Two-layer pointer model of driving style depending on the driving environment

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Abstract

This paper deals with the task of modeling the driving style depending on the driving environment. The model of the driving style is represented as a two-layer mixture of normal components describing data with two pointers: outer and inner. The inner pointer indicates the actual driving environment categorized as "urban", "rural" and "highway". The outer pointer through the determined environment estimates the active driving style from a fuel economy point of view as "low consumption", "middle consumption" and "high consumption". All of these driving styles are assumed to exist within each driving environment due to the two-layer model. Parameters of the model and the driving style are estimated online, i.e., while driving using a recursive algorithm under the Bayesian methodology. The main contributions of the presented approach are: (i) the driving style recognition within each of urban, rural and highway environments as well as in the case of switching among them; (ii) the two-layer pointer, which allows us to incorporate the information from continuous data into the model; (iii) the potential use of the data-based model for other measurements using corresponding distributions. The approach was tested using real data.

Keywords: driving style, driving environment, fuel consumption, two-layer pointer, recursive mixture estimation, mixture-based clustering

1 Introduction

Driving style recognition is a highly desired task in the area of in-vehicle information systems responsible for intelligent vehicle control and fuel efficiency, e.g., (Zhang et al., 2010; Wang and Lukic, 2011; Huang et al., 2012; Xu et al., 2015; Büyükyildiz et al., 2017; Martinez et al., 2017).

According to the definition in (Elander et al., 1993; Lajunen and Özkan, 2011; Sagberg et al., 2015), driving style can be understood as a set of driving habits accumulated by a driver with increasing driving experience. Therefore, the formation of an individual's driving style depends on the environment, where the individual has accumulated the driving experience most of the time. For example, urban inhabitants driving mostly in the city will have to adapt their style for highway driving and naturally do not drive optimally for some time and vice versa, an individual without experience with city congestions will have to spend some time to get used to such driving. It indicates that

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the generally economical driving style of some individual can differ when changing the driving environment. In terms of modeling, it means that the driving style is switching not only among different environments, but it also changes within each of them, i.e., the model can have some kind of a two-layer structure, which is presented in this study.

As noted in (Suzdaleva and Nagy, 2018), driving style is modeled primarily in terms of: (i) safety (Evans, 1996; Sagberg et al., 2015; Eboli et al., 2017), (ii) vehicle dynamics control (Nelles, 2003; Plöchl and Edelmann, 2007; Xu et al., 2015; Bellem et al., 2016), (iii) fuel economy (Ma et al., 2015; Ferreira et al., 2015; Pampel et al., 2015) and (iv) ecology (Sentoff et al., 2015; Rangaraju et al., 2015; Gallus et al., 2017), which are the factors greatly affected by driving style. These factors are closely related to each other. However, their choice influences a construction of a driving style model. In this paper, the driving style is considered from a fuel consumption efficiency point of view depending on the driving environment.

1.1 Related work

Investigating driving style in terms of reducing fuel consumption is discussed in a number of studies. A significant number of publications are aimed at exploring the impact of driving style on fuel consumption. (Lee and Son, 2011) in their study dealt with the correlation analysis of driving style and fuel consumption in a highway environment. (Ma et al., 2015) focused on the influence of driving parameters on the fuel consumption of city buses while accelerating, normal running and decelerating with the help of a vehicle-engine combined model. The correlation between driving style, fuel consumption and cultural factors in two different countries is compared in (Son et al., 2016). The extensive analysis of the factors influencing fuel consumption was presented in (Zhou et al., 2016). (Akena et al., 2017) investigated the influence of operating the vehicle, vehicle dynamics and driver awareness on driving style in terms of fuel consumption with the help of a multi-criterial hierarchical approach. (Faria et al., 2019) combined the analysis of the relationship of driver aggressiveness, fuel consumption and driving environment. Such kind of studies are extremely helpful when choosing suitable variables for a data-based driving style model.

Another group of studies focuses on the classification of driving style regarding fuel efficiency. (Manzoni et al., 2010) dealt with the driving style quantification from a fuel economy point of view using the vehicle longitudinal model. Data mining techniques and neural networks were applied to the driving style classification by (Meseguer et al., 2017), where they analyzed both drivers' and driving data from the vehicle's electronic control unit. The impact of driver behavior on fuel consumption was analyzed and classified by (Peng et al., 2019) using machine learning. In (Javanmardi et al., 2017), the control-based driving style model concerned with fuel economy was proposed. Mental models classifying driving style with respect to fuel efficiency were considered by (Pampel et al., 2015). As the high correlation between driver aggressiveness and fuel consumption was reported, methods of the driving style classification jointly in terms of road safety and fuel economy can be found in (Constantinescu et al., 2010; Ferreira et al., 2015) based on data mining techniques, (Wang et al., 2017) with the help of a semi-supervised support vector machine, (Li et al., 2017) using maneuver transition probabilities to identify high-risk drivers, etc.

Driving style optimization with the aim of reducing fuel consumption was the issue considered by one more category of studies in the discussed field, see, e.g., (Malikopoulos and Aguilar, 2012; Rios-Torres et al., 2018), etc.

As regards modeling driving style depending on the driving environment, (Liessner et al., 2016; Meseguer et al., 2017; Büyükyildiz et al., 2017; Javanmardi et al., 2017; Faria et al., 2019) distinguished driving styles among urban, suburban and highway driving environments. However, despite the considerable number of publications (not limited by the ones mentioned), systematic research, which would lead to the construction of the general probabilistic model of driving style and driving environment was not found. Such a model is expected to be suitable for the description of driving style within each of the environments as well as in the case of switching among them. This motivated us to focus on the research presented.

This paper proposes a construction of the driving style model depending on the driving environment using a two-layer mixture model, which consists of normal components and two pointer models (Kárný et al., 1998). The pointer variable of the outer mixture points to the active driving style, while the mixture components describe the data measured while driving within individual driving styles. The pointer of the inner mixture indicates the actual driving environment with the help of the data described by the inner mixture components. This two-layer construction of the model allows us to take into account the continuous data from the inner mixture for the estimation of the driving style. This idea can be more transparent in Figures 1 and 2. The first one provides a scheme of a mixture model, where the results of modeling are the continuous output variable along with the discrete pointer, which is used for classification.

Figure 1: The graphical representation of a mixture model

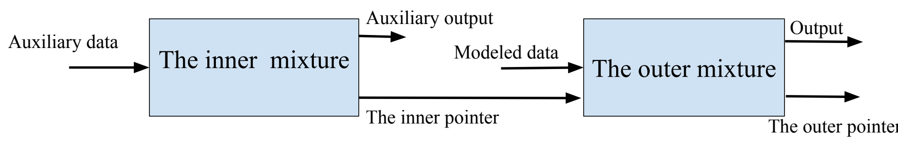

Figure 2: The scheme of a two-layer mixture model

Unlike this scheme, Figure 2 shows the interconnection of two mixture models. The inner mixture (see the left part of the figure) provides values of the inner pointer depending on the data modeled by this part of the model. The name *inner* indicates the fact that this mixture is hidden – it does not model the target data but some auxiliary measurements, which bring information about the modeled ones. The main mixture is the outer one (see the right part of the figure), which describes the target variables to be modeled. It obtains both the inner pointer and the data to be modeled. Thus, Figure 2 indicates that letting the inner pointer be hidden, continuous auxiliary measurements influence the outer pointer used for the classification. In this way, this pointer is the data-dependent, where the data variables are continuous. This structure is used for modeling the driving style depending on the driving environment.

For the estimation of the driving style and driving environment along with mixture parameters, recursive Bayesian algorithms based on (Peterka, 1981; Kárný et al., 1998; Kárný et al., 2006) are used. They were proposed for individual normal models in (Peterka, 1981), categorical models (e.g., for the pointer description) in (Kárný et al., 2006), normal mixtures with the static pointer in (Kárný et al., 1998) and with the dynamic one in (Nagy et al., 2011). The generalization of this approach in the unified form for various types of components was published in (Nagy and Suzdaleva, 2017). The study (Suzdaleva and Nagy, 2018) presented an extension of the mentioned algorithms, where the pointer indicating the driving style was allowed to be dependent on discrete or discretized data. Here, the two-layer mixture model opens a way to introduce the pointer generally dependent on continuous data too via the inner mixture. A straightforward way is to use the logistic regression but it is avoided to keep the recursive estimation, which enables real-time performance of the algorithms. This is one of the most important demands for further development of the adopted methodology applied in this study to the driving style recognition. The main contributions of the presented approach are:

- the driving style detection within each of the urban, rural and highway environments and in the case of driver's switching among them;
- the two-layer pointer, which allows us to incorporate the information from continuous data into the driving style model;
- the potential use of the data-based model for other measurements using corresponding distributions.

The last indicates the possibility of using the approach for driving style detection in terms of safety, ecology or vehicle dynamics control.

1.2 Problem formulation

The problem to be solved in this study can be verbally formulated as follows. Based on data actually measured along with the data history and using the adopted methodology, it is necessary to:

- construct a two-layer mixture model of the driving style dependent on the driving environment;
- estimate recursively parameters both of the mixtures;
- estimate the inner pointer corresponding to the actual driving environment;
- estimate the outer pointer indicating the actual driving style at each time instant;
- validate the model using real measurements.

The general solution to the problem is given in Section 2, which introduces the models used and presents the estimation algorithm. The application of the mentioned algorithm to the driving style estimation is demonstrated in Section 3, which provides the model specification, the initialization of the algorithm, results and a discussion. Conclusions are given in Section 4.

2 Theoretical background

2.1 Two-layer mixture model

Let's consider a multi-modal system, which produces the auxiliary variable z_t and the target variable y_t at discrete time instants $t \in \{1, \ldots, T\}$. Both the variables are continuous and, in general, multi-dimensional. In the considered context of modeling the driving style in terms of fuel efficiency, the target variable y_t is naturally the fuel consumption. The auxiliary data z_t can be any variables, which bring information about y_t.

The system is described by a *two-layer mixture* model of the following structure:

Outer layer: describes the target variable y_t as a mixture of m_c components (in this paper, driving styles).

> **Outer pointer model:** a $m_k \times m_c$ matrix of probabilities, one for each combination of m_k driving environments and m_c driving styles.

Inner layer: models the auxiliary variable z_t as a mixture of m_k components (here, driving environments).

> **Inner pointer model:** a vector of m_k probabilities, one for each category of driving environment.

The specification of the two-layer mixture model and the theoretical background of its estimation is given below.

The outer layer

The outer layer is a mixture of m_c Gaussian components in the form of the probability density functions (pdf)

$$f\left(y_t|\Theta, c_t = i, \psi_t\right) \sim \mathcal{N}_y(\underbrace{\psi_t\theta_i}_{\text{mean}}, \underbrace{r_i}_{\text{variance}}), \text{ where } i \in \{1, 2, \ldots, m_c\}, \tag{1}$$

where $\Theta = \{\theta_i, r_i\}_{i=1}^{m_c}$ are the unknown parameters of the components, ψ_t is the regression vector and c_t is the discrete *pointer* variable (Kárný et al., 1998; Kárný et al., 2006), which points to the active outer component (here, the active driving style) generating the target variable y_t at time t. The outer pointer model describes transitions between values of c_t from the set $\{1, 2, \ldots, m_c\}$ as the conditional probability function (also denoted by pdf)

$$f\left(c_t = i|k_t = j, \alpha\right) = \tag{2}$$

	$c_t = 1$	$c_t = 2$	$\cdots$	$c_t = m_c$
$k_t = 1$	$\alpha_{1\|1}$	$\alpha_{2\|1}$	$\cdots$	$\alpha_{m_c\|1}$
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$k_t = m_k$	$\alpha_{1\|m_k}$	$\cdots$	$\cdots$	$\alpha_{m_c\|m_k}$

where $\alpha_{i|j} \in \alpha$ are the unknown probabilities of the pointer $c_t = i$ under condition that $k_t = j$, where the discrete variable k_t is the pointer of the inner mixture, which points to the active inner component (here, the driving environment) generating the auxiliary variable z_t.

The inner layer

The inner layer represents a mixture of m_k Gaussian components

$$f\left(z_t|\vartheta, k_t = j, \varphi_t\right), \text{ where } j \in \{1, 2, \ldots, m_k\}, \tag{3}$$

where ϑ are the unknown parameters of the components and φ_t is the regression vector. The model of the inner pointer k_t describes transitions between driving environments from the set $\{1, 2, \ldots, m_k\}$ as follows:

$$f\left(k_t = j|\beta\right) = \tag{4}$$

	$k_t = 1$	$k_t = 2$	$\cdots$	$k_t = m_k$
	β_1	β_2	$\cdots$	β_{m_k}

where $\beta = \{\beta_j\}_{j=1}^{m_k}$ are the unknown probabilities of the values of the pointer k_t.

The estimate of the inner pointer k_t (which will be explained later) depends on the auxiliary continuous variables. They influence the pointer k_t, which enters the pointer model of the outer mixture (2). It allows the outer pointer c_t to be influenced by them too via the inner mixture pointer. In this way, the introduced model is a mixture with the mixed-data dependent pointer.

2.2 Clustering with the two-layer mixture

The key point of the cluster analysis with a mixture model (not only the introduced one) is the determination of the active component, i.e., the pointer should be estimated along with mixture parameters. If the clustering should be performed online in real time, the active component is determined at each time instant t. The algorithms used in this area are based on the recursive Bayesian estimation of individual normal regression models in (Peterka, 1981), categorical models (Kárný et al., 2006), mixtures with the static pointer (Kárný et al., 1998) and with the dynamic pointer in (Nagy et al., 2011). This paper develops the adopted methodology for the case of its mixed-data dependent version keeping the unified form of the recursive algorithms.

As it is customary in the mentioned field, the derivation of the algorithm is based on the construction of the joint pdf of all unknown variables to be estimated and the application of the Bayes and chain rules, e.g., (Peterka, 1981). Here, the parameters and pointers both of the mixtures enter the joint pdf, i.e.,

$$\underbrace{f\left(\Theta, c_t = i, \alpha, \vartheta, k_t = j, \beta | D\left(t\right)\right)}_{\text{joint pdf}} \propto \underbrace{f\left(y_t, z_t, \Theta, c_t = i, \alpha, \vartheta, k_t = j, \beta | D\left(t-1\right)\right)}_{\text{via Bayes rule}}, \tag{5}$$

where $D\left(t-1\right)$ denotes all of the historical measurements needed for the estimation including prior knowledge and $D\left(t\right)$ denotes all of the data up to the time instant t, including actual y_t and z_t. Assuming the mutual independence of the parameters, the right hand part of (5) is decomposed using the chain rule and models (1) – (4) as follows:

$$f\left(y_t|\Theta, c_t = i, \psi_t\right) f\left(z_t|\vartheta, k_t = j, \varphi_t\right) f\left(c_t = i|k_t = j, \alpha\right) f\left(k_t = j|\beta\right) \underbrace{f\left(\Theta, \vartheta, \alpha, \beta, |D\left(t-1\right)\right)}_{\text{prior pdfs}}$$

$$\begin{aligned} &= \overbrace{\underbrace{f\left(y_t|\Theta, c_t = i, \psi_t\right)}_{\text{model (1)}} \underbrace{f\left(\Theta|D\left(t-1\right)\right)}_{\text{prior GiW pdf for } \Theta} \underbrace{f\left(c_t = i|k_t = j, \alpha\right)}_{\text{model (2)}} \underbrace{f\left(\alpha|D\left(t-1\right)\right)}_{\text{prior Dir pdf for } \alpha}}^{\text{outer mixture}} \\ &\times \overbrace{\underbrace{f\left(z_t|\vartheta, k_t = j, \varphi_t\right)}_{\text{model (3)}} \underbrace{f\left(\vartheta|D\left(t-1\right)\right)}_{\text{prior GiW pdf for } \vartheta} \underbrace{f\left(k_t = j|\beta\right)}_{\text{model (4)}} \underbrace{f\left(\beta|D\left(t-1\right)\right)}_{\text{prior Dir pdf for } \beta}}^{\text{inner mixture}}, \end{aligned} \tag{6}$$

where GiW denotes the conjugate prior Gauss-inverse-Wishart pdfs used for normal components (Peterka, 1981; Kárný et al., 1998) and Dir denotes the conjugate prior Dirichlet pdfs used for the categorical pointer models according to (Kárný et al., 2006).

The pointer estimates

By integrating the joint pdf (6) over the unknown parameters Θ, ϑ, α and β, the pdf for the estimation of the pointers c_t and k_t is obtained in the following way:

$$\begin{aligned} f\left(c_t = i, k_t = j|D\left(t\right)\right) &\propto \int_{\Theta^*}\int_{\vartheta^*}\int_{\alpha^*}\int_{\beta^*} f\left(y_t, z_t, \Theta, c_t = i, \alpha, \vartheta, k_t = j, \beta|D\left(t-1\right)\right)\, d\beta\, d\alpha\, d\vartheta\, d\Theta \qquad (7) \\ &= \int_{\beta^*} f\left(k_t = j|\beta\right) f\left(\beta|D\left(t-1\right)\right) d\beta \int_{\vartheta^*} f\left(z_t|\vartheta, k_t = j, \varphi_t\right) f\left(\vartheta|D\left(t-1\right)\right) d\vartheta \\ &\times \int_{\alpha^*} f\left(c_t = i|k_t = j, \alpha\right) f\left(\alpha|D\left(t-1\right)\right) d\alpha \int_{\Theta^*} f\left(y_t|\Theta, c_t = i, \psi_t\right) f\left(\Theta|D\left(t-1\right)\right)\, d\Theta, \end{aligned}$$

which for all $i \in \{1, 2, \ldots, m_c\}$ and $j \in \{1, 2, \ldots, m_k\}$ gives the result

$$\aleph\left(\left(\left(M^k \cdot * \hat{\beta}_{t-1}\right) \cdot * M^c\right) \cdot * \hat{\alpha}_{t-1}\right) \equiv W_t, \tag{8}$$

where

- M^k is a column vector of proximities of the inner mixture components obtained by the substitution of the previous-time point estimates of the parameters ϑ and current data item z_t into each component, see (Nagy et al., 2016; Nagy and Suzdaleva, 2017),
- $\hat{\beta}_{t-1}$ is the point estimate of the parameter β obtained from the previous-time statistics of the inner pointer model and it is a m_k-dimensional column vector (Kárný et al., 2006),
- M^c is a row vector of proximities of the outer mixture components obtained similarly by the substitution of the previous-time point estimates of the parameters Θ and current data item y_t into each component,
- $\hat{\alpha}_{t-1}$ is the point estimate of the parameter α obtained by normalizing the previous-time statistics of the outer pointer model and it is a matrix of the dimension $(m_k \times m_c)$ (Kárný et al., 2006; Nagy et al., 2011),
- the denotation $\aleph(\cdot)$ means the normalization to total sum equal to one,
- the denotation W_t is the joint distribution of the pointers c_t and k_t and it is a $(m_k \times m_c)$-dimensional matrix (Nagy et al., 2011; Nagy and Suzdaleva, 2017).

Details about this briefly summarized derivation can be found in the provided sources generally based on (Kárný et al., 1998; Kárný et al., 2006).

Using (8), the weighting vectors for both the mixtures are obtained $\forall i \in \{1, 2, \ldots, m_c\}$ and $\forall j \in \{1, 2, \ldots, m_k\}$. For the inner pointer k_t, the m_k-dimensional weighting vector w_t^k is obtained as the marginal pdf from W_t over columns, i.e.,

$$f(k_t = j | D(t)) = \sum_{i=1}^{m_c} W_{i|j;t} \equiv w_{j;t}^k. \tag{9}$$

The index j of the maximum weight in the vector w_t^k corresponds to the active component of the inner mixture and it is the point estimate of the pointer k_t. This point estimate is substituted as the row number into the joint distribution W_t to obtain the m_c-dimensional weighting vector w_t^c for the outer pointer c_t, i.e.,

$$f(c_t = i | k_t = j, D(t)) = W_{i|k_t;t} \equiv w_{i;t}^c. \tag{10}$$

The point estimate of the outer pointer c_t is obtained using the index of the maximum weight in w_t^c.

The parameter estimates

The obtained weights are used in the recursive updates of the statistics of the posterior parameter pdfs (see (6) and (7)) according to (Kárný et al., 1998; Kárný et al., 2006) as follows.

The statistics update of the Dirichlet pdf for the parameter β

$$\nu_{j;t} = \nu_{j;t-1} + w_{j;t}^k, \ \forall j \in \{1, 2, \ldots, m_k\}, \tag{11}$$

where ν_t denotes the statistics of the model of the pointer k_t as the m_k-dimensional vector with the entries $\nu_{j;t}$ (Kárný et al., 2006).

The statistics update of the Gauss-inverse-Wishart pdf for the parameter ϑ

$$(U_t)_j = (U_{t-1})_j + w_{j;t}^k \begin{bmatrix} z_t \\ \varphi_t \end{bmatrix} \begin{bmatrix} z_t \\ \varphi_t \end{bmatrix}', \tag{12}$$

$$\lambda_{j;t} = \lambda_{j;t-1} + w_{j;t}^k, \tag{13}$$

where $(U_t)_j$ is the information matrix of the GiW pdf of each j-th normal component $f(z_t|\vartheta, k_t = j, \varphi_t)$ and $\lambda_{j;t}$ is the counter (Peterka, 1981; Kárný et al., 1998).

The statistics update of the Dirichlet pdf for the parameter α

$$(\gamma_{i|j})_t = (\gamma_{i|j})_{t-1} + W_{i|j;t}, \tag{14}$$

where γ_t denotes the statistics of the pointer c_t, which is the matrix of the same dimension as the model (2) with the entries $(\gamma_{i|j})_t$. This update was proposed in (Nagy et al., 2011) for the dynamic pointer with the help of approximation based on the Kerridge inaccuracy (Kerridge, 1961). Here, the pointer c_t is not dynamic, but it depends on the inner pointer k_t, which means that the same way of the update can be applied. For the sake of simplicity, here it is used with the approximation similarly to (Kárný et al., 1998).

The statistics update of the Gauss-inverse-Wishart pdf for the parameter Θ

$$\begin{aligned} (V_t)_i &= (V_{t-1})_i + w^c_{i;t} \begin{bmatrix} y_t \\ \psi_t \end{bmatrix} \begin{bmatrix} y_t \\ \psi_t \end{bmatrix}', & (15) \\ \eta_{i;t} &= \eta_{i;t-1} + w^c_{i;t}, & (16) \end{aligned}$$

where similarly to (12)–(13), $(V_t)_j$ is the information matrix of the GiW pdf of each i-th normal component $f(y_t|\Theta, c_t = i, \psi_t)$ and $\eta_{i;t}$ is the counter according to (Peterka, 1981; Kárný et al., 1998). All of the updates start with the initial statistics chosen during the initialization of the estimation, see, e.g., (Kárný et al., 2003; Suzdaleva et al., 2016).

The point estimates of all of the parameters are determined in a standard way for the GiW pdfs (using the partition of the corresponding information matrices) and Dirichlet distributions (by the normalization of the statistics), see again, e.g., (Peterka, 1981; Kárný et al., 1998; Kárný et al., 2006), etc.

2.3 The program scheme of the algorithm

The algorithm presented above can be expressed as the following program scheme. After the offline (preliminary before running) initialization of the estimation (Suzdaleva et al., 2016), i.e., (i) setting the number of components and the initial statistics of the components as well as the pointer models and (ii) computing the point estimates of all of the parameters using the initial statistics, the online estimation includes the following steps for time $t = 1, 2, \ldots$ as long as the new data can be measured.

1. Measure the values of y_t and z_t at time instant t.
2. Construct the proximity vector M^k by substituting the auxiliary data item z_t and the previous point estimates of the parameters ϑ into each normal component (3) of the inner mixture.
3. Construct the proximity vector M^c by substituting the target data item y_t and the previous point estimates of the parameters Θ into each normal component (1) of the outer mixture.
4. Construct the joint matrix weight W_t according to (8) and normalize it to total sum equal to one.
5. Compute the weighting vector w^k_t as the marginal distribution over columns from W_t according to (9).
6. Determine the point estimate of the pointer k_t as the index of the maximum entry in w^k_t and use it as the number of a row in W_t to obtain the weighting vector w^c_t according to (10).

7. Obtain the point estimate of the outer pointer c_t according to the index of the maximum entry of the weighting vector w_t^c.

8. Classify the data according to the estimated value of the outer pointer w_t^c.

9. Perform the update of all of the statistics.

10. Re-compute the point estimates of all of the parameters using the updated statistics.

11. Go to Step 1 and use the point estimates as the initial ones.

This part of the algorithm runs subsequently in a time loop using measured data.

3 Application to driving style estimation

Here, the above general algorithm is applied to the driving style estimation. The main idea is to show that an individual's driving style changes both within the actual driving environment and when changing the environment. For example, some individuals are experienced drivers in a city, but they do not have practical skills on the highway. Therefore, for driving on the highway, they will have to adapt to the environment, which may take some time. In the opposite case, villagers have to adjust their driving to the center of the city, if necessary. The switching of the driving environments influences the driving style as well as the economic efficiency of driving.

Drivers obviously know where they are driving. However, the recognition of the driving style depending on the driving conditions and environment can be decisive for in-vehicle information systems from a fuel consumption optimization' point of view. The estimation with the help of the introduced two-layer model can be a suitable tool in this case. Its application is presented below.

3.1 Model of driving style depending on the driving environment

The normal components of the inner mixture (3) describe the following multi-dimensional auxiliary variable $z_t = [z_{1;t}, z_{2;t}, z_{3;t}, z_{4;t}]'$ within different driving environments:

- $z_{1;t}$ – gas pedal position [%],
- $z_{2;t}$ – brake pedal pressure [bar],
- $z_{3;t}$ – engine speed [rpm],
- $z_{4;t}$ – lateral acceleration in multiples of gravimetric acceleration.

The inner pointer k_t indicates the actual driving environment reduced to three variants:

$$k_t \in \{\text{“urban”}, \text{“rural”}, \text{“highway”}\}.$$

The components of the outer mixture (1) describe the target variable y_t, which is the instantaneous fuel consumption [μl/2s] within different driving styles depending on the environment. The optional multidimensionality of the variable y_t leads to a mere modification of the information matrix in (15) and does not affect the performance of the algorithm. The outer pointer c_t points to the driving style considered from a fuel consumption' point of view and here it has three possible values:

$$c_t \in \{\text{“low consumption”},\ \text{“middle consumption”},\ \text{“high consumption”}\},$$

where all of them are assumed to exist in each of the driving environments.

The approach was validated with the help of experiments in a programming free and open source environment Scilab (www.scilab.org). The aim of the experiments was to demonstrate the recognition of the driving style depending on the estimated driving environments.

3.2 Data collection

The data sets were collected via the CAN bus of a vehicle driven on a chosen route which led through the three explored driving environments, i.e., a city, outside the city and the highway. Drivers repeatedly drove along the route. Each driver was instructed to drive economically, then in a normal way and also with frequent high acceleration.

For the experiments, 7 data sets were taken. Each data set contained 1000 measurements of fuel consumption, gas pedal position, brake pedal pressure, engine speed and lateral acceleration. The variables were measured every 2 seconds, i.e., the average trip duration on the route was 33 minutes.

3.3 Initialization

One of the data sets served for the initialization as prior knowledge. The initialization of the component statistics was performed on the basis of histograms of the individual variables from the prior data set, where the component centers, which cover a significant part of the histograms should be determined.

The histograms can be found in Figure 3. Three of them (gas pedal position, engine speed and fuel consumption) show a distinct multi-modal behavior. However, all of them can be used for the purpose of the initialization. In the figure, the initial centers of the normal components of the gas pedal position can be guessed as the following values: $0, 19, 44$. Similarly, the initial centers of the components of the engine speed that can be initially pre-estimated are $1200, 1500, 1800$. The histograms of the brake pedal pressure and the acceleration exhibit a slightly weaker multi-modal nature. Nevertheless, three components can be initialized for them as well with the following centers: $0.3, 7, 20$ and $-0.6, -0.3, 0$ respectively. The value of 0.3 is explained by the minimum pressure of the brake system.

Using the centers, the initial point estimates of the parameters ϑ for three environments can be set for time $t = 0$

$$(\hat{\vartheta}_0)_1 = [0 \quad 0.3 \quad 1200 \quad -0.6]', \quad (\hat{\vartheta}_0)_2 = [19 \quad 7 \quad 1500 \quad -0.3]', \quad (\hat{\vartheta}_0)_3 = [44 \quad 20 \quad 1800 \quad 0]'. \tag{17}$$

They are substituted into the initial information matrices as follows:

$$(V_0)_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0.3 \\ 0 & 0 & 1 & 0 & 1200 \\ 0 & 0 & 0 & 1 & -0.6 \\ 0 & 0.3 & 1200 & -0.6 & 1 \end{bmatrix}, \quad (V_0)_2 = \begin{bmatrix} 1 & 0 & 0 & 0 & 19 \\ 0 & 1 & 0 & 0 & 7 \\ 0 & 0 & 1 & 0 & 1500 \\ 0 & 0 & 0 & 1 & -0.3 \\ 19 & 7 & 1500 & -0.3 & 1 \end{bmatrix},$$

$$(V_0)_3 = \begin{bmatrix} 1 & 0 & 0 & 0 & 44 \\ 0 & 1 & 0 & 0 & 20 \\ 0 & 0 & 1 & 0 & 1800 \\ 0 & 0 & 0 & 1 & 0 \\ 44 & 20 & 1800 & 0 & 1 \end{bmatrix}. \tag{18}$$

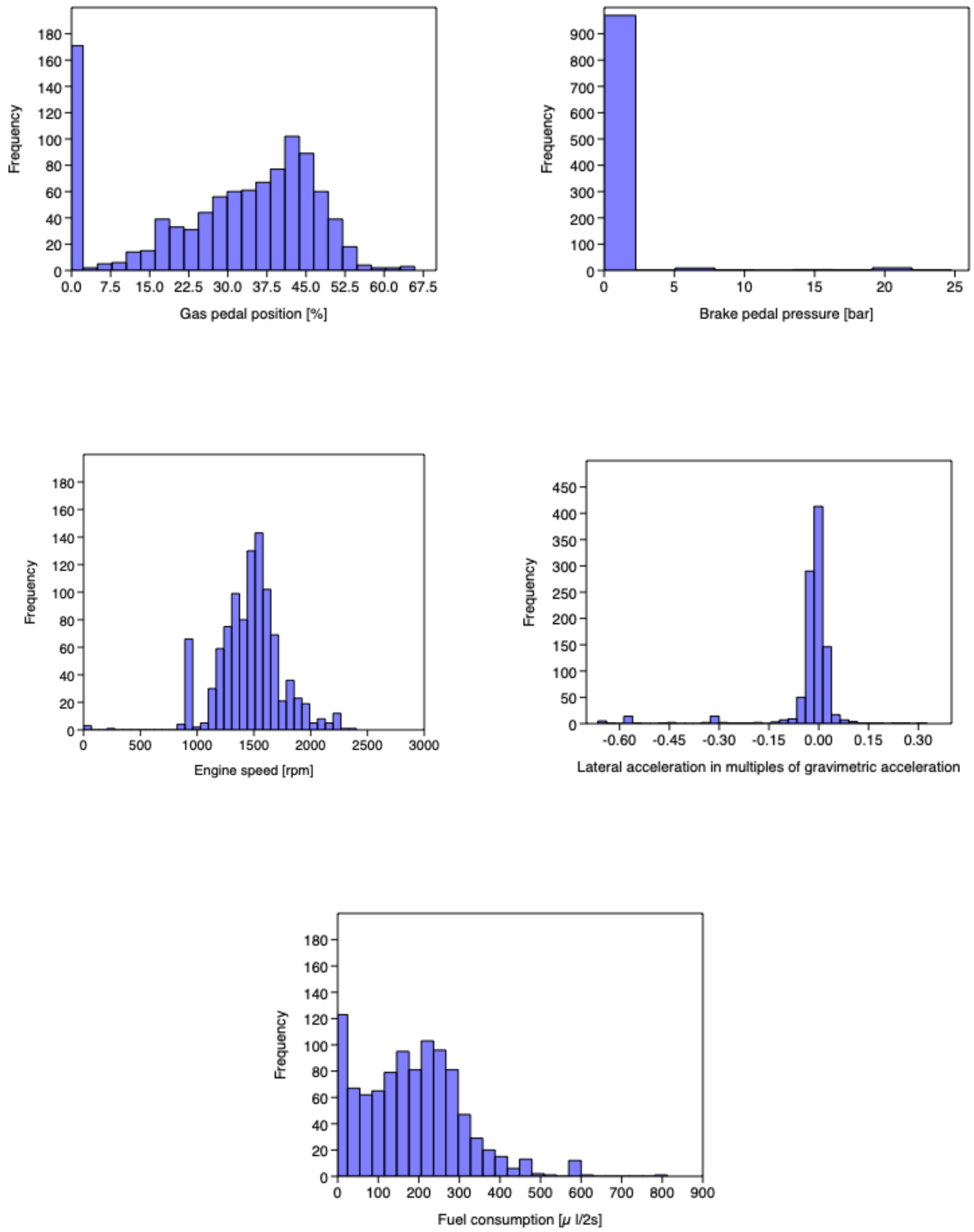

Figure 3: Histograms of gas pedal position, brake pedal pressure, engine speed, acceleration and fuel consumption

In the histogram of the fuel consumption in Figure 3, the initial centers within three driving styles are $0, 250, 600$. Similarly, they are used as the initial point estimates to be substituted into the initial statistics

$$\hat{\theta}_{1;0} = 0, \quad \hat{\theta}_{2;0} = 250, \quad \hat{\theta}_{3;0} = 600, \tag{19}$$

$$(U_0)_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad (U_0)_2 = \begin{bmatrix} 1 & 250 \\ 250 & 1 \end{bmatrix}, \quad (U_0)_3 = \begin{bmatrix} 1 & 600 \\ 600 & 1 \end{bmatrix}. \tag{20}$$

The initial counters $\lambda_{j;0}$ and $\eta_{i;0}$ for all environments and driving styles were set equal to one. The statistics ν_0 and γ_0 both of the pointers were initialized uniformly. The same initialization was used for all of the data sets.

3.4 Results

For the validation of the proposed two-layer model (TLM) and a comparison of the algorithm performance, two alternative variants of the driving style model were chosen. One of them was the mixture with a single pointer (SPM), which describes both the target and auxiliary variables y_t and z_t all together in the data vector. With this model, only driving style is modeled by the pointer without driving environment in the condition. The SPM based driving style estimation is close to that described in (Suzdaleva and Nagy, 2018). Another alternative approach was the joint model of driving style and driving environment (JM) composed of two parts, where (i) first, driving environment was estimated based on the auxiliary variables and then (ii) its point estimate was used along with the fuel consumption in the data vector of target variables for the estimation of driving style. This allowed the model to be segmented by the driving environment to capture all possible inter-dependencies between driving style and driving environment. The validation results are presented below.

Switching the driving environments

Switching the driving environments was estimated with the help of the inner pointer model. The results obtained for one of the data sets which were tested are given in Figure 4. The real switching of the three driving environments on the route (1= "urban", 2 = "rural" and 3 = "highway") received from the data is demonstrated in Figure 4 (left). It can be seen that the chosen route fragment started on the highway, continued outside the city, then passed through the city with a short segment of the country road and finished again in the rural environment. The driving

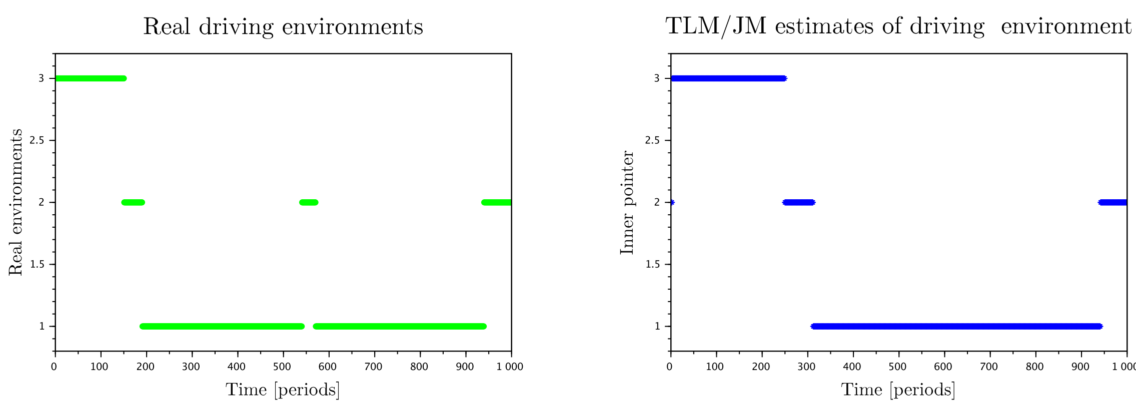

Figure 4: The real driving environment (left) and its recognition using the TLM and JM inner pointers (right)

environment estimation with the help of the inner pointer values can be found in Figure 4 (right). Here, the TLM and JM estimates are naturally identical as they are based on the same approach. The SPM results are not shown, because it was not used for the driving environment description. It can be seen that both the TLM and JM estimates are close to the real values with the exception of locations, where the environment was changing for a short time, see the differences from 200 to 300 time periods as well as around 550. It is explained by the data-based algorithm, which needs some time to actualize the statistics with the current data. The average error of the driving environment estimation was 19.4%. It means that the inner pointer model takes into account the drivers' behavior through the auxiliary variables and is capable of recognizing if their driving corresponds to the current environment. Obviously, the uncertainty, which comes from the behavior of individual drivers cannot be completely covered by the model.

Inner clusters according to driving environments

The gas pedal position and the engine speed can be used to demonstrate the marginal two-dimensional TLM/JM inner clusters of auxiliary variables detected according to the driving environments, see Figure 5. The clusters are clearly visible. This is explained by the pronounced multi-modal behavior of these variables (also shown in their histograms in Section 3.3). The cluster denoted by '$\star$' corresponds to urban driving and shows gas pedal position values in the range from 0 to 65% with an almost uniform distribution of the data between 15 and 50%. The engine speed in this cluster remains up to 1700 rpm. Its upper-bound values are partially overlapped by the cluster denoted by '$\cdot$', which corresponds to rural driving. Within this environment, the gas pedal position values are mostly located from 30 to 55%, while the engine speed values vary from 1700 to 2000 rpm. The highway cluster denoted by '$\times$' includes the gas pedal position values in the range from 40 to 65% and the engine speed with the values from 2000 to 2500 rpm.

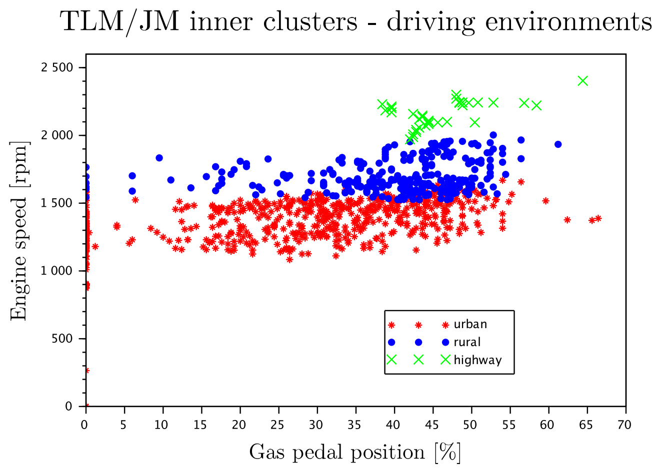

Figure 5: The gas pedal position and the engine speed within three driving environments

The marginal clusters of the brake pedal pressure and lateral acceleration with the rest of the auxiliary variables are not so representative due to their weak multimodality (see Section 3.3). However, all of the auxiliary variables give clearly expressed clusters of driving styles when plotting them in pairs with the fuel consumption. These results are presented in the next section.

Outer clusters according to driving styles

The outer clusters demonstrate driving styles expressing different degrees of the fuel consumption detected by the described algorithm. Each driving style (i.e., "low consumption", "middle consumption" and "high consumption") exists for each driving environment ("urban", "rural" and " highway"), which means that nine driving styles were recognized in the multi-dimensional data space. By plotting the fuel consumption values against the estimated driving styles, their two-dimensional clusters can be visualized only within three values. They are displayed in Figure 6. It can be seen that after the estimation of the driving environments, the driving styles are sharply distinguished regarding the values of fuel consumption.

Unlike the inner clusters in the previous section, here the differences in results of the three compared models can be demonstrated. The outer two-dimensional clusters corresponding to driving styles are shown in Figure 7. The left column of the figure presents the TLM, SPM and JM clusters of the fuel consumption plotted against the gas

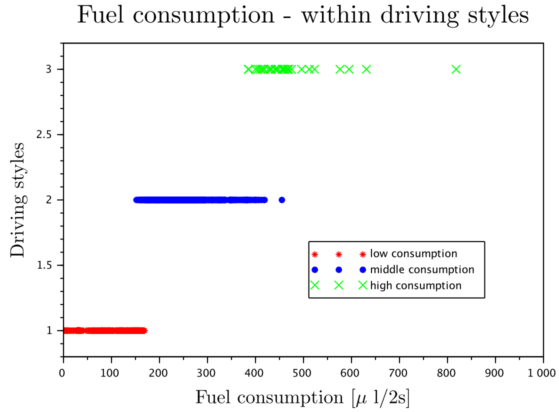

Figure 6: The fuel consumption within three driving styles

pedal position. It is not necessary to use a clustering validity index (Vendramin et al., 2010) to see the significant difference between the clusters detected by the three models. The TLM three driving styles in the top left plot are clearly visible, while the SPM estimation in the middle left plot provides the rather overlapped clusters. This correlates with the single pointer results obtained in (Suzdaleva and Nagy, 2018), where it was noticed that in the considered context, the driving style model requires a higher number of components (seven in (Suzdaleva and Nagy, 2018)), which would be hardly comparable with the proposed TLM approach. The JM driving styles are significantly overlapped, which indicates that the driving style model segmented by the driving environment in the data vector is not beneficial in comparison with the condition in the discrete pointer model, as it is proposed in the TLM approach. Similar comments can be given regarding the right column of Figure 7, where clusters of the fuel consumption and lateral acceleration are presented.

Switching the driving styles within the environments

The above two-dimensional clusters cannot be used for the demonstration of switching the driving styles within individual driving environments. However, this switching can be shown using the evolution of the outer pointer estimates within the urban, rural and highway environments. The left part of Figure 8 (top) demonstrates the switching of the driving styles obtained for one of the data sets using the TLM estimation. The switching is naturally different for each data set, since the pointer estimation depends on individual driver' behavior. For better visibility, a fragment of the route was chosen. The values of $1, 2, 3$ on the y-axis correspond to the low, middle and high consumption driving styles respectively.

In the left top plot, the TLM urban driving style was regularly switching between the low and middle consumption driving styles (values 1 and 2) and occasionally transitions to high consumption were monitored (value 3). While driving outside the city (in the left middle plot), the TLM rural driving style was primarily estimated as economical with the irregular switching to the middle and high consumption styles. In the left bottom plot, the TLM highway driving style was mostly corresponding to the middle fuel consumption. Periodically, it changed to low and occasionally to high fuel consumption.

The right part of Figure 8 (top) demonstrates transitions of the JM driving styles within urban, rural and highway environments. In the right top and right bottom plots, the urban and highway JM driving styles correspond to economical driving for a significant part of the estimation. The transitions directly to high consumption were frequently monitored. In the rural environment (the right middle plot), the JM estimate was either the low or the high fuel consumption driving style.

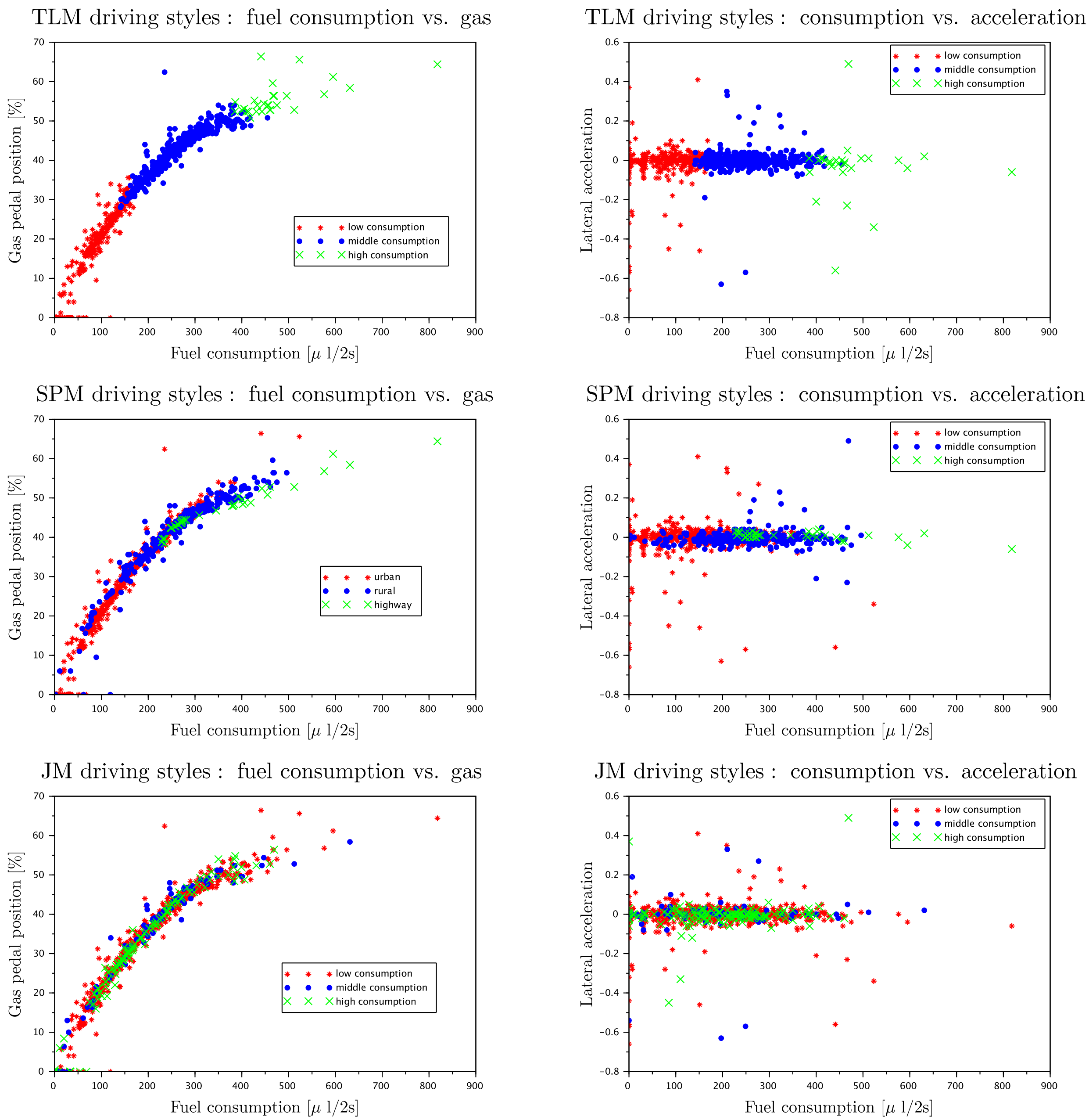

Figure 7: The comparison of the TLM, SPM and JM driving styles

Figure 8 (bottom) displays the SPM driving style switching. It was not conditioned by the driving environment. However, the evolution of the estimation correlates with the inner pointer shown in Figure 4.

Since only a fragment can be visualized, the TLM and JM driving style conditional distributions are presented in Table 1, which demonstrates the probabilities of each driving style depending on the driving environment in percent. It can be seen that the differences in the driving style transitions inside each driving environment are well captured by the TLM estimation, while the JM approach gives almost identical probabilities in the city as well as outside the city, but slightly distinguishing on the highway. For instance, in the urban environment, the TLM middle

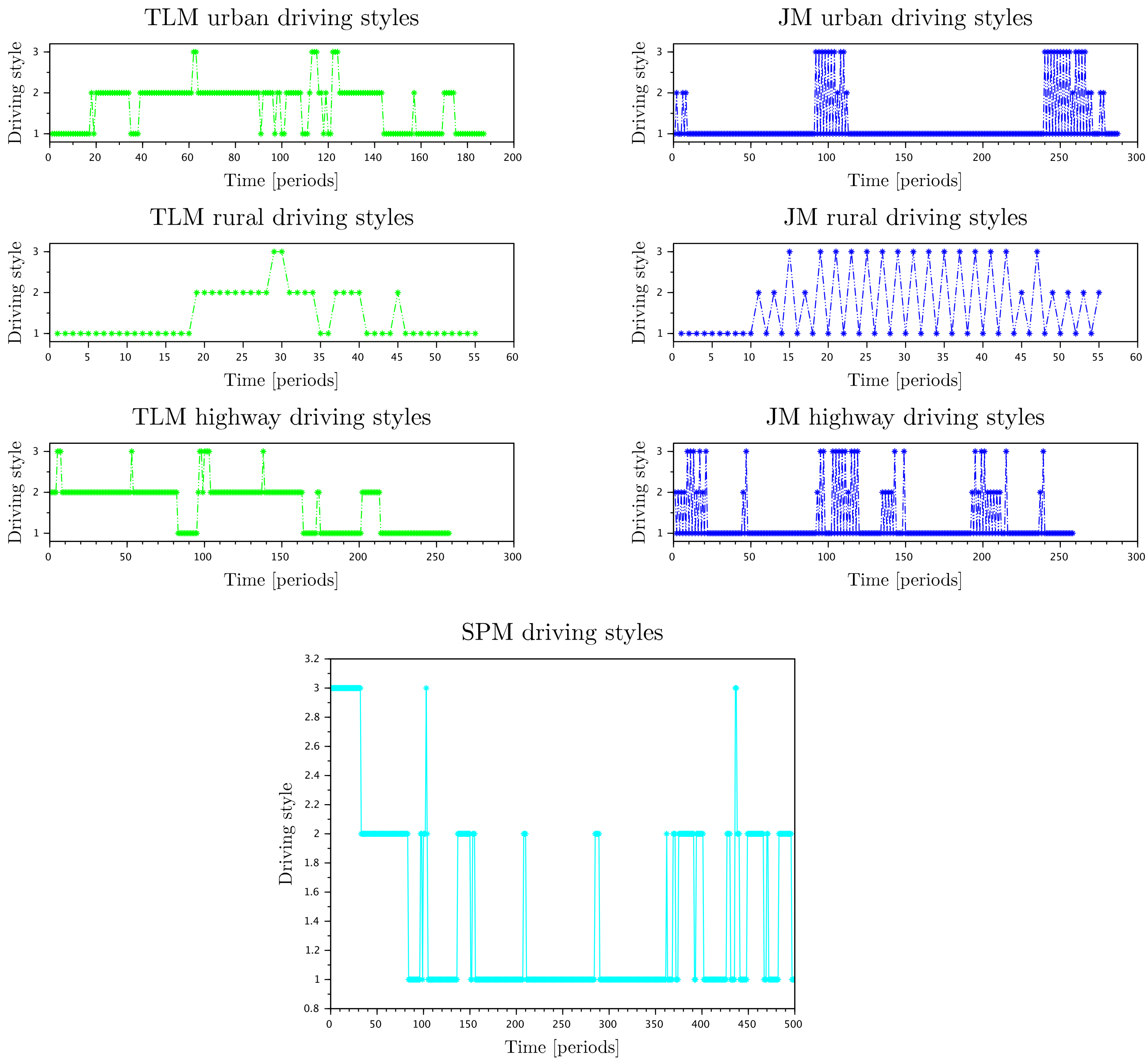

Figure 8: Switching the driving styles in the urban, rural and highway driving environments with the TLM (top left), JM (top right) and SPM (bottom) estimates

consumption driving style has the maximum probability, while in the rural environment it is the TLM economical driving style and so on. Using the JM estimation, the economic driving style has the most significant dominant probability within all of the environments.

The sums of squared deviations between the TLM driving style probabilities in the individual environments were obtained as 215.091, 116.831 and 613.986. In contrast to that, the sums of squares between the JM conditional probabilities were 0.274, 101.4897 and 111.161 respectively, i.e., their variability is much lower than in the TLM case. This confirms the significant difference in the TLM estimation in comparison with JM.

Table 1: The driving style conditional distributions obtained with the TLM and JM estimates

	TLM, [%]	JM, [%]
Urban low consumption	44.976	76.077
Urban middle consumption	53.110	7.655
Urban high consumption	1.914	16.268
Rural low consumption	53.913	75.652
Rural middle consumption	41.739	7.826
Rural high consumption	4.348	16.522
Highway low consumption	36.434	82.946
Highway middle consumption	59.302	8.139
Highway high consumption	4.264	8.915

Fuel consumption prediction

The adopted methodology can be used twofold for clustering and prediction. For the clustering task, static components are rather beneficial, while for the prediction, dynamic components with delayed values in the regression vector are more suitable to obtain a higher accuracy of the results unlike the static components, which follow the estimated regression coefficients around the expectation. Results of the prediction of the target variable, i.e., fuel consumption are demonstrated in Figure 9 (top) in comparison with its real values from a randomly chosen data set. For better visibility, a fragment of 150 time periods of driving on the chosen route is displayed. In the beginning, up until 30 time periods, it corresponds to the highway and further, the driving took place in the rural environment. The predictions of all of the compared models are in the correspondence with the real fuel consumption values. In the figure, the TLM and JM predictions are mostly coinciding, while SPM has more significant deviations during the whole prediction. The differences can be mainly seen in locations where abrupt changes of values are observed, e.g., from 35 to 50 time periods, where both the SPM and JM predictions give negative values, or around 120 time periods.

Figure 9 (bottom) compares box plots of the real data with the TLM, JM and SPM predictions, where the min-max normalization of the values was used. In this figure, the median of the TLM prediction is the closest to the real fuel consumption, while the SPM median is the farthest. This indicates the least difference between the real values and TLM prediction.

For each data set, the normalized root-mean-square error (NRMSE) was computed

$$\mathrm{NRMSE} = \frac{\sqrt{\frac{\sum_{t=1}^{T}(y_t - \hat{y}_t)^2}{T}}}{y_{\max} - y_{\min}}, \tag{21}$$

where $\hat{y}_t$ denotes the prediction at the time instant t and $T = 1000$. The NRMSE averaged over all of the data sets is provided in Table 2 for the three models. The table shows that the TLM has the lowest average NRMSE and the SPM the highest one. However, the difference is insignificant. To complete the comparison, the two-sample paired t-test was used in pairs of the real fuel consumption with each prediction in order to test whether there is a significant difference between their means. The p-values of the tests are given in Table 2. They are higher than the significance level of 0.05, which means that the differences between the real data and predictions are not statistically significant. However, it should be noticed that the SPM p-value only slightly exceeds the significance level, while the TLM and JM p-values are close to 1, with the TLM p-value being the highest among them. This is explained by the worse handling of sharp changes in fuel consumption values in the case of JM and SPM. The data prediction quality was identical in the case of the variables with a distinctive multi-modal behavior, i.e., gas pedal position and engine speed and worse when predicting the brake pedal pressure as well as lateral acceleration. This is caused by the fact that their course is strongly affected by external traffic conditions (pedestrians, traffic, etc.).

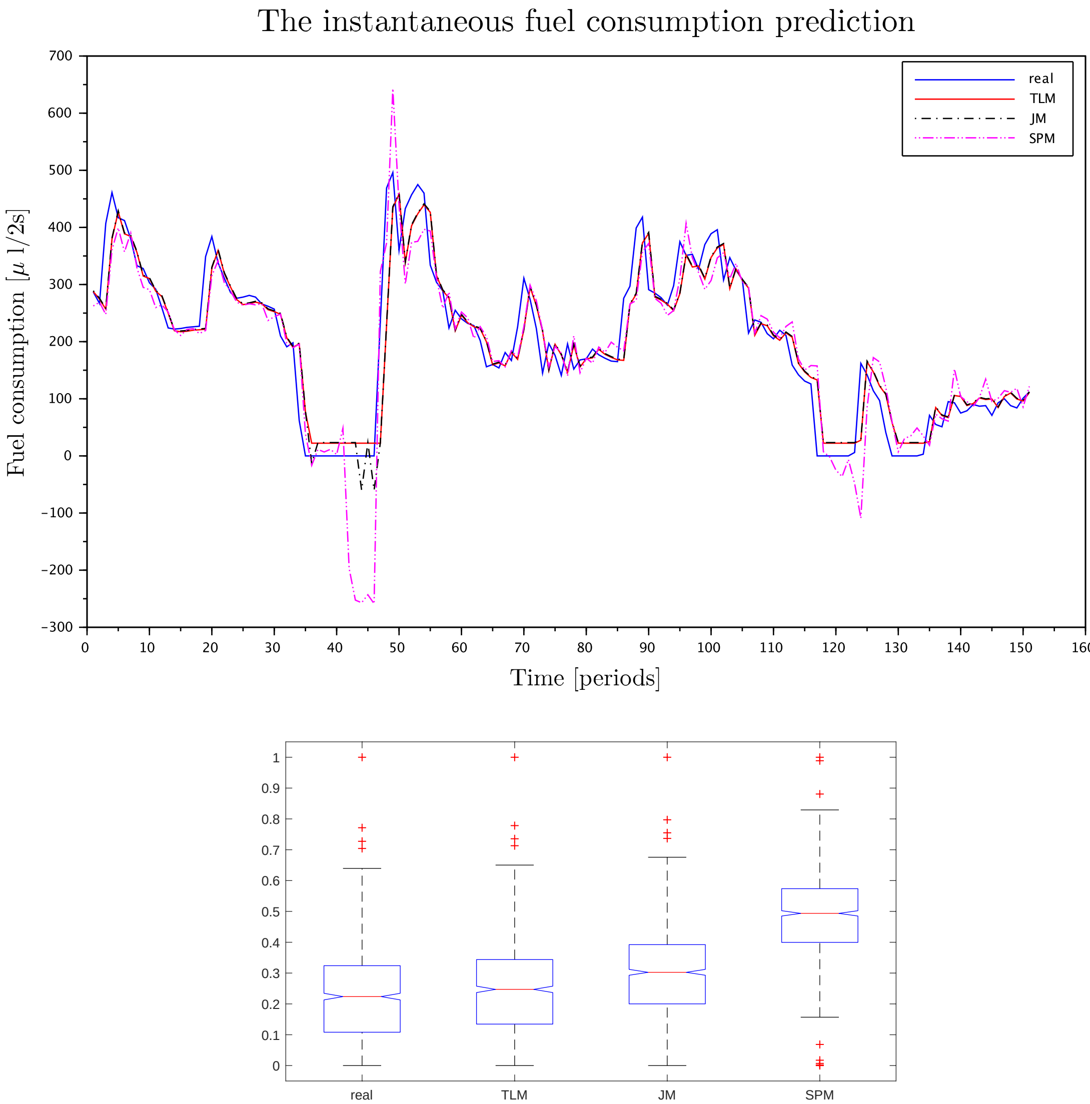

Figure 9: The fuel consumption prediction (top) and box plots (bottom)

In addition, Table 2 also provides the average computational time (CT) of the online estimation of the compared models according to Section 2.3 as well as its standard deviation. The computational time was calculated by the Scilab functions `tic` and `toc` in seconds and averaged over all of the tested data sets with 1000 measurements. The three compared models are based on the recursive Bayesian mixture estimation theory (Kárný et al., 1998; Kárný et al., 2006; Nagy et al., 2011), which enables the one-pass estimation without iterative computations and guarantees the fixed computational time. That is why the average CT of all of the models is very low and fixed with small standard deviations. The SPM has the shortest time, which is explained by the estimation of a single pointer of a single mixture, while both the JM and TLM need to handle with two pointers of two mixtures (JM successively and TLM in two layers) that causes an insignificantly longer duration of computations in thousandths of a second. However, in view of the TLM improving in clustering and prediction, this compromise is acceptable, especially since this CT is guaranteed, shorter than in the case of iterative techniques (see the comparison in (Suzdaleva and Nagy, 2018)) and does not depend on the algorithm convergence.

Table 2: NRMSE, p-values of the paired t-tests, the average CT and its standard deviation

	NRMSE	p-value	Average CT, $[s]$	CT standard deviation
TLM	0.0038935	0.952	0.00122	0.00002
JM	0.0041326	0.899	0.00115	0.00002
SPM	0.0403583	0.054	0.00061	0.00004

3.5 Discussion

The main aim of the presented study was to show the application of the proposed algorithm to the driving style recognition depending on the driving environment. As it is demonstrated in Section 3.4, the aim was successfully accomplished. The considered approach is close to that discussed in (Suzdaleva and Nagy, 2018): both of them are based on recursive algorithms of Bayesian mixture estimation (Kárný et al., 1998; Kárný et al., 2006; Nagy et al., 2011). However, this study brings the further novel extension of the pointer model both from a methodology and application point of view. First, it allows the pointer to be dependent on continuous data by means of the two-layer mixture model through the estimation of the inner pointer. This is extremely suitable for driving style estimation, since, as noted in numerous studies in Section 1.1, driving styles vary depending on different traffic conditions. Here, the traffic conditions were categorized into three driving environments. Thus, in the application field, the proposed approach gives a possibility to get driving style conditioned by the information coming from auxiliary data with the help of incorporating the driving environment as the inner pointer into the model.

The recognition of three driving styles in terms of fuel efficiency, which are switching within each of the three environments, was demonstrated by means of two-dimensional clusters. It means that nine driving styles were recognized in the data sets used for the experiments. The obtained clusters are clearly visible and practically non-overlapping. The comparison with the driving style model (Suzdaleva and Nagy, 2018) based on a single pointer shows the improvement in detecting driving styles as well as in predicting data. Another configuration of the two pointer model was tested as well, which indicated that the segmentation of the driving style model by driving environment is not so successful in clustering, but provides a similar quality of the prediction.

The results of the proposed model show the differences in the driving style evolution in individual environments, which cannot be captured by the single pointer model, identical for all of the environments, and were only slightly indicated by two successive JM pointers. In practice, such differences may mean that the driver has more experience in one of the environments. The regular switching of the driving styles dependent on the environments confirms the suitable configuration of the two-layer model. Such a model is the main contribution of the study, and its updating by the new continuous data along with the data history allows us to perform the cluster analysis while driving and recognize the driving style in a timely manner. The additional contribution of the study concerns the preliminary histogram-based analysis of prior data, which could be used for a choice of other distributions, if necessary.

A practical application of the proposed driving style estimation is expected in the field of in-vehicle information systems, where the recognition of the driving style for a corresponding environment can be beneficial for intelligent vehicle control. The estimation of the driving environment can be critical in case of a GPS signal failure. Moreover, such data can also serve for learning the model up to the case with the known inner pointer based on the discretized signal. However, improvements should be made from a driving environment recognition point of view.

A limitation of the approach is the necessity of using the reproducible statistics of distributions to be updated with new data. It means that for the discussed data-based algorithm, the variables to be involved in the model should be carefully chosen. Preliminary analysis of prior data helps in this task.

4 Conclusion

The paper proposed the two-layer model of driving style depending on the driving environment. The model construction was based on actually measured data along with the available data history and represented a mixture, which was composed of two layers. The inner layer of the model covered the description of driving environment using auxiliary data, while the outer layer operated with target variables and was aimed at modeling driving style in terms of fuel economy conditional on the environment. Gaussian components and discrete pointer models were used within both the layers. For estimating their parameters as well as the inner and outer pointers, the algorithm based on the recursive Bayesian mixture estimation theory was presented, which enables handling with data in real time without iterative computations. Driving style recognition was demonstrated with the help of clusters. The proposed model captured the economical, middle and high fuel consumption driving styles at each time instant within each of urban, rural and highway driving environments, i.e., three driving environments were estimated as the inner pointer and three driving styles as the outer one. It means that nine driving styles were recognized in the available data sets used for the imitation of real driving. The obtained results reported the variability of the fuel efficiency oriented driving style distributions within different environments. The model validation with the help of real data and comparison with theoretical counterparts showed improvements in recognizing driving styles as well as predicting data. In this way, all the subtasks listed in the problem formulation in Section 1.2 were solved to present the novel model of driving style, which is the main contribution of the paper.

One of the strong benefits of the presented model is the possibility to have the pointer dependent on continuous auxiliary data. It was shown that the modeling of a data vector consisting both of the auxiliary and target variables, had provided worse results than in the case of feeding the information from the auxiliary data into the outer layer through the inner pointer. Obviously, it brings less loss of information than in the case of discretization. This is a significant advantage in the task of incorporating the available information into the model.

Among the problems that remain open in the considered context, it is worth mentioning (i) the extension of the methodology up to the mixture of mixed components, which is suitable for a situation when the modeled variables should be described by different distributions; (ii) the use of some distributions that do not have reproducible statistics and need a special approximation; (iii) the multi-step prediction of the outer pointer for a pre-set inner pointer, which would enable the driving style prediction for a predefined route. Testing the approach using a more representative drivers' sample in different traffic situations could be also beneficial for further research.

In addition, it should be mentioned that the inner pointer is not limited by its application for the determination of driving environment. Using suitable data, it can be used for indicating the emissions level, which would enable us to model the driving style jointly in terms of the fuel consumption and ecology. Another potential application can be expected in the case of using data not only from a vehicle, but also from a driver as, e.g., in (Yang et al., 2018). In this case, the inner pointer model could describe abrupt changes in the driver's health state, tiredness, aggressiveness, etc. Such combination of the data described by suitable distributions could serve for the driving style recognition regarding road safety.

Acknowledgements

This work has been supported by the project SILENSE, project number ECSEL 737487 and MSMT 8A17006.

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