N = m$, the decay rate of LF based on this theorem is equal to $\sqrt[m]{(1 - \alpha)}$. So, by choosing a big N, the decay rate steps down which ends in bigger ADT. Since one of the possible solutions based on N-step ahead LF approach is the set of symmetric matrices and the ADT constraint that assess the monotonically decreasing of LF ($N = 1$), our results must relax the ADT constraint.

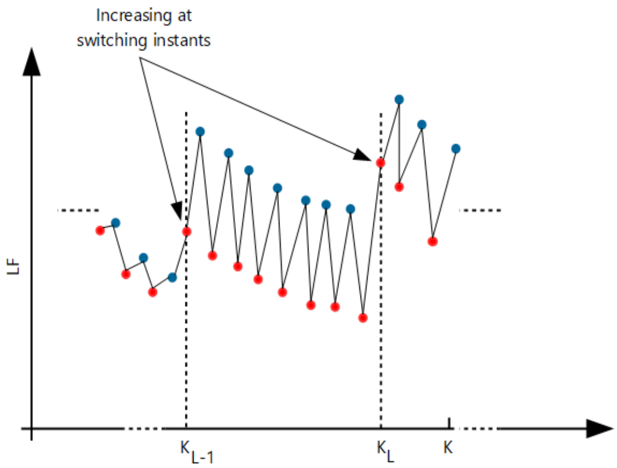

FIGURE 1. Lyapunov function evolution under switching.

Now, the corrected form of the previous theorem and the proof are presented.

Theorem 2: Given constants $0 < \alpha < 1$, $\mu_Q \geq 1$ and $\mu_P \geq 1$, the switched system (1) without uncertainties and disturbance is GUAS, if there exist a set of symmetric matrices $Q_{i,f}$ ($f \in \mathbb{N}_{1,N-1}$) and $P_i > 0$ such that for $N = 1$,

$$A_i^T P_i A_i - \alpha P_i \leq 0 \tag{3}$$

for $N = 2$

$$A_i^T P_i A_i - \alpha Q_{i,1} \leq 0$$
$$A_i^T Q_{i,1} A_i - \alpha P_i \leq 0 \tag{4}$$

for $N \geq 3$

$$A_i^T P_i A_i - \alpha Q_{i,N-1} \leq 0$$
$$A_i^T Q_{i,N-f+1} A_i - \alpha Q_{i,N-f} \leq 0, \quad f \in \mathbb{N}_{2,N-1}$$
$$A_i^T Q_{i,1} A_i - \alpha P_i \leq 0 \tag{5}$$

for all $i, j \in \Omega$

$$P_i \leq \mu_p P_j$$
$$Q_{i,f} \leq \mu_Q P_i \tag{6}$$

and the ADT of the switching signal $\sigma(k)$ satisfies

$$\tau_a > \tau_a^* = -\ln\left(\mu_P \mu_Q\right) \big/ \ln \alpha \tag{7}$$

then the underlying system is GUAS.

Proof: Consider a class of quadratic LF given by

$$V_i(x_k) = x_k^T P_i x_k.$$

Taking the N-step time difference of the above LF, we have

$$\Delta V_{i,N}(x_k) = V_i(x_{k+N}) - V_i(x_k).$$

If the following inequality holds, the above N-step time difference of LF is negative and the stability of each subsystem is guaranteed based on N-step ahead Lyapunov function approach presented in [9].

$$\Delta V_{i,N}(x_k) \leq \left(\alpha^N - 1\right) V_i(x_k) \tag{8}$$

Also, if condition (8) holds, we have

$$V_i(x_{k+N}) - \alpha^N V_i(x_k) \le 0 \tag{9}$$

This inequality means that the LF is decreasing with the pre-defined decay rate α^N in N steps.

Let

$$V_i'(x_{k+f}) = x_{k+f}^T Q_{i,f} x_{k+f} \tag{10}$$

Adding and subtracting some terms to and from (9), we have

$$\left\{ \begin{array}{c} V_i(x_{k+N}) - \alpha V_i'(x_{k+N-1}) + \alpha V_i'(x_{k+N-1}) - \alpha^2 V_i'(x_{k+N-2}) \\ + \alpha^2 V_i'(x_{k+N-2}) - \cdots + \alpha^{N-1} V_i'(x_{k+1}) - \alpha^N V_i(x_k) \end{array} \right\} \le 0$$

It can be easily derived that if the following conditions are satisfied, inequality (9) holds and each subsystem is stable with a predefined decay rate of LF.

$$\begin{cases} V_i(x_{k+N}) - \alpha V_i'(x_{k+N-1}) \le 0 \\ V_i'(x_{k+N-1}) - \alpha V_i'(x_{k+N-2}) \le 0 \\ \quad\vdots \\ V_i'(x_{k+1}) - \alpha V_i(x_k) \le 0 \end{cases} \tag{11}$$

Employing $x_{k+f} = A_i x_{k+f-1}$ and (10), it is easy to infer inequalities (11) from conditions (3), (4) or (5) when $N = 1$, $N = 2$, or $N \ge 3$, respectively.

Also, if inequalities (11) hold, from the first inequality we have

$$V_{\delta(k)}(x_k) \le \alpha \left(x_{k-1}^T Q_{i,1} x_{k-1} \right).$$

And by considering the second inequality, we have

$$V_{\delta(k)}(x_k) \le \alpha^2 \left(x_{k-2}^T Q_{i,2} x_{k-2} \right).$$

Repeating the procedure by considering other inequalities, the following inequalities are derived.

$$\begin{aligned} V_{\delta(k)}(x_k) &\le \alpha \left(x_{k-1}^T Q_{i,1} x_{k-1} \right) \\ V_{\delta(k)}(x_k) &\le \alpha^2 \left(x_{k-2}^T Q_{i,2} x_{k-2} \right) \\ &\vdots \\ V_{\delta(k)}(x_k) &\le \alpha^{N-1} \left(x_{k-N+1}^T Q_{i,N-1} x_{k-N+1} \right) \\ V_{\delta(k)}(x_k) &\le \alpha^N V_{\delta(k)}(x_{k-N}) \\ V_{\delta(k)}(x_k) &\le \alpha^{N+1} \left(x_{k-N-1}^T Q_{i,1} x_{k-N-1} \right) \\ &\vdots \end{aligned} \tag{12}$$

By substituting $Q_{i,f} \le \mu_Q P_i$ into (12), we obtain

$$\begin{aligned} V_{\delta(k)}(x_k) &\le \alpha \left(x_{k-1}^T Q_{i,1} x_{k-1} \right) \le \alpha \mu_Q V_{\delta(k)}(x_{k-1}) \\ V_{\delta(k)}(x_k) &\le \alpha^2 \left(x_{k-2}^T Q_{i,2} x_{k-2} \right) \le \alpha^2 \mu_Q V_{\delta(k)}(x_{k-2}) \\ &\vdots \\ V_{\delta(k)}(x_k) &\le \alpha^N V_{\delta(k)}(x_{k-N}) \le \alpha^N \mu_Q V_{\delta(k)}(x_{k-N}) \\ V_{\delta(k)}(x_k) &\le \alpha^{N+1} \left(x_{k-N-1}^T Q_{i,1} x_{k-N-1} \right) \\ &\le \alpha^{N+1} \mu_Q V_{\delta(k)}(x_{k-N-1}) \\ &\ddots \end{aligned}$$

From the above inequalities, it can be concluded that

$$V_{\delta(k)}(x_k) \le \alpha^{k-k_l} \mu_Q V_{\delta(k)}\left(x_{k_l}\right). \tag{13}$$

Since $P_i \le \mu_P P_j$, at the switching instant k_l we have

$$V_{\delta(k)}(x_k) \le \alpha^{k-k_l} \mu_P \mu_Q V_{\delta(k_l)}\left(x_{k_l}\right)$$

By repeating this procedure for earlier time steps, we obtain,

$$\begin{aligned} V_{\delta(k)}(x_k) &\le \alpha^{k-k_l+1} \mu_P \mu_Q^2 V_{\delta(k_l)}\left(x_{k_l-1}\right) \\ V_{\delta(k)}(x_k) &\le \alpha^{k-k_l+2} \mu_P \mu_Q^2 V_{\delta(k_l)}\left(x_{k_l-2}\right) \\ &\vdots \\ V_{\delta(k)}(x_k) &\le \alpha^{k-k_{l-1}} \mu_P^2 \mu_Q^2 V_{\delta(k_{l-1})}\left(x_{k_{l-1}}\right) \\ &\vdots \\ V_{\delta(k)}(x_k) &\le \alpha^{k-k_{l_0}} \mu_P^{N_0} \mu_Q^{N_0} V_{\delta(k_{l_0})}\left(x_{k_{l_0}}\right). \end{aligned}$$

The last inequality can be written as

$$\begin{aligned} V_{\delta(k)}(x_k) &\le \left(\alpha^{\frac{k-k_{l_0}}{N_0}} \right)^{N_0} \mu_P^{N_0} \mu_Q^{N_0} V_{\delta(k_{l_0})}\left(x_{k_{l_0}}\right) \\ V_{\delta(k)}(x_k) &\le \left(\mu_P \mu_Q \alpha^{\frac{k-k_{l_0}}{N_0}} \right)^{N_0} V_{\delta(k_{l_0})}\left(x_{k_{l_0}}\right) \end{aligned}$$

If the ADT satisfies (7), one has

$$\begin{aligned} \mu_P \mu_Q \alpha^{\tau_a} &< \mu_P \mu_Q \alpha^{\tau_a^*} = \mu_P \mu_Q \alpha^{-\ln(\mu_P \mu_Q)/\ln \alpha} \\ \mu_P \mu_Q \alpha^{\tau_a} &< e^{\ln\left(\mu_P \mu_Q \alpha^{-\ln(\mu_P \mu_Q)/\ln \alpha} \right)} = e^0 = 1 \end{aligned}$$

Consequently, by considering (3), (4) or (5) and (7), we have $V_{\delta(k)}(x_k) < V_{\delta(k_{l_0})}\left(x_{k_{l_0}}\right)$ and the GUAS of system (1) without uncertainties and disturbance is established and the proof is completed.

Now, by considering the aforementioned momentous aspect, the problem of robust state feedback H_∞ controller (Theorem 3 in [8]) is revisited as follows.

Theorem 3: Given scalars $0 < \alpha < 1$, $\mu_Q \ge 1$ and $\mu_P \ge 1$, the switched system (1) under the controller $u_k = M_i G^{-1} x_k$ is robustly GUAS with an l_2-gain γ, if there exist a set of symmetric matrices $Q_{i,f}$ ($f \in \mathbb{N}_{1,N-1}$) and $P_i > 0$, matrices G and M_i of appropriate dimension, and constant positive values ε_i and γ such that for $N = 1$,

$$\begin{bmatrix} -\alpha P_i & * & * & * & * \\ 0 & -\gamma^2 I & * & * & * \\ A_i G + B_i M_i & B_{iw} & -G^T - G + P_i + \varepsilon_i S_i S_i^T & * & * \\ C_i G + D_i M_i & 0 & 0 & -I & * \\ H_i G + N_i M_i & 0 & 0 & 0 & -\varepsilon_i I \end{bmatrix} \le 0$$

TABLE 1. Optimized attenuation levels versus the decay rate parameter α.

α	0.99	0.964	0.9	0.8	0.7	0.6	0.3	0.2	0.13	0.12
Optimal l_2-gain γ when N=1	0.4067	0.4247	0.4821	0.7155	1.0338	1.3061	3.5176	6.5746	93.7199	NA
Optimal l_2-gain γ when N=3	0.3825	0.4067	0.4728	0.6903	1.0057	1.2456	2.7891	5.1281	82.1821	15678

for $N = 2$

$$\begin{bmatrix} -\alpha Q_{i,1} & * & * & * & * \\ 0 & -\gamma^2 I & * & * & * \\ A_iG + B_iM_i & B_{iw} & -G^T - G + P_i + \varepsilon_i S_i S_i^T & * & * \\ C_iG+D_iM_i & 0 & 0 & -I & * \\ H_iG + N_iM_i & 0 & 0 & 0 & -\varepsilon_i I \end{bmatrix} \leq 0$$

$$\begin{bmatrix} -\alpha P_i & * & * & * & * \\ 0 & -\gamma^2 I & * & * & * \\ A_iG + B_iM_i & B_{iw} & -G^T - G + Q_{i,1} + \varepsilon_i S_i S_i^T & * & * \\ C_iG+D_iM_i & 0 & 0 & -I & * \\ H_iG + N_iM_i & 0 & 0 & 0 & -\varepsilon_i I \end{bmatrix} \leq 0$$

for $N \geq 3$

$$\begin{bmatrix} -\alpha Q_{i,N-1} & * & * & * & * \\ 0 & -\gamma^2 I & * & * & * \\ A_iG+B_iM_i & B_{iw} & -G^T - G+P_i +\varepsilon_i S_i S_i^T & * & * \\ C_iG+D_iM_i & 0 & 0 & -I & * \\ H_iG+N_iM_i & 0 & 0 & 0 & -\varepsilon_i I \end{bmatrix} \leq 0$$

$$\begin{bmatrix} -\alpha Q_{i,f-1} & * & * & * & * \\ 0 & -\gamma^2 I & * & * & * \\ A_iG+B_iM_i & B_{iw} & -G^T - G+Q_{i,f} +\varepsilon_i S_i S_i^T & * & * \\ C_iG+D_iM_i & 0 & 0 & -I & * \\ H_iG+N_iM_i & 0 & 0 & 0 & -\varepsilon_i I \end{bmatrix} \leq 0$$

$$\begin{bmatrix} -\alpha P_i & * & * & * & * \\ 0 & -\gamma^2 I & * & * & * \\ A_iG+B_iM_i & B_{iw} & -G^T - G+Q_{i,1} +\varepsilon_i S_i S_i^T & * & * \\ C_iG+D_iM_i & 0 & 0 & -I & * \\ H_iG+N_iM_i & 0 & 0 & 0 & -\varepsilon_i I \end{bmatrix} \leq 0$$

for all $i, j \in \Omega$

$$P_i \leq \mu_p P_j$$
$$Q_{i,f} \leq \mu_Q P_i$$

and the ADT of the switching signal $\sigma(k)$ satisfies

$$\tau_a > \tau_a^* = -\ln\left(\mu_P \mu_Q\right)/\ln\alpha \tag{14}$$

Proof: Similar to the proof of Theorem 3 in [8] and by considering the main issue discussed in Theorem 2, the proof is omitted.

III. ILLUSTRATIVE EXAMPLE

In this section, the effectiveness of robust state feedback H_∞ controller for $N = 3$ is demonstrated through a numerical example. Consider a discrete model of a modified population ecological system with parameters and uncertainties given in [8]. Choosing $\mu_P = 5$ and $\mu_Q = 1$, the optimized attenuation level versus α as the decay rate parameter is tabulated in Table 1. NA means that no controller can be designed using the corresponding theorem which can guarantee the robust stability of system with an optimal l_2-gain γ. As shown in Table 1, the optimized attenuation level for each decay rate based on N-step ahead LF is smaller than the optimized attenuation level based on traditional LF. Furthermore, the proposed approach ends in more relaxed minimal ADT constraint when the attenuation level is fixed. For instance, when $\gamma = 0.4067$ the minimal ADT constraint for $N = 1$ and $N = 3$ are $\tau_a^* = 160.138$ and $\tau_a^* = 43.897$, respectively.

Altogether, it can be concluded that our proposed approach ends in more relaxed results; however, more inequalities are gained which render higher computational cost.

IV. CONCLUSION

In summary, this note resolves the main drawback of stability analysis and synthesis of switched systems based on N-step ahead LF approach with ADT switching. Achieving better performance in sense of exogenous disturbance attenuation and relaxed ADT constraint with higher value of N is illustrated through a numerical example.

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