OPTIMIZATION OF COSTS OF PREVENTIVE MAINTENANCE

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Abstract: In various fields of real life, many interesting optimization problems appear. The present contribution deals with optimization of maintenance of a technical device. Namely, both the period of maintenance and its level are controlled, the costs are compared with the cost caused by the device failure and necessary repair after it. We consider a variant the Kijima model assuming that the consequence of such a repair is the decrease of 'virtual' age of the object. The main objective is to formulate a proper stochastic objective function evaluating the costs of given maintenance strategy and then to present an optimization method for selected characteristics of the costs distribution.

Keywords: reliability, degradation model, maintenance, stochastic optimization.

1 INTRODUCTION

In reliability analysis, the models of imperfect repairs are mostly based on the reduction of the hazard rate, either directly or indirectly (by shifting the virtual age of the system). If the state of the system is characterized by its degradation, the repair degree can be connected with the reduction of the degradation level. We shall concentrate mainly to selection of certain repair schemes, their consequences, and possibilities of an 'optimal' repair policy leading to the hazard rate stabilization and costs minimization. The organization of the paper is the following: First, general models of repairs will be recalled. Then, a variant of the Kijima II type model for preventive repair [6] will be considered and its scheme applied to the case of a model with degradation process. Repair then will be connected with the reduction of the level of degradation. Finally, a solution searching for optimal repair parameters will be demonstrated on an artificial example.

2 BASIC REPAIR MODELS

Let us first recall briefly the most common schemes of repair of a repairable component and the relationship with the distribution of the time to failure (cf. [1]). The renewal means that the component is repaired completely, fully (e.g. exchanged for a new one) and that, consequently, the successive random variables – times to failure – are distributed identically and independently. The resulting intensity of the stream of failures is defined as

$$h(t) = lim_{d \to 0^+} \frac{P(failure\, occurs\, in\, [t, t+d))}{d}.$$

Its integral (i.e. cumulated intensity) is then $H(t) = E[N(t)] = \sum_{k=0}^{\infty} k \cdot P(N(t) = k)$, where $N(t)$ is the number of failures in (0,t].

2.1 Models of partial repairs

There are several natural ways how the models of complete repairs can be widen to repairs incomplete. One of basic contribution is in the paper [6]. Let F be the distribution function of the time to failure of a new system. Assume that at each time the system fails, after a lifetime T_n from the preceding failure, a maintenance reduces the *virtual age* to some value $V_n = y, y \in [0, T_n + V_{n-1}]$ immediately after the n-th repair ($V_0 = 0$). The distribution of the n-th failure-time T_n is then

$$P[T_n \leq x | V_{n-1} = y] = \frac{F(x+y) - F(y)}{1 - F(y)}.$$

M. Kijima then specified several sub-models of imperfect repairs. Denote by A_n the degree of the n-th repair (a random variable taking values between 0 and 1). Then in Model I the n-th repair cannot remove the damages incurred before the (n-1)th repair, $V_n = V_{n-1} + A_n \cdot T_n$. On the contrary, the Model II allows for such a reduction of the virtual age, namely $V_n = A_n \cdot (V_{n-1} + T_n)$. Special cases contain the perfect repair model with $A_n = 0$, minimal repair model, $A_n = 1$, and frequently used variant with constant degree $A_n = A$. Naturally, there are many other approaches, e.g. considering a randomized degree of repair, the regressed degree (based on the system history), accelerated virtual ageing, change of hazard rate etc., see for instance [3], [1], [5]), or [7].

2.2 A variant of Kijima model of preventive maintenance

Let us recall the following simple case of the Kijima II model with constant degree δ of virtual age reduction, and assume that it is used for the description of consequence of preventive repairs. Further, let us assume that after the failure the system is repaired just minimally, or that the number of failures is much less than the number of preventive repairs. Let Δ be the (constant) time between these repairs, V_n, V_n^* the virtual ages before and after $n-$th repair. Hence:

$$V_n = V_{n-1}^* + \Delta \text{ and } V_n^* = \delta \cdot V_n.$$

If we start from time 0, then $V_1 = \Delta,\ V_1^* = \delta\Delta,\ V_2 = \delta\Delta + \Delta = \Delta(\delta+1), V_2^* = \Delta(\delta^2+\delta),\ V_3 = \Delta(\delta^2 + \delta + 1)$ etc. Consequently, $V_n \to \frac{\Delta}{1-\delta}$, i.e. it 'stabilizes'.

Now, let us consider a variant, in which the reduction of "virtual age" means just reduction of the failure rate to the level corresponding to virtual age. I.e., for each δ and Δ there is a limit meaning that the actual intensity of failures $h(t)$ 'oscillates' between $h_0(\frac{\delta\Delta}{1-\delta})$ and $h_0(\frac{\Delta}{1-\delta})$, where $h_0(t)$ is the hazard rate of the time-to-failure distribution of the non-repaired system.

Simultaneously, the cumulated intensity increases regularly through intervals of length Δ by $dH = H(\frac{\Delta}{1-\delta}) - H(\frac{\delta\Delta}{1-\delta})$, i.e. 'essentially' with the constant slope $a = dH/\Delta$. Figure 1 shows graphical illustration of such a stabilization in the case that the hazard rate $h_0(t)$ increases exponentially.

Example: Let us consider the Weibull model, with $H_0(t) = \alpha \cdot \exp^{\beta}$, ($\beta > 1$, say). In that case

$$dH = \alpha\Delta^{\beta} \frac{1-\delta^{\beta}}{(1-\delta)^{\beta}} \text{ and } A = \alpha\Delta^{\beta-1} \frac{1-\delta^{\beta}}{(1-\delta)^{\beta}}.$$

As special cases, again the perfect repairs with $\delta = 0$, minimal repairs with $\delta \sim 1$, and the exponential distribution case with $\beta = 1$ can be considered.

Remark 1. If the model holds (with constant times between repairs Δ) it is always possible to stabilize the intensity by selecting the upper value of H^* and repair always when $H(t)$ reaches this value. Then $V_n = V = H^{-1}(H^*)$, $V_n^* = \delta V_n$ again, and the interval between repairs should be $\Delta = V(1 - \delta)$.

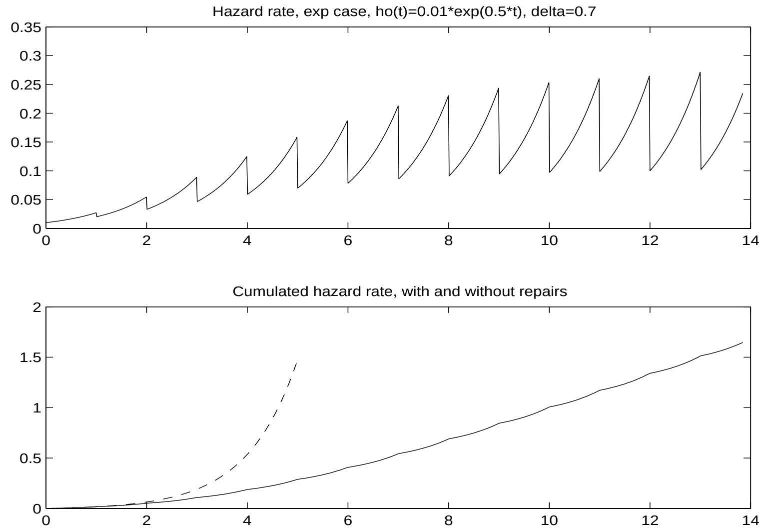

Figure 1: Case of exponentially increasing $h_0(t) = 0.01 \cdot \exp(0.5 \cdot t), \delta = 0.7, \Delta = 1$. Above: Intensity after repairs. Below: Cumulated intensity with repairs (full) and without (dashed curve).

3 TOWARDS OPTIMAL MAINTENANCE STRATEGY

Let us consider the stabilized case, as in Figure 1, and assume that the failures are much less frequent than preventive repairs, then there quite naturally arises the problem of selection of δ to given repair interval Δ (or optimal selection of both). By optimization we mean here the search for values yielding the minimal costs of repairs, which has a sense especially in the case when the repairs after failures are too expensive.

Let C_0 be the cost of failure (and its repair), $C_1(\delta, \Delta)$ the cost of the preventive repair. Then the mean costs to a time t can be written as

$$C \approx C_0 \cdot E(N(t)) + \frac{t}{\Delta} \cdot C_1(\delta, \Delta),$$

where $E(N(t))$ is the mean number of failures up to t, which actually equals $H(t)$, the cumulated intensity of failures under our repairs sequence. The proportion $\frac{t}{\Delta}$ is the number of preventive actions till t. The problem is the selection of function C_1, it should reflect the extent of repair. It leads to the idea to evaluate the level of system degradation and to connect the repair with its reduction.

3.1 Maintenance as a reduction of system degradation

Let us therefore consider a function $S(t)$ (or a latent random process) evaluating the level of degradation after a time t of system usage. In certain cases we can imagine $S(t) = \int_0^t s(u)du$ with $s(u) \geq 0$ is a stress at time u. We further assume that the failure occurs when $S(t)$ crosses a random level X. Recall also that the cumulated hazard rate $H(t)$ of random variable T, the time to failure, has a similar meaning, namely the failure occurs when $H(t)$ crosses a random level given by $Exp(1)$ random variable.

As $T > t <=> X > S(t)$, i.e. $\bar{F}_0(t) = \bar{F}_X(S(t))$, where $\bar{F}$ denote survival functions, then

$$H_0(t) = -log\bar{F}_X(S(t)).$$

We can again consider some special cases, for instance:

– $X \sim Exp(1)$, then $H_0(t) = S(t)$,

– $S(t) = c \cdot t^d$, $d \geq 0$, and X is Weibull (a, b), then T is also Weibull $(\alpha = ac^b, \beta = b + d)$, i.e. $H_0(t) = \alpha \cdot t^\beta$.

Let us now imagine that the repair reduces $S(t)$ as in the Kijima II model, to $S^*(t) = \delta \cdot S(t)$. In the Weibull case considered above we are able to connect such a change with the reduction of virtual time from t to some t^*: $S(t^*) = S^*(t) => t^* = \delta^{\frac{1}{d}} \cdot t$, so that the virtual time reduction follows the Kijima II model, too, with $\delta_t = \delta^{\frac{1}{d}}$. As it has been shown, each selection of δ, Δ leads (converges) to a stable ('constant' intensity) case.

For other forms of function $S(t)$, e.g. if it is of exponential form, $S(t) \sim e^{ct} - 1$, such a tendency to a constant intensity does not hold. Nevertheless, it is possible to select convenient δ and Δ, as noted in Remark 1.

3.2 Degradation as a random process

In the case we cannot observe the function $S(t)$ directly, and it is actually just a latent factor influencing the lifetime of the system, it can be modelled as a random process. What is the convenient type of such a process? There are several possibilities, for instance:
1. $S(t) = Y \cdot S_0(t)$, $Y > 0$ is a random variable, $S_0(t)$ a function.
2. Diffusion with trend function $S_0(t)$ and $B(t)$-the Brown motion process, $S(t) = S_0(t) + B(t)$.
3. $S(t)$ cumulating a random walk $s(t) \geq 0$.
4. Compound Poisson process and its generalizations, see for instance [4].

Though the last choice, sometimes connected also with the "random shock model", differs from the others, because its trajectories are not continuous, we shall add several remarks namely to this case. The compound point process is the following random sum

$$S(t) = \sum_{T_j < t} Y(T_j) = \int_0^t Y(u) dN_s(u)$$

with the counting (mostly Poisson) process $N_s(t)$ yielding the random times T_j and random variables $Y(t) > 0$ giving the increments. Let λ be the intensity of Poisson process, μ, σ^2 the mean and variance of increments, then it holds

$$ES(t) = \int_0^t \lambda(u) \cdot \mu(u) du,$$

$$var(S(t)) = \int_0^t \lambda(u) \cdot (\mu^2(u) + \sigma^2(u)) du.$$

Again, let us assume that the failure occurs when the process $S(t)$ crosses a level x. Then $S(t) < x <=> t < T$, therefore $\bar{F}_0(t) = F_{S(t)}(x)$, where $\bar{F}_0(t)$ denotes again the survival function of the time to failure and $F_{S(t)}(x)$ is the compound distribution function at t. If X is a random level, then the right side has the form $\int_0^\infty F_{S(t)}(x) dF_X(x)$. The evaluation of the compound distribution is not an easy task, nor in the simplest version of compound Poisson process. There exist approximations (derived often in the framework of the financial and insurance mathematics, see again [4]). Another way consists in random generation.

4 PARTIAL MAINTENANCE OPTIMIZATION

What occurs when, as in the preceding cases, the repairs reducing degradation, with degree δ, are applied in regular time intervals Δ? It is assumed that when we decide to repair, then we are able to observe actual state of $S(t)$. Random generation shows that the system then has the tendency to stabilize the intensity of failures, as in Figure 1.

We can now return to the 'cost optimization' problem. Function $C_1(\delta, \Delta)$ can now be specified for instance as $C1 \cdot (dS(t))^\gamma + C_2$, where $dS(t) = S(t)(1 - \delta) = S(t_{end}) - S(t_{init})$. Here

C_1 and C_2 are constants, the later evaluating a fixed cost of each repair. Of course, a proper selection of costs and function $C1$ in real case is a matter of system knowledge and experience. We performed several randomly generated examples, with different variants of the objective function (which was stochastic), with the goal to find optimal maintenance parameters, in the sense of minimization of costs (i.e. their mean, or median, or other quantile).

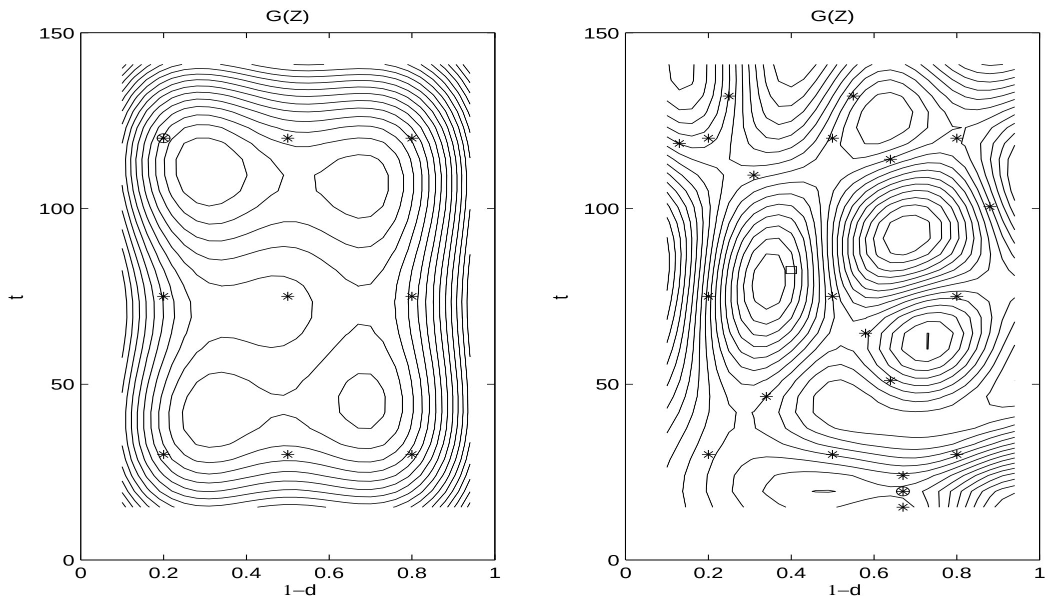

Figure 2: Example of optimal maintenance: Initial phase of search (left), state of search after 12 iterations (right); $1-\delta$ on horizontal, τ on vertical axis.

4.1 Example of optimal maintenance

Let us again assume, in a Kijima II model of non-complete repair, that the device is repeatedly repaired in its virtual age τ with a degree $1-\delta$, which means that after repair the virtual age of the device is $\delta \cdot \tau$. Then the parameter Δ of inter-maintenance times equals $(1-\delta)\cdot\tau$.

In the example it is assumed that the Kijima model concerns to preventive repairs, meanwhile after the failure the device has to be renewed completely. We are given the costs of renewal, C_0, and of preventive repair, $C_1(\delta,\tau)$. It is due the problem assumptions that the objective can be formulated as to maximize, over τ and δ, selected characteristics of random objective function $\varphi(T,\delta,\tau)$ equal to proportion of the time to renewal to the costs to renewal. Here T is the random time to failure of the device. This proportion equals

$$\varphi(T,\delta,\tau) = \frac{T}{C_0} \quad \text{with probability} \quad P(T \le \tau),$$

$$\varphi(T,\delta,\tau) = \frac{\tau + \tau\cdot(1-\delta)\cdot(k-1) + T_k}{C_0 + k\cdot C_1} \text{ with } P(T > \tau)\cdot P(T_1 > \tau)^{k-1}\cdot P(T_1 \le \tau),$$

where $T_1 = \{T | T > \tau\cdot\delta\}$ and k is the number of preventive repairs before the failure. It is due the fact revealed in sect. 2.2 and shown in Figure 1, that the hazard rate stabilizes and after each preventive action the conditional distribution above is (approximately) the same.

The direct evaluation of objective function is not easy, moreover, it is strongly non-concave. Therefore, the distribution of variable $Y(\delta,\tau) = \varphi(T,\delta,\tau)$, for different δ,τ, is obtained ‚empirically' by random generation, its characteristics then as sample characteristics. The choice could be the mean, median, or certain quantiles.

For numerical illustration we selected $T \sim \text{Weibull}(a = 100, b = 2)$, with survival function $\overline{F}(t) = \exp\Big(-\big(\frac{t}{a}\big)^b\Big)$, $\mathrm{E}T \sim 89$, $\mathrm{std}(T) \sim 46$. Further, the costs $C_0 = 40$, $C_1 = 2 + ((1-\delta)\cdot\tau)^\gamma$, $\gamma = 0.2$. Such a selection of C_1 corresponds to case when the degradation $S(t) \sim t$, in the sense of previous discussion, value 2 stands for fixed costs. We decided to maximize the $\alpha = 0.1$ quantile of distribution of $\varphi(T, \delta, \tau)$. Optimal parameters were found with the aid of the Bayes optimization method (cf. [2]) using 2-dimensional Gauss process as an approximation of the 10% quantile of the objective function. Such a choice says that (roughly) with 90% probability the value of $\varphi(T, \delta, \tau)$ will be larger than found maximal value.

Figure 2 shows the results. The procedure started from its Monte Carlo generation in 9 points showed in the left plot. Maximum is denoted by a circle, its value was 0.876. The plot contains also contours of resulting Gauss process surface. The right plot shows the situation after 12 iterations. It is seen how the space was inspected, maximal value was stabilized around 1.124, the corresponding point $(1-\delta \sim 0.7, \tau \sim 20)$ is again marked by a circle.

5 CONCLUSION

In the present paper, first, several variants of the Kijima II model were presented, relating the maintenance degree to the reduction of the followed technical object degradation. The main objective then was to show how such models can be connected with maintenance costs evaluation, and, finally, with stochastic optimization problem. One example of such a task was formulated in detail and solved, with the aid of Bayes optimization approach, though other procedures of randomized search are applicable as well.

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