In defence of Higher-Level Plural Logic: drawing conclusions from natural language

Berta Grimau[1] (ID)

Abstract

Plural Logic is an extension of First-Order Logic which has, as well as singular terms and quantifiers, their plural counterparts. Analogously, Higher-Level Plural Logic is an extension of Plural Logic which has, as well as plural terms and quantifiers, higher-level plural ones. Roughly speaking, higher-level plurals stand to plurals like plurals stand to singulars; they are pluralised plurals. Allegedly, Higher-Level Plural Logic enjoys the expressive power of a simple type theory while committing us to nothing more than the austere ontology of First-Order Logic. Were this true, Higher-Level Plural Logic would be a useful tool, with various applications in philosophy and linguistics. However, while the notions of plural reference and quantification enjoy widespread acceptance today, their higher-level counterparts have been received with a lot of scepticism. In this paper, I argue for the legitimacy of Higher-Level Plural Logic by providing evidence to the effect that natural languages contain higher-level plural expressions and showing that it is likely that they do so in an indispensable manner. Since the arguments I put forward are of the same sort advocates of Plural Logic have employed to defend their position, I conclude that the commonly held view that Plural Logic is legitimate, but not so its higher-level plural extensions is untenable.

✉ Berta Grimau
  grimau@utia.cas.cz

1   Institute of Information Theory and Automation, Czech Academy of Sciences, Pod Vodárenskou věží 4, 182 08 Prague 8, Czech Republic

1 Introduction

After being a subject of debate for more than three decades, Plural Logic (PL) has now been widely adopted as part of the philosophical toolbox. PL is an extension of First-Order Logic which, as well as the usual singular terms, predicates and quantifiers, has their plural analogues. Plural terms refer plurally to the objects first-order singular terms refer to singularly (i.e. they refer to many objects at once), plural quantifiers bind plural variables and plural predicates take plural terms as arguments.

PL is a First-Order Logic in the sense that it only has quantification into term position. However, it goes beyond First-Order Logic in the sense that it has the expressive power of Monadic Second-Order Logic (M2OL)—with PL we can express the idea that an object has a certain property by saying that the object in question is among some objects (intuitively, the objects which fall within the extension of the relevant property). This was observed by Boolos (1984) and Boolos (1985) and it is indeed one of the key reasons why PL has been seen as an attractive device: it gives us the expressive power of M2OL while avoiding some of its most objectionable putative features, such as its high ontological costs (i.e. to properties, sets or classes).

In fact, many of the applications of PL turn on this feature. For example, some philosophers have employed PL to support their nominalist views.[1] Moreover, because of its ontological innocence, PL would have a better claim than M2OL to count as genuinely logical, thus having applications in the logicist field.[2] However, PL only has these advantages if it is taken at face value, that is, if it is given a formal interpretation according to which plural terms denote more than one object at once. The view that this interpretation of PL is legitimate is what I call 'Pluralism'. In this paper, I take Pluralism as a starting point, so I do not argue for it. Moreover, I assume familiarity with the formal presentation of PL.[3]

Some authors believe that PL can be extended by adding higher-level plural terms, quantifiers and predicates to it, thereby obtaining Higher-Level Plural Logic (HLPL).[4] Crucially, second-level plurals do not stand in predicate position with respect to plurals, but rather stand to them as the latter stand to singulars. Roughly speaking, they are the result of pluralising the plural idiom. Analogously for third-level plurals. And so on. Importantly, HLPL is said to be more expressive than PL and to inherit some of its key characteristics—most crucially, its ontological innocence. It is because of this alleged gain in expressive power free of any additional ontological costs that higher-level plural reference has attracted some attention in the recent literature on plurals. To illustrate with an example, just as plural language allows us to go from *cat* to *cats*,

---

[1] For example, Black (1971) uses it to support his eliminativist view about sets, Boolos (1984), Boolos (1985) and Uzquiano (2003) use it in the framework of an eliminativist view of classes and Hossack (2000) uses it in an eliminative project about complex objects. Moreover, a notable eliminativist project that employs plurals is that of Lewis (1991). He endorses a theory of sets where set membership is reduced to singleton-set membership plus mereological fusion of singletons and where polyadic relations are reduced to plural reference to ordered pairs. Thus his aim is not to eliminate sets altogether, but rather to replace the (in his view) mysterious notion of set membership with a stock of allegedly more transparent notions.

[2] See, for instance, Boccuni (2011, 2013).

[3] For a survey of the philosophical debate on plurals and a description of a simple logic of plurals, see Linnebo (2017).

[4] For instance, Rayo (2006) and Oliver and Smiley (2016).

a higher-level plural language takes us even further by allowing us to talk of what in an extension of English would be captured by terms like *catses*, *catseses*, etc. Informally speaking, just as *the cats* denotes a plurality of cats, *the catses* would denote a plurality of pluralities of cats.[5]

But why may one be interested in a higher-level extension of PL in the first place? There are at least three areas where higher-level plurals have promising applications. To begin with, HLPL could be of use in nominalist endeavours in which the use of PL were insufficient. For instance, a logic containing all finite levels of plural variables and quantifiers would be equi-interpretable with a simple type theory, thereby showing that we can enjoy the expressive power of the latter while being committed solely to the values of its singular first-order variables. Linnebo and Rayo (2012) consider this application in their comparison of type theory and ZF set theory.[6] Alternatively, one could employ HLPL to replace talk of sets. The idea would be to replace all singular reference to sets of individuals with plural reference, singular reference to sets of sets with second-level plural reference, and so on. Oliver and Smiley (2016, Chap. 15) point towards an application along these lines.

Another area where HLPL promises to deliver results is in the interpretation of polyadicity. As mentioned above, PL can be used to interpret only the monadic fragment of Second-Order Logic. This is enough for some applications, but not for others. A field where we could benefit from a plural treatment of the polyadic fragment of Second-Order Logic is neo-logicism. PL could be used to develop a strengthened version of neo-Fregeanism by replacing second-order expressions with plural ones. However, if one proceeds via Hume's Principle, interpreting the monadic fragment of Second-Order Logic will not suffice. There are various ways to go about replacing its polyadic fragment, but most of them have limitations which make them unsuitable for neo-Fregean purposes.[7] However, higher-level plural reference lends itself to a simple strategy to tackle polyadicity. Roughly speaking, one can encode pairs by employing one of the usual set-theoretic definitions of ordered pair in its higher-level plural version, thus modelling polyadic relations as higher-level pluralities.[8]

Finally, one may want to employ HLPL as a tool for natural language analysis. For example, Cotnoir (2013) proposes a semantics which makes use of higher-level plurals for generalised identity claims—including one–many, one–one and many–

---

[5] I will occasionally speak of 'pluralities' in this paper. However, note that I will use the term 'plurality' as a convenient shorthand for 'objects'. This is common practice in this debate and is necessary due to the expressive limitations of English.

[6] Even though they ultimately argue against the claim that the higher-level plural reading carries a substantial advantage.

[7] For instance, one could assume the existence of ordered pairs, but this would be at odds with the purpose of the neo-Fregean programme by not only presupposing the existence of a plethora of entities without providing a further story as to why this is legitimate, but also by presupposing the existence of entities which are suspiciously set-theoretic in nature. Another option would be to use a pairing function, but this strategy would be problematic too, since the existence of a pairing function makes specific demands on the size of one's background ontology, as argued by Shapiro and Weir (2000).

[8] The idea would be to express a definition analogous to that of, for instance, Hausdorff as follows:
$\langle a, b \rangle := ((a, 1), (b, 2))$, where $((a, 1), (b, 2))$ is meant to represent a second-level plurality which consists of the plurality of a and 1, on the one hand, and the plurality of b and 2, on the other. See Grimau (2018, Chap. 7) for a proposal along these lines.

many identity statements—and Nicolas (2008) argues that higher-level plurals should be employed in the semantics of mass nouns.

Nevertheless, perhaps unsurprisingly, HLPL has received a lot of scepticism.[9] The very intelligibility of the notion of higher-level plural reference has been questioned, partly on the basis that natural language does not seem to contain any such device. Moreover, it has been argued, even if there were natural language higher-level plural expressions, they would be eliminable in favour of singular or plural ones.

In this paper I argue that, on closer inspection, such widespread scepticism is unjustified by defending the primitive intelligibility of natural language higher-level plurals. If successful, this should serve to legitimise to some extent HLPL.

My arguments should be especially convincing to the pluralist, since the guiding principles behind them are those most often appealed to in order to argue for Pluralism. In other words, as we will see, the main reasons to adhere to Pluralism provide support in turn for the intelligibility of higher-level plural reference and the legitimacy of HLPL (understood at face value). Consequently, if correct, my arguments will put the pluralist in the following dialectic position: either she gives up on her scepticism towards HLPL or she abandons her support for PL. Otherwise, her views are likely to suffer from an instability, since some of the methodological premises behind them will be prone to be in tension with one another.

The plan is as follows. In Sect. 2, I present Pluralism and I identify the main principles underlying the position (this will allow us, later on, to reach the conclusion just sketched). In Sect. 3, I describe the language of HLPL, its proof-theoretic profile and its semantics. In Sect. 4, I present the main objections to HLPL. Finally, in Sect. 5, I turn to a defence of HLPL by offering replies to those objections. I conclude by taking stock and assessing what has been achieved.

2 E pluribus plures

Plural terms are referring expressions which, intuitively, denote various objects at once. For instance, *the member states of the European Union*, *Serena and Venus*, *Harry's parents* and *the Outer Hebrides* are all natural language plurals.[10]

The semantics of formal plural languages can be classified according as to whether they adhere to this intuition and assign plural terms more than one referent under a single interpretation (pluralist semantics) or else they depart from it and take plural terms to denote a single object (singularist semantics). Another way of looking at this distinction is by noting that the pluralist believes that plural expressions can figure

---

[9] We find this scepticism explicitly expressed, for instance, in Simons (1982), Lewis (1991), Uzquiano (2004), Rumfitt (2005), McKay (2006) and Ben-Yami (2013).

[10] In the literature on PL it is often assumed that singular reference is a limiting case of plural reference, since plural terms may happen to denote a single thing. On this basis, formal plural terms are often interpreted as possibly denoting a single thing. In this article, for the sake of simplicity, I will not adhere to this assumption at the formal level. For us, a plural term will denote more than one thing and thus will serve to regiment the natural language phenomenon of strict plural reference. Moreover, Oliver and Smiley (2016) further liberalise the notion by allowing formal plural terms to fail to denote. Since allowing for vacuous reference would also significantly complicate the formalism and none of my arguments hinge on this choice, I discard this possibility as well.

in a formal language and be interpreted at face value (that is, she thinks that seman-
tic homophonicity with respect to plural reference and quantification is legitimate),
whereas the singularist accepts the use of plurals in a formalism only insofar as they
are analysed away, as denoting a single object, in the semantics. Whether one goes one
way or the other turns on whether one accepts natural language plurals themselves as
legitimate devices with which to carry out semantic theorising.

Singularist accounts differ on which kind of collectivising entity —i.e. an entity that
has members (or other constituents)—a plural term denotes: set-theoretic singularism
assigns plural terms sets of objects from the first-order domain,[11] property singularism
assigns them properties of the objects from the first-order domain[12] and mereological
singularism, mereological sums thereof.[13] Since the acceptance of Pluralism is the
starting point of this paper, I leave these aside.[14]

Pluralists such as Boolos (1984), Lewis (1991), Yi (2005), McKay (2006) and
Oliver and Smiley (2016) take the ubiquity of plural expressions in natural languages
as lending support to their incorporation in a formalism. This is an important idea for
them, often confronted with the criticism that plurals are not clear enough to be taken
at face value and thus to be employed in semantics.[15] For them, plurals, used widely
and competently by speakers, are as clear as any category of expressions can be and
thus as capable of figuring in the language in which a semantic theory is formulated.

However, more needs to be said in order to justify Pluralism, since the mere possi-
bility of using plurals in semantic theorising is not sufficient to legitimise our doing
so. To see this, note that the use of uninterpreted plural idioms in the metalanguage of
our theories carries a certain cost with it: one needs to be willing to accept that there is
a new (primitive) way to refer to objects of the domain, the plural way. This is a con-
ceptual cost and in order to show that it is worth it, we need to engage in cost-benefit
analysis by showing that alternative paths would have higher costs. At this point a
common strategy has been to argue for the impossibility to eliminate plurals from
natural language. Here 'eliminability' can be understood in two senses: in the sense
of paraphrasing away and in the sense of semantically analysing away. Pluralists have
tried to show that both strategies ultimately run into trouble. To this end, a variety of
considerations have been used. Without intending to give an exhaustive list, let me
mention three of them.

In the first place, pluralists have been moved by the idea that our semantic theoris-
ing should respect and account as much as possible for the speaker's (mostly implicit)
knowledge of her own linguistic practices. In other words, attributing speakers mis-
takes regarding their linguistic understanding should be a strategy of last resort when
analysing natural language. I will call this general methodological principle FACE
VALUE.

---

[11] This has been endorsed in Landman (1989a, b) and Schwarzschild (1996).

[12] Properties can be seen as collectivising entities in the sense that they gather together all of their instances. One such proposal can be found in Florio (2010).

[13] This line can be found in Link (1983, 1998).

[14] A survey can be found in Florio (2014).

[15] This criticism is raised, for instance, in Shapiro (1991, pp. 225–226) and Shapiro (1993, pp. 471–472).

Although this methodological assumption is controversial, it can be motivated by a less controversial observation: our semantic theories are decided on the basis of facts having to do with the speaker's use of language. For example, if our semantics of vague predicates says that vague predicates denote fuzzy sets of objects, there should be facts concerning how speakers employ vague predicates that decide this. Since the speakers' linguistic knowledge is very likely to be based on their own linguistic behaviour, the fact that formal semantics operates as just described justifies, to some extent, FACE VALUE.

Admittedly, the principle of FACE VALUE is rather vague as it stands. In order to clarify it, let me point to a specific point where it has played a relevant role in the debate around plurals: it has been used to argue against singularist analyses that carry with them implausible ontological commitments (in the sense that speakers would find them counter-intuitive). For example, commitment to abstract entities, such as (possibly) sets, arising from a discourse not explicitly about this kind of entities.[16]

Nevertheless, this principle has important limitations. Let me illustrate this with an example. Hewitt (2012) argues that we should accept a new primitive form of reference: reference to several things in an order (and with repetition). For instance, this kind of reference could be at play in sentences like *The first five Fibonacci numbers are* 0, 1, 1, 2, *and* 3*, respectively*. Despite the fact that the resulting analysis fulfills FACE VALUE, Florio and Nicolas (2015) have argued that the linguistic phenomena at play in this kind of statements is better explained otherwise, based on the fact that the alternative analysis they propose has a broader scope and accounts for the phenomenon in full generality. This example demonstrates that speakers' intuitions often spring from too local a perspective; when considered in the context of a wider linguistic fragment, they are seen to lead us astray. This is one of the reasons why speakers' intuitions may be unreliable when it comes to semantic analysis and thus why FACE VALUE must be adopted only as a secondary principle, helping us decide between accounts which otherwise are equally adequate.

Note that arguments based on ontological considerations need not proceed via FACE VALUE, but can also be based on certain forms of metaphysical NOMINALISM. For example, if one has independent reasons to believe that proper classes do not exist, this can be used in an argument for Pluralism in the context of regimenting the axioms of set theory, in particular the axioms of Replacement and Separation, which involve quantification over non-set-sized collections of sets.[17]

Finally, another important principle at play in this debate is that of ABSOLUTE GENERALITY. This is the idea that sometimes our language demands absolutely general interpretations. In other words, whenever a speaker intends to speak of a general domain, such as the domain of absolutely everything (as is common in, for instance, philosophical theorising), she indeed manages to do so and thus our semantics must be capable of accounting for this fact.[18] In this sense, a set-theoretic analysis is easily seen to be defective, since it imposes cardinality restrictions on the domain. By contrast,

---

[16] See, for example, Boolos (1984, p. 65).

[17] This argument can be found in Boolos (1984, pp. 65–66) and Uzquiano (2003, p. 68).

[18] This line of defence can be found, for example, in Lewis (1991, p. 68), but note that this argument relies on the further assumption that the all-encompassing domain is not indefinitely extensible (see Rayo and Uzquiano (2006, pp. 4–6) for an overview of this topic).

we can account for absolutely general readings by means of a plural regimentation, since it makes available the domain consisting of the plurality of everything. Note that one can see the principle of ABSOLUTE GENERALITY as a specific consequence of FACE VALUE, since it is only tenable if one takes seriously the fact that speakers sometimes appear to speak of non-set-sized domains of objects. Thus endorsement of FACE VALUE and ABSOLUTE GENERALITY often go hand in hand. However, it is worth distinguishing them, since the latter has played a very salient role in the current debate.

3 Beyond plurals

Some pluralists believe that PL can be extended by adding higher-level plural terms and quantifiers to it and interpreting them at face value. As mentioned above, second-level plurals stand to plurals (in this context, 'first-level plurals') as the latter stand to singulars (analogously for higher levels). If legitimate, they would be the result of iterating the step from the singular to the plural.[19] I refer to the view that HLPL is legitimately interpreted at face value as 'Higher-Levellism'.

Interestingly, pluralists are divided when it comes to the legitimacy of the notion of higher-level plural reference. Some, like Rayo (2006) and Oliver and Smiley (2016) have embraced it and developed formal systems which incorporate it. By contrast, others, for instance Lewis (1991), McKay (2006) and Uzquiano (2004), have argued against its availability.

Moreover, not all authors who admit higher-level plurals see them as good news. Some authors have appealed to the fact that PL is extendible to HLPL to argue against the alleged advantages of PL. Most notably, Linnebo (2003) and Linnebo and Rayo (2012). The view that higher-level plural quantification is bad news for Pluralism turns on the observation that once an infinite hierarchy of higher-level plurals is accepted, the very same objections raised against type theory with respect to its capacity to capture absolutely general discourse (and hence with respect to its logicality) can be turned against HLPL. Briefly, since we do not have quantification across levels, we seem unable to formalise some of our metatheoretic commitments.[20] Suppose we were to lift the type restriction on our quantifiers. Then we would obtain a theory with a single type of terms ranging over all higher-level pluralities, which could be easily accused of being a mere notational variant of set theory, thus arousing suspicions of a lack of logicality. Either way, the move from plurals to higher-level plurals will be seen as an unwelcome development by some. I will not take a stance with respect to this debate here. The view I argue for is that, regardless of whether one judges it positively or negatively, the acceptance of PL inevitably leads to the acceptance of its higher-level extensions. For better or for worse, it is an all or nothing matter.

---

[19] The idea that higher-level plurals are the result of iterating semantic pluralisation is expressed by Hazen (1997, p. 247) and Linnebo and Nicolas (2008, p. 186), among others.

[20] See Linnebo (2006) for a development of this view with respect to type theory.

3.1 HLPL

As far as I am aware, only three higher-level plural formal languages have been developed to date: Rayo (2006)'s, Florio (2014)'s and Oliver and Smiley (2016)'s. The main difference between them is that the first two have infinitely many types of terms, whereas the third one makes use of a single type of term that can denote individuals, first-level pluralities, second-level pluralities and so on. The language I present in what follows is closer to Rayo's and Florio's proposal in this respect.[21]

Language $\mathcal{L}_{HLPL}$ is an extension of the language of PL (which is in turn an extension of the language of First-Order Logic with identity). $\mathcal{L}_{HLPL}$ has, as well as singular and plural variables and constants, those of higher levels.[22] We take terms to be superscripted letters: $x^k, y^k \ldots c^k, d^k \ldots$ (for $0 \leq k$). Under this notation, singular terms are superscripted with 0 and plurals terms, with 1. Moreover, we have non-logical n-adic (for $1 \leq n$) predicates $P_n^{k_1,...,k_n}$, $Q_n^{k_1,...,k_n} \ldots$ that take terms of level k_i (for $0 \leq k_i$) in each of their positions, a logical predicate of singular identity = and a logical predicate of higher-level plural membership $\prec$ (to be read 'is/are among' depending on the case) which takes terms of consecutive levels in its positions. We also have a universal quantifier $\forall$ binding variables of any level and the logical connectives $\rightarrow$ and $\neg$.[23]

$\mathcal{L}_{HLPL}$ has the following formulas (I use expressions of the form t^i and u^i, where $0 \leq i$, as placeholders for terms of level i):

   $t^0 = u^0$ is a formula.
   $t^k \prec t^{k+1}$ is a formula.
   $P_n^{k_1,...,k_n}(t^{k_1}, \ldots, t^{k_n})$ is a formula.
   If ϕ and ψ are formulas, so are $\neg\phi$ and $(\phi \rightarrow \psi)$.
   If ϕ is a formula, so is $\forall x^k \phi$.
Finally, higher-level plural identity is defined as:
   $t^k = u^k := \forall x^{k-1}(x^{k-1} \prec t^k \leftrightarrow x^{k-1} \prec u^k)$

Conjunction, disjunction, the biconditional and the existential quantifier are defined in the usual way.

Proof theory We start with the standard deductive system for First-Order Logic with identity (minus the axioms for the quantifiers, which are redundant in the present

---

[21] Even though I do not have a strong preference for either option, I take the typed route since it captures better the intuition that the ascent from the singular to the plural and from the basic plural to higher levels is a matter of expressive rather than ontological expansion.

[22] I limit the hierarchy to finite levels for simplicity.

[23] Note that we do not have any mechanism of complex term formation, such as a rule to form definite descriptions out of non-logical predicates or to form lists out of terms. Although these would allow HLPL to regiment natural language more accurately (and thus might be necessary for applications of HLPL in natural language or ordinary reasoning analysis), I leave them out for the sake of simplicity, since nothing that I discuss in this article turns on the availability or lack thereof of these complex terms. (see Oliver and Smiley (2016) for a language which allows for definite description formation).

framework). To these, we add the axioms and rules for the higher-level plural quantifiers. All of the following hold for $0 \leq k$.

(HLP-UI) *Higher-Level Plural Universal Instantiation*
$\forall x^k \phi(x^k) \rightarrow \phi(t^k)$, where t^k is free for x^k in $\phi(x^k)$.

(HLP-UG) *Higher-Level Plural Universal Generalisation*
From $\phi \rightarrow \psi(x^k)$ infer $\phi \rightarrow \forall x^k \psi(x^k)$, provided x^k does not occur free in ϕ or in any premise of the deduction.

The rest of the axioms are analogous to those of a basic Plural Logic:

(HLP-C) *Higher-Level Plural Comprehension*
$\exists x^k \exists y^k (\phi(x^k) \wedge \phi(y^k) \wedge \neg(x^k = y^k)) \rightarrow \exists x^{k+1} \forall x^k (x^k \prec x^{k+1} \leftrightarrow \phi(x^k))$, where ϕ is a formula of $\mathscr{L}_{\text{HLPL}}$ that contains x^k and possibly other variables free but contains no occurrence of x^{k+1}.

(HLP-NV) *Higher-Level Plural Non-Vacuity*
$\forall x^k \exists x^{k-1} \exists y^{k-1} (x^{k-1} \prec x^k \wedge y^{k-1} \prec x^k \wedge \neg(x^{k-1} = y^{k-1}))$

(HLP-Ext) *Higher-Level Plural Extensionality*
$\forall x^k \forall y^k (\forall x^{k-1}(x^{k-1} \prec x^k \leftrightarrow x^{k-1} \prec y^k) \rightarrow (\phi(x^k) \leftrightarrow \phi(y^k)))$, where y^k is free for x^k in $\phi(x^k)$.

A higher-levellist semantics

A model M of $\mathscr{L}_{\text{HLPL}}$ is a triple $\langle d^1, P^1, I \rangle$, where d^1 are some individuals (i.e. the domain of interpretation), P^1 are some properties and relations (understood as higher-order entities[24] over which neither singular, nor basic plural, nor higher-level plural quantifiers can range) and I is a function which assigns appropriate individuals from d^1, pluralities of any level of individuals from d^1 or higher-order entities from P^1 to the non-logical vocabulary.[25] A variable assignment s is a function from the variables to the individuals in d^1 or pluralities of any level formed from them. Note that I am using d^1 and P^1 as plural terms, the former is first-order and the latter, second-order. For some of the applications presented in Sect. 1, we do not need $\mathscr{L}_{\text{HLPL}}$ to include non-logical predicates and thus our models need not have higher-order entities. I include them here for generality.

As expected, singular terms denote single individuals, plural terms denote pluralities thereof, second-level plural terms denote pluralities of pluralities thereof, and so on.[26] I will employ the metalinguistic term 'plurality of level n' for lack of a better idiom

---

[24] I will remain neutral as to the specific nature of these entities, since this issue is irrelevant in the present context.

[25] The ordered triples acting as models are not ordinary ones: they require that some of their members be pluralities. However, this is unproblematic in light of the fact that we can code these unorthodox ordered pairs using techniques already available to us, as shown in Linnebo and Rayo (2012, pp. 304–306).

[26] In this article I rule out the possibility of referring to mixed higher-level pluralities, that is, pluralities consisting of, say, a single individual on the one hand and a first-level plurality on the other (intuitively the denotation of e.g. *Rafa Nadal and the Williams sisters*). Given that none of the arguments put forward in the present article depend on this and keeping in mind that they could be accounted for by complicating the formalism (i.e. adopting a logic analogous to cumulative type theory; see Linnebo and Rayo (2012) for details), I leave mixed higher-level pluralities aside.

in English, but, just as in the basic plural case, it should not be taken at face value. In particular, 'plurality of level n' is not a singular term picking out a special kind of object, but it stands in lieu of an nth-level plural term, which is unavailable in English (e.g. *objectses*).

Interpretation of terms and predicates

For each variable x^k, $s(x^k)$ is an individual from d^1 or a plurality of level k of individuals from d^1.

For each constant c^k, $I(c^k)$ is an individual from d^1 or a plurality of level k of individuals from d^1.

For each predicate P_n, $I(P_n)$ is a property or relation of adicity n from P^1.

Satisfaction

Let us write $M \models_s \phi$ for 'M and s satisfy ϕ'. We write $I_s(\phi)$ for the value that the interpretation function or the variable assignment assign to ϕ, depending on the case—i.e. if ϕ is a variable, $I_s(\phi)=s(\phi)$; if it is a non-logical term or predicate, $I_s(\phi)=I(\phi)$. We define a relation of satisfaction with respect to a model and an assignment implicitly as follows:

$M \models_s t^0 = u^0$ iff $I_s(t^0) = I_s(u^0)$.

$M \models_s t^k \prec t^{k+1}$ iff $I_s(t^k)$ is/are among $I_s(t^{k+1})$.

$M \models_s P_n^{k_1,...,k_n}(t^{k_1}, ..., t^{k_n})$ iff $I_s(P_n^{k_1,...,k_n})$ holds of $(I_s(t^{k_1}), ..., I_s(t^{k_n}))$.

$M \models_s \forall x^k \phi$ iff for every assignment s' which is an x^k-variant of s[27] $M \models_{s'} \phi$.

$M \models_s \neg\phi$ iff it is not the case that $M \models_s \phi$.

$M \models_s \phi \rightarrow \psi$ iff $M \models_s \psi$ or it is not the case that $M \models_s \phi$.

Truth, logical truth and logical consequence are defined as usual.

The main metatheoretical results concerning PL transfer to HLPL. Neither PL nor HLPL are compact.[28] As a consequence, they are not strongly axiomatisable. That they are also not weakly axiomatisable follows from the fact that PL can serve to finitely axiomatise a version of Peano Arithmetic which is categorical. Thus, by Gödel's theorem, it follows that the logical truths of the underlying logic are not effectively enumerable.

4 Against Higher-Levellism

The fact that higher-level plural reference has been looked at with apprehension should not come as a surprise; as we have seen, were it legitimate, it could help settle some important debates. Quite a lot appears to be at stake, so many are wary of the optimism of some. Lewis, in the context of discussing Boolos' plural understanding of second-order quantification, put it bluntly:

[Boolos' view] hints that the third, fourth, and higher orders cannot be far behind but what might plurally plural quantification be? (Infinite blocks of plural

---

[27] An x^k-variant of s is an assignment that only differs from s at most in what it assigns to x^k.

[28] Yi (2006, pp. 262–264) and Oliver and Smiley (2016, Chap. 13).

quantifiers?—That will be only a skimpy third order, and no start at all on the fourth). (Lewis 1991, pp. 70–71)[29]

The idea behind the sceptics' bafflement towards higher-level plurals seems to be that, since a plurality is always a plurality of things, pluralities would need to be things themselves in order to be collectable into other pluralities. But this would be at odds with their alleged ontological innocence.[30]

Opponents of Higher-Levellism typically raise the complaint that higher-level plural reference is inherently unintelligible to us: it is impossible for us to make sense of and thus properly utilise it. The allegation of unintelligibility is often supported by the claim that there are no higher-level plurals in natural languages.

The view that higher-level plurals are not to be found in natural languages is very widespread. It is explicitly expressed, for example, in Lewis (1991), Uzquiano (2004) and Ben-Yami (2013). In fact, even some advocates of HLPL claim that there are no higher-level plurals in natural language. For instance, this view is held by Hazen (1997), Linnebo (2003) and Rayo (2006).[31]

What can be said in response to this criticism? First of all, the higher-levellist may push the view that the legitimacy of a formal language should not hinge on the contingent fact that natural languages happen (or not) to display a certain linguistic phenomenon, but only on the fact that we can indeed achieve a good grasp thereof. She may add that one shall get a clear enough understanding of HLPL by simply mastering its use.[32] Little can be said against this stance. By the lights of the higher-levellist, who claims to have a clear grasp of the notions involved, the face value semantic approach is unproblematic. The discussion at this point mirrors the debate around higher-order logic and its semantics. Given that it is not clear that natural languages contain higher-order quantifiers, the legitimacy of the homophonic move in that context has been questioned too. Prima facie, the discussion differs from the one to be had concerning the use of homophonic semantics for some of the logical connectives, for first-order quantification or for plural quantification. In all these cases, there appears to be a widespread agreement among speakers as to the meaning of the metalinguistic expressions, which can be found and appear to be indispensable in English and many other natural languages.

Be that as it may, I believe the higher-levellist can do better than this: she can show that higher-level plural expressions are indeed found in natural languages. I will argue for this claim in Sect. 5.1.

However, as explained above with respect to Pluralism, showing that a certain class of expressions is found in natural language does not suffice to establish the claim that

---

[29] Authors have used different terminology to refer to higher-level plurals. These are some of the terms that have been used in the literature: 'perplurals' (Hazen 1997; McKay 2006), 'pluplurals' (Rosen and Dorr 2002; Simons 2016), 'plurally plurals' (Hossack 2000; McKay 2006; Rumfitt 2005; Uzquiano 2004), 'hyperplurals' (Cotnoir 2013), 'superplurals' (Oliver and Smiley 2016; Rayo 2006).

[30] See Ben-Yami (2013, pp. 82–83), Rumfitt (2005, p. 13) and Simons (1982, pp. 192–193) for some sceptical reactions. Note that Simons seems to have recently retracted this view, as can be seen in Simons (2016).

[31] Rayo only claims that English, in particular, does not contain such devices.

[32] See Rayo (2006, p. 227) for an endorsement of this approach. This position would be akin to the view endorsed in Williamson (2003, p. 459) with respect to higher-order logic.

those expressions should be interpreted at face value, since doing so carries a certain cost. As in the plural case, this is a conceptual cost: one needs to accept the legitimacy of a new primitive form of reference, higher-level plural reference. Just as before, in order to establish this stronger claim with respect to higher-level plurals, one needs to show that alternative treatments are unsuitable and a way to achieve this is to show that natural language higher-level plurals are indispensable. Recall that I distinguished two different senses in which one may try to eliminate plurals: paraphrasing them away or semantically analysing them away. Accordingly, the view that natural language higher-level plurals are dispensable may be endorsed in either of these two senses.

Firstly, it might be argued that we can always paraphrase away apparent higher-level plurals in favour of expressions involving only plural or singular reference. This has been endorsed by Black (1971) and McKay (2006). Secondly, it might be argued that even if those expressions are not eliminable in this sense, they can be interpreted in ways that show that they are not really higher-level plurals. That is, that we can correctly semantically analyse them in a non-homophonic fashion, thereby eliminating them from the metalanguage used in their semantics. In the philosophical literature, Ben-Yami (2013) has made a proposal along these lines in terms of what he calls 'articulated reference'. In linguistics, perhaps unsurprisingly, almost all authors are of this opinion. In particular, linguists hold one of two views. One current of thought has it that apparent higher-level plural terms are plural terms under a cover-reading. More precisely, sentences involving apparent higher-level plural subjects are analysed as having basic plural subjects and predicates which are evaluated with respect to a cover of the (plural) reference of their subject.[33] This view has been advocated in Gillon (1987, 1992) and Schwarzschild (1996).[34] The other line of thought stems from the sum-based account of plurals of Link (1984) and has been further developed and modified by Landman (1989a, b). According to this proposal, higher-level plurals denote sums of groups, where groups are not sums, but a special sort of atomic individuals related to sums via two functions: firstly, an injective and multivalued function, group formation, which takes us from sums to groups and, secondly, a non-injective function of membership-specification which brings us back to sums.[35]

Thus, in order for the higher-levellist to strengthen her position, she needs to argue against both of these approaches. She needs to show that what appear to be natural language higher-level plurals are neither paraphrasable away nor semantically analysable away, i.e. that the cost to pay for those paraphrases/analyses is too high. I turn to this line of defence in Sect. 5.2.

---

[33] In its set-theoretic version, a cover of a set a (such as the denotation of a plural term, under set-theoretic singularism) is a set of non-empty subsets of a, where every member of a belongs to some such subset. In its sum-based version, a cover of a sum s (such as the denotation of a plural term, under sum-based singularism) is a set of sums whose fusion is s.

[34] Although neither Gillon nor Schwarzschild endorse their semantics in the context of HLPL, Linnebo and Nicolas (2008) note its potential relevance for the present topic.

[35] In this article I will mostly be using the expression 'group' in its non-technical informal sense. Whenever I use it in the sense just described, I will make it explicit.

5 For Higher-Levellism

5.1 Cross-linguistic evidence

In what follows, I argue that not only is there evidence of the presence of higher-level plurals in languages other than English, but they appear to be present in English as well.

5.1.1 In English

The English expressions which have a prima facie claim to being higher-level plural come, at least, in four different forms.

Lists of plurals   Firstly, we find lists of plural NPs. E.g. *the Gordons, the Stewarts and the MacLeods* and *the students and their lecturers*. And, given that we are assuming that lists of referring expressions themselves are referring expressions, nested lists of singulars must be considered among these. E.g. *Serena and Venus, and Rafa and Roger*.[36] A prima facie reason to think that lists of plurals are higher-level plurals is that in their nested form they are the result of iterating the syntactic operation of list formation (one of the operations by which one may form a plural term), which suggests that an analogous semantic operation is being iterated as well.[37]

Admittedly, this is based merely on an intuition about the possible semantic effect of a syntactic operation and, thus, of itself does not carry much weight. But, as we will see in detail later, this intuition receives support from the existence of sentences whose predicate seems to apply not to the result of joining the pluralities denoted by each component of the list, but instead to those pluralities considered separately.

(1) *The pigs and the cows are equally loud.*

For instance, there is a reading of (1) according to which the pigs, taken together, are as loud as the cows are, taken together. This kind of sentences suggest that list formation sometimes conveys the existence of various groupings as the relevant subjects of a predication, rather than always acting as a flat operation such as set-theoretic union or mereological fusion.

Plural definite descriptions built from collective predicates   Secondly, we have plural definite descriptions which are formed with a collective predicate—a predicate which, while holding of some objects, may fail to hold of each of them separately (e.g. *are siblings*, *despise each other* and *are numerous*). This kind of restriction facilitates a reading of the resulting phrases according to which they do not simply denote some objects fulfilling a certain condition, but all the pluralities of objects which do so separately.[38] For example, *the numbers whose product is larger than 25, the*

---

[36] I use commas (possibly followed by a conjunction) in order to indicate where one nested list ends and another one begins. This rather artificial notation is more naturally captured by intonation in spoken language.

[37] According to this understanding of lists, nested lists of plurals would be third-level plurals. E.g. *the cat lovers and their cats, and the dog lovers and their dogs*.

[38] See Oliver and Smiley (2016, Chap. 8), who call them 'plurally exhaustive descriptions'.

specialists competing for the same jobs and *the authors of multi-volume classics in logic* fall within this category.

Under this interpretation *the numbers whose product is larger than 25* would denote all the numbers which jointly satisfy the property of having a product larger than 25; thus it would denote numbers organised in different pluralities.

Plurals accompanied by certain appositive phrases Thirdly, we find plural terms followed by an appositive phrase explicitly describing a certain internal structure of the objects being denoted. For example, *the kids, organised in two groups* or *the shoes, divided by colour*. The appositive marks a certain division of the plurality which, seemingly, allows us to speak of them higher-level plurally.

Definite descriptions with a pseudo-singular head noun Finally, we find terms in the form of definite descriptions which have a pseudo-singular[39] as their head noun. E.g. *the ugliest pairs of shoes in this shop* and *my favorite teams*.

A pseudo-singular term is a syntactically singular, but semantically plural term. Thus, just as *catses* would be a pluralisation of *cats* by an iterated application of the plural suffix, *pairs* would be a pluralisation of *pair* by the same operation. The fact that some semantically plural terms happen to be syntactically singular allows for the process of plural suffix application to be applied to plurals.

Moreover, pseudo-singulars would allow us to easily form plurals of the third level and beyond. Of course, we cannot iterate syntactic pluralisation any further without abandoning the confines of good English, but we can use list formation: *these couples and those couples*. These terms are similar to the nested lists of plurals considered above, but enjoy more naturalness due to the pseudo-singularisation involved.

Despite having been embraced by some advocates of PL, pseudo-singularity is a controversial notion. Prima facie evidence for its legitimacy is given by the fact that pseudo-singulars behave in a grammatically hybrid way in many languages. For example, in English, Spanish, Catalan, French and Latvian, they admit plural override: while they are usually accompanied with singular determiners, they admit a plural verb and/or plural anaphoric reference back to them.[40] However, what exactly does plural override establish is open to discussion. For example, the existence of a plural anaphora linked to a syntactically singular term does not necessarily show that the term refers plurally. More generally, anaphora does sometimes refer to novel entities. For instance, in *John shouted and this annoyed Mary*, the demonstrative pronoun appears to refer to a fact which we have no independent reason to believe has been referred to by any preceding expression.[41] Moreover, some linguists have argued that group nouns like *pair* or *team* refer singularly, rather than plurally. For example, de Vries (2017) argues that these expressions denote single individuals, because certain distributivity patterns

---

[39] The notion of pseudo-singularity comes from Oliver and Smiley's work on plurals. See Oliver and Smiley (2016, pp. 305–306).

[40] Some languages only display the anaphoric form of plural override. For instance, in French and Latvian that is the only way in which we find the phenomenon.

[41] I am grateful to an anonymous referee for this example and, more generally, for pointing out this issue.

fail for them, demonstrating that the compositional semantics of the sentence does not have access to the members of the relevant group.[42]

Nevertheless, recall that the conclusion I aim to establish in this article is the conditional claim that if one accepts PL, one should also be willing to accept HLPL. Since pseudo-singularity has played an important role in the defence of PL, I will assume, for the sake of the argument, that it is indeed legitimate.

5.1.2 In other languages

Whether these English expressions are higher-level plurals is nonetheless doubtful. First, their surface form does not display what would be the paradigmatic morphology of higher-level plurals in English: an iterated application of the plural suffix. Second, some (e.g. lists) are thought to be easily eliminable from discourse and thus not really substantial parts thereof. Finally, some crucially involve the controversial notion of pseudo-singularity. Below I argue that these terms are not eliminable, thus providing additional support to the claim that they should be taken as higher-level plurals. But before doing that, I show that although English may not contain clear-cut cases of higher-level plural reference, other languages do.

Even though it is almost unanimously assumed that paradigmatic higher-level plurals are not found in any natural language, little seems to have been done to investigate whether the empirical data supports this hypothesis. As far as I know, in the literature on HLPL, Linnebo (2017) is the only one who has addressed this issue. He has pointed out that Icelandic contains a group of expressions that appear to be higher-level plural.[43] But, in fact, the empirical literature on grammatical number suggests that Icelandic is not the only language that contains higher-level plurals. Moreover, some of the phenomena present in these languages are even more interesting for us, since they offer evidence of more general forms of higher-level plural reference. Unsurprisingly, none of the authors reporting these forms refers to them as 'higher-level plurals'. This is because formal semanticists interpret basic plurals singularly to begin with. However, they do interpret them as being, semantically, the plural of the plural[44] and both the reports of native speakers and the morphology of the expressions support the hypothesis that they are indeed natural language higher-level plurals.

Icelandic   As pointed out by Linnebo, the higher-level plurals found in Icelandic are numeral phrases, that is, NPs whose determiner is numerical. More specifically, Icelandic's first four cardinal numbers have both a singular and a plural form and they can be combined with nouns in both of these forms:

---

[42] For arguments in favour of the atomicity of the reference of group nouns like *pair* or *team* see also Barker (1992), Link (1984), Landman (1989a), Schwarzschild (1996) and Winter (2002).

[43] This was also noted, although not specifically in relation with HLPL, by Jespersen (1924, p. 189).

[44] For instance, Corbett (2000) talks of 'semantically composing plural on plural'.

Value	Singular	Plural
1	*einn*	*einir*
2	*tveir*	*tvennir*
3	*þrír*	*þrennir*
4	*fjórir*	*fernir*

When the plural form is combined with a common noun, we obtain expressions which reportedly pick out pluralities of the objects being referred to by the head noun—as many as the original singular number. Note that they do not pick out any kind of plurality, but a specific kind: pairs of objects. For example, whereas *einn skór* translates as 'one shoe', *einir skór* means 'one pair of shoes'. Analogously, *tveir skór* means 'two shoes' (not necessarily forming a pair) and *tvennir skór* means 'two pairs of shoes'. And so forth. The limitation to pairs is distinctive of the Icelandic case.

The plural numerical determiners allow us to talk about pairs of shoes as second-level pluralities rather than as first-level pluralities of individual objects. What is distinctive about these expressions, compared to the analogous English ones, is that they involve no mention of anything else other than shoes. That is, while in the English case in order to show that these phrases are higher-level plural, we need to further argue that nouns like *pair* are semantically plural, in the case of Icelandic we can skip this step.[45]

Finnish Like Icelandic, Finnish contains plural numeral phrases. However, unlike Icelandic, it contains plural forms of all cardinals, not only the first four.[46] These are a few examples:

Value	Singular	Plural
1	*yksi*	*yhdet*
2	*kaksi*	*kahdet*
3	*kolme*	*kolmet*
10	*kymmenen*	*kymmenet*
50	*viisikymmentä*	*viidetkymmenet*
100	*sata*	*sadat*
1000	*tuhat*	*tuhannet*

When the plural numerals of Finnish precede plural nouns, the resulting NPs usually mean something like '*n* pluralities of ...', where *n* corresponds to the original singular numeral determiner. The most uncontroversial cases involve things that come naturally in certain groupings, such as pairs—i.e. hands, socks, eyes. However, things which do not typically come in any specific groupings can also be denoted by these phrases. For example, *kahdet paperit* means 'two pluralities of papers'.

---

[45] Ben-Yami (2013, pp. 85–86) raises the objection that 'the translations of the Icelandic phrases in fact disagree with the use Linnebo would like to make of them'. This is because they do make use of an expression like *pair*. However, this objection appears to put the cart before the horse. English translations cannot play the role Ben-Yami intends them to play here, since they cannot help us identify a non-English expression as higher-level plural, given the expressive limitations of English. If this were a valid criterion of identification, the outcome of the investigation would be decided from the start.

[46] According to Hurford (2003), Estonian is 'to a large extent' similar to Finnish in this respect. I focus on Finnish for simplicity.

Just as it occurred with Icelandic, there is no way to denote pluralities of objects in the plural simpliciter, but a specific number thereof must always be specified. This is not the case with the next language I shall consider.

Interestingly for the higher-levellist, Finnish appears to have plural forms of other determiners as well:

English	Singular	Plural
'pair', 'couple'	*pari*	*parit*
'a few'	*muutama*	*muutamat*
'many'	*moni*	*monet*

Hurford (2003) argues that, despite appearances, only the first one is interpreted by native speakers as a higher-level plural. Be that as it may, the presence of a single higher-level plural determiner in a natural language is a step forward for the higher-levellist. The quantifier we form by attaching *parit* to a common noun is a pluralised plural quantifier—that is, a higher-level plural quantifier. So not only do we find higher-level plural reference, but also higher-level plural quantification in natural languages.

Khamtanga   Khamtanga, a Cushitic language, also contains a form of higher-level plural. An important difference with respect to Icelandic and Finnish is that the higher-level plural terms present in Khamtanga are not numeral phrases (thus specifying the number of groups of objects being denoted), but phrases with an inflection indicating reference to various things as divided into any number of pluralities.[47]

Khamtanga	*ieferā*	*iefír*	*iefírt*
English	'child'	'children'	'crowds of children'
Khamtanga	*lálā*	*lal*	*lálāle*
English	'bee'	'bees'	'swarms of bees'

Finally and for the sake of completeness, let me discuss the controversial case of the Celtic language Breton. This language has been reported as containing higher-level plurals by Jespersen (1924) and Corbett (2000). According to Corbett (2000), Breton has two different sorts of higher-level plurals. Firstly, terms formed from pluralising a dual, a term that denotes exactly two objects. In Breton, a dual noun is a noun prefixed with *daou* (masc.) or *diou* (fem.). This prefix is obligatory with the nouns which have it, which typically are nouns denoting parts of the body or clothing that come in pairs. When we pluralise the dual we seem to obtain a higher-level plural expression picking out various pairs of objects. For example:

Breton	*lagad*	*daoulagad*	*daoulagadoù*
English	'eye'	'(two) eyes'	'pairs of eyes'

Secondly, Breton has a more liberal form of higher-level plural, since it allows composition of plural suffixes:[48]

---

[47] While the following examples were reported in Appleyard (1987, p. 252), they were originally recorded a whole century before, in Reinisch (1884). Appleyard reports that even though in his study he found similar forms, they had evolved into mere alternative first-level plural forms. It would not be surprising if the distinction had been lost today.

[48] In this example, the first plural suffix (*e*) is irregular and the second one (*où*) is regular.

Breton	*bugel*	*bugale*	*bugaleoù*
English	'child'	'children'	'groups of children'

However, two qualifications are in order. Firstly, Corbett (2000, pp. 36–37) and Acquaviva (2008, p. 260) report that the composed form of the plural is not available for all nouns in Breton and that, even when available, its interpretation is far from being predictable. Secondly and most importantly, against the opinion of Jespersen and Corbett, Acquaviva (2008, pp. 234–265) argues (in my view, convincingly) that cases of double pluralisation in Breton are not really cases of double plural inflection, since the plural suffixes involved play two different roles: lexical/derivational and grammatical/inflectional.[49] It follows that Breton is akin to English, since it simply contains the plural of lexicalized terms such as *pair* or *group*.

Overall, even though it is a rare linguistic phenomenon, there is evidence of the presence of expressions in various languages which both display a paradigmatic higher-level plural morphology and are reported by native speakers to be understood as one would expect higher-level plurals to be understood. If this kind of evidence were not enough to settle this issue, it is not clear what could be. So, even though alternative analyses of these natural language expressions cannot be ruled out at this point, I conclude that, at the very least, it is not evidently true that there are no such things as natural language higher-level plurals and, in fact, the evidence suggests that the contrary is the case.

5.2 The indispensability of higher-level plurals

Let us now turn to the other line of defence available to the higher-levellist: arguing for indispensability.

Any account which eliminates natural language higher-level plurals (either by paraphrasing them away or by semantically analysing them away) would undermine HLPL as follows: the advocate of HLPL must employ informal, unanalysed, higher-level plurals in her semantics if HLPL is to be something genuinely new; but if those primitive higher-level plurals stand for something else (e.g. plural expressions denoting sets of individuals), then nothing has been really gained after all—the semantics of $\mathscr{L}_{\text{HLPL}}$ would be best regimented in a plural language, which would show that HLPL is simply PL (with its models augmented with, for example, a domain of sets).

Nevertheless, in the first place, it is important to note that even if the sceptic were right that higher-level plurals can be dispensed with in natural languages, this would not mean that they should be. Thus although, if they worked, these objections would weaken the position of the higher-levellist, they would not be conclusive. Moreover, I believe the higher-levellist can indeed argue that natural language higher-level plurals cannot be dispensed with, that any attempt to do so runs into trouble. In what follows I develop this line of argumentation.

Due to my inability to properly discuss the expressions belonging to the other languages considered in the previous subsection, I discuss dispensability only for English expressions. However, I am not assuming that an analysis of the expressions belonging to one language would necessarily transfer to those of another language.

---

[49] I wish to thank an anonymous referee for pointing this out to me.

The idea is simply that if we show that the English expressions are ineliminable, (i) this will suggest that similar arguments can be provided for the expressions of the other languages and (ii) this will strengthen the prima facie evidence that the English (and, as a consequence of (i), the non-English) expressions discussed above are indeed higher-level plurals.

I will mainly focus on lists of terms in my examples for ease of exposition, since their interpretations tend to be less context-dependent than those of the other kinds of terms we saw in Sect. 5.1.1.

5.2.1 Against paraphrasing away

The discussion as to whether higher-level plurals are paraphrasable away mirrors the analogous debate around plural reference. In that debate, the singularist proposes strategies to paraphrase plurals away and her opponent comes up with counter-examples. I take the role of the opponent here.

Two main strategies for syntactically eliminating higher-level plurals have been proposed in the literature. The first one is what I call FLATTENING and it consists in paraphrasing them away in favour of plural expressions which do not pick out sets or other collectivising objects, but rather the very same individuals the original expression appears to denote (in other words, they do not involve any new predicates).[50] The second strategy is what I call REIFICATION and it consists in paraphrasing higher-level plural terms away by appealing to collectivising entities.[51] In what follows I argue that whereas the paraphrasability objection would have some force if paraphrases of the former kind were always available, this is not the case. And when we need to recur to REIFICATION the objection loses its appeal, since the cost of this kind of paraphrases is too high.

FLATTENING allows one to paraphrase statements involving non-collective predicates. For instance, (2) can be paraphrased as the conjunctive statement (3) thereby eliminating any apparent higher-level plural reference:

(2) *The cat lovers and the dog lovers indulge their pets.*

(3) *The cat lovers indulge their pets and the dog lovers indulge their pets.*

Moreover, a similar move is available when it comes to collective predicates built from dyadic relations. For example, in (4), the collective monadic predicate *despise each other* can be replaced with the dyadic predicate *despise* by means of a conjunctive paraphrase:

(4) *The cat lovers and the dog lovers despise each other.*

(5) *The cat lovers despise the dog lovers and vice versa.*

However, as argued in what follows, things are usually not so simple and in many cases one needs to rely on REIFICATION. Consider the following sentence:

---

[50] See Black (1971, p. 633) for one such proposal.

[51] This has been proposed by McKay (2006, p. 138).

(6) *The students and the lecturers met in adjacent rooms.*

Under the reading intended here (i.e. the students and the lecturers met in two different rooms which were next to each other),[52] the predicate of (6) applies to the students and the lecturers taken together, rather than to each of these groups separately, hence we cannot simply distribute the predicate down to each component of the subject list. Thus, it seems that, unless we appeal to a paraphrase that takes *adjacent* as a defined notion, the only way to eliminate the list of plurals is by turning it into a list of singulars and using REIFICATION:

(7) *The group of students and the group of lecturers met in adjacent rooms.*

This paraphrase eliminates the list of plurals in favour of a list of singular group-denoting terms. However, from the pluralist point of view there are problems with this strategy. Firstly, if this strategy were adopted, the pluralist would need to explain why (8) should not be paraphrased as (9) as well:

(8) *The students met.*

(9) *The group of students met.*

In other words, REIFICATION motivates a similar move with respect to basic plurals. Paraphrasing them away along these lines would result in a more homogeneous analysis. But this would be in clear tension with the view assumed here, Pluralism, according to which natural language plurals should be understood as plurally referring expressions.

Moreover, the acceptance of (7) as a paraphrase of (6) leads to the acceptance of an entailment relation that is prima facie counter-intuitive. A sentence that is apparently about people entails a sentence that is about groups. In other words, by asserting something about people I appear to be forced into asserting the existence of some collectivising objects. This seems wrong. Even though this is far from being uncontroversial, the claim that paraphrasing a sentence as another one that explicitly invokes more ontology is unacceptable is a widely held assumption by pluralists and, as explained in Sect. 2, it can be seen as a consequence of FACE VALUE. Moreover, this move will be unwelcome by any pluralists who endorse some form of NOMINALISM which affects groups.

Let me give another example of the same sort:

(10) *The members of each party compete for the same seats.*

This sentence can be understood as saying that any group of people who are members of a certain party competes against any other such group for the same seats (in an assembly such as a parliament). Thus, under this reading, the predicate cannot be distributed down to each group of party members. Therefore, once again, it seems that REIFICATION is called for.

---

[52] To see that this reading is available imagine (6) as being uttered in a context in which some lecturers met to discuss the results of the final evaluation, while some students met to plan an end-of-the-year party, and the person who utters (6) is explaining why the lecturers complained that the students were too loud.

A more interesting example is given by a pluralised version of the Geach–Kaplan sentence. Recall, the original sentence is:

(11) *Some critics admire only one another.*

As proved by Kaplan, (11) cannot be correctly regimented in First-Order Logic.[53] This follows from the fact that its second-order formalisation,

(12) $\exists X(\exists x\,Xx \wedge \forall u \forall v(Xu \wedge A(u,v) \rightarrow Xv \wedge u \neq v))$

with '$u = 0 \vee u = v + 1$' instead of '$A(u,v)$', is true in all and only the non-standard models of arithmetic.[54] This can never be the case for a first-order sentence, hence the non-firstorderisability result. Thus (11) seems to require a second-order regimentation. However, it would be counter-intuitive if the truth value of (11) were to depend on the existence of some set-like entities (as a second-order reading is typically seen to demand), against FACE VALUE. Fortunately, we can instead use a PL regimentation:

(13) $\exists x^1(\forall u^0 \forall v^0(u^0 \prec x^1 \wedge Au^0v^0 \rightarrow v^0 \prec x^1 \wedge u^0 \neq v^0))$.

Which, under a pluralist reading, presupposing a domain of critics, makes the sentence true simply in virtue of the existence of some suitably related individuals.

Now, a pluralised version of (11) would be:

(14) *Some couples admire only one another.*

This is a higher-level plural sentence provided one accepts that *couple* is a pseudo-singular term. That *couple* functions, at least in some contexts, as a pseudo-singular receives support from the fact that, in many languages, it is affected by plural override. Moreover, the claim that in this particular context it does so receives further support from the following inference, where we observe that predication of *admire* of a single couple demands plural agreement: *The Carters only admire the Obamas, and vice versa. Therefore, some couples admire only one another*. However, whether *couple* really acts as a pseudo-singular in this context will be called into doubt by some. For example, one may analyse the proposed inference as capitalising on well-known facts about couples (e.g. that they have members), instead of being an instance of existential generalisation.[55] Nevertheless, importantly for us, some pluralists are sympathetic with this view and thus should be moved by the present argument.

The reason why this example is more interesting than (6) and (10) is that, unlike in those cases, we now know for sure that there is no paraphrase available for (14) via FLATTENING. If we were to find a plural paraphrase for (14), this paraphrase could be turned into a singular paraphrase of the original Geach–Kaplan sentence (11) by substituting *couple* with *critic*. But drawing from the result of non-firstorderisability, we know that this is impossible.

---

[53] This is reported in Quine (1950, sec. 38).

[54] See Boolos (1984, p. 57, fn7) for details.

[55] And see also the discussion of pseudo-singularity in general in Sect. 5.1.1.

An alternative paraphrase is given by (15), which is obtained via REIFICATION:

(15) *There is a set such that if a couple belongs to it, then if it admires another couple, then the latter also belongs to it.*

This paraphrase appeals crucially to the existence of a certain set and, again, it appears that (14) does not entail (15) from the point of view of ordinary speakers, against FACE VALUE. Moreover, this paraphrase would be at odds with set-theoretic NOMINALISM, which some pluralists adhere to. Thus the cost to pay for this paraphrase is exactly analogous to that of (7).

As pointed out by Yi (2005) (in another context, with an analogous example), the fact that this result is counter-intuitive is made more obvious when we observe that

(16) *This couple and that couple admire only one another.*

clearly entails (14), but not (15), since (16) can be paraphrased as a basic plural sentence:

(17) *This couple admire only that couple, and vice versa.*

(17) does not imply the existence of a set of couples, but it implies (14). Therefore (14) does not entail the existence of such set either.

Another difficulty with REIFICATION has to do with the principle of ABSOLUTE GENERALITY. The problem is that for terms with non-set-sized denotations there is not even a candidate set to be invoked in paraphrases via REIFICATION. For example, consider:

(18) *The cardinals, the ordinals and the transitive sets overlap.*[56]

We cannot easily apply FLATTENING to (18). A natural choice of paraphrase via FLATTENING would be:

(19) *The cardinals overlap the ordinals, the ordinals overlap the transitive sets, and the transitive sets overlap the cardinals.*

But (19) is not equivalent to (18) (think about the scenario in which each conjunct in (19) is true and yet nothing is a cardinal, an ordinal and a transitive set). Yet another possibility would be to paraphrase it as:

(20) *Some item is one of the cardinals, one of the ordinals and one of the transitive sets.*

However, as pointed out by Linnebo and Nicolas, this paraphrase would make *overlap* a defined term, '[b]ut it seems extremely plausible that 'overlap' can be taught to a child as a lexical atom and is in fact primitive' (Linnebo and Nicolas 2008, p. 194). Thus unless we are willing to take *overlap* as a defined notion, we appear to be forced to use REIFICATION. A naive move would be:

---

[56] An analogous example is given in Linnebo and Nicolas (2008) and I follow their discussion. However, they do not tackle the issue of non-set-sized denotations.

(21) *The set of cardinals, the set of ordinals and the set of transitive sets overlap.*

But, of course, (21) suffers from reference failure. One may alternatively paraphrase it as:

(22) *The class of cardinals, the class of ordinals and the class of transitive sets overlap.*

While avoiding reference failure, this move results in the need for further sorts of REIFICATION strategies (since we can now construct an ordinary sentence with a higher-level plural denoting too many classes to form a class, which would lead to a paraphrase in terms of super-classes, and so on). The need for such a variety of paraphrasing strategies (on top of the ones we had already seen) further supports the conclusion that higher-level plurals are indispensable at the object-language level—natural language higher-level plural expressions are best taken seriously.

5.2.2 Against analysing away

Some theorists think that, even if they cannot be paraphrased away, higher-level plural expressions can be given a semantics which shows that they are not higher-level plural.

In Sect. 4, I mentioned three such proposals: Ben-Yami's articulated reference analysis, cover-based semantics and group-based semantics. In this section, I mainly focus on the first proposal and leave the other two aside, since both of those proposals have features which are seen as deal-breakers by many pluralists. Firstly, under a common view of ontological commitment, they commit one to the existence of groups[57] or sets, which goes against FACE VALUE and some versions of NOMINALISM. Secondly, they impose limitations of size on apparent higher-level plurals, against ABSOLUTE GENERALITY.

Let me briefly justify the latter claim. First, as mentioned in Sect. 4, cover-based semantics comes in two different versions: one built on top of a set-based semantics and one built on top of a sum-based semantics. In either case, however, covers are limited to set-sized higher-level denotations, which leads to the limitation of size issues which have typically concerned pluralists.

Things are not quite so straightforward with group-based theories. However, on closer examination, they seem to suffer from analogous shortcomings, since, given some plausible assumptions, there is a paradox analogous to Russell's affecting groups. Let us call the atomic individuals making up the sum from which a certain group is formed 'the members' of the group. Thus, a non-self-membered group is such that it is not an atomic individual making up the sum from which it is formed. It is obvious that there are some non-self-membered groups. Now consider the sum of all the non-self-membered groups. If we can form a group from any sum, paradox ensues in a familiar way. There are, of course, a multitude of ways in which one can go about blocking this paradox. The point, however, is that groups appear to give rise to a discussion parallel to the one regarding sets; thus the kind of concerns pluralists raise against standard model-theoretic semantics are lurking around the corner in the case of group-based semantics.

---

[57] In what follows, I use 'group' in its technical sense.

Let us turn to the articulated reference proposal. According to Ben-Yami (2013), the notion of higher-level plural reference should be rejected and an alternative account of the meaning of apparent natural language higher-level plurals should be adopted—one which characterises them as articulated terms. An articulated term is defined as a term which refers to some objects by virtue of containing other referring expressions that refer to some of those objects (possibly to one of them).[58]

Note that when Ben-Yami talks of 'an expression containing another one' he is referring to the relation between a list and its items. Hence, *the philosophers and the linguists* would be an articulated term, as it refers to a bunch of people by virtue of containing terms that refer to sub-pluralities of them. Thus articulated terms are semantically plural terms which display a specific kind of syntax. This is important, as it is supposed to show that there is no need for a new mode of reference: articulated reference is plural reference. One reason why articulated reference is considered as being semantically relevant is that it gives rise to failures of substitutivity of co-referring expressions. One of the examples Ben-Yami uses to demonstrate this is:

(23) *My children, your children and her children played against each other.*[59]

This sentence must be understood as describing a three-way game. Ben-Yami points out that even in a context where *My children, your children and her children* denoted the same children as *The boys and the girls*, substitutivity of one for the other in (23) could lead to a shift in truth value.

Although I agree with Ben-Yami that accounting for these failures of substitutivity is central in the semantic analysis of these terms, I think that the notion of articulated reference is problematic for two reasons.

Firstly, I believe that there is a certain unclarity in the characterisation of the notion of articulated reference and that, once elucidated, the proposal runs into trouble. According to Ben-Yami, 'articulation does not affect the semantic value of the articulated expression' (Ibid., p. 91). This is crucial in order to guarantee that articulated terms are semantically classified as plurals. Although articulation is not supposed to affect the semantic value of the articulated term, it is nonetheless supposed to affect the semantic value of the sentence where the term occurs, as shown by the failures of substitutivity just mentioned. But it is unclear how this is achieved. One way in which I can make sense of this claim is that whatever semantic rule explains those failures, it is one that is applied to the denotations of the components of a list and which gives rise not to another denotation, but rather to a string enumerating the denotations of the argument terms of what is interpreted in turn as a polyadic predicate—i.e. lists are not analysed as functions from terms to terms, but from terms to strings.[60] To see this note that if list formation gave rise to terms, then either their denotation would be simply plural (in which case the failures of substitutivity would remain unexplained) or it would be a new kind of denotation (against the claim that articulated reference is

---

[58] See Ben-Yami (2013, p. 89).

[59] An analogous example was originally proposed in Linnebo and Nicolas (2008).

[60] See Oliver and Smiley (2016, Chap. 10) for more details on these two analyses of lists.

just a case of plural reference). However, if this is right, then articulated terms are not referring expressions at all, contrary to Ben-Yami's treatment of them.[61]

Secondly, the notion of articulated reference seems to be extensionally inadequate. Let us leave the previous problem aside and assume that articulated terms are referring expressions (and not strings), for the sake of the argument. According to this analysis, all and only lists of terms get classified as articulated terms. I believe this makes the notion too restrictive: there are other types of terms which are relevantly equivalent to articulated terms which are not counted amongst them—thus making the notion semantically uninteresting.

This follows from the fact that other types of terms (i.e. the ones enumerated in Sect. 5.1.1), which behave exactly like lists of plurals with respect to the failures of substitutivity mentioned above do not fall under the articulated type of referring expressions. This appears to be even a problem by Ben-Yami's lights, since he acknowledges that those terms should be understood in the same way.[62] However, he thinks the notion is extendible to cover these too:

> The application of the concept of articulated reference to definite descriptions [...] is an *extension* of that concept as introduced above. [...] because ['the joint authors of multivolume treatises on logic'] applies to particulars two by two (in this case), the definite description behaves under predication the way the articulated noun phrase 'the authors of Principia Mathematica and the authors of Grundlagen der Mathematik' does. (Ibid., p. 98)

But the fact that Ben-Yami needs to explain this extension in terms of the predicate *being a joint author* collectively holding of several individuals suggests that an underlying semantic account is in order, one that supersedes articulated reference.

6 Conclusion: the instability of pluralist scepticism towards higher-level plurals

I believe the empirical data and the arguments presented here show that there is not as much room for scepticism towards HLPL as typically thought. Unsurprisingly, there is still some. Firstly, there is a form of scepticism that remains completely unmoved: the view that higher-level plurals intuitively denote sets of sets (or some other collectivising entities) and that, given that surface linguistic form is often misleading with respect to its semantic analysis, they should be analysed as such. Similar claims have been raised against Pluralism.[63] But, of course, such claims fall short of an argument—they simply assert an intuition (and there is bound to be a standoff of intuitions in this kind of conversation). More importantly, the sceptic can reject some of the basic tenets employed in the arguments put forward (most notably, FACE

---

[61] Moreover, the data involving failures of substitutivity is compatible with articulation falling on the side of the semantic value of the terms which have it (and a fortiori with treating them as referring expressions). I believe more needs to be said to justify placing articulation outside of their semantic interpretation.

[62] See Ben-Yami (2013, p. 97).

[63] See, for instance, Resnik (1988, p. 77).

VALUE, NOMINALISM or ABSOLUTE GENERALITY), if not tout court, in the contexts relevant to this debate. However, if the arguments presented here are correct, there is an interesting way in which they leave less room for scepticism: the advocate of PL cannot reject HLPL without violating her own strictures, since she typically does endorse these principles. In other words, I conclude, against a commonly held opinion, that the prospects of HLPL are analogous to those of PL.

Unsurprisingly, many advocates of PL accept some, but not all of the ideas on which the arguments presented here rely and thus will go along with some, but not all of them. For example, I pointed out that the notion of pseudo-singularity is controversial and, of course, its rejection is not in conflict with the pluralist position; thus some pluralists will not be moved by the arguments that involve this notion. Nevertheless, I have offered arguments starting from a variety of views held by pluralists, each of which independently supports the acceptance of HLPL. Thus, I submit that the (typical)[64] pluralist is ultimately compelled to accept higher-level plurals—with more or less strength, depending on how many of the basic principles appealed to she endorses.

Recall that the availability of higher-level plural extensions of PL will be seen as good news by some and bad news by others. The good news is that there is an increase in expressive power; the bad news is that with it a suspicion of a loss of logicality arises. I hope to have shown that, be that as it may, PL and HLPL stand or fall together.

References

Acquaviva, P. (2008). *Lexical plurals: A morphosemantic approach*. Oxford: Oxford University Press.

Appleyard, D. L. (1987). A grammatical sketch of Khamtanga-I. *Bulletin of the School of Oriental and African Studies, University of London*, *50*(2), 241–266.

Barker, C. (1992). Group terms in English: Representing groups as atoms. *Journal of Semantics*, *9*(1), 69–93.

Ben-Yami, H. (2013). Higher-level plurals versus articulated reference, and an elaboration of salva veritate. *Dialectica*, *67*(1), 81–102.

Black, M. (1971). The elusiveness of sets. *Review of Metaphysics*, *24*(4), 614–636.

Boccuni, F. (2011). On the consistency of a plural theory of Frege's Grundgesetze. *Studia Logica*, *97*(3), 329–345.

Boccuni, F. (2013). Plural logicism. *Erkenntnis*, *78*(5), 1051–1067.

Boolos, G. (1984). To be is to be a value of a variable (or to be some values of some variables). *Journal of Philosophy*, *81*(8), 430–449.

Boolos, G. (1985). Nominalist platonism. *Philosophical Review*, *94*(3), 327–344.

Corbett, G. G. (2000). *Number. Cambridge textbooks in linguistics*. Cambridge: Cambridge University Press.

Cotnoir, A. J. (2013). Composition as general identity. In K. Bennett & D. W. Zimmerman (Eds.), *Oxford studies in metaphysics* (Vol. 8, p. 294-322). Oxford: Oxford University Press.

Florio, S. (2010). The semantics of plurals: A defense of singularism. Ph.D. thesis, Ohio State University.

---

[64] Naturally, the possibility of endorsing PL for reasons not considered in this article cannot be ruled out, but, to the best of my knowledge, the main reasons appealed to in the literature in defence of PL have been examined.

Florio, S. (2014). Semantics and the plural conception of reality. *Philosophers' Imprint*, *14*(22), 1–20.

Florio, S., & Nicolas, D. (2015). Plural logic and sensitivity to order. *Australasian Journal of Philosophy*, *93*(3), 444–464.

Gillon, B. S. (1987). The readings of plural noun phrases in English. *Linguistics and Philosophy*, *10*(2), 199–219.

Gillon, B. S. (1992). Towards a common semantics for English count and mass nouns. *Linguistics and Philosophy*, *15*(6), 597–639.

Grimau, B. (2018). From plurals to superplurals: In defence of higher-level plural logic. Ph.D. thesis, Universty of Glasgow.

Hazen, A. P. (1997). Relations in Lewis' framework without atoms. *Analysis*, *57*(4), 243–248.

Hewitt, S. (2012). The logic of finite order. *Notre Dame Journal of Formal Logic*, *53*(3), 297–318.

Hossack, K. (2000). Plurals and complexes. *British Journal for the Philosophy of Science*, *51*(3), 411–443.

Hurford, J.R. (2003). The interaction between numerals and nouns. In *Noun Phrase Structure in the Languages of Europe, Volume 20-7 of Empirical Approaches to Language Typology*.

Jespersen, O. (1924). *The philosophy of grammar*. Crows Nest: Allen & Unwin.

Landman, F. (1989a). Groups, I. *Linguistics and Philosophy*, *12*(5), 559–605.

Landman, F. (1989b). Groups, II. *Linguistics and Philosophy*, *12*(6), 723–744.

Lewis, D. (1991). *Parts of classes*. New York: Blackwell.

Link, G. (1983). The logical analysis of plurals and mass terms: A lattice-theoretic approach. In P. Portner & B. H. Partee (Eds.), *Formal semantics—The essential readings* (pp. 127–147). New York: Blackwell.

Link, G. (1984). Hydras: On the logic of relative clause constructions with multiple heads. In F. Landman & F. Veltman (Eds.), *Varieties of formal semantics: Proceedings of the fourth Amsterdam colloquium*. Dordrecht: Foris Publications.

Link, G. (1998). *Ten years of research on plurals—Where do we stand?* (pp. 19–54). Dordrecht: Springer.

Linnebo, O. (2003). Plural quantification exposed. *Noûs*, *37*(1), 71–92.

Linnebo, O. (2006). Sets, properties, and unrestricted quantification. In G. Uzquiano & A. Rayo (Eds.), *Absolute generality*. Oxford: Oxford University Press.

Linnebo, O. (2017). Plural quantification. In E. N. Zalta (Ed.), *Stanford encyclopedia of philosophy*. Stanford: The Metaphysics Research Lab, Stanford University.

Linnebo, O., & Nicolas, D. (2008). Superplurals in English. *Analysis*, *68*, 186–197.

Linnebo, O., & Rayo, A. (2012). Hierarchies ontological and ideological. *Mind*, *121*(482), 269–308.

McKay, T. J. (2006). *Plural predication*. Oxford: Oxford University Press.

Nicolas, D. (2008). Mass nouns and plural logic. *Linguistics and philosophy*, *31*(2), 211–244.

Oliver, A., & Smiley, T. (2016). *Plural logic: 2nd edition, revised and enlarged* (2nd ed.). Oxford: Oxford University Press.

Quine, W. V. O. (1950). *Methods of logic*. Harvard: Harvard University Press.

Rayo, A. (2006). Beyond plurals. In A. Rayo & G. Uzquiano (Eds.), *Absolute generality* (pp. 220–54). Oxford: Oxford University Press.

Rayo, A., & Uzquiano, G. (2006). *Absolute generality*. Oxford: Oxford University Press.

Reinisch, L. (1884). *Die Chamirsprache in Abessinien*. New York: Gerold.

Resnik, M. D. (1988). Second-order logic still wild. *Journal of Philosophy*, *85*(2), 75–87.

Rosen, G., & Dorr, C. (2002). Composition as a fiction. In R. Gale (Ed.), *The Blackwell companion to metaphysics* (pp. 151–174). New York: Blackwell.

Rumfitt, I. (2005). Plural terms : Another variety of reference? In J. L. Bermúdez (Ed.), *Thought, reference, and experience: Themes from the philosophy of gareth evans* (pp. 84–123). Oxford: Clarendon Press.

Schwarzschild, R. (1996). *Pluralities*. Berlin: Springer.

Shapiro, S. (1991). *Foundations without foundationalism: A case for second-order logic*. Oxford: Oxford University Press.

Shapiro, S. (1993). Modality and ontology. *Mind*, *102*(407), 455–481.

Shapiro, S., & Weir, A. (2000). 'Neo-logicist' logic is not epistemically innocent. *Philosophia mathematica*, *8*(2), 160–189.

Simons, P. (1982). Number and manifolds. In B. Smith (Ed.), *Parts and moments: Studies in logic and formal ontology* (pp. 160–98). Berlin: Philosophia.

Simons, P. (2016). *The ontology and logic of higher-order multitudes* (pp. 55–69). Oxford: Oxford University Pres.

Uzquiano, G. (2003). Plural quantification and classes. *Philosophia Mathematica*, *11*(1), 67–81.

Uzquiano, G. (2004). Plurals and simples. *The Monist*, *87*(3), 429–451.

de Vries, H. (2017). Two kinds of distributivity. *Natural Language Semantics*, *25*(2), 173–197.

Williamson, T. (2003). Everything. *Philosophical Perspectives*, *17*(1), 415–465.

Winter, Y. (2002). Atoms and sets: A characterization of semantic number. *Linguistic Inquiry*, *33*(3), 493–505.

Yi, B. (2005). The logic and meaning of plurals. Part I. *Journal of Philosophical Logic*, *34*(5–6), 459–506.

Yi, B. (2006). The logic and meaning of plurals. Part II. *Journal of Philosophical Logic*, *35*(3), 239–288.