On Experimental Part of Behavior under Ambiguity

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Abstract

People are risk-takers, risk-averse, or neutral. In the literature, one can find experiments illustrating the ambiguity aversion of human decision-makers. Recently, a personal coefficient of ambiguity aversion has been introduced. We have decided to measure the coefficient and its stability during the time. In this paper, we describe performed experiments and their structure to launch a discussion of possible design weaknesses or to suggest other methods of measuring it.

1 Introduction

When designing new methods of artificial intelligence, it is necessary to know how people behave when they have to make decisions and do not have enough information to do so. In this case, the fact of whether the decision-maker is either risk-taker or risk-averse has a huge influence.

It is generally accepted that the basis for the normative decision-making theory was laid by [25], and (ten years later) [18] who has developed the idea that the behavior of a decision-maker can be modeled with the help of subjective probability and utility functions. Such a subjective setting explains why different decision-makers accept different decisions but all of them, if they are rational, use the same decision criterion: all of them maximize the *expected utility.* Since that time, many papers presenting situations with human decision-makers not following this principle, have been published. Let us mention here just some of these papers like [14, 1, 9, 4] (the last one introduces the famous Ellsberg's example/paradox, which is generally accepted as evidence for ambiguity aversion), and especially the papers introducing the prospect theory [15, 16] (Nobel prize 2002). The phenomenon of ambiguity aversion is connected with the fact that human decision-makers do not like ignorance. They usually prefer uncertainty connected with randomness than total ignorance.

We believe that the ambiguity aversion phenomenon [4, 5, 6, 14, 13] is closely connected to the fact that classical probability theory has difficulties with representing ignorance, or vagueness [19]. And it is this shortcoming, which causes why some decision theorists - using probability theory as the main theoretical tool - consider human decision-making behavior paradoxical. The importance of a "more powerful" tool for decision-making starts to be obvious to a wide range of users. E.g. [8] argued that traditional approaches to decision-making based on expected utility maximization are out of their depth in the area of environmental policy, *"as they force us to act as if we know things that we know we do not"*.

The goal of our current research is to find a way to create mathematical models proving the same ambiguity aversion as human decision-makers. The method of our research is described in detail in our other paper in these proceedings that contains the preliminary results from performed experiments [11]. This paper tries to describe the way how the experiments were performed and how they evolve during the time. It also brings a description of the testing tool - a web-based application.

2 Belief functions

The theory of belief functions [19, 3, 21] (and it does not matter at this moment whether we consider Dempster-Shafer theory of evidence, or if we understand belief function as a generalization of a probability theory based on the concept of credal sets) was designed to describe situations under vagueness and/or ignorance. Therefore, there is no surprise that situations described by Allais, or Ellsberg can be well represented in this theory. This is also the reason why we have decided to apply the theory of belief functions to model subjective human decision-making under ambiguity (as suggested already by Thomas [24])

The idea of representing the knowledge in the form of belief function is not new. Nevertheless, it is of great importance to have a tool how to compute the expected utility from it. To do so, we use the approach suggested in [12, 10]). We assume that the reader is familiar with at least the foundations of this approach. Therefore, we introduce just the notation used in this paper.

The theory of belief functions [19] can be interpreted as a generalization of probability theory [7], or within another nonadditive uncertainty theory having a possibility to represent situations that are connected with the terms like vagueness, ignorance, or ambiguity. The same role that is played by a probability distribution (measure) in probability theory can be played by several functions in the theory of belief functions. In our brief exposition, we will do just with three of them: basic probability assignment, belief and plausibility functions.

Suppose X is a random variable with state space Ω_X. Let 2^{Ω_X} denote the set of all *non-empty* subsets of Ω_X. A *basic probability assignment* (BPA) m for X is a function $m : 2^{\Omega_X} \to [0, 1]$ such that

$$\sum_{\mathsf{a} \in 2^{\Omega_X}} m(\mathsf{a}) = 1.$$

The subsets $\mathsf{a} \in 2^{\Omega_X}$ such that $m(\mathsf{a}) > 0$ are called *focal* elements of m. An important example of a BPA for X is the vacuous BPA for X, denoted by ι_X, such that $\iota_X(\Omega_X) = 1$. It corresponds to total ignorance. If all focal elements of m are singletons (one-element subsets) of Ω_X, then we say m is *Bayesian*. In this case, m is equivalent to a probability distribution.

In the theory of belief functions, the fact that $\mathbf{a} \subseteq \Omega$, for which $|\mathbf{a}| > 1$, is a focal element for BPA m, expresses our ignorance regarding how the probability mass $m(\mathbf{a})$ is distributed among the elements of set $\mathbf{a}$. For example, suppose $\Omega = \{x_1, x_2\}$, and BPA m is defined as follows: $m(\{x_1\}) = 0.2$, $m(\{x_2\}) = 0.3$, $m(\{x_1, x_2\}) = 0.5$. This m represents the knowledge that the probability of x_1 is at least 0.2 and at most 0.7, and the probability of x_2 is at least 0.3 and at most 0.8. We know nothing more, nothing less.

As said above, the information in a BPA m can be equivalently represented by corresponding *belief* and *plausibility* functions Bel_m and Pl_m, respectively that are defined as

$$Bel_m(\mathsf{a}) = \sum_{\mathsf{b} \in 2^{\Omega_X} : \, \mathsf{b} \subseteq \mathsf{a}} m(\mathsf{b}), \qquad Pl_m(\mathbf{a}) = \sum_{\mathbf{b} \in 2^{\Omega} : \, \mathbf{b} \cap \mathbf{a} \neq \emptyset} m(\mathbf{a}),$$

for all $\mathsf{a} \in 2^{\Omega_X}$. Notice that it is obvious that for all $\mathbf{a} \in 2^{\Omega}$, $Bel(\mathbf{a}) \leq Pl(\mathbf{a})$. If $Bel(\mathbf{a}) = Pl(\mathbf{a})$ then we are sure that the probability of $\mathbf{a}$ equals this value. Otherwise, the larger difference $Pl(\mathbf{a}) - Bel(\mathbf{a})$ the more ambiguity about the value of the probability of $\mathbf{a}$. This follows from the fact that like Bayesian BPA corresponds to a unique probability distribution, a non-Bayesian BPA m corresponds to the following convex set of probability distributions on Ω called a *credal set* of BPA m ($\mathcal{P}$ denote the set of all probability distributions on Ω):

$$\mathcal{P}(m) = \left\{ P \in \mathcal{P} : \sum_{x \in \mathsf{a}} P(x) \geq Bel_m(\mathsf{a}) \text{ for } \forall \mathsf{a} \in 2^{\Omega} \right\}.$$

3 Decision making

Savage's decision-making theory [18] is based on the computation of expected utility. To this end, we need a respective (subjective) probability distribution. In our approach, we follow the same basic idea, but to compute a value of decision criterion we do not use a subjective probability distribution but another function that, from the mathematical point of view, manifests properties of a *superadditive capacity*. Such a function is deduced from the credal set $\mathcal{P}(m)$ in two steps described below [13]. In the first step, we select a probability distribution, which in a way represents the knowledge from the considered credal set – this is done by a probability transform – and by a subsequent subjective reduction that models an ambiguity aversion of a decision-maker.

In the belief function theory, there are several methods how to find a mapping that assigns a probability distribution to each BPA [2]. As examples, let us mention just two of them: well-known *Maximum entropy* element of $\mathcal{P}(m)$, and *pignistic transform*, advocated by [22, 23], defined by the formula

$$Bet\_P_m(x) = \sum_{\mathbf{a} \in 2^{\Omega} : x \in \mathbf{a}} \frac{m(\mathbf{a})}{|\mathbf{a}|}.$$

Notice that the importance of the latter transform is stressed by the fact that, as it was recently proved by [17], it coincides with the famous Shapley value [20] known from the game theory.

As already explained, the maximization of the expected utility does not correspond to the human way of decision-making. Therefore, we do not use a probability distribution to compute an expected value. We assume that the ambiguity aversion makes a decision-maker to underestimate the probabilities of some events. The greater ambiguity, the greater underestimation. Therefore we reduce a probability P_m (got by some of the above-introduced probability transforms) to get a personalized weights

$$r_{m,\alpha}(x) = (1 - \alpha) P_m(x) + \alpha Bel_m(\{x\}),$$

where the coefficient $\alpha \in [0, 1]$ reflects the level of the ambiguity aversion of a considered decision-maker. Notice, that the amount of reduction depends not only on the ambiguity aversion coefficient α but also on the amount of ignorance associated with the state x. If we are certain about the probability of state x, it means that $P_m(x) = Bel_m(\{x\})$, the corresponding probability is not reduced: $r_{m,\alpha}(x) = P_m(x)$. On the other hand, the maximum reduction is achieved for the states connected with maximal ambiguity, i.e., for the states for which $Bel_m(\{x\}) = 0$.

4 Experiments

People are different. Some are risk-takers, some are risk-averse. We believe that the behavior can be modeled using the theory of belief functions and that each decision-maker has different strength of ambiguity aversion – possibly expressible using coefficient α defined above. We even found some people with negative coefficient using our experiments - they are risk-takers. Using the following experiments we would like to prove or disapprove whether the ambiguity aversion is a personal characteristic of the decision-maker and whether it is possible to measure it.

As far as we know, nobody has studied an inter-temporal behavior of an individual decision-maker under different scenarios concerning ambiguity aversion up to now. Therefore, though keeping the anonymity of the experimental individuals, we have designed our experiments to analyze the behavior of a decision-maker facing different problems, plus we plan to test one problem in different experimental sessions (e.g., half a year afterward). This was however not done so far. This should also testify whether the coefficient of ambiguity aversion (mentioned in the previous section) is a personal characteristic of a decision-maker.

What is important, no personal data are collected and stored. All persons participating the behavioral testing are identified by their identifiers (a sequence of characters of their personal choice). To be able to answer some statistical questions, the only information of personal character are age, sex, and education. The main information stored mirror the behavior of experimental persons in specified situations of a gamble.

4.1 Experiment design

Discussions with psychologists have revealed some interesting insights. People behave differently in real situations comparing to a presented hypothetical situation. How to achieve real behavior in a laboratory environment? One option is to use the concept of money by putting the participant's own money into the experiment. The fact of using their own money is crucial – the behavior when playing with artificial money or money belonging to someone else is different. Similarly, to minimize the influence of the ordering of the individual tasks, we have to give them to each participant in different random order.

Participation in the experiment is rewarded. People will receive 50CZK but this amount is paid before the experiment and it is emphasized that fact that this money belongs to them and they can keep it (and use it in the experiment as well). Using this we expect that the participants will feel that they play with your own money.

The experiment is realized in the form of a lottery. Imagine an urn with colored balls. Colors are known, information about the number of balls of each color varies according to the actual lottery. In total there are about 12 different scenarios. Players will receive all known information about the urn content and must decide which color to bet on and how much to bet. Depending on the amount of the bet, they participate in a real lottery. If the player guesses the color of the draw, he/she wins 100CZK.

The participants can use the money they received as a reward for participation as an input "capital" for the lottery games they are participating in. Of course, one can bet more than 50CZK in the sum to increase the change of winning. After betting, the lottery is played and any eventual winnings are paid out.

Because the participants receive game situations in a different order, the lotteries cannot be played immediately. The participants have to go through all the situations and bet on all games. Then the betting is closed and the lotteries are performed in reality. To do so, we have a real urn and real balls. Following the description of each situation, we randomly fill the urn and then one of the participants randomly drawn a ball. Note that this part is no longer important for our needs and we do not collect information about winnings and losses. Nevertheless, it is vital for the real-life feeling of the participants.

4.2 Typical session

Typically, the session is organized as follows. After an introduction of the goal of the research, the participants are asked to select their identifier. The sessions are usually held in a computer laboratory so that each person answers the questions using a keyboard.

The assistant then gives out an information leaflet with the following text:

4.3 Information letter

*Dear participants of **Decision Making under Uncertainty** experiment.*

When designing new methods of artificial intelligence, it is necessary to know how people's behavior changes when they face varying degrees of lack of information. Therefore, we sincerely thank you for your help in participating in this experiment. Please, accept 50 CZK as a little reward for this help and also as an input "capital" for several lottery games on which the experiment is based. The assistant will reward you within the next few minutes.

*Please note that this is a statistical **anonymous survey**. We do not collect or store your personal information. Nevertheless, we would like to know if there is a difference in the behavior of men and women, students and mature managers. Therefore, we ask you for information about your sex, age, and education. Since we would like to know if you are always behaving the same (or alike), we would like to welcome you to participate in experiments repeatedly, so please also sign up with a nickname that you will remember for the next time.*

After you start your computer and sign in with your nickname, your computer presents you with a variety of situations. In each of them, you can participate in the draw and win 100 CZK.

How the given lotteries differ? For each lottery you will learn some (even incomplete) information about the contents of the lottery urn in which there are colored balls:

- *You will always find out whether balls that can appear in the urn are of three colors (black, white, yellow) or six colors (black, white, yellow, red, green, blue).*
- *You can learn how many balls are in the urn (but you don't have to).*
- *You can know the exact number of balls of one (or several) colors used in the draw (but you don't have to). In almost all situations, however, you will lack*

information on the ratio of other colors. In this case, one of the colors maybe not presented in the urn at all. For example, if you only know that there are eight balls in the urn of six possible colors, and just one of them is red, then maybe only balls of two colors are presented in the urn. Or you can imagine an urn with five balls only, each of six possible colors. Of course, one color may be missing at all.

For each lottery you must specify:

- *Which color is the winning color for you?*
- *How much you are willing to bet to participate in the game.*

When the betting is finished by all participants, all the draws will be realized (we cannot realize them during the data collection because the individual situations are presented to you in random order). Therefore, when you fill in the data on your computer, think well about each situation, because you have a chance to win, but also to lose real money. Only a part of you is involved in each draw. The bigger the amount you bet, the more chance you will participate in the draw. On the other hand, you also risk this amount if your color is not drawn. You are no longer allowed to withdraw from the game during the draw. If the computer selects you in the game, it will deduct the money you bet and, in case of a win, it will credit you 100 CZK. Be aware that lotteries are designed so that the vast majority of you have a big chance of winning (even over 300 CZK). However, some of you will lose (although losing more than the 50 CZK you received as an entry reward is unlikely - even though it has already happened). Any winnings and losses are settled with the assistant after the experiment.

After everyone has read the information letter, the web-based application is launched and participants go through different lotteries and answer questions about selected color and bet. Recall that each participant receives lotteries in random order to minimize the impact of the order on experiment results. In the following example, you can see a typical lottery from the experiment.

In the experiment, we want to estimate the aversion to uncertainty. To calculate it, we need to get the maximum amount the player is willing to bet on his chosen color. So, we want to push him into the highest bet he is still willing to make. We do this by limiting the number of participants in a real lottery. At the beginning of the experiment, it is announced that the number of participants in each real game will be limited to about a third of all participants, mainly based on the amount staked. More precisely, the greater part of the players is selected based on the amount staked (the larger the bet, the more likely you are to play a real game). The smaller part is then selected randomly.

So if you don't bet enough money in the game, it's highly likely that you won't participate in real games at all. So you can neither to win nor to lose.

4.4 Example

Figure 1 illustrates the design of the application. Because we expect to have Czech participants only, the application is in Czech only so far. The description of the lottery can be translated as follows:

Experiment

Task 1/8 - (situation no. 9)

Urn A

Number of balls:

- *total number:* ***9***
- *number of colors:* ***6***
- *red balls:* ***exactly one***
- *numbers of balls of other colors are unknown*

Question

Choose a color. If the randomly drawn ball has the color you have selected, you win 100 CZK. How much are you maximally willing to pay to take part in the lottery?

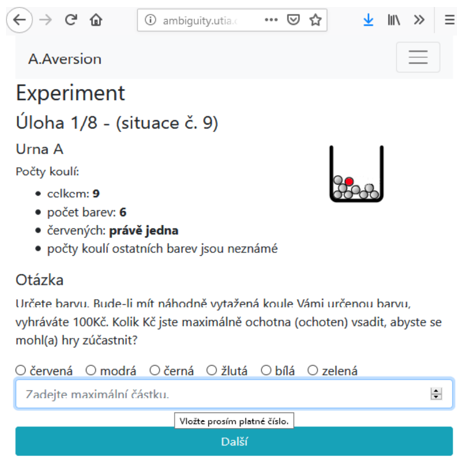

Figure 1: Screenshot from the application

Then you can see the list of six colors and a text input form for your bet. You submit your bet and color by clicking on the button. Then the next task is shown. The lottery is illustrated by a sketch of an urn with 9 balls. Exactly one of them is red and the rest is gray to emphasize that we have no information about their colors.

5 Experiences

During the first few experiments, we have discovered several problems in the experiment settings.

- The game labeled as **Rn**, which contains several lotteries, was split into separate lotteries in the first experiment. The results of the experiment were confusing and inconsistent across participants. It seems that people are not able to keep in mind the individual variants of this game and therefore the associated game Rn was created. The variants are next to each other.
- In the second experiment, we did not sufficiently emphasize the fact that only a part of the participants with the highest bet will participate in the draw. This led to the printed manual you saw in the previous section.

ID	Ignorance Red	Ignorance Color	Uniform 30 Red	Uniform 30 Color	One red in 5	One red in 6	One red in 7	One red in 8	One red in 9	One red in 10	One red in 11	One red in 12	Ellsberg One color	Ellsberg Two colors
1	10	10	10	10	5	5	5	5	5	5	5	5	15	15
2	5	7	16	10	1	1	1	1	1	1	1	1	8	17
3	3	1	16	16	20	16	13	12	11	10	9	8	33	66
4	0	0	20	20	10	0	0	0	0	0	0	0	10	30
5	1	1	1	20	1	1	1	1	1	1	1	1	1	10
6	10	10	16	16	19	15	14	12	10	10	10	10	32	20
7	16	15	16	11	6	10	6	2	5	3	2	7	15	11
8	10	20	10	12	15	8	4	5	5	5	5	5	20	20
9	3	6	20	17	10	5	5	4	3	2	1	1	23	15
10	3	3	16	15	19	15	14	12	11	9	9	8	33	33
11	1	0	5	5	5	10	0	0	0	0	0	0	5	5
12	6	2	0	9	2	6	1	1	1	1	1	1	10	13
13	14	14	14	14	19	14	14	14	15	15	15	15	24	24
14	1	3	19	2	16	8	11	8	7	8	9	5	26	7

Figure 2: An overview of bets on selected colors in one experiment.

- In another experiment, we came across the fact that some of the participants were playing some sort of alternative game, trying to guess how much the others would bet and bet accordingly.

- During standard experiments with students, the average profit was around CZK 130 per person. An interesting situation occurred in an experiment carried out during the traditional seminar of our institute. In this case, perhaps no one left the experiment with some winnings. Some participants lost more than 150 CZK. We are not sure about the reasons for this to happen, however, education does not seem to be an asset. Thanks to the knowledge of probability theory, the participants bet amounts corresponding to the probability of drawing the color. Unfortunately, this is not a good approach even in the long run, because you win as much as you loose only. Moreover, the draw is done only once in our case.

  Above that, even in the case of Ellsberg's case of variant E2 with a chance to win of $\frac{2}{3}$ in the case of choosing black color, the red color was finally drawn.

6 Experiment application

As already mentioned above, we created a web-based application for the betting. You can find it at http://ambiguity.utia.cas.cz/. It is a simple application written in PHP scripting language that is especially suited to web development. Data are stored in MySQL database that has 6 tables. The structure of the database is illustrated by Figure 3.

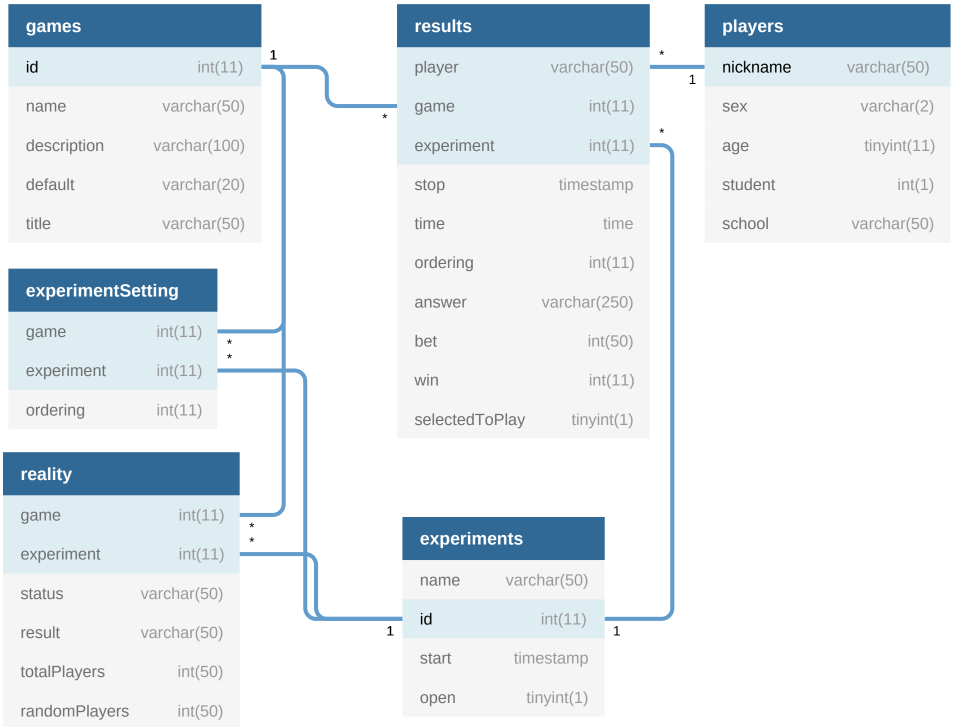

Figure 3: Database structure

- `players` - the list of participants. The table has 5 columns: nickname, sex, age, student (Yes/No), school
- `games` - the list of possible situations in the system. The table has 5 columns including unique integer identifier, name, and description
- `experiments` - the list of experiments. The table has 4 columns including unique integer identifier, name, date, and a flag whether the experiment is running (Yes/No)
- `lotterySetting` - it specifies the games(lotteries) assigned to each experiment
- `results` - table with bets and answers. It has 10 columns: game, player, time needed to finish the task, in which ordering the question arrived, etc.

7 Conclusion

Work on experiments is still ongoing. However, it is already clear that it is possible to measure a personal coefficient expressing something like ambiguity aversion and the answers seem to be consistent with this coefficient. In the future, we plan to conduct further experiments with the same participants to see where the coefficient changes over time. We are also planning to add more variants and other situations containing ambiguities.

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