# The arbitrage inconsistencies of implied volatility extraction in connection to calendar bandwidth

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**Abstract**

Options are often priced by Black and Scholes model by using artificial (and unobserved) volatility implied by option market prices. Since many options do not have their traded counterparts with the same maturity and moneyness, it is often needed to interpolate the volatility values. The general procedure of implied volatility extraction from market prices and subsequent smoothing can, however, lead to inconsistent values or even arbitrage opportunities. In this paper, a potential arbitrage area is studied in connection with the calendar bandwidth construction.

**Key words**

Option pricing, implied volatility, arbitrage opportunity, calendar bandwidth, bandwidth size.

**JEL Classification:** C46, E37, G17, G24

## 1. Introduction

Despite the well-known deficiencies of the famous Black and Scholes model (1973), it is still used for pricing of options, especially those with low liquidity or even at the OTC markets. Notwithstanding, the Black and Scholes model is used indirectly – we take the market price of liquid option, invert the Black and Scholes formula, obtain a volatility (ie. implied volatility), put it into the formula by setting the parameters of illiquid option and get the price.

Thus, if we are able to extract the implied volatility curve (options with the same maturity, but different strikes) or surface (options with different strikes, as well as maturities) from market prices of liquid options, we can use them to price the illiquid options or even options exotic, which we can trade only OTC. Such options, however, mostly differs in moneyness (ration of forward price and the strike) and maturity in comparison with traded options.

Hence, the implied volatilities must be extrapolated. This procedure should be performed carefully, since there exist several conditions on the price of call and put options, that must be fulfilled. Otherwise an arbitrage opportunity can arise, ie. riskless profit higher than common riskless return.

In this paper, we extend our previous analysis, see e.g. Kopa and Tichý (2014), Kopa et al. (2015) and Tichý et al. (2015) by studying the impact of the bandwidth size on the estimated state price densities and their arbitrage area of implied volatility surface. Basically, we proceed in line with Benko et al. (2007), since we apply relatively classic approach of local polynomial smoothing techniques and study the bandwidth selection process in more details on relatively recent data of DAX option prices (December 2011).

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We proceed as follows. In the following section we briefly review the problem of option pricing (Tichý, 2011) and we also provide some basic facts about the implied volatility modelling. We procced with introduction of the data (DAX options data) and finally, an implementation of implied volatility extraction and state price density calculation for the calendar bandwidth is provided.

## 2. Option valuation and the concept of implied volatility

Options are nonlinear types of financial derivatives, which give the holder the right (but not the obligation) to buy the underlying asset in the future (at maturity time) at prespecified exercise price. Simultaneously, the writer of the option has to deliver the underlying asset if the holder asks. Options can be classified due to a whole range of criterions, such as counterparty position (short and long), maturity time and possibility of exercising (*European* v. *American*), complexity of the payoff function (*plain vanilla* v. *exotic*), etc. The basic features are *the underlying asset* ($S$), which should be specified as precisely as possible (it is important mainly for commodities), *the exercise price* ($K$), and *the maturity time* ($T$).

The pricing in line of Black and Scholes model (1973) is crucially dependent on the payoff function type. For example, assuming the payoff function of plain vanilla call and the normal distribution we get the valuation formula as follows (*BS model for vanilla call*):

$$f_{call}^{vanilla}(\tau, S, K, r, \sigma) = S\, F_N\,(d_+) - e^{-r\tau}\, K\, F_N\,(d_-).$$

Here, $S$ is the underlying asset price at the valuation time ($t$) and it is supposed to follow log-normal distribution, $\tau$ is the time to maturity, $r$ is riskless rate valid over $\tau$, $\sigma$ is the volatility expected over the same period, both per annum, $F_N(x)$ is distribution function for standard normal distribution and

$$d_\pm = \frac{\ln\left(\frac{S}{K}\right) + \left(r \pm \sigma^2/2\right)\tau}{\sigma\sqrt{\tau}}.$$

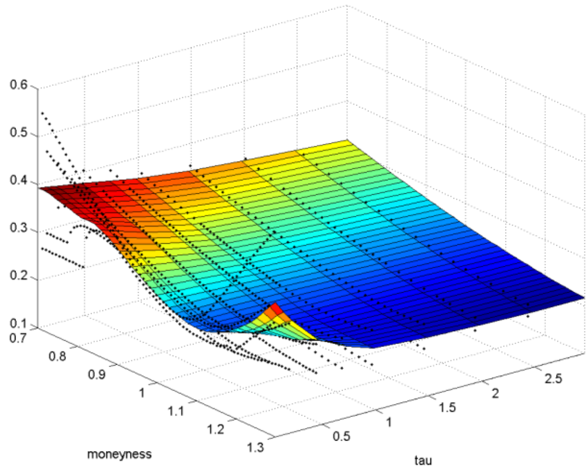
If the price of some options is available from the market, we can invert the formula to obtain the implied volatility, ie. the number that makes the formula equal to market price. Among the important works, whose authors analyzed the impact of implied volatility on option price, belongs, besides others Dupire (1994), who formulated a process followed by the underlying asset price in dependency on the moneyness and maturity, and Rubinstein (1994), who formulated a discrete time model, the implied binomial tree.

## 3. Implementation

In this section, similarly to our previous research, we present the analysis using a dataset of all the options on DAX listed on 30 December 2011 with a large set of maturities and strikes. We have selected a given day due to quite high number of observed prices of options. Options on DAX, a German stock market index, were selected in order to overcome the difficulties with the estimation of the dividend yield (compare with options on common stocks). As concerns the riskless interest rate, we have applied the procedure suggested in Tichý et al. (2014), which is based on extracting of this quantity from observed prices.

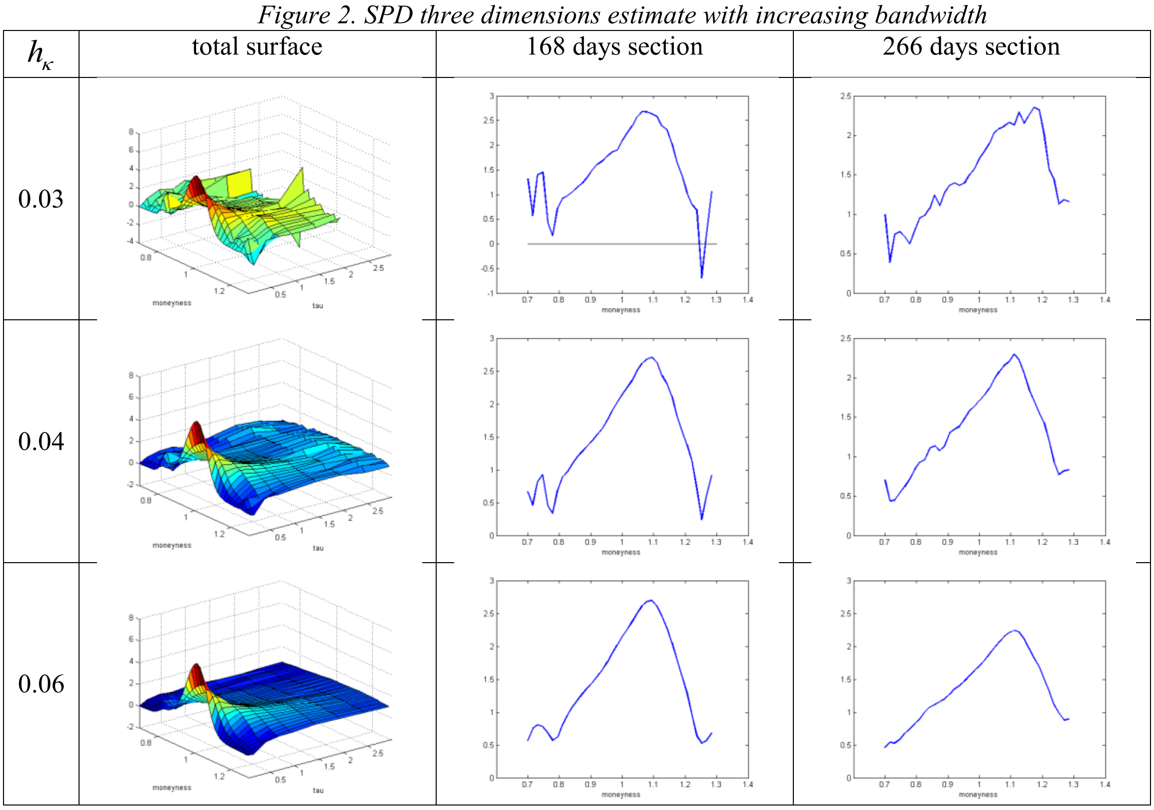
First, following Benko et al. (2007) we compute the unconstrained estimation of the IV surface. In Figure 1 we show the estimation with Epanechnikov kernel function, for moneyness bandwidth $h_\kappa = 0.04$ and for maturity (calendar) bandwidth $h_\tau = 1$. The historical data are represented by black dots and easily show missing liquidity of far ITM/OTM options. The complex picture is well described by the estimated surface. The IV smirk (or a similar structure) is clear for very small maturities and becomes less noticeable as the maturity increases.

*Figure 1. 3D estimation of implied volatility for DAX options*




Our subsequent analysis differs against the approach adopted by Benko et al. (2007), since we use a non-fixed calendar bandwidth. These results are documented in Figure 2 assuming $h_\tau = 0.4$ for $\tau \le 0.5$, $h_\tau = 0.6$ for $0.5 < \tau < 1$ and $h_\tau = 1$ for $\tau \ge 1$. Such approach should respect the differences in the market volume for various maturities.

*Figure 2. SPD three dimensions estimate with increasing bandwidth*




The computations are done with Epanechnikov kernel function and with three representative moneyness bandwidths $h_\kappa = 0.03, 0.04, 0.06$. If we compare these results with those obtained for fixed calendar bandwidth (see, eg. Tichý et al., 2015) we can notice that the arbitrage measure seems to be smaller for any choice of moneyness bandwidth. We document this behaviour in Figure 2. Most probably, it is due to the fact that with respect to the risk attitude of the traders and investors, most of the arbitrage generally occurs for long maturities so a smaller bandwidth does not include those maturities in the estimations. On the other hand, with a large fix calendar bandwidth the estimation for the shorter maturities are in some way disturbed by the turbulence that persist for the long maturities.

Finally we compute also the Calendar arbitrage measure as the volume of negative first derivative of total variance, see Benko et al. (2007). In this case we do not observe any violation of Calendar arbitrage free condition. We demonstrate it in Figure 4 where we show that indeed the total variance is strictly increasing in the calendar (maturity) direction for all moneyness values.

*Figure 6. Arbitrage Measure for fixed calendar bandwidth (blue line, top) versus Arbitrage Measure for increasing calendar bandwidth (red line, bottom)*

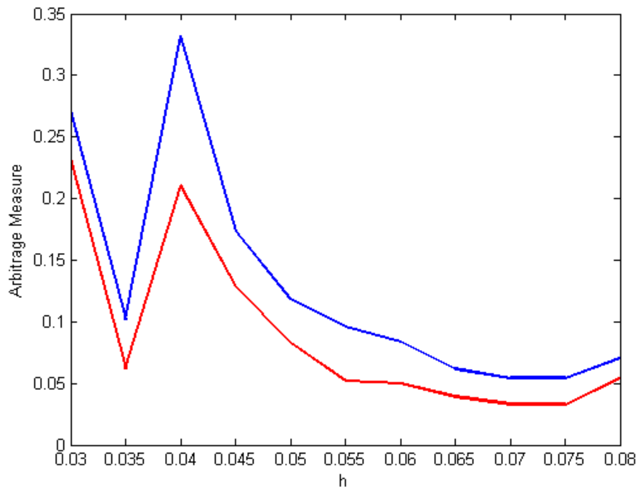
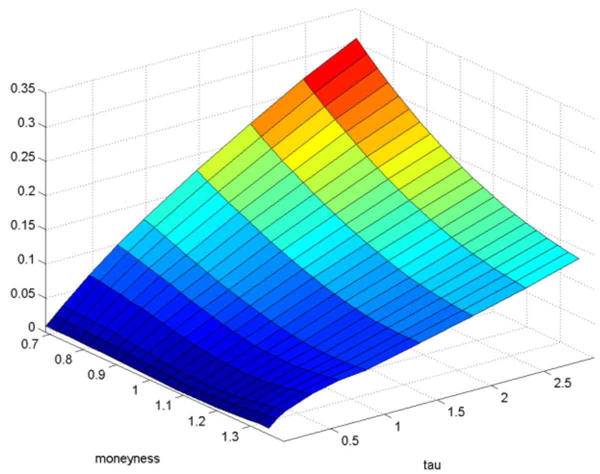



*Figure 7. Total variance for maturity and moneyness*




## 4. Conclusion

In order to price illiquid or exotic options that are not traded at option markets, there is no way but to use the implied volatilities. In this paper we have extended our previous research on option implied volatilities and analysed the impact of the calendar bandwidth selection and we have documented clear impact of its choice with respect to different behaviour of prices and implied volatilities for various maturities and potentially also the risk attitude of the investors and traders.

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