



Commodity futures and market efficiency

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ABSTRACT

We analyze the market efficiency of 25 commodity futures across various groups—metals, energies, soft commodities, grains and other agricultural commodities. To do so, we utilize the recently proposed Efficiency Index to find out that the most efficient among all of the analyzed commodities is heating oil, closely followed by WTI crude oil, cotton, wheat, and coffee. On the other end of the ranking scale we find live cattle and feeder cattle. The efficiency is also found to be characteristic for specific groups of commodities, with energy commodities being the most efficient and other agricultural commodities (composed mainly of livestock) the least efficient groups. We also discuss contributions of long-term memory, fractal dimension and approximate entropy to the total inefficiency. Last but not least, we come across the nonstandard relationship between the fractal dimension and the Hurst exponent. For the analyzed dataset, the relationship between these two variables is positive, meaning that local persistence (trending) is connected to global anti-persistence. We attribute this behavior to specifics of commodity futures: they may be predictable over a short term and locally, but over a long term they return to their fundamental prices.

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1. Introduction

Efficient market hypothesis (EMH) has been a cornerstone of financial economics for decades and it has been brought to the forefront by the influential paper of Fama (1970), summarizing the empirical findings based on efficient market hypotheses by Fama (1965) and Samuelson (1965). Even though the actual definitions differ – the former study builds on a random walk definition and the latter on a martingale definition – the qualitative consequences are the same: the efficiency of a market originates in the impossibility of systematic control of the market, usually in the form of above-average risk-adjusted returns. Fama (1991) later subdivided the efficiency hypothesis into three forms – weak, medium and strong – which vary by the different information sets taken into consideration, and all are based on inclusion of the information sets in market prices. The weak-form EMH says that all past price movements (and associated statistics) are already reflected in the market prices. Prediction of market movements based on historical time series (technical analysis) is thus not possible for this form. The medium-form EMH states that all publicly available information is already contained in the prices, while the strong-form EMH adds all (even privately) available information. The medium form thus discards fundamental analysis and the strong form eliminates even

insiders from making a profit. Evidently, a weaker form of EMH is always a subset of a stronger form. Even though EMH has been repeatedly disparaged both empirically and theoretically (Cont, 2001; Malkiel, 2003), and even more so after the Global Financial Crisis broke out in 2007/2008, its validity remains an open issue, yet still it persists in standard textbooks on financial economics (Elton et al., 2003).

Comparison of efficiency across various assets has been discussed in different studies. In a series of papers, Di Matteo et al. (2003, 2005) and Di Matteo (2007) study long-term memory and multi-scaling of a wide portfolio of stock indices, foreign exchange rates, Treasury rates and Eurodollar interbank interest rates using various estimators of long-term memory. They show that stock indices of more developed countries are also more efficient yet showing weak signs of anti-persistence (properties of long-term memory are described in detail in the Methodology section), finding no deviations from EMH for any of the analyzed maturities of Eurodollar and Treasury rates. For US dollar exchange rates, the authors find diverse results with no evident pattern connecting the exchange rate efficiency level with geographical or geopolitical properties. In another series of papers, Cajueiro and Tabak (2004a,b,c, 2005) compare stock market indices from different continents, finding that the US and Japanese markets are the most efficient whereas the Asian and Latin American ones are revealed to be the least efficient. Lim (2007) studies non-linear dependencies, their evolution in time and connection to market efficiency for a set of stock markets. The author finds the US market to be the most efficient, followed by Korea, Taiwan and Japan. On the other end of the ranking scale lie Malaysia, Chile and Argentina. Zunino et al. (2010) utilize the complexity–entropy causality plane to rank stock market

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indices to show that the emerging markets are less efficient than the developed ones, as one would expect. The difference is attributed to a lower entropy value and a higher complexity of the emergent markets. Kristoufek and Vosvrda (2013) introduce the Efficiency Index and come up with a ranking of stock market indices, finding that the most efficient markets are located in Western Europe, USA and Japan, whereas the least efficient markets are situated in Latin America and Asia.

However, to the best of our knowledge, proper attention has not been given to a comparison of the efficiency of commodity markets. In this paper, we analyze futures markets for a wide range of commodities – energy, metals, and various agricultural commodities – and compare their efficiency using the Efficiency Index proposed by Kristoufek and Vosvrda (2013). The paper is structured as follows. Section 2 covers literature dealing with the efficiency of commodities. Section 3 describes the methodology in detail. Section 4 describes the analyzed dataset and gives the results. Section 5 is the conclusion. We show that efficiency is related to a type of commodity (energy commodities being the most efficient and other agricultural commodities being the least efficient). In addition, we find a non-standard relationship between the local and global properties of the series: most of the series show local persistence, yet they are globally mean-reverting. The series thus follow quite strong local trends but over a long term, they return to their fundamental value.

2. Literature review

Testing the market efficiency in commodity markets has a long history. Roll (1972) examines the commodity price index and argues that the market is inefficient due to significant serial correlations among its returns. Danthine (1977) disputes such claims and shows that the violation of the standard martingale condition does not imply inefficiency in the commodity spot markets with support of risk aversion and no arbitrage opportunities. Gjolberg (1985) analyzes oil spot prices at the Rotterdam market, rejects the efficiency hypothesis and constructs a profitable trading rule for daily, weekly and monthly price changes. Panas (1991) studies the Rotterdam oil market as well, together with the Italian market, and based on leptokurtic monthly price changes, he rejects the markets' efficiency. Herbert and Kreil (1996) examine the US spot (cash) and futures markets for natural gas and find these to be inefficient. They argue that such inefficiency is caused by the specific structure of the US gas markets.

More recently, Tabak and Cajueiro (2007) analyze the efficiency of Brent and WTI crude oil using the rescaled range analysis and show that the markets are becoming more efficient in time. Alvarez-Ramirez et al. (2008) study the auto-correlation structure of the crude oil process using the detrended fluctuation analysis. They show that the market is efficient over a long term, but the auto-correlation structure leads to rejection of the efficiency over a short term. Alvarez-Ramirez et al. (2010) further inspect the crude oil markets using lagged detrended fluctuation analysis and argue that multi-scaling and deviations from the random walk behavior cause the spot prices to be inefficient. The research on evolution of efficiency in time is further extended by Wang and Liu (2010) where the authors study short-, medium- and long-term efficiency for various scales within the detrended fluctuation analysis approach. They show that the WTI crude oil becomes more efficient in time for all three of the analyzed scales. Also using the detrended fluctuation analysis, Wang et al. (2011) argue that WTI crude oil spot and futures are not efficient for time scales shorter than one month. Crude oil markets (Brent and WTI) are also analyzed by Charles and Darné (2009), who use the variance ratio tests to show that the Brent market is weak-form efficient but the WTI market is not, while providing some discussion about effects of deregulation on the markets.

Lee and Lee (2009) study four energy commodities – coal, oil, gas, and electricity – using panel data stationarity tests to uncover that

none of the studied markets is efficient in the strict stationarity sense. Lean et al. (2010) study WTI crude oil spot and futures prices using mean-variance and stochastic dominance approaches, finding no arbitrage opportunities between spot and futures prices while the findings are robust for various sub-periods and critical events. Narayan et al. (2010) study the long-term relationship between spot and futures prices of gold and oil. They find that investors use the gold market to hedge against inflation, and – more importantly for our purposes – the crude oil market predicts the gold market and vice versa, implying inefficiency.

Wang and Yang (2010) study high-frequency futures data of crude oil, heating oil, gasoline, and natural gas using various non-linear models. For heating oil and natural gas, the authors find market inefficiencies which are profound mainly during the bull market conditions. Gebre-Mariam (2011) focuses on the US natural gas market (spot and futures) finding no arbitrage opportunities for daily prices but in general, the author claims that the markets can be seen as efficient only for contracts with approximately a month to maturity. Martina et al. (2011) utilize entropy approaches to WTI crude oil spot prices and find various cycles in its prices. Entropy is also applied by Ortiz-Cruz et al. (2012) who again study daily WTI prices, finding the market to be efficient with two episodes of inefficiency connected to the US recessions in the early 1990s and late 2000s. The authors stress that deregulation of the market has helped to improve its efficiency.

Zunino et al. (2011) apply information theory methods (specifically the permutation entropy and permutation statistical complexity) to the commodity markets for purposes of efficiency ranking, finding silver, copper and cotton to be the most efficient commodities. Wang et al. (2011) study the gold market using the multifractal detrended fluctuation analysis to show that the market, especially after 2001, becomes more efficient in time. Kim et al. (2011b) use the random matrix theory and network analysis to show that stock and commodity markets are well decoupled, except for oil and gold showing signs of inefficiency. Kim et al. (2011a) then focus on the Korean agricultural market using the detrended fluctuation analysis, finding anti-correlated series with strong volatility clustering that leans toward inefficiency.

From these selected papers, it is evident that analysis of the efficiency of commodity markets is fruitful with many approaches to the topic. However, the studies usually focus on a single (or a pair of) efficiency measure(s) to test whether the specific markets are or are not efficient. Moreover, the analysis is usually strongly focused on a single commodity or a small group of commodities. Here, we contribute to the literature by applying various efficiency measures on a wide portfolio of commodities ranging from energy and agricultural (with several subgroups) commodities to metals. Moreover, we utilize the efficiency measure introduced by Kristoufek and Vosvrda (2013) to rank the commodities according to their efficiency.

3. Methodology

An efficient market can be defined in several ways. The main distinction has its roots back in 1965 when Fama (1965) and Samuelson (1965) used different definitions—a random walk and a martingale, respectively. We stick to the martingale definition of efficiency because it is less restrictive. Based on this definition, we assume that the returns of a financial asset are serially uncorrelated and with finite variance for the efficient market situation. Such a simple definition allows us to use various measures of market efficiency, which are described in this section. Eventually, we refer to the Efficiency Index which takes these statistics into consideration and it helps to rank different assets according to their efficiency while using various dynamic properties of the time series under study.

3.1. Long-term memory

Long-term memory (long-range dependence) series are characterized by values in the (even distant, in theory infinitely distant) past

influencing the present and future values. These processes are typically described with the long-term memory parameter H (Hurst exponent) which ranges between $0 \leq H < 1$ for stationary invertible processes. The midpoint, $H = 0.5$, holds for uncorrelated (or in general short-term memory) processes, i.e., processes of the efficient market. For $H > 0.5$, the processes are positively correlated with long-term memory and are usually referred to as persistent. These processes systematically follow local trends while still remaining stationary. For $H < 0.5$, we have long-term memory processes with negative correlations—anti-persistent processes. Such processes switch direction more often than a random process does.

More formally, the long-term memory processes are defined in both time and frequency domains. In the time domain, they are connected to a power-law decaying auto-correlation function. For the auto-correlation function $\rho(k)$ with time lag k , the decay is characterized as $\rho(k) \propto k^{2H-2}$ for $k \rightarrow +\infty$. In the frequency domain, the spectrum $f(\lambda)$ with frequency λ of the long-range dependent process diverges at the origin so that $f(\lambda) \propto \lambda^{1-2H}$ for $\lambda \rightarrow 0+$. These definitions further lead to non-summable auto-correlations and diverging covariance of partial sums of the process for the persistent series. Such properties are used in various estimators of parameter H . For comparison of both time and frequency domain estimators, see Beran (1994), Taqqu et al. (1995), Taqqu and Teverovsky (1996), Robinson (1995), Geweke and Porter-Hudak (1983), Di Matteo et al. (2003), Di Matteo (2007), Barunik and Kristoufek (2010) and Teverovsky et al. (1999). Out of these estimators, we opt for the local Whittle and GPH estimators (explained later in this section) which are suitable for short time series with a possible weak short-term memory (Taqqu and Teverovsky, 1996; Taqqu et al., 1995), which can easily bring a bias into the time domain estimators (Kristoufek, 2012; Teverovsky et al., 1999). Moreover, these estimators have well-defined asymptotic properties—they are consistent and asymptotically normal estimators (Beran, 1994; Geweke and Porter-Hudak, 1983; Phillips and Shimotsu, 2004; Robinson, 1995).

3.1.1. Local Whittle estimator

The local Whittle estimator (Robinson, 1995) is a semi-parametric maximum likelihood estimator utilizing a likelihood function of Künsch (1987) and focusing only on a part of the spectrum $f(\lambda)$ near the origin. The full parametric specification is thus not necessary, and one does not need to assume any specific underlying long-term memory model but only a model with a spectrum divergent at origin. This way, the estimator does not take into consideration high frequencies and it is in turn resistant to the short-term memory bias. As an estimator of the spectrum of series $\{x_t\}$, we use the periodogram defined as $I(\lambda_j) = \frac{1}{T} \sum_{t=1}^T \exp(-2\pi i t \lambda_j) x_t$ with $j = 1, 2, \dots, m$ where $m \leq T/2$ and $\lambda_j = 2\pi j/T$. The local Whittle estimator is defined as

$$\hat{H} = \arg \min_{0 \leq H < 1} R(H), \quad (1)$$

where

$$R(H) = \log \left(\frac{1}{m} \sum_{j=1}^m \lambda_j^{2H-1} I(\lambda_j) \right) - \frac{2H-1}{m} \sum_{j=1}^m \log \lambda_j. \quad (2)$$

The local Whittle estimator is consistent and asymptotically normal, specifically

$$\sqrt{m}(\hat{H} - H^0) \rightarrow_d N(0, 1/4). \quad (3)$$

3.1.2. GPH estimator

Unlike the local Whittle estimator, the GPH estimator, named after the authors Geweke and Porter-Hudak (1983), is based on a fully

functional specification of the underlying process as the fractional Gaussian noise implying a specific spectral form:

$$\log f(\lambda) \propto -(H-0.5) \log(4 \sin^2(\lambda/2)). \quad (4)$$

Again, the spectrum is estimated using the periodogram, and the Hurst exponent is estimated using the ordinary least squares on

$$\log I(\lambda_j) \propto -(H-0.5) \log(4 \sin^2(\lambda_j/2)). \quad (5)$$

The GPH estimator is consistent and asymptotically normal (Beran, 1994), specifically

$$\sqrt{T}(\hat{H} - H^0) \rightarrow_d N(0, \pi^2/6). \quad (6)$$

The GPH estimator is thus asymptotically infinitely more efficient than the local Whittle estimator. However, this is true only if the underlying process is in fact the fractional Gaussian noise. In financial and economic time series this is frequently not the case, as the processes are mostly a combination of short-term (such as autoregressive moving average – ARMA – processes of various specifications) and long-term memory (such as the aforementioned fractional Gaussian noise of fractionally integrated ARMA) processes. In this case, the GPH estimator becomes biased. To avoid the bias, the GPH estimator is based only on a part of the spectrum (periodogram) close to the origin similar to the local Whittle estimator. The regression in Eq. (5) is then applied only for a part of the periodogram based on the same parameter m as for the local Whittle estimator¹.

3.2. Fractal dimension

Contrary to the long-term memory, which can be seen as a characteristic of global dependence and correlation structure, the fractal dimension D can be taken as a measure of local memory of the series as it is a measure of roughness of the series (Kristoufek and Vosvrda, 2013). As specific parts of a series can be different with regard to their roughness or smoothness, the series can be locally serially correlated even though, on a global level, the correlations might vanish and are not necessarily observable or detectable.

For a univariate series, the fractal dimension ranges between $1 < D \leq 2$. For self-similar processes, the fractal dimension is tightly connected to the Hurst exponent (long-term memory) of the series so that $D = 2 - H$. In economic terms, this can be understood as a perfect transmission of a local behavior (fractal dimension) to a global behavior (long-term memory). However, this relationship usually does not hold perfectly for the financial series so that both D and H give different insights into the dynamics of the series, making it worth studying them separately.

In general, $D = 1.5$ holds for an uncorrelated series with no local trending or no local anti-correlations and thus it is also a value of D for the efficient market. For a low fractal dimension, $D < 1.5$, the roughness of the series is lower than for an uncorrelated process; hence we observe local trending and the series is said to be locally persistent. Conversely, a high fractal dimension, $D > 1.5$, characterizes a series rougher than the uncorrelated one, which is connected to local anti-persistence, i.e., such series are negatively auto-correlated locally. For purposes of the Efficiency Index introduced later in this section, we utilize Hall–Wood and Genton estimators (Gneiting and Schlather, 2004; Gneiting et al., 2010). Statistical properties of these estimators are well documented by Gneiting et al. (2010).

¹ In our analysis, we apply $m = T^{0.6}$ (Phillips and Shimotsu, 2004).

3.2.1. Hall–Wood estimator

The Hall–Wood estimator (Hall and Wood, 1993) is a box-counting procedure which utilizes scaling of absolute deviations between steps. Formally, we have

$$A(l/n) = \left[\frac{l}{n} \sum_{i=1}^{\lfloor l/n \rfloor} |x_{il/n} - x_{(i-1)l/n}| \right] \quad (7)$$

representing the absolute deviations. Using the definition of the fractal dimension (Gneiting and Schlather, 2004; Gneiting et al., 2010), the Hall–Wood estimator is given by

$$\widehat{D}_{HW} = 2 - \frac{\sum_{l=1}^L s_l - \bar{s} \log(A(l/n))}{\sum_{l=1}^L (s_l - \bar{s})^2} \quad (8)$$

where $L \geq 2$, $s_l = \log(l/n)$, and $\bar{s} = \frac{1}{L} \sum_{l=1}^L s_l$. To minimize potential bias Hall and Wood (1993) propose using $L = 2$ so that we obtain the estimate of the fractal dimension \widehat{D}_{HW} as

$$\widehat{D}_{HW} = 2 - \frac{\log A(2/n) - \log A(1/n)}{g^2} \quad (9)$$

3.2.2. Genton estimator

Gneiting and Schlather (2004) and Gneiting et al. (2010) propose a moment estimator method based on the robust variogram of Genton (1998). The variogram is defined as

$$V_2(l/n) = \frac{1}{2(n-l)} \sum_{i=1}^n (x_{i/n} - x_{(i-l)/n})^2, \quad (10)$$

and the Genton estimator is obtained as

$$\widehat{D}_G = 2 - \frac{\sum_{l=1}^L (s_l - \bar{s}) \log(V_2(l/n))}{2 \sum_{l=1}^L (s_l - \bar{s})^2} \quad (11)$$

where again $L \geq 2$, $s_l = \log(l/n)$ and $\bar{s} = \frac{1}{L} \sum_{l=1}^L s_l$. Davies and Hall (1999), and Zhu and Stein (2002) again suggest to use $L = 2$ to reduce the potential bias so that the estimate \widehat{D}_G reads

$$\widehat{D}_G = 2 - \frac{\log V_2(2/n) - \log V_2(1/n)}{2 \log 2} \quad (12)$$

3.3. Approximate entropy

Entropy can be considered a measure of complexity of the considered system. The systems with high entropy can be characterized by no information flows and are thus random up to uncertainty and conversely, the series with low entropy can be seen as deterministic (Pincus and Kalman, 2004). The efficient market can be then seen as the one with maximum entropy; and the lower the entropy, the less efficient the market is. For purposes of the Efficiency Index, we need an entropy measure which is bounded. Therefore, we utilize the approximate entropy introduced by Pincus (1991).

Let m be a positive integer and let r be a positive real number. For a time series $\{u_1, u_2, \dots, u_T\}$, with length T , let us form a sequence of vectors $X_1, X_2, \dots, X_{T-m+1}$ in R^m where $X_i = (u_i, u_{i+1}, \dots, u_{i+m-1})$. Using the Takens metrics of distance

$$d[X_i, X_j] = \max(|u(i+k-1) - u(j+k-1)|), \quad (13)$$

Table 1
Analyzed commodities.

Full name	Short name	Type
CBOT Corn C1	Corn	Grains
CBOT Oats O1	Oats	Grains
CBOT Rough Rice RR1	Rough rice	Grains
CBOT Soybean Meal SM1	Soybean meal	Grains
CBOT Soybean Oil BO1	Soybean oil	Grains
CBOT Soybeans S1	Soybeans	Grains
CBOT Wheat W1	Wheat	Grains
CME Feeder Cattle FC1	Feeder cattle	Other agriculturals
CME Lean Hogs LN1	Lean hogs	Other agriculturals
CME Live Cattle LC1	Live cattle	Other agriculturals
CME Lumber LB1	Lumber	Other agriculturals
COMEX Copper HG1	Copper	Metals
COMEX Gold GC1	Gold	Metals
COMEX Silver SI1	Silver	Metals
ICE Brent Crude Oil B1	Crude oil (Brent)	Energy
ICE Cocoa CC1	Cocoa	Softs
ICE Coffee KC1	Coffee	Softs
ICE Cotton No. 2 CT1	Cotton	Softs
ICE Orange Juice	Orange juice	Softs
ICE Sugar No. 11 SB1	Sugar	Softs
NYMEX Crude Oil CL1	Crude oil (WTI)	Energy
NYMEX Heating Oil HO1	Heating oil	Energy
NYMEX Natural Gas NG1	Natural Gas	Energy
NYMEX Palladium PA1	Palladium	Metals
NYMEX Platinum PL1	Platinum	Metals

and defining a characteristic function $\chi_i^m(r)$ as a number of instances in which $d[X_i, X_j] \leq r/(T-m+1)$ for each $1 \leq i \leq N-m+1$, we define

$$\Phi^m(r) = \frac{1}{T-m+1} \sum_{i=1}^{T-m+1} \log[\chi_i^m(r)] \quad (14)$$

which is further used in

$$ER_m = \lim_{r \rightarrow 0} \lim_{T \rightarrow \infty} [\Phi^m(r) - \Phi^{m+1}(r)]. \quad (15)$$

Table 2
Results.

Commodity	AE	\widehat{D}_{HW}	\widehat{D}_G	H_{LW}	H_{GP}	EI
Cocoa	0.9728	1.4665	1.4605	0.3542	0.3367	0.1594
Coffee	0.9680	1.4948	1.4606	0.4575	0.4665	0.0469
Copper	0.8264	1.5613	1.4974	0.6205	0.6992	0.1843
Corn	0.9015	1.4592	1.4299	0.5241	0.4858	0.0744
Cotton	0.9564	1.4702	1.4564	0.5057	0.4735	0.0439
Crude oil (Brent)	0.8919	1.5307	1.5084	0.5620	0.5986	0.0988
Crude oil (WTI)	0.9427	1.5243	1.4987	0.5466	0.4499	0.0309
Feeder cattle	0.3857	1.3498	1.3166	0.5751	0.3882	0.3500
Gold	0.5759	1.5161	1.4707	0.4278	0.4067	0.2277
Heating oil	0.9568	1.4943	1.4916	0.5081	0.4592	0.0280
Lean hogs	0.7081	1.3894	1.3584	0.3795	0.4256	0.2161
Live cattle	0.4527	1.4206	1.3773	0.4433	0.4306	0.2985
Lumber	1.0040	1.4301	1.4428	0.4278	0.3603	0.1236
Natural gas	1.1140	1.5246	1.4781	0.5210	0.5204	0.0607
Oats	0.9365	1.3926	1.3696	0.4105	0.2364	0.2152
Orange juice	0.8770	1.4266	1.3899	0.4126	0.3399	0.1659
Palladium	1.0230	1.4266	1.4210	0.5625	0.5970	0.1109
Platinum	0.7443	1.4686	1.4845	0.5535	0.5465	0.1393
Rough rice	0.8525	1.4278	1.4181	0.4512	0.4635	0.1149
Silver	0.8515	1.5161	1.4914	0.4685	0.4448	0.0861
Soybean meal	0.8861	1.4448	1.4328	0.4878	0.4548	0.0884
Soybean oil	0.7286	1.4735	1.4364	0.5330	0.5307	0.1465
Soybeans	0.7649	1.4900	1.4745	0.5266	0.5173	0.1209
Sugar	0.9759	1.4786	1.4818	0.5543	0.5505	0.0573
Wheat	0.9133	1.5129	1.4829	0.4626	0.5117	0.0453

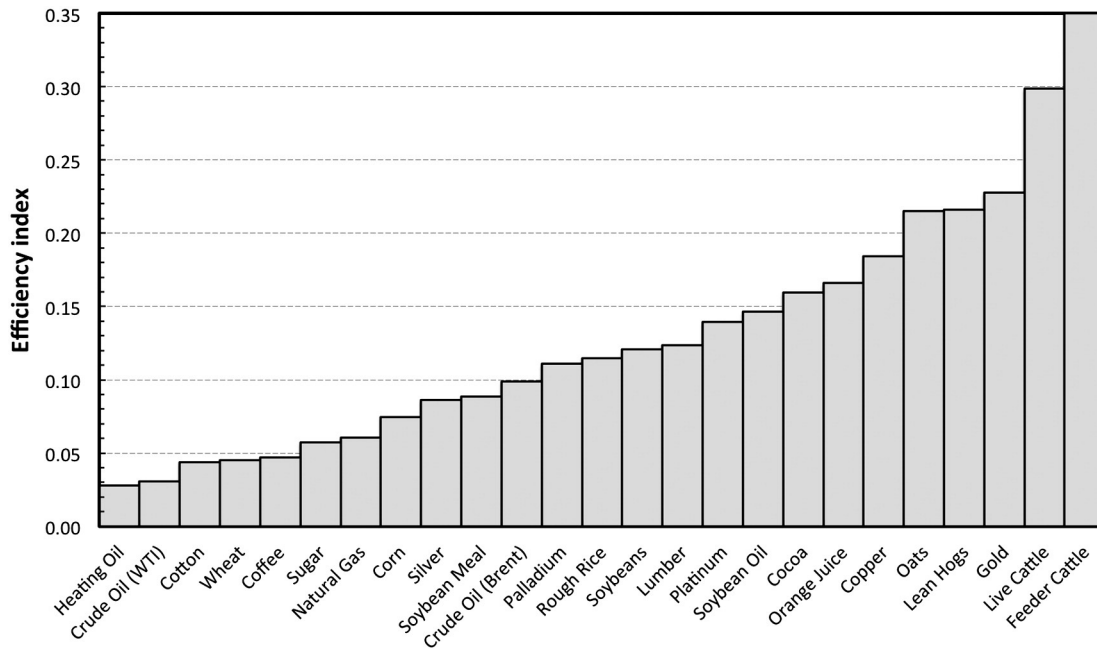


Fig. 1. Efficiency Index of commodity futures. Commodities are sorted from the most efficient (left) to the least efficient (right). Some clustering based on a commodity type, which is shown in Fig. 2 is already evident.

The approximate entropy (ApEn) is then defined as

$$ApEn = \lim_{m \rightarrow \infty} ER_m. \quad (16)$$

Since r can be seen as a discriminating factor for the distance measured by the Takens metrics and m is the number of elements whose closeness is measured, the approximate entropy measures a degree to which different segments of the series follow similar patterns. For a random process consisting of independent random variables with identical uniform distributions, the approximate entropy converges to $-\log(r/\sqrt{3})$ for all m (Pincus, 1991). For a completely deterministic process, the entropy goes to 0. Therefore, we can rescale the approximate entropy so that $0 \leq ApEn \leq 1$, where 0 characterizes a completely deterministic process and 1 a completely uncertain process characteristic for the efficient market². In turn, it can be utilized in the Efficiency Index, a definition of which follows.

3.4. Capital market efficiency measure

Kristoufek and Vosvrda (2013) introduce the Efficiency Index (EI), defined as

$$EI = \sqrt{\sum_{i=1}^n \left(\frac{\hat{M}_i - M_i^*}{R_i} \right)^2}, \quad (17)$$

where M_i is the i th measure of efficiency, \hat{M}_i is an estimate of the i th measure, M_i^* is an expected value of the i th measure for the efficient market and R_i is a range of the i th measure. In words, EI is simply a distance from the efficient market situation. Here, we base the index on three measures of market efficiency—Hurst exponent H with an expected value of 0.5 for the efficient market ($M_H^* = 0.5$), fractal dimension D with an expected value of 1.5 ($M_D^* = 1.5$), and the approximate entropy

² Note that the utilized estimator is a biased estimator of the true approximate entropy as well documented by Pincus (1995). Nevertheless, we opt to include it in the Efficiency Index as it controls for a different type of inefficiency than the fractal dimension and long-term memory do and it is the only entropy measure which is well bounded and thus scalable to a closed interval.

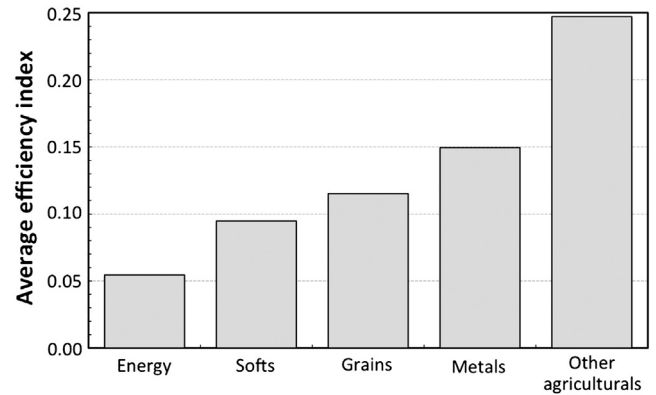


Fig. 2. Average Efficiency Index for groups of commodities. Groups are sorted from the most (left) to the least (right) efficient ones. Energy commodities are the most efficient ones whereas the other agricultural commodities are the least efficient ones. Note that the latter group is formed mainly by the livestock commodities.

with an expected value of 1 ($M_{AE}^* = 1$). The Hurst exponent is taken as an average of the GPH and the local Whittle estimates. In the same way, the fractal dimension is set as an average of the Hall–Wood and Genton estimates. For the approximate entropy, we utilize the estimate described in the corresponding section. The approximate entropy needs to be rescaled as it ranges between 0 and 1 with the efficient market of $ApEn = 1$. We thus have $R_{AE} = 2$ and $R_D = R_H = 1$.

4. Data description and results

We analyze daily prices of front futures, i.e., futures with the earliest delivery, of 25 commodities in the period between January 1, 2000 and July 22, 2013³. The dataset contains four energy types (Brent crude oil, WTI crude oil, heating oil, and natural gas), five metals (copper, gold, silver, palladium, and platinum), seven grains (corn, oats, rough rice, soybean meal, soybean oil, soybeans and wheat), five soft commodities (cocoa, coffee, cotton, orange juice, and sugar) and four other agricultural

³ The time series were obtained from <http://www.quandl.com> server on July 23, 2013.

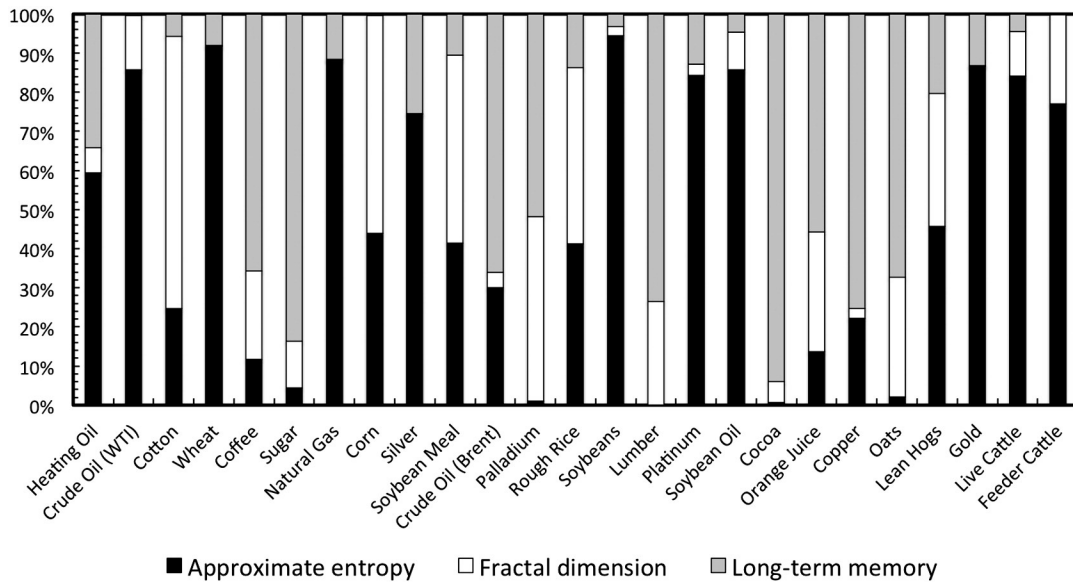


Fig. 3. Contribution to inefficiency I. Commodities are sorted according to their efficiency with respect to Fig. 1. For about half of the futures, the approximate entropy is the dominant inefficiency source. For the others, the long-term memory part is dominant. Fractal dimension usually only forms a smaller part of the inefficiency.

commodities (feeder cattle, lean hogs, live cattle, and lumber) futures from the Chicago Board of Trade (CBOT), Chicago Mercantile Exchange (CME), InterContinentalExchange (ICE), New York Mercantile Exchange (NYMEX), and its division Commodity Exchange (COMEX), which are summarized in Table 1. We analyze logarithmic prices $S_{i,t} = \log P_{i,t}$, where $P_{i,t}$ is the price of futures i at time t , for the fractal dimension and logarithmic returns $r_{i,t} = S_{i,t} - S_{i,t-1}$ for the long-term memory and approximate entropy. The returns of all the analyzed futures are stationary according to ADF (Dickey and Fuller, 1979) and KPSS (Kwiatkowski et al., 1992) tests (we do not report the p -values here).

The estimated values of Hurst exponents, fractal dimensions and approximate entropies are summarized in Table 2. We observe that a fractal dimension below 1.5, which indicates local persistence, is characteristic for the majority of commodities. These series are thus locally trending. This is most evident for feeder cattle, lean hogs, and live cattle, i.e., mostly livestock futures. On the contrary, the energy commodities – namely both crude oils and natural gas – are close to a fractal dimension of 1.5 and, as such, they do not show any signs of local inefficiencies. For the long-term memory part, most of the futures are below 0.5 indicating anti-persistence which translates into a mean-reversion of prices, something that is not commonly observed for stocks, stock indices and exchange rates which are characterized by a unit-root behavior. The strongest anti-persistence is seen for cocoa, oats, and orange juice. Nonetheless, there is a portion of commodities which shows signs of persistence. These are copper, palladium, platinum, and sugar. Cotton and natural gas get the closest to the value of the efficient market. For the approximate entropy, several values are close to 1 for the efficient market⁴—lumber, sugar, and heating oil. The most complex, and thus the least efficient, series include feeder cattle and live cattle.

Putting these results together, we arrive at the Efficiency Index and efficiency ranking values which are graphically represented in Fig. 1. The most efficient of the commodities turns out to be heating oil, closely followed by WTI crude oil. Cotton, wheat, and coffee come after these with a similar level of efficiency. The ranking is then supplemented by other commodities, the efficiency of which increases quite steadily across the ranking. Feeder cattle is the least efficient commodity in this set, quite closely followed by live cattle. The livestock futures thus seem to be rather inefficient compared to the others. Connected to this finding, we also show an average Efficiency Index value for

commodities according to their type. In Fig. 2, we can see that the energy futures are the most efficient followed by soft commodities, grains, and metals. By far the least efficient group consists of the other agricultural commodities, i.e., feeder cattle, lean hogs, live cattle, and lumber. This is well in hand with the observations about very inefficient livestock futures.

In Figs. 3 and 4, we decompose the Efficiency Index into its parts. In Fig. 3, the actual futures ranked according to the Efficiency Index are represented, and in Fig. 4, these are sorted according to their type to better see possible patterns and regularities. We observe that, for about half of the futures, the approximate entropy is the dominant inefficiency source. Interestingly, it is the most important part for both the most and the least efficient commodities. For the others, the long-term memory part is dominant. Fractal dimension usually only forms a smaller part of the inefficiency; it provides a larger contribution only for wheat. When we look at the whole groups of commodities, we observe that for energy, grains and other agricultural commodities, the approximate entropy forms an important or even a dominant part for most of them. For grains, fractal dimension creates a significant part for three of the group. For soft commodities, the long-term memory is the most important of the inefficiency contributors. And for metals, the evidence is mixed.

Fig. 5 then illustrates a relationship between fractal dimension and Hurst exponent. For self-similar processes, it holds that $D = 2 - H$. In economic terms, self-similar processes are characteristic by translating the local properties into the global ones. Therefore, for a locally persistent process with $D < 1.5$, this translates into the global persistence with $H > 0.5$, and vice versa. However, we do not observe such a relationship for the analyzed commodities. Actually, the dependence is reversed so that the fractal dimension increases with increase of the Hurst exponent. This is in contrast with the results for stock indices (Kristoufek and Vosvrda, 2013). Nonetheless, such result can be well explained by characteristics of commodity futures—locally (or in the short term), the changes in futures prices are partially predictable, but globally, the prices return to their fundamental values.

5. Conclusions

We have analyzed the market efficiency of 25 commodity futures across various groups—metals, energies, soft commodities, grains, and

⁴ Several values even reach a value above 1 due to the finiteness of the sample.

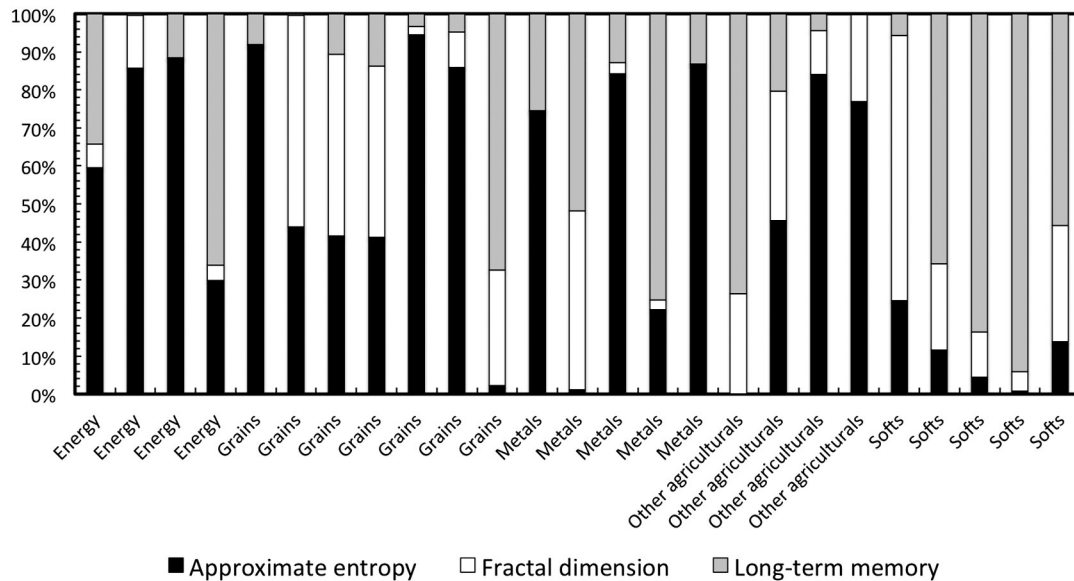


Fig. 4. Contribution to inefficiency II. Commodities are sorted according to their group. For energy, grains and other agricultural commodities, the approximate entropy forms an important or even a dominant part for most of them. For grains, fractal dimension creates a significant part for three of the group. For soft commodities, the long-term memory is the most important of the inefficiency contributors. And for metals, the evidence is mixed.

other agricultural commodities. To do so, we have utilized the recently proposed Efficiency Index to find that the most efficient of all the analyzed commodities is heating oil, closely followed by WTI crude oil, cotton, wheat and coffee. On the other (least efficient) end of the ranking scale, we have detected live cattle and feeder cattle. The efficiency also seems to be characteristic for specific groups of commodities—energy commodities have been found the most efficient, followed by soft commodities, grains, and metals, whereas the other agricultural commodities (formed mainly of livestock) form the least efficient group. Apart from that, we have also discussed the contributions of the long-term memory, fractal dimension and approximate entropy to the total inefficiency. We have discovered that the contribution is type-dependent as well, even though the regularities are not strongly pronounced. Last but not least, we have come across the nonstandard relationship between the fractal dimension and the Hurst exponent. For the analyzed dataset, the relationship between these two is positive, meaning that local persistence (trending) is connected to global anti-persistence. We attribute this to specifics of commodity futures which might be predictable over a short term and locally, but over a long term they return to their fundamental price, which differs from the results found for stock indices (Kristoufek and Vosvrda, 2013).

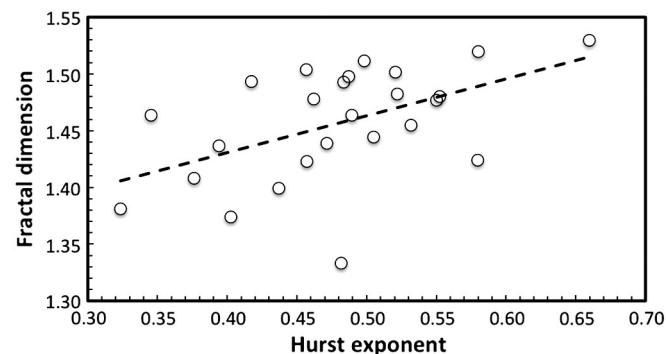


Fig. 5. Relationship between Hurst exponent and fractal dimension. For self-similar processes, we expect $D = 2 - H$, i.e., a negative slope. The dashed line represents the least squares fit uncovering a positive relationship between D and H .

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