Preliminary Results from Experiments on the Behavior under Ambiguity

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Abstract

In the literature, some experiments proving that human decision-makers manifest an ambiguity aversion are described. In our knowledge, no one has studied a possibility to measure the strength of this aversion and its stability in time. The research, we have recently started to realize should find out answers to these and similar questions. The goal of this paper is to present some preliminary results to initiate a discussion that should help us to modify either the process of data collection and/or the analysis of the collected data.

1 Introduction

One of the goals of the research project GAČR 19-06569S is to find out how to construct normative models manifesting the same ambiguity aversion as human decision-makers. This term is used when speaking about the behavior, which is irrational if "rationality" means the behavior in agreement with the Savage's postulates formulated in his famous book [10]. The term is connected with the fact that human decision-makers do not like ignorance; they usually prefer uncertainty connected with a random experiment to total ignorance. The difference will be clear when describing the lotteries, which we use to test the behavior of experimental persons.

One of the first authors who experimentally studied this phenomenon was Ellsberg [4], and so it is not surprising that the behavior is often connected with the term *Ellsberg's paradox*. His experiments were often repeated [1, 5] but, in our knowledge, nobody made the experiments to measure the strength of ambiguity aversion. And this is why, during the first year of the above-mentioned project, we realize several experiments, the results of which should help us to characterize the concept of subjective ambiguity aversion. We want to find out to what extent we can rely upon our starting assumptions:

- In analogy to risk aversion, the ambiguity aversion is also a personal characteristic; not all decision-makers are influenced by this phenomenon in the same way.
- To some extent, it is possible to measure the strength of ambiguity aversion for individual human decision-makers.

Nevertheless, even if the above-stated assumptions are not declined, currently nobody knows to what extent the strength of the ambiguity aversion of a decision-maker depends also on the type of a decision task, and to what extent it is stable in time. All these are the open questions we are planning to study within the project mentioned above. As the starting point of our experimental research, we have designed the experiments, in which volunteers are asked to describe their behavior in several situations.

All the considered situations are formulated in the form of lotteries, in which the participants have a chance to win 100 CZK. At each of the situation, the content of a lottery drum is partially described, and the participants are asked to decide how much they are maximally willing to pay to be allowed to take part in the specified lottery. Details from the organization of these experiments are described in another paper presented at this conference. Here we just say that the following 14 situations are presented to experimental persons.

F1 The drawing urn contains 30 balls, five of each of the following colors: red, black, yellow, white, green, and azure. How much are you maximally willing to pay to take part in the lottery in which you win 100 CZK if the randomly drawn ball is red?

F2 The drawing urn contains 30 balls, five of each of the following colors: red, black, yellow, white, green, and azure. How much are you maximally willing to pay to take part in the lottery in which you choose a color and get 100 CZK if the randomly drawn ball is of the color of your choice?

I1 The drawing urn contains 30 balls, they may be of the following colors: red, black, yellow, white, green, and azure. You know nothing more, you even do not know how much colors are present in the urn. How much are you maximally willing to pay to take part in the lottery in which you win 100 CZK if the randomly drawn ball is red?

I1 The drawing urn contains 30 balls, they may be of the following colors: red, black, yellow, white, green, and azure. You know nothing more, you even do not know how much colors are present in the urn. How much are you maximally willing to pay to take part in the lottery in which you choose a color and get 100 CZK if the randomly drawn ball is of the color of your choice?

Rn This represents 8 lotteries for n= 5, 6, 7, . . ., 12. The drawing urn contains n balls, each of which is either red, or black, or yellow, or white, or green, or azure. You know that one and only one of them is red, nothing more. You even do not know how many colors are present in the urn. How much are you maximally willing to pay to take part in the lottery in which you choose a color and get 100 CZK if the randomly drawn ball is of the color of your choice?

E1 The drawing urn contains 15 red, black and yellow balls, you know that exactly 5 of them are red, you do not know the proportion of the remaining black and yellow balls. How much are you maximally willing to pay to take part in the lottery in which you choose a color and get 100 CZK if the randomly drawn ball has the color of your choice?

E2 The drawing urn contains 15 red, black and yellow balls, you know that exactly 5 of them are red, you do not know the proportion of the remaining black and yellow balls. How much are you maximally willing to pay to take part at the lottery in which you choose a color and get 100 CZK if the randomly drawn ball is either yellow or of the color of your choice?

2 Uncertain Knowledge representation

Considering the situations **F1** and **F2**, the knowledge can fully be described by a uniform probability distribution. Denoting the corresponding state space (i.e., a set of possible outcomes of a random draw) $\Omega = \{red, black, white, yellow, green, azure\}$ ($\Omega = \{r, b, w, y, g, a\}$ for short), for the uniform probability distribution $P_u(r) = P_u(b) =, \ldots, P_u(a) = \frac{1}{6}$. Notice that, due to additivity of probabilities, we also know (for example) that $P_u(\{r, g\}) = \frac{1}{3}$, and $P_u(\{b, y, a\}) = \frac{1}{2}$. Generally, for $\mathbf{a} \subseteq \Omega$, $P_u(\mathbf{a}) = \frac{|\mathbf{a}|}{6}$. It is also clear that from the situations introduced in the previous section, only the situations **F1** and **F2** can fully be described by probability distributions. For the description of the remaining situations we have to use another theoretical instrument.

2.1 Belief Functions

Consider the situation **Rn** describing One-red-ball example with n balls in a drawing drum. In this case we know only the probability $P_{\varrho,n}(r) = \frac{1}{n}$. We do not know the probabilities of other colors. But, again thanks to additivity of probability, we know that $P_{\varrho,n}(\{b, w, y, g, a\}) = P_{\varrho,n}(\Omega \setminus \{r\}) = 1 - \frac{1}{n}$. And this is the information that can be used to define a belief function. It is the information, which allows us to define the basic notion from this theory, so called *basic probability assignment*.

Since there is abundant literature on belief function theory (e.g., [11, 3, 12], and the papers introducing the models discussed in this paper [9, 8]), we presume that the reader is familiar with at least the foundations of this approach. Therefore, we introduce just the notation used in this paper.

The fundamental notion is that of a basic probability assignment (bpa), which describes all the information about the considered situation at our disposal. It is a function[1] $m : 2^\Omega \to [0, 1]$, such that $\sum_{\mathbf{a}\in 2^\Omega} m(\mathbf{a}) = 1$ and $m(\emptyset) = 0$.

For bpa m, $\mathbf{a} \in 2^\Omega$ is said to be a *focal element* of m if $m(\mathbf{a}) > 0$. In what follows we will consider the following two special classes of bpa's representing the extreme situations:

- m is said to be *vacuous* if $m(\Omega) = 1$, i.e., m has only one focal element, Ω. A vacuous bpa is denoted by m_ι. It represents total ignorance, i.e., it represents the situations **I1** and **I2**.

- m is said to be *Bayesian*, if all its focal elements are singletons, i.e., for Bayesian bpa m, $m(\mathbf{a}) > 0$ implies $|\mathbf{a}| = 1$. Bayesian bpa's represent exactly the same knowledge as probability functions. As all focal elements of a Bayesian bpa m are singletons, we can define probability distribution P_m for Ω such that
$$P_m(x) = m(\{x\}) \tag{1}$$
for all $x \in \Omega$. Thus, Bayesian bpa's represent in our examples situations **F1** and **F2**.

Exactly the same knowledge that is expressed by a bpa m can also be expressed by a *belief function*, and by *plausibility function*.

$$Bel_m(\mathbf{a}) = \sum_{\mathbf{b}\in 2^\Omega:\, \mathbf{b}\subseteq\mathbf{a}} m(\mathbf{b}). \tag{2}$$

$$Pl_m(\mathbf{a}) = \sum_{\mathbf{b}\in 2^\Omega:\, \mathbf{b}\cap\mathbf{a}\neq\emptyset} m(\mathbf{b}). \tag{3}$$

In this paper we take advantage of the fact that for each bpa there exists a *credal set*, which is a convex set of probability distributions P on Ω defined as follows ($\mathcal{P}$ denotes the set of all probability distributions on Ω):

$$\mathcal{P}(m) = \left\{ P \in \mathcal{P} : \sum_{x\in \mathsf{a}} P(x) \geq Bel_m(\mathsf{a}) \text{ for } \forall \mathsf{a} \in 2^\Omega \right\}.$$

Notice that P_m defined by Equation (1) for a Bayesian bpa m is such that $\mathcal{P}(m) = \{P_m\}$, and that $\mathcal{P}(m_\iota) = \mathcal{P}$. From Equations (2) and (3), it can easily be deduced that for all $P \in \mathcal{P}(m)$
$$Bel_m(\mathsf{a}) \leq P(\mathsf{a}) \leq Pl_m(\mathsf{a}),$$
for all $\mathsf{a} \in 2^\Omega$. Thus, if $Bel(\mathbf{a}) = Pl(\mathbf{a})$ then we are sure that the probability of $\mathbf{a}$ equals $Bel(\mathbf{a})$. Otherwise, the larger the difference $Pl(\mathbf{a}) - Bel(\mathbf{a})$, the more uncertain we are about the value of the probability of $\mathbf{a}$. Using the terminology

[1] As usually, 2^Ω denote a set of all subsets of Ω.

of Srivastava [15], the greater this difference, the more ambiguity one has for the event (set of states) **a**.

The last notion we need in this paper is that of a famous *pignistic transform*, which was introduced in [16] and for decision making strongly advocated by Philippe Smets [13, 14]):

$$Bet\_P_m(x) = \sum_{\mathsf{a} \in 2^\Omega : x \in \mathsf{a}} \frac{m(\mathsf{a})}{|\mathsf{a}|}. \tag{4}$$

Notice, it defines for each bpa m a probability distribution, which is from the corresponding credal set $\mathcal{P}(m)$.

2.2 Measuring Strength of Ambiguity

The proposed way of measuring the strength of individual ambiguity aversion is based on the following mental model.

Consider situations **I1** and **F1** (or equivalently **I2** and **F2**). Usually (and it is confirmed also in our experiments) people are willing to pay more to take part in the lottery **F1** than in the lottery **I1**. This well known, seemingly paradoxical phenomenon, can hardly be explained by different subjective utility functions or by different subjective probability distributions. To explain this fact, we accepted a hypothesis that humans do not use their personal probability distributions but just *capacity functions* that do not sum up to one [6, 17]. Roughly speaking, the subjective probability of drawing a red ball is $\frac{1}{6}$ in the case that the person knows that the number of balls of all colors are the same in the drum. However, the respective "subjective probability" in the case of lack of knowledge is $\varepsilon < \frac{1}{6}$. *The lack of knowledge psychologically decreases the subjective chance of drawing the selected color – it decreases the subjective chance of success.* Thus, while we can accept that in situation **F2** the decision-maker considers that the probabilities of individual colors are $\frac{1}{6}, \frac{1}{6}, \ldots, \frac{1}{6}$, in situation **I2** these "subjective probabilities" are only $\varepsilon, \varepsilon, \ldots, \varepsilon$. Assuming this decrease is linear with the subjective strength of ambiguity, we can measure it by a personal coefficient of ambiguity α, which can be expressed, in case that the person is willing to pay a CZK in situation **F1** and b CZK in situation **I1**, by the following simple formula

$$\alpha = \frac{a-b}{a}. \tag{5}$$

The higher this coefficient, the stronger the aversion. Namely, if the person is willing to pay a CZK when her expected probability of success is $\frac{1}{6}$ (situation **F1**), then, in case of the decreased probability of success, which is $(1-\alpha) \cdot \frac{1}{6}$ (in case of **I1**), she is willing to pay

$$(1-\alpha) \cdot a = (1 - \frac{a-b}{a}) \cdot a = a - (a-b) = b.$$

Let us, now, show how this personal coefficient of ambiguity influences behavior of an experimental decision maker in situations **Rn**.

As we have already said at the beginning of Section 2.1, in situation **Rn**, the content of the drawing drum is described by bpa $m_{\varrho,n}$ given as follows:

$$m_{\varrho,n}(\mathbf{a}) = \begin{cases} \frac{1}{n}, & \text{if } \mathbf{a} = \{r\}; \\ \frac{n-1}{n}, & \text{if } \mathbf{a} = \{b, g, o, y, w\}; \\ 0, & \text{otherwise,} \end{cases}$$

and the corresponding belief function is $Bel_{m_{\varrho,n}}(\{x\}) = 0$ for all $x \in \{b, g, o, y, w\}$, and $Bel_{m_{\varrho,n}}(\{r\}) = \frac{1}{n}$.

For the sake of simplicity let us accept here the Smets' advice [14] saying that for decision making one should compute the expected value using the pignistic transform (for a survey of other probabilistic transforms see [2], and for more discussion on the problem of a probabilistic transfrom selection see [7]), which is

$$Bet\_P_{m_{\varrho,n}}(x) = \begin{cases} \frac{1}{n}, & \text{if } x = r; \\ \frac{n-1}{5n}, & \text{for } x \in \{b, g, o, y, w\}. \end{cases}$$

If there were not for the ambiguity, we should use it directly for the computation of the expected winnings. However, in our approach, we have to decrease it using the personal coefficient of ambiguity aversion α. We have to decrease it at each point of Ω proportionally to the strength of the ambiguity connected with the considered point. Realize, that $Bet\_P_{m_{\varrho,n}}(x) - Bel_{m_{\varrho,n}}(\{x\}) \geq 0$, and equals 0 if and only if $Pl_{m_{\varrho,n}}(x) = Bel_{m_{\varrho,n}}(\{x\})$. Thus, if $Bet\_P_{m_{\varrho,n}}(x) = Bel_{m_{\varrho,n}}(\{x\})$, we know the respective probability exactly, we do not have any ambiguity about its value. However, the greater the difference $Bet\_P_{m_{\varrho,n}}(x) - Bel_{m_{\varrho,n}}(\{x\})$, the greater the ambiguity, and therefore we have to decrease probabilities $Bet\_P_{m_{\varrho,n}}(x)$ accordingly. After the decrease, they do not sum up to one, any more, and therefore we call them *reduced weights*, and compute them according to the following formula:

$$r_{m,\alpha}(x) = (1-\alpha)Bet\_P_m(x) + \alpha Bel_m(\{x\}). \tag{6}$$

Thus, in situation **Rn** we get:

$$r_{m_{\varrho,n},\alpha}(x) = \begin{cases} \frac{1}{n}, & \text{if } x = r; \\ (1-\alpha) \cdot \frac{n-1}{5n}, & \text{for } x \in \{b, g, o, y, w\}. \end{cases}$$

Considering (for the sake of simplicity just two) gain functions $g^{\mathrm{r}}(x)$, and $g^{\mathrm{w}}(x)$ (corresponding to betting on red and white color, respectively), the total subjective rewards are as follows. When betting on red it equals

$$R_{m_{\varrho,n},\alpha}(r) = \frac{1}{n} g^{\mathrm{r}}(r) + \sum_{x \in \Omega : x \neq r} \frac{(1-\alpha)(n-1)}{5n} g^{\mathrm{r}}(x) = \frac{100}{n},$$

Table 1: One Red Ball Example: Total subjective reward as a function of the coefficient of ambiguity aversion α, and the number of balls n.

		$R_{m_{\varrho,n},\alpha}(\mathrm{w})$						
n	$R_{m_{\varrho,n},\alpha}(\mathrm{r})$	α=0	α=0.1	α=0.2	α=0.3	α=0.4	α=0.5	α=0.6
5	20.00	16.00	14.40	12.80	11.20	9.60	8.00	6.40
6	16.67	16.67	15.00	13.33	11.67	10.00	8.33	6.67
7	14.29	17.14	15.43	13.71	12.00	10.29	8.57	6.86
8	12.50	17.50	15.75	14.00	12.25	10.50	8.75	7.00
9	11.11	17.78	16.00	14.22	12.44	10.67	8.89	7.11
10	10.00	18.00	16.20	14.40	12.60	10.80	9.00	7.20
11	9.09	18.18	16.36	14.55	12.73	10.91	9.09	7.27
12	8.33	18.33	16.50	14.67	12.83	11.00	9.17	7.33

and analogously, for betting on white

$$R_{m_{\varrho,n},\alpha}(\mathrm{w}) \quad = \quad \frac{1}{n} g^{\mathrm{w}}(r) + \sum_{x \in \Omega : x \neq r} \frac{(1-\alpha)(n-1)}{5n} g^{\mathrm{w}}(x) = \frac{100(1-\alpha)(n-1)}{5n}.$$

Some of the values of these functions are tabulated in Table 1. From this table we see that, for example, a person with $\alpha = 0.4$ should bet on red color for $n \leq 9$, because for these $R_{m_{\varrho,n},\alpha}(r) > R_{m_{\varrho,n},\alpha}(x)$ $(x \neq r)$, and bet on any other color for $n \geq 10$, because for these n, $R_{m_{\varrho,n},\alpha}(r) \leq R_{m_{\varrho,n},\alpha}(x)$ $(x \neq r)$. This means that for $n \leq 9$, it is subjectively more advantageous to bet on the red color. In the next section we say that the *computed breaking point* of such a person is 10.

We conclude this section mentioning that the description of the reduced function for the Ellsberg's examples is more complicated, because the gain function for the situation **E2** equals 100 for two values. In this case, we have to consider both pignistic transform and reduced weights functions as mappings on 2^{Ω}. Since we do not necessarily need it in the rest of this paper, we do not describe it here and refer the interested reader to [9, 8].

3 Results from Experiments

At the time of preparation of this paper, we have data from 49 respondents. Naturally, not all the respondents undertook the task with the same responsibility. It can be seen, among others, from the time, which they needed for finishing the task. In average, the respondents needed 5 minutes, and 19 seconds, but two of them finished the whole task in less then one minute (33 and 36 seconds). A similarly irresponsible attitude may be expected from the respondents who were willing to bet just one CZK (or 0 CZK) for all twelve situations. Naturally, for correct statistical data processing we should clean the data, and delete these obviously misleading responses. Because we do not have enough data and we do not have criteria how to detect misleading data, for this preliminary discussion we keep all the data as they were collected.

3.1 First Glance Comments

The reader certainly noticed that in situations **F1** and **F2** (in the same way as in situations **I1** and **I2**) the participants have the same information about the content of the drawing drum. The difference is just that in **F1** the winning color is predetermined (red), while in **F2** the participant determines the winning color herself. We included both of them into the battery of the considered situations, because we were not sure whether the participants would not suspect the organizers to exclude red color from the drawing drum in case that the winning color (red) is predetermined. This suspicious appeared false. The total amount of money bet in **I1** was 292 CZK, while in **I2** they altogether bet 290 CZK (for **F1** and **F2** the total amounts were 584 and 557 CZK, respectively).

The only observation, which surprised us at the first glance, concerns the behavior of the respondents in situations **E1** and **E2**. The reader familiar with the famous Elsberg's paper [4] already noticed that these situations were designed to repeat the Eslberg's experiment. Let us briefly recollect his example.

In [4] (pp. 653–654), Ellesberg considers the situation with a drawing drum containing 30 red balls and 60 black or yellow balls, the latter in unknown proportion. With this drum, Ellsberg considers two experiments. The first experiment (which we repeat as **E1** in our study) finds out whether people prefer betting on red or black ball, in the case they get the reward if the ball of the respective color is drawn at random. According to his observations, "very frequent pattern of response is that betting on red is preferred to betting on black". This corresponds also with our results, in which 36 (out of all 49) respondents bet on black color. In the second Ellsberg's experiment (simulated in our experiments as **E2**), a decision-maker can bet on red and yellow, or on black and yellow. Again, the participant gets the reward in case that the randomly drawn ball is of one of the selected colors. In this case, the Ellsberg's observation is that " betting on black and yellow is preferred to betting on red and yellow", which is not in the agreement with the results we have achieved. In our case, only 16 participants betted on black color.

3.2 Coefficient of Ambiguity α

Let us turn our attention to what can be said about the coefficient of ambiguity on the basis of the considered preliminary data. As a starting point, we computed this coefficient according to Formula (5) for all respondents. Having two pairs of situations, we computed two such coefficients; one from the bets in situations **F1** and **I1**, the other from bets in situations **F2** and **I2**. The situations are submitted to the participants in a random order, so it is quite interesting to what extent the two coefficients differ from each other. The results are depicted in Figure 1. Each point corresponds to one respondent (or several, if both coefficients coincide for

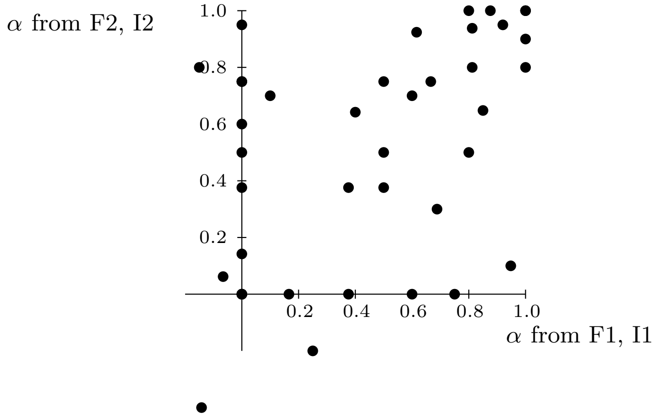

Figure 1: Comparison of coefficients α computed from bets in **F1**, **I1** and **F2**, **I2**.

several respondents), who took part in our experiments. The coordinates of each point are the respective coefficients α. From this figure, we see several unexpected facts. First, a few participants are exhibiting *ambiguity inclination*; their coefficient of ambiguity α is negative. Second, there is not a small part of participants, who manifest the ambiguity aversion just in one pair of situations (either in **F1**, **I1**, or in **F2**, **I2**) – see the points on the axes. For only a small number of participants, both coefficients are close to each other. Naturally, we have only a small amount of data (some of which should be removed because of the reasons mentioned above), so we cannot make any final conclusions. Therefore, in what follows, we consider just one coefficient of ambiguity, which is computed from sum betted together in **F1** and **F2**, and the sum betted in **I1** and **I2** together. To simplify the next exposition, let us call these coefficients the *joint coefficients of ambiguity*.

Going back to situations **Rn**, and assuming that the joint coefficient α expresses the strength of the ambiguity aversion of the individual respondents, we can, using Table 1, estimate the breaking point, i.e., the number of balls when the participants start betting on another color than the red one. We compute it for each experimental person using her personal joint coefficient of ambiguity. Comparing the computed breaking point with that, which can be read from data, we have found out that for half of the respondents (more precisely, for 25 out of 49) the breaking point computed from the models does not differ from the actual breaking point by more than one.

It is worth mentioning that from three respondents with negative joint coefficient of ambiguity, one did not bet on red color even for $n = 5$, and another betted on blue color already for $n = 6$. Thus, these two respondents displayed their ambiguity inclination even when reacting in situations **Rn**. Again, even this surprising result must be taken with a great care because of a small amount and not cleaned data.

Though the amount of money, the respondents are maximally willing to pay to take part in lotteries, is not in the center of our interest, the question is whether

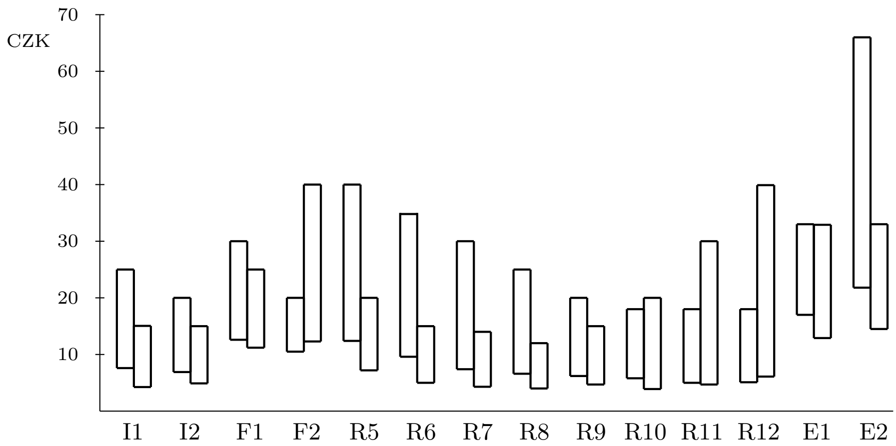

Figure 2: Maximal and average bets on individual lotteries.

these numbers should be taken into consideration when evaluating the quality of data. For example, is it meant seriously, if an experimental person claims that she is willing to pay $5, 8, 10, 15, 20, 30, 40$ CZK in situations **R5**, **R6**, ..., **R12**, respectively? Some irrational behavior of respondents can also be read from Figure 2, in which each situation (lottery) is described with two boxes. Left-hand box corresponds to those 25 respondents, for which the breaking point from lotteries R5 – R12 does not differ from that computed using the joint coefficient α by more than one. The right-hand box is computed from data of the rest of 24 respondents. The lower edge of each box shows the average of the amounts the respondents are willing to pay for taking part in the lottery, the upper edge shows the maximal value (it does not have the sense to depict the minimum because of the above-mentioned respondents stating that they are willing to pay just 1 CZK (or 0 CZK) in all situations).

4 Conclusions

In the paper, we described the experiments we are realizing to better understand the concept of individual subjective ambiguity aversion. The analysis of first data arises more questions than answers, and this is the main reason, why we present this paper at the Czech-Japan seminar. We want to initiate the discussion that should help us to find answers to the following questions:

- Is it possible to minimize the number of respondents replying the questions without thinking?
- Should the data be cleaned before their processing?

- If yes, what criteria should be used to clean the data?

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