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Output-Feedback Model Predictive Control for Systems under Uniform Disturbances

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***Abstract*—The paper deals with an output-feedback model predictive control (MPC) for discrete-time systems influenced by bounded disturbances. The proposed MPC combines a state-space design and a state estimation. The state estimates are obtained by a specific uniform Bayesian filter. It provides an evident disturbance attenuation in the estimated state. The MPC design considers a quadratic cost function that incorporates penalties on the tracking error, on the actuation effort and on the system output increments. The theoretical results are completed by illustrative examples using a dynamic model of a parallel kinematic machine as a controlled system.**

***Index Terms*—control design, output feedback, position control, robot control, state estimation, linear systems, Bayes methods, recursive estimation, uncertainty, stochastic systems**

I. Introduction

The state-space based model predictive control (MPC) [1] is frequently used in industrial applications. There, states are often unmeasurable. It leads to the output-feedback MPC and the need for a state estimator. Further, the control process is frequently influenced by disturbances that are related to the model inaccuracy and to unmeasured noises. The mentioned disturbances are often bounded. MPC is not naturally robust against disturbances. This problem can be solved using a suitable state estimator.

There are enough papers that address the output-feedback MPC with bounded disturbances. A representative sample is presented below. In the paper [2], a robust MPC controller is proposed. A robustness is guaranteed through a specific robust Kalman filter. The robustness of the control is tunable. The controller is tested on a servomechanism system. In the paper [3], a robust MPC for a linear polytopic uncertain system with bounded disturbance is proposed. The control law is based on the pre-specified state estimator using the estimation error bound. The paper [4] combines MPC and set-membership state estimation techniques for controlling linear systems with unknown but bounded disturbances subject to hard input and state constraints. The paper [5] proposes an output-feedback MPC scheme for the case of stabilising control for linear discrete-time systems incorporating a set-valued estimator based on a fixed finite number of recent measurements. The proposed MPC scheme is illustrated by a numerical example. The paper [6] proposes a controller that consists of a state estimator and a tube based robust predictive control law. A single tube directly bounds the worst case difference between the real and predicted behaviour.

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The given overview is a motivation for our research oriented to the output-feedback MPC for a specific class of mechanical systems where bounded disturbances occur frequently. Considering a class of industrial stationary robots-manipulators, i.e. mechanical systems, a measurement of their outputs is usually influenced by disturbances having physically bounded uncertainties. The outputs are predominantly positions both longitudinal and angular. Corresponding velocities are incorporated in unmeasurable states, complemented possibly by accelerations and jerks. Thus, considering that only system outputs are available instead of full measurement of the system state in the combination of high-dynamic systems such as robots or generally mechatronic systems, the output-feedback MPC is proposed as a powerful and flexible way that is computationally-achievable in real time.

In the previous paper of authors, [7], an output-feedback MPC for motion control of the mentioned robotic systems was proposed. A time-varying state-space robot model influenced by a bounded uncertainty with unknown bounds was considered. The state and noise parameter estimation was performed on a moving window. Estimated states were used for updating state-dependent elements in the robot model and for control design itself. Estimated noise parameters are employed in advanced tuning of control parameters, namely penalisation matrices.

In this paper, we aim to improve the results of [7] with respect to the smoother control actions and a better output stabilisation. To achieve this aim, we propose an output-feedback MPC scheme that uses an alternative state estimator and an extended cost function for control design.

The paper is organized as follows. In Section II, the control problem is formulated including the used theoretical background and notation. The proposed output-feedback MPC design is explained in Section III. In Section IV, a dynamic model of the considered robotic system is presented. The model is used as a controlled system for the proposed MPC scheme in several illustrative experiments. Section V concludes the paper.

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II. Problem Setup

This section explains the used notation, introduces a state-space model with uniform disturbances and the Bayesian state estimation, and formulates an output-feedback MPC problem.

A. Notation

Throughout the paper, we consider column vectors and denote them by lowercase letters, e.g. z. Then, z_k denotes the value of a vector variable z at a discrete-time instant $t \in \{1, \cdots, \overline{t}\}$; $z_{t;i}$ is the i-th entry of z_t; $\underline{z}$ and $\overline{z}$ are lower and upper bounds on z, respectively. $\hat{z}$ denotes an estimate of z. The symbol $f(\cdot|\cdot)$ denotes a conditional probability density function (pdf); names of arguments distinguish respective pdfs; no formal distinction is made between a random variable, its realisation and an argument of the pdf. $\mathcal{U}_z(\underline{z}, \overline{z})$ denotes a multivariate uniform distribution of z, $\underline{z} \leq z \leq \overline{z}$, inequalities are meant entrywise.

B. State-Space Model with Uniform Disturbances

We introduce a linear state space model with uniform disturbances (LSU model) in the form

$$x_t = \underbrace{Ax_{t-1} + Bu_{t-1}}_{\tilde{x}_t} + \nu_t, \quad \nu_t \sim \mathcal{U}_\nu(-\rho, \rho) \quad (1)$$
$$y_t = \underbrace{Cx_t}_{\tilde{y}_t} + n_t, \quad n_t \sim \mathcal{U}_n(-r, r)$$

where y_t is an observable output, u_t is a control input, x_t is an unobservable system state, A, B, C are the known model matrices, $\tilde{x}_t$ and $\tilde{y}_t$ correspond to the mean values of x and y; ν_t and n_t are independent and identically distributed state and observation disturbances, they are uniformly distributed with known parameters ρ and r, respectively.

C. Bayesian Filtering

Within the considered Bayesian framework [8], a controlled system is described by:

$$\text{prior pdf:} \quad f(x_0) \quad (2)$$
$$\text{time evolution model:} \quad f(x_t|x_{t-1}, u_{t-1}) \quad (3)$$
$$\text{observation model:} \quad f(y_t|x_t) \quad (4)$$

Bayesian filtering consists of the evolution of the posterior pdf $f(x_t|d(t))$ where $d(t)$ is a sequence of observed data records $d_t = (y_t, u_t)$, $d_0 \equiv u_0$. The evolution of $f(x_t|d(t))$ is described by a two-steps recursion that starts from the prior pdf $f(x_0|u_0) \equiv f(x_0)$:

– time update that reflects the evolution $x_{t-1} \to x_t$:

$$f(x_t|d(t-1)) = \int_{x^\star_{t-1}} f(x_t|u_{t-1}, x_{t-1}) f(x_{t-1}|d(t-1))\, \mathrm{d}x_{t-1} \quad (5)$$

– data update that incorporates information about data d_t:

$$f(x_t|d(t)) = \frac{f(y_t|x_t) f(x_t|d(t-1))}{\int_{x^\star_t} f(y_t|x_t) f(x_t|d(t-1)) \mathrm{d}x_t} \quad (6)$$

D. MPC Problem

In this paper, an output-feedback MPC problem is considered. The problem includes a state estimation with bounded disturbances and a specific state-based MPC design, [1]. A consideration of bounded disturbances enables MPC design to use state estimates close to real physical bounds which is difficult when an unbounded normal distribution is used. The state estimates serve, besides MPC design itself, also for the updating elements in the model of controlled system representing nonlinear dynamics of considered robot, see details in Section IV-A. Resulting implementation (algorithm sequence) of indicated MPC problem is described in Section III-C.

III. Main results

A. Bayesian Filtering of LSU model

The LSU model (1) defined in Subsection II-B can be equivalently described, using pdf notation (3) and (4), as follows

$$f(x_t|u_{t-1}, x_{t-1}) = \mathcal{U}_x(\tilde{x}_t - \rho, \tilde{x}_t + \rho) \quad (7)$$
$$f(y_t|x_t) = \mathcal{U}_y(\tilde{y}_t - r, \tilde{y}_t + r). \quad (8)$$

State estimation of LSU model (7), (8) with the prior pdf (2) using (5) and (6) leads to a very complex form of posterior pdf. In [9], an approximate Bayesian state estimation of this model is proposed. The presented algorithm provides the evolution of the uniformly distributed posterior pdf $f(x_t|d(t))$ in two sequential steps:

1. Time update – The time update (5) starts at $t = 1$ with $f(x_{t-1}|d(t-1)) = f(x_0) = \mathcal{U}_{x_0}(\underline{x}_0, \overline{x}_0)$. Being χ the indicator function, it holds

$$f(x_t|d(t-1)) \approx \prod_{i=1}^{\ell} \frac{\chi(\underline{m}_{t;i} - \rho_i \leq x_{t;i} \leq \overline{m}_{t;i} + \rho_i)}{\overline{m}_{t;i} - \underline{m}_{t;i} + 2\rho_i} =$$
$$= \prod_{i=1}^{\ell} \mathcal{U}_{x_{t;i}}(\underline{m}_{t;i} - \rho_i, \overline{m}_{t;i} + \rho_i) = \mathcal{U}_{x_t}(\underline{m}_t - \rho, \overline{m}_t + \rho), \quad (9)$$

where $\underline{m}_t = [\underline{m}_{t;1}, \ldots, \underline{m}_{t;\ell}]'$, $\overline{m}_t = [\overline{m}_{t;1}, \ldots, \overline{m}_{t;\ell}]'$,

$$\underline{m}_{t;i} = \sum_{j=1}^{\ell} \min(A_{ij}\underline{x}_{t-1;j} + B_i u_{t-1}, A_{ij}\overline{x}_{t-1;j} + B_i u_{t-1}), \quad (10)$$
$$\overline{m}_{t;i} = \sum_{j=1}^{\ell} \max(A_{ij}\underline{x}_{t-1;j} + B_i u_{t-1}, A_{ij}\overline{x}_{t-1;j} + B_i u_{t-1}),$$

A_{ij} means the term on the i-th row and the j-th column of A, ℓ is the size of x.

2. Data update – According to (6), we process the observation y_t as $y_t - r \leq Cx_t \leq y_t + r$ (see (8)) by the Bayes rule together with the prior (9) from the time update. The resulting uniform pdf posses a support in the form of polytope. It is approximated by a uniform pdf with an orthotopic support

$$f(x_t|d(t)) \approx \mathcal{U}_{x_t}(\underline{x}_t, \overline{x}_t). \quad (11)$$

The proposed approximation is based on a minimising of Kullback-Leibler divergence of two pdfs [9].

The point state estimate $\hat{x}_t$ corresponds to the centre of the orthotope in (11)

$$\hat{x}_t = \frac{\underline{x}_t + \overline{x}_t}{2}. \tag{12}$$

B. Model Predictive Control Design

• *Cost function and equations of predictions* – The behaviour of a control process is influenced by the choice of the cost function. In this paper, considering positional predictive algorithm, a quadratic cost function balances control errors, i.e. differences between predicted outputs $\hat{y}_{t+j}$ and given references w_{t+j}, against amount of input energy given by control vector u_{t+j-1} and, in additon, against the output increments $\Delta y = \hat{y}_{t+j} - \hat{y}_{t+j-1}$. The used cost function has the following form:

$$\begin{aligned} J_t =& \sum_{j=1}^{N} \{ \|Q_{yw}(\hat{y}_{t+j} - w_{t+j})\|_2^2 \\ &+\|Q_{\Delta y}(\hat{y}_{t+j} - \hat{y}_{t+j-1})\|_2^2 + \|Q_u u_{t+j-1}\|_2^2 \} \\ =& \Big\{ (\hat{Y}_{t+1} - W_{t+1})^T Q_{YW}^T Q_{YW} (\hat{Y}_{t+1} - W_{t+1}) \\ &+ \Delta\hat{Y}_{t+1}^T Q_{\Delta Y}^T Q_{\Delta Y} \Delta\hat{Y}_{t+1} + U_t^T Q_U^T Q_U U_t \Big\} \end{aligned} \tag{13}$$

where N is the prediction horizon that equals to the control horizon, $\|.\|_2^2$ means the squared quadratic norm; $\hat{Y}_{t+1}$ are predictions with respect to unknown overall vector U_t of control actions u_{t+j-1}:

$$\hat{Y}_{t+1} = \left[\hat{y}_{t+1}^T, \cdots, \hat{y}_{t+N}^T\right]^T = F\hat{x}_t + G\,U_t \tag{14}$$

$$U_t = \left[u_t^T, \cdots, u_{t+N-1}^T\right]^T \tag{15}$$

$$F = \begin{bmatrix} CA \\ \vdots \\ CA^{N-1} \\ CA^N \end{bmatrix}, G = \begin{bmatrix} CB & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ CA^{N-2}B & \cdots & CB & 0 \\ CA^{N-1}B & \cdots & CAB & CB \end{bmatrix} \tag{16}$$

and $\Delta\hat{Y}_{t+1}$ uses the state-space model (1) as follows:

$$\begin{aligned} \hat{x}_{t+1} - \hat{x}_t = \Delta\hat{x}_{t+1} &= A(\hat{x}_t - \hat{x}_{t-1}) + B(u_t - u_{t-1}) \\ \Delta\hat{y}_{t+1} &= C\Delta\hat{x}_{t+1} = CA\hat{x}_t + CBu_t - C\hat{x}_t \\ \Delta\hat{y}_{t+2} &= \hat{y}_{t+2} - \hat{y}_{t+1} = (CA^2 - CA)\hat{x}_t \\ &\quad + (CAB - CB)u_t + CBu_{t+1} \\ &\vdots \\ \Delta\hat{y}_{t+N} &= (CA^N - CA^{N-1})\hat{x}_t + \cdots \\ &\quad + (CAB - CB)u_{t+N-2} + CBu_{t+N-1} \end{aligned}$$

Thus

$$\Delta\hat{Y}_{t+1} = \left[\Delta\hat{y}_{t+1}^T, \cdots, \Delta\hat{y}_{t+N}^T\right]^T = F_\Delta\,\hat{x}_t + G_\Delta\,U_t \tag{17}$$

$$F_\Delta = \begin{bmatrix} CA - C \\ \vdots \\ CA^{N-1} - CA^{N-2} \\ CA^N - CA^{N-1} \end{bmatrix} \tag{18}$$

$$G_\Delta = \begin{bmatrix} CB & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ CA^{N-2}B - CA^{N-3}B & \cdots & CB & 0 \\ CA^{N-1}B - CA^{N-2}B & \cdots & CAB - CB & CB \end{bmatrix}$$

Furthermore W_{t+1} represents a sequence of references

$$W_{t+1} = \left[w_{t+1}^T, \cdots, w_{t+N}^T\right]^T \tag{19}$$

and Q_{YW}, $Q_{\Delta Y}$ and Q_U are penalisation matrices defined as follows

$$Q_\diamond^T Q_\diamond = \begin{bmatrix} Q_*^T Q_* & & 0 \\ & \ddots & \\ 0 & & Q_*^T Q_* \end{bmatrix} \left| \begin{array}{l} \text{subscripts } \diamond, *: \\ \diamond \in \{YW, \Delta Y, U\} \\ * \in \{yw, \Delta y, u\} \end{array} \right. \tag{20}$$

• *Minimization Procedure* – Optimality criterion is generally defined as follows

$$\begin{aligned} &\min_{U_t} J_t\,(\hat{Y}_{t+1}, W_{t+1}, U_t) \\ &\text{s.t. state space model (1)} \\ &\qquad \text{state estimates } \hat{x}_t \end{aligned} \tag{21}$$

The involved quadratic cost function J_t (13) can be written in square-root form:

$$J_t = \mathbb{J}_t^T \mathbb{J}_t \tag{22}$$

where square-root $\mathbb{J}_t$ of the cost function J_t is as follows

$$\begin{aligned} \mathbb{J}_t &= \begin{bmatrix} Q_{YW} & 0 & 0 \\ 0 & Q_{\Delta Y} & 0 \\ 0 & 0 & Q_U \end{bmatrix} \begin{bmatrix} \hat{Y}_{t+1} - W_{t+1} \\ \Delta Y \\ U_t \end{bmatrix} \\ &= \begin{bmatrix} Q_{YW}F\hat{x}_t + Q_{YW}GU_t - Q_{YW}W_{t+1} \\ Q_{\Delta Y}F_\Delta\,\hat{x}_t + Q_{\Delta Y}G_\Delta\,U_t \\ Q_U U_t \end{bmatrix}. \end{aligned} \tag{23}$$

Considering minimization of the square-root $\mathbb{J}_t$ as a specific solution of least-squares problem then let us take into account the following system of algebraic equations:

$$\begin{bmatrix} Q_{YW}\, G \\ Q_{\Delta Y}\, G_{\Delta} \\ Q_U \end{bmatrix} U_t = \begin{bmatrix} Q_{YW}\,(W_{t+1} - F\hat{x}_t) \\ Q_{\Delta Y}\,(-F_{\Delta}\,\hat{x}_t) \\ 0 \end{bmatrix} \tag{24}$$

or

$$\begin{bmatrix} Q_{YW}\, G & Q_{YW}\,(W_{t+1} - F\hat{x}_t) \\ Q_{\Delta Y}\, G_{\Delta} & Q_{\Delta Y}\,(-F_{\Delta}\,\hat{x}_t) \\ Q_U & 0 \end{bmatrix} \begin{bmatrix} U_t \\ -I \end{bmatrix} = 0 \tag{25}$$

The over-determined system (24) or (25) respectively can be written in condensed general form (26). It can be transformed to another form (27) by orthogonal-triangular decomposition [10] and solved for unknown U_t

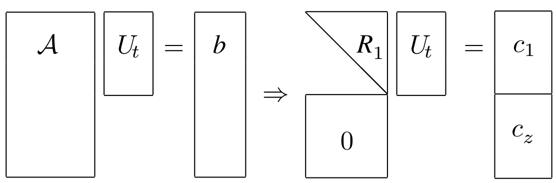
$$\mathcal{A}U_t = b \tag{26}$$

$$Q^T\mathcal{A}U_t = Q^T b \ \text{ assuming that } \ \mathcal{A} = QR$$

$$R_1 U_t = c_1 \tag{27}$$

where Q^T is an orthogonal matrix that transforms matrix $\mathcal{A}$ into upper triangle R_1.

It is indicated by the following equation diagram



(28)

Vector c_z represents a loss vector, Euclidean norm $||c_z||$ of which equals to the square-root of the optimal cost function minimum, i.e. scalar value $\sqrt{J_t}$, where $J_t = c_z^T c_z$. For control, only the first elements corresponding to u_t are used from computed vector U_t, i.e. $u_t = MU_t$, where matrix M is defined as $M = [I_{n_u}, 0_{n_u}, \cdots, 0_{n_u}]$, n_u is dimension of vector of control actions u_t.

C. Output-Feedback MPC under Uniform Disturbances

This subsection summarises the proposed output-feedback MPC scheme. We assume that the model of a controlled system, generally nonlinear, can be converted into a linear state space model with the state-dependent model matrices and that the state estimation uses the model (1). The corresponding block diagram is shown in Fig. 1. Note that the mentioned nonlinear model and its conversion is described in Sec. IV-A. The data flow in Fig. 1 considering this specific model is indicated by the following algorithm sequence:

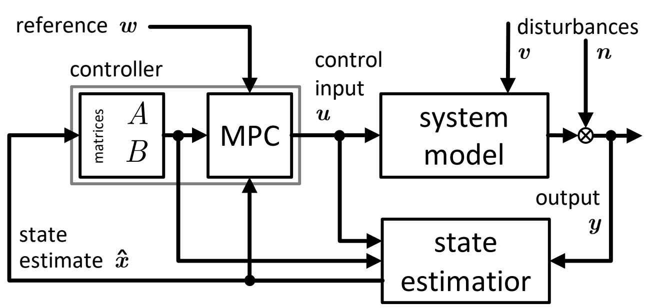


Fig. 1. Block diagram of output-feedback MPC.

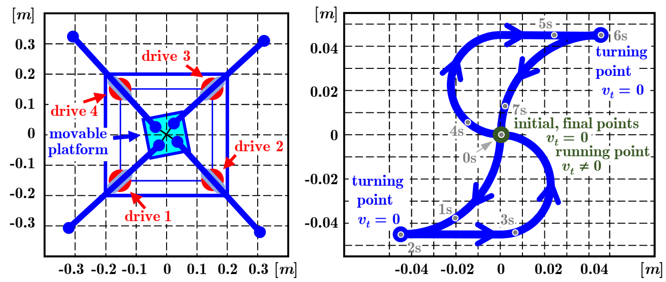


Fig. 2. Robot wireframe and used testing trajectory

Initialisation:

i. assign the initial state $\hat{x}_0$ and control u_0

ii. set $t := 1$, $\bar{t} \geq 1$

iii. load the reference trajectory $w_1, w_2, \ldots, w_{\bar{t}}$

iv. initialise nonlinear continuous model (29)

v. set r and ρ for LSU model (1)

vi. set N, Q_{yw}, $Q_{\Delta y}$ and Q_u in (13)

On-line phase:

1. update the model matrices A_t, B_t in (30)
2. compute the control input u_t (see Sec. III-B)
3. simulate a new state of model (29) in $t+1$
4. set time $t := t+1$
5. measure disturbed system output y_t
6. estimate the state $\hat{x}_t$ (see Sec. III-A)
7. if $t < \bar{t}$, go to 1.

End, result evaluation.

IV. Experiments

This section demonstrates the proposed output-feedback MPC applied to the motion control of a parallel kinematic machine (PKM) represented by the model of the machine dynamics.

A. Description of Controlled System

The selected PKM represents the planar parallel robot-manipulator [11] with four inputs (torques) and three outputs (tool center point (TCP) positions x_{TCP} and y_{TCP} and rotation angle ψ_{TCP} of robot movable platform around axis z), see the left part of Fig. 2.

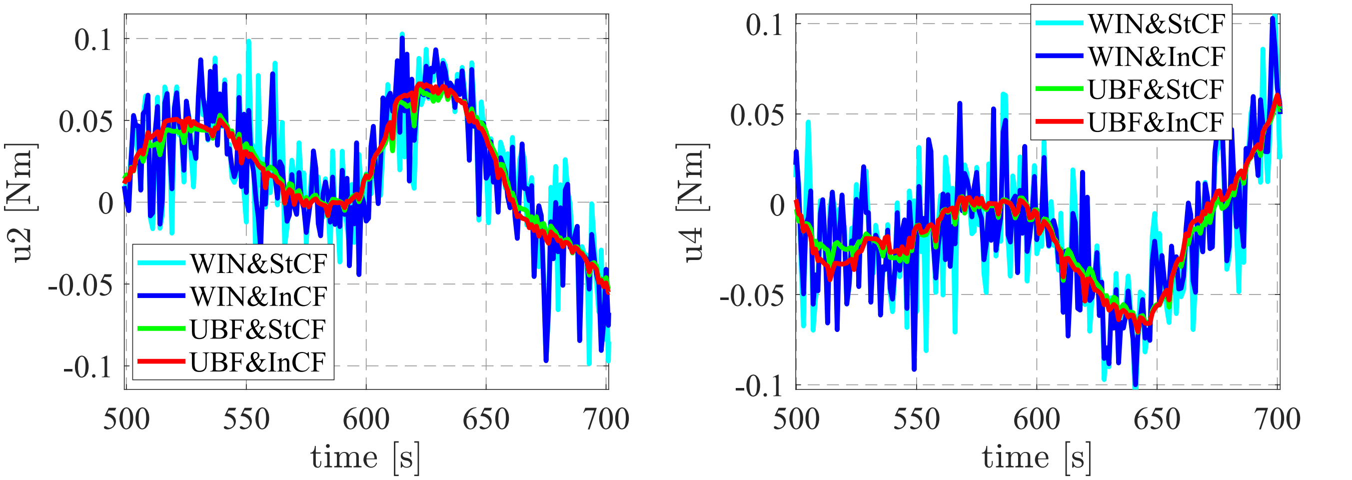


Fig. 3. Experiment (I): Comparison of a zoomed part of control input u_2 (on the left) and u_4 (on the right) for WIN & StCF (cyan), WIN & InCF (blue), UBF & StCF (green) and UBF & InCF (red)

The ideal mathematical-physical dynamic model of PKM,

$$\ddot{y} = f(y, \dot{y}) + g(y)\, u \tag{29}$$

is derived using Lagrange equations [12]. It can be transformed into the discrete-time linear-like state-space model

$$\begin{aligned} x_{t+1} &= A_t\, x_t + B_t\, u_t \\ y_t &= C\, x_t \end{aligned} \tag{30}$$

The transformation uses a specific decomposition technique, keeping $A(x)\,x = [\dot{y}^T, f(y,\dot{y})^T]^T$ and $B(x) = [0, g(y)^T]^T$. The elements of model matrices A_t and B_t of a discretized model depend on a current system state $x_t = [y_t^T, \dot{y}_t^T]^T$. This state corresponds to the system output y and its time derivative $\dot{y}$ in the discrete time instants $\tau = t\, T_s$, where T_s is a sampling period, $t = 1, 2, \ldots$ i.e. $x_t = x(\tau)|_{\tau = t\, T_s}$: $A(x_t) \to A_t$ and $B(x_t) \to B_t$. The update of nonlinear model of robot dynamics (29), decomposition and discretisation (30) are repeated in each time instant t.

In the proposed MPC design, the matrices A_t and B_t are considered to be constant i.e., $A_t \to A$ and $B_t \to B$, within one optimisation step, see (16) and (18).

Further, we consider that the states and outputs are influenced by additive bounded disturbances.

Under above mentioned assumptions, the deterministic model (30) converts into the stochastic model (1) whose matrices A and B are updated in each time step. That model is subsequently used for the state filtering as described in Section III-A. The respective state estimates (12) are then utilised both for the control design (see Sec. III-B) and for the next update of A_t, B_t during the transformation of (29).

B. Experiment Setup

The controlled system is simulated by (29) and a uniform noise is added to the output. The state estimates $\hat{x}_t$ are obtained by (12) using the model (1) with the noise bounds set as follows: $\rho = 10^{-6}[m, m, rad, m\,s^{-1}, m\,s^{-1}, rad\,s^{-1}]^T$, $r = 10^{-3}[m, m, rad]^T$. The control parameters in (13) are set as follows: $N = 10$; $Q_{yw} = I$, $Q_{\Delta y} = c\,I$, $c \in \{0; 3\}$, $Q_u = 10^{-2}\, I$, I is the identity matrix of the appropriate order.

In the presented experiments, we compare the performance of the proposed MPC scheme—the state estimation by uniform Bayesian filter (UBF) from Sec. III-A and the incremental cost function (13) with Q_{yw}, Q_u and $Q_{\Delta y}$, shortly (InCF)—with the performance of the MPC scheme presented in [7] where the state estimation is performed on a moving window (WIN) [13] and the standard cost function (StCF) is used that correspond to (13) with $Q_{\Delta y} = 0$. Also, the combinations of UBF with StCF and WIN with InCF are examined.

The quality of the control process is evaluated by the visual comparison of the results and by the using a root mean square error ($RMSE$) between outputs y_t and references w_t:

$$RMSE_i = \sqrt{\frac{1}{\bar{t}} \sum_{t=1}^{\bar{t}} (y_{t;i} - w_{t;i})^2}, \; i = \{1, 2, 3\}. \tag{31}$$

We perform the following experiments: (I) the robot moves along the whole reference trajectory as depicted in Figure 2 and (II) the robot moves from the start point to the first turning point, then stops and it is required to stay in this position.

C. Results and Discussion

The experiments show that the proposed MPC with UBF outperforms the previously proposed MPC with WIN from the control inputs point of view, see Fig. 3. There, the behaviour of two control inputs is demonstrated. Using UBF filter, the control action are significantly smoother. Note that the remaining two control inputs behaves in a similar way.

The numerical comparison of $RMSE_i$ values for experiment (I) is presented in Table I. The results for both UBF and WIN filter are comparable. The InCF brings the bigger values for both filters.

The results of experiment (II) show that the UBF filter (both with InCF and StCF) provides a better output stabilisation,

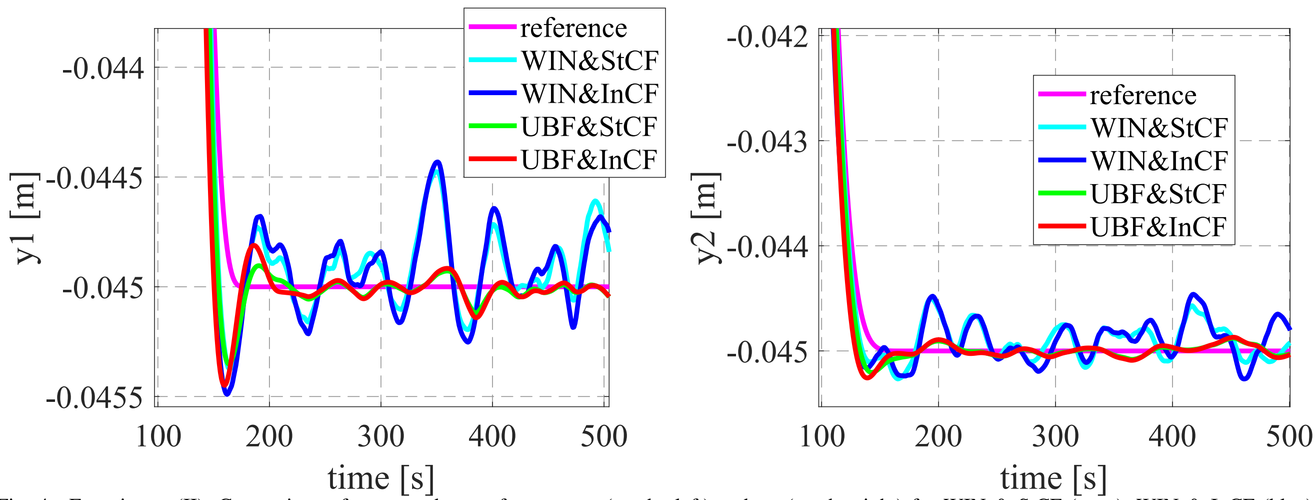


Fig. 4. Experiment (II): Comparison of a zoomed part of output y_1 (on the left) and y_2 (on the right) for WIN & StCF (cyan), WIN & InCF (blue), UBF & StCF (green) and UBF & InCF (red) with the reference (magenta).

TABLE I
EXPERIMENT (I): $RMSE_i$ (31) FOR THE VARIOUS COMBINATION OF FILTERS AND COST FUNCTIONS.

i	UBF&InCF	UBF& StCF	WIN& InCF	WIN& StCF
1	$1,01.10^{-3}$	$0,69.10^{-3}$	$1,05.10^{-3}$	$0,73.10^{-3}$
2	$0,94.10^{-3}$	$0,71.10^{-3}$	$0,96.10^{-3}$	$0,72.10^{-3}$
3	$0,71.10^{-3}$	$0,71.10^{-3}$	$0,64.10^{-3}$	$0,66.10^{-3}$

see Fig. 4, comparing to MPC with WIN filter. This is important e.g. when the PKM has to stop.

V. CONCLUSION

The paper proposes a novel solution to the output-feedback MPC considering bounded state and output disturbances. The proposed filter provides the state estimates that are used both for control design and also for the update of state dependent model matrices. The cost function (13) could be reduced further in (21), if parallel observations of a related state sequence are available, such as in a multi-sensor environment. Bayesian knowledge transfer between such uniformly modelled state-space processes has been reported recently in [14].

Comparing to the previous work of authors [7], the proposed control scheme with UBF estimator provides significantly smoother control actions and a better output stabilisation. The adding of penalties of the output increment does not influence the control process significantly. Nevertheless, the WIN estimator [13] would be useful in cases when the disturbance bounds are unknown as it provides not only the state estimates but also the estimates of the noise bounds.

The proposed solution considers an unconstrained positional MPC. The overshoot of possible constraints is prevented by the appropriate design of reference trajectory and its suitable time parametrisation [15].

The following research will concentrate on a deeper analysis of a proposed control scheme. An alternative choice of the point state estimate will be investigated. Namely, the proposed UBF provides state estimates in the form of uniform distribution (11), with only its mean (12) processed in the current control design. However, the fully Bayesian state inference (11) provides the opportunity for fully probabilistic design of the control, based possibly on optimal transfer between multiple filters [14].

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