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EXPERIMENTAL COMPARISON OF TRAFFIC FLOW MODELS ON TRAFFIC DATA

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Abstract

Despite their deficiencies, continuous second-order traffic flow models are still commonly used to derive discrete-time models that help traffic engineers to model and predict traffic flow behaviour on highways. We briefly overview the development of traffic flow theory based on continuous flow-density models of Lighthill-Whitham-Richards (LWR) type, that lead to the second-order model of Aw-Rascle. We will then concentrate on widely-adopted discrete approximation to the LWR model by Daganzo's Cell Transmission Model. Behaviour of the discussed models will be demonstrated by comparing the traffic flow prediction based on these models with real traffic data on the southern highway ring of Prague.

1. Introduction

Management systems for highway traffic have existed since 1970s. These complex systems consist of different decision-making tools that address the management of pavement and bridges, public transport, congestion and safety, or traffic data monitoring. In this paper we will concentrate on numerical aspects of three mathematical models that can be used to predict traffic flow behaviour for highway management purposes.

Traffic flow models can be divided into four basic groups according to the level of detail that the model attempts to implement. The most widely employed class of traffic flow models are probably *macroscopic models* that disregard individual vehicles and consider the highway traffic to be an equivalent of compressible fluid flow. As these models are commonly used to predict and control (manage) the highway traffic, the role of such models is crucial for the success of any management action: A good model provides relatively accurate predictions of the future and it is computationally as simple as possible.

In the rest of our paper we will evaluate three possible traffic flow models that may be considered to be good candidates for modelling of highway traffic, and we will examine their performance in predicting highway traffic flow.

2. Macroscopic traffic models

A macroscopic traffic model incorporates traffic flux q [veh/hr],[1] traffic density ϱ [veh/km] and velocity v [km/hr], and describes the so-called *fundamental diagram of traffic flow*. Such a model can be used to predict the behaviour of a road system when applying control or management actions (e.g. ramp metering or speed limits).

Let us first study the number of vehicles N_1 and N_2 entering and leaving a road segment of length Δx metres during Δt seconds. Consider a hypothetical situation where a build-up of vehicles ($N_2 < N_1$) occurs. The change in the flow rate q is $\Delta q = \Delta N/\Delta t$ and the change in vehicle density ϱ is $\Delta \varrho = -\Delta N/\Delta x$.

As the vehicles inside the segment have no possibility to exit the road, vehicles are conserved, with ΔN denoting the number of vehicles inside the segment. Therefore,

$$\Delta q \Delta t = \Delta N = -\Delta \varrho \Delta x \quad \Leftrightarrow \quad \frac{\Delta \varrho}{\Delta t} + \frac{\Delta q}{\Delta x} = 0.$$

This justifies the following relationship for continuous $q(x,t)$ and $\varrho(x,t)$:

$$\frac{\partial \varrho}{\partial t} + \frac{\partial q}{\partial x} = 0 \qquad \text{or} \qquad \partial_t \varrho + \partial_x q = 0. \tag{1}$$

Continuous macroscopic traffic flow models are typically derived from this equation, by introducing a form of speed-influenced density. The most prominent type of the first-order models that result from the direct application of the above equation is the LWR model, described in the next section.

2.1. Lighthill-Whitham-Richards

Lighthill-Whitham-Richards (LWR) model [8, 10] is a first-order model that results from a direct application of the conservation law (1) where the flow rate is function of velocity $v(\varrho)$

$$q(x,t) = v(\varrho(x,t)) \cdot \varrho(x,t).$$

The speed is typically expressed as

$$v(\varrho) = v_{\mathrm{f}}\,(1 - \frac{\varrho}{\varrho_{\mathrm{jam}}}),$$

where v_{f} is the free-flow speed of solitary vehicles, and ϱ_{jam} denotes so-called *jam density* of the road, that is the maximum possible density of vehicles in the moment when the traffic flow has completely stopped due to traffic jam.

The model is quite simple and numerically stable and even today is often used to study traffic flow phenomena that occur on highways or in road tunnels. According to critical studies [5, 6], the model provides results that correspond well with the theory of kinematic waves, and its output is consistent with empirically observed fundamental diagram data. However, this simple model is unable to capture certain phenomena that occur in everyday traffic, like stop-an-go waves or travel speed adaptivity.

[1] The unit “veh” denotes a *unit vehicle*, an average vehicle that makes it possible to disregard the heterogeneity of traffic flow. Also known as PCE, *passenger car equivalent*.

2.2. Second-order fluid approximations

The inability of LWR-class models to capture more complex traffic flow phenomena led to creation of more elaborate models. These models use an additional set of equations to introduce a relation similar to conservation of momentum in fluids, in the hope that this additional level of detail would lead to a more detailed level of description.

Two seminal works of Payne [9] and Whitham [11] emerged, and sparked a great deal of effort resulting in numerous publications of so-called PW-type flow models, introducing variations and extensions and proposing different numerical schemes. However, 20 year later Daganzo [5] demonstrated that these "higher order" approaches are not appropriately constructed and lead to unrealistic results.

The only continuous traffic flow model of second order that is currently still being studied is due to Aw and Rascle [1]. This AR-type model[2] addresses most of the previous flaws of PW-type models. It takes the composite form of two first-order models,

$$\partial_t \varrho + \partial_x (v\varrho) = 0$$
$$\partial_t (v + p(\varrho)) + v\partial_x \big(v + p(\varrho)\big) = 0$$

where the pressure function of vehicle density $p(\varrho)$ is smooth and increasing.

An AR-type model can be quite conveniently solved using the central upwind scheme [7, 2]. However, as we will see in our experiments, special attention has to be paid to selecting appropriate space- and time-steps.

3. Cell Transmission Model (CTM)

Daganzo in [3] introduced the CTM, where he simplified the first-order models by using a piecewise-linear approximation of the fundamental diagram, depicted in Figure 1. CTM replaces the original LWR state equation (1) by a set of affine functions

$$q = \min\left(v\varrho, q_{\max}, w(\varrho_{\text{jam}} - \varrho)\right).$$

The follow-up paper [4] examines the evolution of traffic on a highway segment divided into I consecutive cells numbered starting at the upstream end of the road, $i = 1, 2, \ldots, I$. The segments are homogeneous and their length is set equal to the distance traveled by typical vehicle in light traffic in one clock tick (time step k of constant length Δt).

The cell transmission model is based on a recursion where the cell occupancy at step $k+1$ equals its occupancy at step k, plus the inflow and minus the outflow,

$$n_i[k+1] = n_i[k] + y_i[k] - y_{i+1}[k], \tag{2}$$

where the flow from cell $i-1$ to i during the time interval k is assumed to be

$$y_i[k] = \min\{n_{i-1}[k-1], Q_i[k], N_i[k] - n_i[k]\}, \tag{3}$$

[2]Note that AR in this paper is not related to autoregressive models.

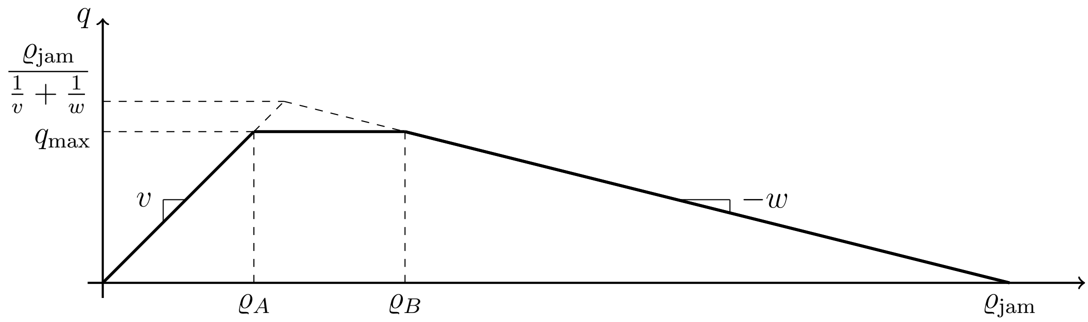

Figure 1: An approximation of the fundamental diagram suggested by Daganzo [3].

where $Q_i[k]$ is the capacity flow into i for time interval t, and $N_i[k] - n_i[k]$ is the amount of empty space in cell i at time step k. Cell occupancies are updated for each step of the clock during the simulation.

4. Experiments

In order to demonstrate the behaviour of all three discussed models, we have tested the prediction capabilities using the data from the southern leg of the Prague Ring (SOKP) section from km 20.1 to km 17.0. We fed the measurements, provided by detectors at km 20.1, as a boundary condition into our models, and used the models to predict the traffic at km 17.0. The predicted data were then compared with the measurements provided by detectors.

The basic parameters for the simulation were the length of a segment $\Delta x = 150\,\text{m}$, and the time step $\Delta t = \Delta x / v_{\text{f}}$. The free flow speed v_{f} has been identified from the measured data as $v_{\text{f}} = 115\,\text{km/h}$, implying $\Delta t \approx 4.7\,\text{s}$. Jam density $\varrho_{\max}$ is given by an average length of a passenger vehicle $d_{\text{avg}} = 6\,\text{m}$ as $\varrho_{\max} = 1000/d_{\text{avg}} = 166\,\text{veh/km}$. Maximum vehicle flow is given by the theoretical speed limit of the highway, which is 130 km/h.

When numerically solving a partial differential equation using a method based on finite differences, a necessary condition of stability of the solution is provided by Courant–Friedrichs–Lewy (CFL) condition [7]. This condition arises if explicit time integration schemes are used for the numerical solution. As a consequence, the time step of such a scheme must be less than a certain time, otherwise the simulation will produce incorrect results. While the rounded time-step $\Delta t = 5\,\text{s}$ is an acceptable value for first-order LWR-type models (even if it violates the CFL condition), integrating an AR-type model with such a large time step does not converge to a plausible solution. The higher-order model unfortunately requires a shorter time step fulfilling the CFL condition. Hence, for an AR-type model, $\Delta t = 0.5\,\text{s}$ has been used.

The results of all three models are compared in Figure 2. We can see that the second-order AR-type model has still issues in following the general trends recogniz-

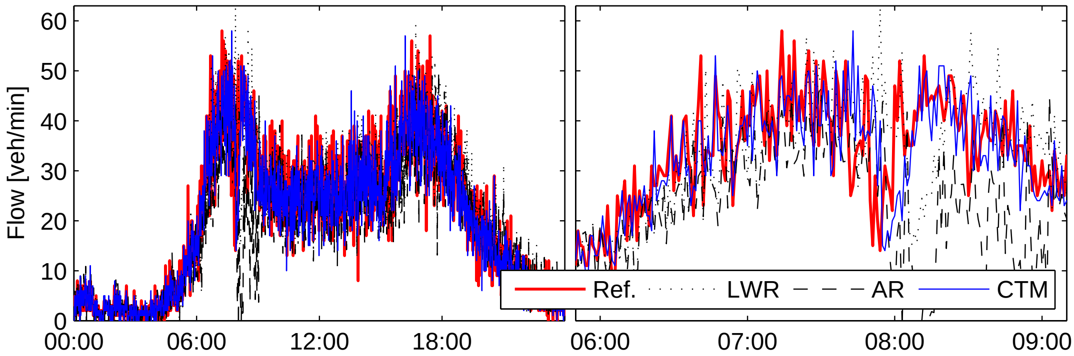

Figure 2: Comparison of predictions of the LWR, AR, and CTM models with the reference flow obtained by measurements. Left: data for one day of traffic, right: the same data between 6:00 and 9:00.

Model	Δt [s]	Steps	Time [s]	MSE	$\max \epsilon_r$ [%]
LWR	5	18017	42	59.58	42%
AR	0.5	180167	1122	80.09	101%
CTM	4.7	18880	25	27.79	14%

Table 1: Comparison of all models on real time data. MSE denotes the mean squared error of the prediction, ϵ_r is the relative prediction error.

able in the traffic data. This is especially visible in the right panel of Figure 2 for times between 8:00 and 9:00. The most probable reason for this anomaly is the higher sensitivity of AR-type models to repetitive changes in the boundary conditions. Most important observation, however, can be found in Table 1 which summarizes the computational times and errors of the models: From the practical point of view the test demonstrates that the AR-type model is almost useless due to the necessary small time-step and resulting long computational time. A simple CTM scheme that resembles cellular automata beats even the simple LWR model in both computational speed and accuracy. Again, our assumption is that the continuous nature of the underlying model is disturbed by the time-variable boundary conditions.

5. Conclusions

We have demonstrated three different traffic flow models and their performance on real-world traffic flow data. Our experiment shows that from the practical point of view, Daganzo's CTM, a simple compartment model based on piecewise linear approximation of the fundamental diagram of traffic flow, provides best results in both accuracy and computational speed. In theory, a continuous higher-order model of AR-type should be able to address traffic phenomena that the CTM is unable to capture, however, the higher order model is significantly less numerically stable. The need for strict fulfillment of the CFL stability condition results in tenfold decrease of the original time-step, rendering the whole model unsuitable for practical application.

The whole Matlab package can be downloaded from the website of the corresponding author at `http://staff.utia.cas.cz/prikryl/panm17.zip`.

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