Capital market efficiency in the Ising model environment: Local and global effects

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Abstract. Financial Ising model is one of the simplest agent-based models (building on a parallel between capital markets and the Ising model of ferromagnetism) mimicking the most important stylized facts of financial returns such as no serial correlation, fat tails, volatility clustering and volatility persistence on the verge of non-stationarity. We present results of Monte Carlo simulation study investigating the relationship between parameters of the model (related to herding and minority game behaviors) and crucial characteristics of capital market efficiency (with respect to the efficient market hypothesis). We find a strongly non-linear relationship between these which opens possibilities for further research. Specifically, the existence of both herding and minority game behavior of market participants are necessary for attaining the efficient market in the sense of the efficient market hypothesis.

Keywords: Ising model, efficient market hypothesis, Monte Carlo simulation.

JEL classification: G02, G14, G17
AMS classification: 91G60, 91G70

1 Introduction

Agent-based models (ABM) have attracted much attention in economics and finance in recent years [10, 12, 21] as they describe the reality better than simplified models of traditional economics and finance. The crucial innovation lies in assuming a boundedly rational economic agent [20, 18] instead of a perfectly rational representative agent with homogeneous expectations [16, 14]. In these models, agents make decision without utility maximization but usually using simple heuristics. The resulting systems are majorly driven endogenously, i.e. without exogenous shocks forcing the dynamics.

In finance, the founding contributions were laid by Brock and Hommes models [3, 4] characteristic by strategy-switching agents and possible bifurcation dynamics. Here, we focus on one of the simplest ABMs built on a parallel between ferromagnetism and market dynamics, i.e. the Ising model adjusted for financial economics. In the model, economic agents participating in the market are spins of a magnet. In the same way as the spins, the agents are influenced by (make their decisions based on) their neighbors, or agents with similar beliefs, but also by the overall market sentiment and activity. Such model has been shown to mimic the basic financial stylized facts successfully [2]. We focus on the model parameters and how they influence price and returns dynamics in the optics of the efficient market hypothesis. The attention is given to finding a combination of parameters which yields an efficient market or dynamics close to it. We show that the effects of parameters are more complicated than one might expect and their influence is apparently non-linear. This opens further research options which are shortly discussed as well.

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2 Methodology

In this section, we provide a brief introduction to the Ising model adjusted for financial markets and we shortly discuss the essence of the efficient market hypothesis.

2.1 Ising model

As a representative of the agent-based models applied to finance and financial economics, we opt for a simple Ising model adjusted for financial markets as proposed by Bornholdt [2]. The model builds on a combination of the standard Ising model of ferromagnetism (with local field interactions) [11] and a minority game behavior of market agents [1, 5]. Financial market is represented by a square lattice (usually with torus-like neighborhoods) with a side of N, i.e. with N^2 elements representing market agents. These elements are referred to as spins due to their magnetization of either $+1$ or -1. This spin orientation is translated into a financial market as either a buy or a sell signal (decision), respectively. The spin orientation of agent i for a time period t is labelled as $S_i(t)$. For each agent i, the local field $h_i(t)$ for a time period t is defined as

$$h_i(t) = \sum_{j=1}^{N} J_{ij} S_j(t) - \alpha C_i(t) \frac{1}{N} \sum_{j=1}^{N} S_j(t). \tag{1}$$

The first term is defined as a local Ising Hamiltonian with neighbor interactions J_{ij}. This is the reference to the standard Ising model. The second term represents the minority game dynamics as it depends on the total magnetization of the system $M(t) \equiv \frac{1}{N}\sum_{j=1}^{N} S_j(t)$ at time t with sensitivity α. $C_i(t)$ gives the strategy of spin i. Orientation of spin i at time $t+1$ is given as

$$\begin{aligned} S_i(t+1) &= +1 \quad \text{with} \quad p = [1+\exp(-2\beta h_i(t))]^{-1}, \\ S_i(t+1) &= -1 \quad \text{with} \quad 1-p, \end{aligned}$$

which is directly connected to Eq. 1 with an additional sensitivity β, which is parallel to the inverse temperature of the original Ising model.

The strategy term $C_i(t)$ is given as a general term in Eq. 1 which can be further specified. A popular choice is to highlight the minority game behavior of the spin by allowing the strategy to change with respect to the total magnetization and the spin's own orientation. This specification also allows for more strategy types. Bornholdt [2] proposes the following dynamics:

$$C_i(t+1) = -C_i(t) \text{ if } \alpha S_i(t) C_i(t) \sum_{j=1}^{N} S_j(t) < 0 \tag{2}$$

A simple alternative is to keep the strategy spin update immediately, which reduces the local field equation to

$$h_i(t) = \sum_{j=1}^{N} J_{ij} S_j(t) - \alpha S_i(t) \left| \frac{1}{N} \sum_{j=1}^{N} S_j(t) \right|, \tag{3}$$

i.e. it does not depend on the strategy of any spin at all.

The price dynamics of the system is extracted directly from the magnetization dynamics so that

$$\log P(t) = M(t) \equiv \frac{1}{N} \sum_{j=1}^{N} S_j(t). \tag{4}$$

2.2 Efficient market hypothesis

Efficient market hypothesis (EMH) has been a cornerstone of modern financial economics for decades. Even though its validity has been challenged on many fronts, it still remains the firm theoretical basis of the financial economics theory [6, 15]. In the fundamental paper, Fama [8] summarizes the empirical validations of the theoretical papers of himself [7] and Samuelson [17]. The theory is revised and made clearer in Fama's 1991 paper [9] where the market efficiency is split into three forms based on availability of information. From mathematical standpoint, the historical papers [7, 17] are more important as they provide specific model forms of an efficient market. Specifically, Fama [7] connects the (logarithmic) price process of an efficient market to a random walk and Samuelson [17] specifies it as a martingale. Implications for the statistical properties of the returns process of the efficient market are straightforward. For the former, the returns are expected to be serially uncorrelated and follow the Gaussian (normal) distribution, which implies independence. For the latter, only the serial uncorrelatedness is implied. We thus have two straightforward implications of the market efficiency – normally distributed (for the random walk definition) and serially uncorrelated (for both random walk and martingale definition) returns – which we use in the simulations presented in the next section.

3 Results and Discussion

3.1 Simulation setting

We are interested in the ability of the Ising model defined between Eqs. 1-4 to meet the criteria attributed to the efficient capital market, i.e. normality and serial uncorrelatedness of returns. To test these, we use the Shapiro-Wilk test [19] and Ljung-Box test [13], respectively.

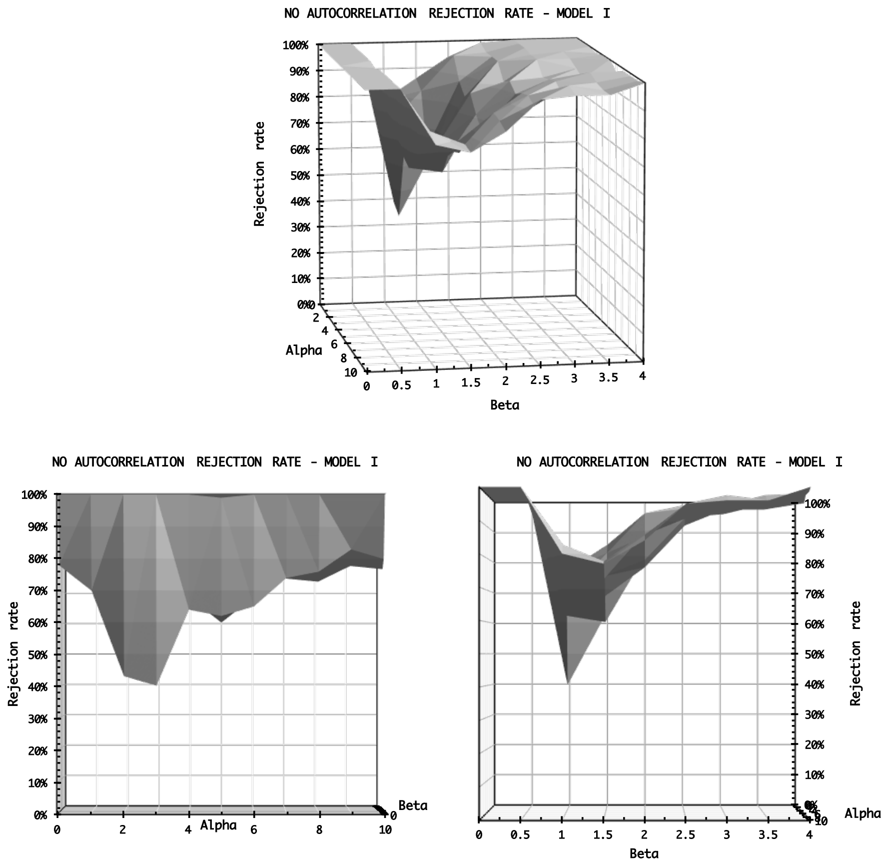

Figure 1: Rejection rates of no serial correlation hypothesis for Model I according to Eq. 1. Parameter α varies between 0 and 10 with a step of 1, and parameter β between 0 and 4 with a step of 0.5. Other parameters are set at $T = 1000$ and $N = 25$, neighborhood interactions J_{ij} are set to the nearest neighbors and the spin itself with a weight of 1, and 0 otherwise. We provide a 3D view as well as focusing on parameters separately.

There are two crucial parameters in the model – α and β – which can influence the prices and returns dynamics emerging from the model. We vary these two parameters and study how it influences the rejection rate of normality and uncorrelatedness with respective tests. In other words, we are interested in a proportion of times these tests reject (with a significance level of 0.10) market efficiency of series generated by the financial Ising model with specified parameters. Based on findings of previous research [2], we manipulate α between 0 and 10 with a step of 1 and β between 0 and 4 with a step of 0.5. We fix the time series length $T = 1000$ and the number of agents in the market to $N^2 = 25^2 = 625$. The neighborhood influence J_{ij} is set equal to 1 for the nearest neighbors and the spin's own position (five spins in total), and 0 otherwise. For each setting, we perform 100 simulations. Two specifications are studied – Model I given by Eq. 3, i.e. with fixed strategy spins, and Model II given by Eq. 1, i.e. with variable strategy spins. The code in R is available upon request.

3.2 Main findings

The findings are summarized in Figs. 1 and 2 for Model I and Model II, respectively. The 3D charts summarize the results (rejection rates) for simulations described in the previous section. Before turning to these, it needs to be noted that for the normality testing, only the case when $\beta = 0$ gives the rejection rates around 10% whereas for $\beta > 0$, normality is rejected practically always. This is true for both specifications of the model and regardless the values of parameter α. These are thus not represented graphically.

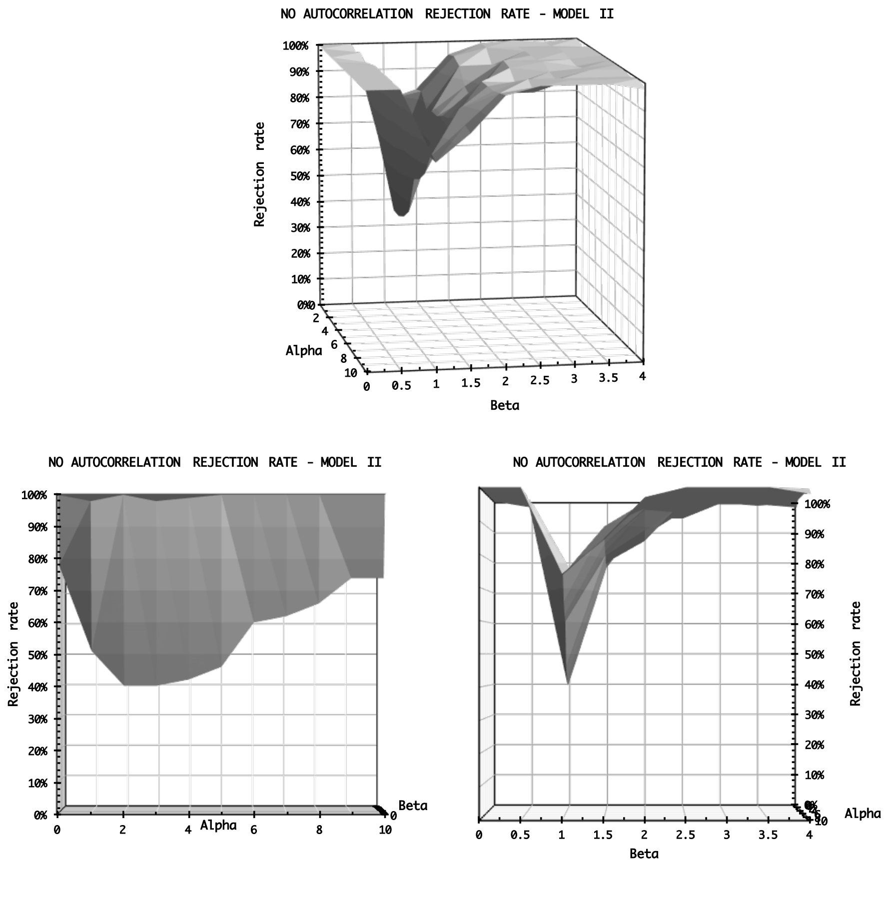

Figure 2: Rejection rates of no serial correlation hypothesis for Model II according to Eq. 3. Parameter α varies between 0 and 10 with a step of 1, and parameter β between 0 and 4 with a step of 0.5. Other parameters are set at $T = 1000$ and $N = 25$, neighborhood interactions J_{ij} are set to the nearest neighbors and the spin itself with a weight of 1, and 0 otherwise. We provide a 3D view as well as focusing on parameters separately.

We now turn to the tests of uncorrelatedness. For both models, we find a strongly non-linear dependence between rejection of no serial correlation hypothesis and the model parameters. For the sensitivity to the global magnetization (parameter α), we find a minimal rejection rate of approximately 40% at $\alpha = 3$ for Model I and at $\alpha = 2$ for Model II. For $\alpha = 0$, the rejection rate is around 80% for both models, and the same is true for the other boundary of $\alpha = 10$. The serial correlation dynamics thus emerges both for no reaction to the total magnetization, i.e. avoiding the influence of the overall market situation, and for a strong minority game behavior. There is thus no simple outcome such that a minority game behavior induces a serial correlation structure or the other way around. Such structure emerges for both extremes and market gets closer to efficiency for a setting in between.

Qualitatively similar results are found for the β parameter, i.e. the sensitivity to the local field. The minimal rejection rate is found at $\beta = 1$ for both models. For $\beta < 1$, the no serial correlation hypothesis is rejected practically always. The relationship between β and the rejection rate is smoother for $\beta > 1$ but still the rejection rate gets very close to 100% for $\beta > 3$ for Model I and $\beta > 2$ for Model II.

These preliminary results suggest the following. First, there is no simple linear relationship between market efficiency and model parameters. This poses a problem for policy makers potentially trying to get the market closer to efficiency as there is no simple answer to this endeavor. Second, which is tightly connected to the first, more detailed (smoother) simulations need to be undertaken to find a more precise efficient setting. And third, inclusion of the strategy spin plays no important role for this task.

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