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# Measurement of common risks in tails: A panel quantile regression model for financial returns☆

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ABSTRACT

We investigate how to measure common risks in the tails of return distributions using the recently proposed panel quantile regression model for financial returns. By exploring how volatility crosses all quantiles of the return distribution and using a fixed effects estimator, we can control for otherwise unobserved heterogeneity among financial assets. Direct benefits are revealed in a portfolio value-at-risk application, where our modeling strategy performs significantly better than several benchmark models. In particular, our results show that the panel quantile regression model for returns consistently outperforms all competitors in the left tail. Sound statistical performance translates directly into economic gains.


## 1. Introduction

Many studies document cross-sectional relations between risk and expected returns, generally measuring a stock's risk as the covariance between its return and a certain factor. In this laborious search for appropriate risk factors, volatility has played a central role in explaining expected stock returns for decades (French et al., 1987; Harvey et al., 2016; Feng et al., 2019). The most recent efforts explore increasingly available datasets and make measurement of ex post volatility more precise than ever before. In turn, these measures can be used for a more precise identification of market risk. Although predictions about expected returns are essential for classical asset pricing, little is known about the potential of the factors to precisely identify extreme tail events of the returns distribution. More importantly, even less is known about commonalities between more assets in this respect. Our research attempts to contribute in this direction.

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Asset pricing models typically describe the decision making process of an economic agent with help of the expected utility function. However, the concept of expected utility may be too restrictive to deliver satisfactory descriptions of the real behavior of agents. Therefore, number of researchers strive to incorporate heterogeneity into dynamic economic models that include agents who maximize their stream of future quantile utilities (Chambers, 2007; Rostek, 2010; de Castro and Galvao, 2019). We contribute to these efforts by developing a panel quantile regression model for financial returns that is able to control for otherwise unobserved heterogeneity among financial assets and allows us to exploit common factors in volatility that directly affect future quantiles of returns. In a sense, we revisit a large body of literature connecting volatility with the cross-section of returns, as by construction, we model the tail events of the conditional distributions via volatility.

Since the seminal work of Koenker and Bassett Jr. (1978), quantile regression models have been increasingly used in many disciplines. In finance, Engle and Manganelli (2004) are among the first to use quantile regression to develop the conditional autoregressive value-at-risk (CAViaR) model and capture conditional quantiles of the financial returns. Baur et al. (2012) use quantile autoregressions to study conditional return distributions, and Cappiello et al. (2014) detect comovement between random variables with a time-varying quantile regression. Žikeš and Baruník (2016) show that various volatility measures are useful in forecasting quantiles of future returns without making assumptions about underlying conditional distributions. The resulting semiparametric modeling strategy captures conditional quantiles of financial returns well in a flexible framework of quantile regression. Moving towards a multivariate framework and concentrating on interrelations between quantiles of more assets, White et al. (2015) pioneer the extension. Different streams of multivariate quantile regression-based literature concentrate on the analysis using factors (Chen et al., 2018; Ando and Bai, 2020).[1] From a theoretical point of view, Giovannetti (2013) derives an asset pricing model in which the equity premium is no longer based on the covariance between return and consumption. Instead, Giovannetti (2013) argue that under optimism, higher volatility can be connected to a high chance of high returns leading to increased prices, hence decreasing expected returns, and vice versa under pessimism. Based on Choquet utility functions, Bassett et al. (2004) show that pessimistic optimization can be formulated as a linear quantile regression problem and can lead to optimal portfolio allocation.

In this respect, the work by Žikeš and Baruník (2016) is important because it provides a link between future quantiles of financial return distributions and their past variation. As the financial sector is highly connected and comovements in asset prices are common, there is a need for the proper identification of dependencies in joint distributions. In the classical mean-regression framework, Bollerslev et al. (2018) show that the realized volatility of financial time series shares many commonalities. In the quantile regression setup, however, there is no similar study that attempts to uncover information captured in the panels of volatility series. Moreover, to the best of our knowledge, there is no study that estimates the conditional distribution of returns in a multivariate setting that explores ex post information in volatility.

In this paper, we contribute to the literature by introducing a panel quantile regression model for financial returns that allows us to measure common risk factors in tails of the return distributions. Our model utilizes all the advantages offered by panel quantile regression and financial market datasets. In particular, we can control for otherwise unobserved heterogeneity among financial assets and reveal common factors in volatility that have direct influence on the future quantiles of returns. To the best of our knowledge, this is one of the first applications of panel quantile regressions using a dataset in which the time dimension $T$ is much greater than the cross-sectional dimension $N$, i.e., $T >> N$. As a result, we can obtain estimates of quantile-specific individual fixed effects that represent the idiosyncratic part of market risk.

In an empirical application, we hypothesize that our model will deliver more accurate estimates compared to currently established methods. Moreover, these estimates translate into better forecasting performance of the panel quantile regression model for financial returns. In addition, using a fixed effect estimator, we disentangle overall market risk into systematic and idiosyncratic parts. The actual performance of our model is tested in a portfolio value-at-risk (VaR) forecasting exercise using dataset consisting of 29 constituents of the S&P 500 Index during period from July 1, 2005 to December 31, 2015. For robustness reasons, we evaluate forecasts from both statistical and economic perspectives. In the statistical comparison, we further distinguish between the absolute and relative performance of the given model. The economic comparison then studies the performance of the models in terms of optimal portfolio.

The results suggest that our panel quantile regression model for financial returns is dynamically correctly specified. Moreover, it dominates the benchmark models in the economically important quantiles (5%, 10% or 95%). Overall, we find that according to statistical comparisons, none of the benchmark models is able to outperform our model consistently. Furthermore, our model provides direct economic gains according to economic comparison.

The remainder of this paper is organized as follows. Section 2 provides details about risk measures computed from the high-frequency data. Section 3 introduces panel quantile regression model for financial returns. Section 4 describes benchmark models and the forecasting exercise, including economic and statistical evaluation criteria. In Section 5 and Section 6 we discuss our results, and Section 7 concludes.

[1] Panel quantile methods are useful in other areas of economics as well. They are applied mostly in labor economics (Toomet, 2011; Dahl et al., 2013; Billger and Lamarche, 2015), banking and economic policy analysis (Klomp and De Haan, 2012; Covas et al., 2014), economics of education (Lamarche, 2008, 2011), energy and environmental economics (You et al., 2015; Zhang et al., 2015), and international trade (Dufrenot et al., 2010; Foster-McGregor et al., 2014; Powell and Wagner, 2014).

## 2. Risk measurement using high-frequency data

In this section, we discuss the so called "realized measures" that are used later for measurement of risk using high-frequency data. Let us assume that the efficient logarithmic price process $p_{i,t}$ of $i = 1, \dots, N$ assets evolves over time $0 \leq t \leq T$ according to the following dynamics:

$$dp_{i,t} = \mu_{i,t}dt + \sigma_{i,t}dW_{i,t} + dJ_{i,t}, \tag{1}$$

where $\mu_{i,t}$ is a predictable component, $\sigma_{i,t}$ is a cadlag process, $W_{i,t}$ is a standard Brownian motion, and $J_{i,t}$ is a jump process.

The volatility of the logarithmic price process can be measured by quadratic return variation, which can be decomposed into integrated variance (IV) of the price process and the jump variation (JV):

$$QV_{i,t} = \underbrace{\int_{t-1}^{t} \sigma_{i,s}^2 ds}_{IV_{i,t}} + \underbrace{\sum_{l=1}^{L_{i,t}} \kappa_{i,t,l}^2}_{JV_{i,t}}, \tag{2}$$

where $L_{i,t}$ is the total number of jumps during day $t$ and $\sum_{l=1}^{L_{i,t}} \kappa_{i,t,l}^2$ represents the magnitude of the jumps. As shown by Andersen et al. (2003), the realized variance estimator can be simply constructed by squaring intraday returns:

$$\widehat{RV}_{i,t} = \sum_{k=1}^{M_{i,t}} \left(\Delta_k p_{i,t}\right)^2, \tag{3}$$

where $\Delta_k p_{i,t} = p_{i,t-1+k/M_{i,t}} - p_{i,t-1+(k-1)/M_{i,t}}$ is a discretely sampled vector of $k$-th intraday log-returns of $i$-th asset in $[t-1, t]$, with $M$ intraday observations. Moreover, the realized variance estimator converges uniformly in probability to $QV_{i,t}$ as the sampling frequency goes to infinity, as follows:

$$\widehat{RV}_{i,t} \underset{M_{i,t}\to\infty}{\overset{p}{\to}} \int_{t-1}^{t} \sigma_{i,s}^2 ds + \sum_{l=1}^{L_{i,t}} \kappa_{i,t,l}^2.$$

Building on the concept of realized variance, Barndorff-Nielsen and Shephard (2004a) and Barndorff-Nielsen and Shephard (2006) introduce the bipower variation (BPV) estimator that is robust to jumps and thus able to consistently estimate $IV_{i,t}$. Furthermore, Andersen et al. (2011) adjust the original estimator, which helps render it robust to certain types of microstructure noise:

$$\widehat{IV}_{i,t}^{BPV} = \mu_1^{-2} \left(\frac{M_{i,t}}{M_{i,t}-2}\right) \sum_{k=3}^{M_{i,t}} |\Delta_{k-2} p_{i,t}||\Delta_k p_{i,t}|,$$

where $\mu_\alpha = E(|Z^\alpha|)$, and $Z \sim N(0,1)$. Having an estimator of $IV_{i,t}$ in hand, jump variation can be consistently estimated[2] as a difference between the realized variance and the bipower variation:

$$\left(\widehat{RV}_{i,t} - \widehat{IV}_{i,t}^{BPV}\right) \underset{M_{i,t}\to\infty}{\overset{p}{\to}} \sum_{l=1}^{L_{i,t}} \kappa_{i,t,l}^2.$$

For many financial applications, not only the magnitude of the variation but also its sign is important. Therefore, Barndorff-Nielsen et al. (2010) introduce an innovative approach for measuring negative and positive variation in data called realized semivariance (RSV). They show that realized variance can be decomposed to realized downside semivariance ($RS_{i,t}^-$) and realized upside semivariance ($RS_{i,t}^+$):

$$RV_{i,t} = RS_{i,t}^+ + RS_{i,t}^-,$$

where $RS_{i,t}^+$ and $RS_{i,t}^-$ are defined as follows:

$$\widehat{RS}_{i,t}^+ = \sum_{k=1}^{M_{i,t}} \left(\Delta_k p_{i,t}\right)^2 I\left(\Delta_k p_{i,t} > 0\right) \overset{p}{\to} \frac{1}{2} IV_{i,t} + \sum_{l=1}^{L_{i,t}} \kappa_{i,t,l}^2 I\left(\kappa_{i,t,l} > 0\right) \tag{4}$$

$$\widehat{RS}_{i,t}^- = \sum_{k=1}^{M_{i,t}} \left(\Delta_k p_{i,t}\right)^2 I\left(\Delta_k p_{i,t} < 0\right) \overset{p}{\to} \frac{1}{2} IV_{i,t} + \sum_{l=1}^{L_{i,t}} \kappa_{i,t,l}^2 I\left(\kappa_{i,t,l} < 0\right). \tag{5}$$

[2] Asymptotic behavior and further details on the estimator can be found in Barndorff-Nielsen and Shephard (2006).

Consequently, the negative and positive semivariance provides information about variation associated with movements in the tails of the underlying variable. Similar to Patton and Sheppard (2015) and Bollerslev et al. (2020), we use negative semivariance as a proxy for the bad state of the returns and positive semivariance as an empirical proxy for the good state of the underlying variable.

Since correlation is inevitably important in portfolio applications, and we use it later in our portfolio VaR application, we also define the realized covariance estimator (Barndorff-Nielsen and Shephard, 2004b) as:

$$\widehat{\Sigma}_t = \sum_{k=1}^{M} \left(\mathbf{\Delta_k p_t}\right)\left(\mathbf{\Delta_k p_t}\right)^\top,$$

where $\mathbf{\Delta_k p_t} = (\Delta_k p_{1,t}, \ldots, \Delta_k p_{N,t})^\top$ is a vector containing intraday log-returns of $N$ individual assets.

## 3. Panel quantile regression model for financial returns

Having briefly described realized measures that we need for model construction, we now propose simple linear models for the cross-section of quantiles of future returns. We base our model in a recent theoretical endeavor to move from expected values to quantiles, thereby understanding heterogeneity in asset prices. Based on the risk preferences of quantile maximizers defined by Manski (1988) and Rostek (2010), de Castro and Galvao (2019) develop a dynamic model of rational behavior under uncertainty, in which an agent maximizes streams of future quantile utilities. This is in sharp contrast to the mainstream literature where it is assumed that the decision-making process is driven by maximization of the expected utility. In the spirit of Bassett et al. (2004), the general version of our model can be viewed as a linear asset pricing equation:

$$Q_{r_{i,t+1}}\left(\tau|v_{i,t}, F_t, \alpha_i(\tau)\right) = \underbrace{\alpha_i(\tau)}_{\text{Unobserved Heterogeneity}} + \underbrace{v_{i,t}^\top \beta(\tau)}_{\text{Idiosyncratic Risk}} + \underbrace{F_t^\top \gamma(\tau)}_{\text{Common Risk}}, \quad \tau \in (0,1), \tag{6}$$

where $r_{i,t+1} = \sum_{k=1}^{M_{i,t+1}} \Delta_k p_{i,t+1}$ are logarithmic daily returns, $\alpha_i(\tau)$ represents individual fixed effects that account for unobserved heterogeneity, $v_{i,t}$ are measures of quadratic variation as defined in the previous section and account for the firm-specific (idiosyncratic) risk, and $F_t$ represents exogenous common risk that is the same for all stocks. This model enables us to study the influence of the various sources of risk on the specific quantiles of future returns. Furthermore, in the case of $\gamma(\tau) \neq 0$ for a given $\tau$, the model allows us to capture the common risk factors in the tails.

While equation (6) accommodates many possible specifications, we are interested in the set of following models. In the first set of model specifications, quantiles of the return series depend on the unobserved heterogeneity and idiosyncratic risk measured by one of the three realized measures[3]:

- *PQR-RV* model defined as:

$$Q_{r_{i,t+1}}\left(\tau|RV_{i,t}^{1/2}, \alpha_i(\tau)\right) = \alpha_i(\tau) + \beta_{RV^{1/2}}(\tau) RV_{i,t}^{1/2}, \tag{7}$$

- *PQR-RSV* model defined as:

$$Q_{r_{i,t+1}}\left(\tau|RS_{i,t}^{+\,1/2}, RS_{i,t}^{-\,1/2}, \alpha_i(\tau)\right) = \alpha_i(\tau) + \beta_{RS^{+1/2}}(\tau) RS_{i,t}^{+\,1/2} + \beta_{RS^{-1/2}}(\tau) RS_{i,t}^{-\,1/2}, \tag{8}$$

- *PQR-BPV* model defined as:

$$Q_{r_{i,t+1}}\left(\tau|BPV_{i,t}^{1/2}, JV_{i,t}^{1/2}, \alpha_i(\tau)\right) = \alpha_i(\tau) + \beta_{BPV^{1/2}}(\tau) BPV_{i,t}^{1/2} + \beta_{JV^{1/2}}(\tau) JV_{i,t}^{1/2}. \tag{9}$$

In the second set of model specifications, we study the role of the ex ante measure of market volatility, i.e., the VIX index, which we consider to be a good proxy for the common exogenous factor. These specifications will measure the direct influence of the common market factor once we control for asset-specific volatility and unobserved heterogeneity:

- *PQR-RV-VIX* model defined as:

$$Q_{r_{i,t+1}}\left(\tau|RV_{i,t}^{1/2}, VIX_t, \alpha_i(\tau)\right) = \alpha_i(\tau) + \beta_{RV^{1/2}}(\tau) RV_{i,t}^{1/2} + \gamma_{VIX}(\tau) VIX_t, \tag{10}$$

- *PQR-RSV-VIX* model defined as:

$$Q_{r_{i,t+1}}\left(\tau|RS_{i,t}^{+\,1/2}, RS_{i,t}^{-\,1/2}, VIX_t, \alpha_i(\tau)\right) = \alpha_i(\tau) + \beta_{RS^{+1/2}}(\tau) RS_{i,t}^{+\,1/2} + \beta_{RS^{-1/2}}(\tau) RS_{i,t}^{-\,1/2} + \gamma_{VIX}(\tau) VIX_t, \tag{11}$$

[3] The realized measures we are using as regressors have been previously generated and generally may cause a generated regressors problem. In our setup, however, the variances are computed via bootstrapping, which should correct for the problem.

- *PQR-BPV-VIX* model defined as:

$$Q_{r_{i,t+1}}\left(\tau|BPV_{i,t}^{1/2},JV_{i,t}^{1/2},VIX_t,\alpha_i(\tau)\right)=\alpha_i(\tau)+\beta_{BPV^{1/2}}(\tau)BPV_{i,t}^{1/2}+\beta_{JV^{1/2}}(\tau)JV_{i,t}^{1/2}+\gamma_{VIX}(\tau)VIX_t. \quad (12)$$

Details of these specifications are described in Section 6. Generally, equation (6) can be easily extended by other exogenous variables, such as the factors used in Fama and French (1993). The results of analysis when the risk factors are the three Fama-French (FF3) factors and market volatility proxied by realized volatility of the S&P 500 Index (SPX) are presented in the Online Appendix C.

### 3.1. Estimation

In our work, we concentrate on commonalities in the quantiles of several return series. To obtain parameter estimates of the general model defined in equation (6), we use panel quantile regression as introduced in Koenker (2004), who proposed a penalized fixed effects estimator as a general way of estimating quantile regression models in the panel data framework. Subsequently, Lamarche (2010) studied a penalized quantile regression estimator, and Galvao (2011) introduced a fixed effects model for dynamic panels. Galvao and Montes-Rojas (2010) moreover showed that bias in dynamic panels can be reduced using a penalty term. Furthermore, Canay (2011) introduced a simple two-step approach to the estimation of panel quantile regressions and showed consistency and asymptotic normality of the proposed estimator. Other influential works developing the theory of panel quantile methods include Harding and Lamarche (2009), Kato et al. (2012), Harding and Lamarche (2014), Galvao and Montes-Rojas (2015), Galvao and Wang (2015), Galvao and Kato (2016), and Graham et al. (2018).

Although the literature devoted to panel quantile estimators is growing and many interesting alternatives have been introduced, to estimate the parameters of our models we use the non-penalized fixed effects estimator of Koenker (2004). We refrain from including a penalty in the main analysis since our time dimension is much higher than the cross-sectional dimension and Galvao and Montes-Rojas (2010) show that the increasing time dimension reduces the usefulness of the shrinkage method.[4] The advantage of this approach is the ability to account and control for unobserved heterogeneity among financial assets, which will yield more precise quantile-specific estimates. As a consequence, these estimates will directly translate into better forecasting performance. Moreover, this approach can be used to obtain precise estimates of the VaR, which is a commonly used financial industry risk measure. In the VaR application, panel data will utilize all the favorable properties of the standard time series. In addition, the cross-sectional dimension will help us to account for common shocks among the assets.

To obtain the parameter estimates, we solve the following optimization problem:

$$\min_{\alpha_i(\tau),\beta(\tau),\gamma(\tau)}\sum_{t=1}^{T}\sum_{i=1}^{N}\rho_\tau\left(r_{i,t+1}-\alpha_i(\tau)-v_{i,t}^\top\beta(\tau)-F_t^\top\gamma(\tau)\right), \quad (13)$$

where $\rho_\tau(u)=u\,(\tau-I(u<0))$ is the quantile loss function (Koenker and Bassett Jr., 1978).

In equation (13), we further consider individual fixed effects to have distributional effects, and we concentrate on each quantile separately rather than solving the optimization problem through several quantiles simultaneously. In contrast, Koenker (2004) and the vast majority of the theoretical and applied works consider $\alpha_i$ to have a pure location shift effect on the conditional quantiles. This restriction is a consequence of the structure of the usual panel datasets where the cross-sectional dimension is much larger than the time dimension.[5] This problem is not so severe in our application since the majority of assets have a long history and thus consist of thousands of observations. Moreover, analysis of the specific quantiles is essential for many financial applications, including the popular VaR, in which we are most often interested in finding a 1-day 5% VaR or 10-day 1% VaR, as historically recommended by the Basel Committee on Banking Supervision in 2004.

## 4. Competing models and evaluation

In the previous section, we introduce the panel quantile regression model for financial returns, which is used in the applied part of the paper to analyze empirical data.[6] In this section, we describe alternative approaches that can be viewed as direct competitors to our model. The benchmarks in our work include the popular and widely used RiskMetrics model, which is the industry standard for risk evaluation in high-dimensional problems, and two applications of the univariate quantile regression model for financial returns.

[4] Discussion about the penalized fixed effect estimator is presented in Online Appendix B.

[5] As detailed in Koenker (2004), it is not advisable to estimate $\tau$-specific $\alpha_i$ in problems with small/medium $T$.

[6] In the Online Appendix D. we present results of the analysis using simulated data.

### 4.1. RiskMetrics

Based on the exponentially-weighted moving average, J.P. Morgan Chase in 1996 introduced a new methodology for accessing financial risk, called RiskMetrics. It is considered to be the baseline benchmark model for numerous financial applications. For our benchmark purposes, we adopt the specification in its original form as defined in Longerstaey and Spencer (1996) with the decay factor $\lambda_{RM}$ set to 0.94. For logarithmic daily returns, $r_{i,t} \sim N\left(0, \sigma^2_{i,t}\right)$ with conditional variance $\sigma^2_{i,t}$ and conditional covariance between assets $i$ and $j$ at time $t+1$:

$$\sigma_{ij,t+1} = \lambda_{RM}\sigma_{ij,t} + (1-\lambda_{RM})r_{i,t}r_{j,t},$$

the RiskMetrics dictates quantile function has the following form:

$$Q_{r_{i,t+1}}(\tau|\sigma_{i,t+1}) = \varphi_\tau \sigma_{i,t+1}, \tag{14}$$

where $\varphi_\tau$ is the $\tau$-quantile of the standard normal distribution.

### 4.2. Univariate quantile regression model for financial returns

As already mentioned, Žikeš and Baruník (2016) introduce an elegant framework for modeling and obtaining forecasts of the conditional quantiles of financial returns in a univariate setting. They propose modeling quantiles of return series according to:

$$Q_{r_{i,t+1}}(\tau|v_{i,t}, z_t) = \alpha_i(\tau) + v_{i,t}^\top \beta_i(\tau) + z_t^\top \gamma_i(\tau), \tag{15}$$

where $v_{i,t} = \left(\widehat{QV}^{1/2}_{i,t},\ \widehat{QV}^{1/2}_{i,t-1}, \dots, \widehat{IV}^{1/2}_{i,t}, \widehat{IV}^{1/2}_{i,t-1}, \dots, \widehat{JV}^{1/2}_{i,t}, \widehat{JV}^{1/2}_{i,t-1}, \dots\right)$ are components of quadratic variation and $z_t$ is a vector of weakly exogenous variables. Estimates are obtained by minimizing the following objective function:

$$\min_{\alpha_i(\tau),\beta_i(\tau),\gamma_i(\tau)} \frac{1}{T}\sum_{t=1}^{T} \rho_\tau\left(r_{i,t+1} - \alpha_i(\tau) - v_{i,t}^\top \beta_i(\tau) - z_t^\top \gamma_i(\tau)\right), \tag{16}$$

where $\rho_\tau(u) = u(\tau - I(u < 0))$ is the quantile loss function defined in Koenker and Bassett Jr. (1978). The application of the model in a multivariate setting is further described in the following subsection.

### 4.3. Forecasting exercise and forecast evaluation

To evaluate the performance of our panel quantile regression model for financial returns, we conduct a forecasting exercise in which we study portfolio VaR from statistical and economic points of view. We decided to concentrate on both statistical and economic evaluations to obtain a complete picture of the behavior of our model. Moreover, good statistical performance might not necessarily translate into economic gains. Therefore, to make our results robust, we apply two statistical and two economic evaluation criteria.

In the statistical comparison, we focus on the absolute and relative performance of the considered models in an equally-weighted portfolio setup. By focusing on an equally-weighted portfolio, we refrain from specifying complicated weighting schemes that might affect the overall performance.

In the economic comparison, we study the efficient frontier of the VaR-return trade-off and the global minimum VaR portfolio (GMVaRP). As both approaches by definition attempt to find the optimal weights of the assets, we are no longer using an equally-weighted portfolio here.

#### 4.3.1. Portfolio VaR

VaR is an elegant way of quantifying the risk of an investment. Its simplicity makes it popular in the financial industry because it provides a single number that represents the potential loss that can be incurred at a certain probability level during a predefined period of time. Using VaR as the only risk measure, however, has some limitations. VaR is generally not a coherent risk measure because it violates the subadditivity criteria (Artzner et al., 1999). However, Daníelsson et al. (2013) show that under reasonable assumptions, the VaR might be subadditive. We use a VaR framework because the forecasts we obtain from the panel quantile regression model for returns are by definition semi-parametric VaRs.[7] Moreover, we are not attempting to introduce new measures of financial risk[8]; rather, we aim to show the accuracy of our model in the standard setup.

[7] According to Jorion (2007, p. 17) "Value-at-risk describes the *quantile* of the projected distribution of gains and losses over the targeted horizon." Since the VaR is a quantile of returns and we model quantiles of returns directly by panel quantile regressions, we obtain semi-parametric VaR estimates.

[8] Recently, the Basel Committee on Banking Supervision has promoted the move from VaR to expected shortfall as the primary risk measurement for banks.

We next examine the VaR framework itself. Generally, there are two main approaches to calculating VaR: (semi)parametric estimation and historical simulation. We concentrate on the parametric approach because it enables us to compare forecasts from several benchmark models.

The original parametric VaR (Longerstaey and Spencer, 1996) is defined as:

$$VaR_{i,t} = \varphi_\tau \sigma_{i,t}, \tag{17}$$

where $\varphi_\tau$ is the $\tau$ quantile of the standard normal distribution and $\sigma_{i,t}$ is the volatility of asset $i$. If we would like to study the VaR of the portfolio instead of the individual assets, $\sigma_{i,t}$ is replaced by the portfolio volatility $\sigma_{P,t}$. Under the assumption of multivariate normality, $\sigma_{P,t}$ is calculated as:

$$\sigma_{P,t} = \sqrt{w_t^\top \Sigma_t w_t},$$

where $\Sigma_t$ is the covariance matrix and $w_t$ is the vector of asset weights at time $t$. We can therefore calculate the percentage VaR ($\%VaR_t$) of the given portfolio as:

$$\%VaR_{P,t} = sgn(\varphi_\tau)\sqrt{\varphi_\tau^2 w_t^\top \Sigma_t w_t}, \tag{18}$$

where $\text{sgn}(\varphi_\tau)$ is the *sign* function defined as:

$$sgn(\varphi_\tau) = \begin{cases} -1 & \text{if} \quad \varphi_\tau < 0 \\ 0 & \text{if} \quad \varphi_\tau = 0 \\ 1 & \text{if} \quad \varphi_\tau > 0 \end{cases}$$

Since it is common practice to report VaR as an absolute value, i.e., a positive number, we stick to this convention and rewrite equation (18) as:

$$\%VaR_{P,t} = \sqrt{\varphi_\tau^2 w_t^\top \Sigma_t w_t}. \tag{19}$$

We can rewrite equation (19) in terms of the VaR of the individual assets as:

$$\%VaR_{P,t} = \sqrt{(w_t^\top \odot \%VaR_t^\top)\Omega_t(w_t \odot \%VaR_t)}, \tag{20}$$

where $\%VaR_t$ is a vector of individual percentage VaR estimates, $\Omega_t$ denotes the correlation matrix, and $\odot$ is the Hadamar product. Alternatively, we can also write it as:

$$\%VaR_{P,t} = \sqrt{\sum_{i=1}^{N}(w_{i,t}\%VaR_{i,t})^2 + 2\sum_{i=1}^{N-1}\sum_{j=i+1}^{N} w_{i,t}w_{j,t}\%VaR_{i,t}\%VaR_{j,t}\rho_{ij,t}},$$

where $w_{i,t}$ is the weight of asset $i$, $\%VaR_{i,t}$ is the percentage VaR of the $i$th asset, and $\rho_{ij,t}$ represents the correlation between assets $i$ and $j$.

In the forecasting exercise, we study the portfolio VaR performance of the four benchmark model specifications: RiskMetrics, panel quantile regression (PQR) model for financial returns, univariate quantile regression (UQR) model for financial returns, and portfolio version of the univariate quantile regression (portfolio UQR) model for financial returns. For the calculation of portfolio VaR using the RiskMetrics approach, we apply equation (19), where $\Sigma_t$ is the covariance matrix obtained from RiskMetrics and $\varphi_\tau$ is a cut-off point of a standard normal distribution at a given quantile $\tau$.

In the case of PQR and UQR, forecasts of quantiles of return series are considered to be a semiparametric *%VaR*. The correlation matrix $\Omega_t$ is obtained from the realized covariance matrix estimate, $\Sigma_T$, as:

$$\Omega_t = \big(diag(\Sigma_t)\big)^{-1/2}\Sigma_t\big(diag(\Sigma_t)\big)^{-1/2},$$

equation (20) therefore can be used for the VaR calculation.

In contrast to previous approaches, portfolio UQR is calculated in a different fashion. We first create portfolio returns and portfolio volatility series using individual returns and the correlation structure obtained from the realized covariance matrix, $\Sigma_t$, as:

$$r_{P,t} = w_t^\top r_t$$

and

$$\sigma_{P,t} = \sqrt{w_t^\top \Sigma_t w_t},$$

where $r_{P,t}$ and $\sigma_{P,t}$ are portfolio return and portfolio volatility at time $t$, respectively, and $r_t$ is a vector of individual returns at time $t$. Series $r_{P,t}$ and $\sigma_{P,t}$ are further modeled using the UQR model, and the forecasts of the quantiles of the portfolio return series are considered to be a semiparametric percentage portfolio VaR.

*4.3.2. Statistical evaluation*

In the portfolio VaR statistical comparison, we study absolute performance, which tells us whether a model is dynamically correctly specified (i.e., we study goodness-of-fit) and relative performance in which we compare PQR and benchmark models against each other. For the absolute performance evaluation, we use a modified version of the dynamic quantile test (Engle and Manganelli, 2004), referred to as the CAViaR test by Berkowitz et al. (2011). Similar to Berkowitz et al. (2011), we define a "hit" variable as:

$$hit_{t+1} = \begin{cases} 1 & if \quad r_{P,t+1} \le Q_{r_{P,t+1}}(\tau) \\ 0 & otherwise \end{cases},$$

i.e., $hit_{t+1}$ is a binary variable that takes a value of 1 if the conditional quantile of portfolio returns is violated and 0 otherwise. The hit series of a dynamically correctly specified series should be i.i.d Bernoulli distributed with parameter $\tau$ as follows:

$$hit_{t+1} \sim iid(\tau, \tau(1-\tau)).$$

By construction, *hit* is a binary variable; therefore, Berkowitz et al. (2011) proposed testing the hypothesis of correct dynamic specification using logistic regression $\mathbb{P}\left(hit_t = 1\right) = \Lambda(\Theta_t) = \frac{e^{\Theta_t}}{1+e^{\Theta_t}}$, where $\Theta_t = c + \sum_{d=1}^{n} \beta_{1d} hit_{t-d} + \sum_{d=1}^{n} \beta_{2d} Q_{r_{P,t-d+1}}(\tau)$. We use a likelihood ratio test to verify the null hypothesis that $\beta$s are equal to zero and $\mathbb{P}(hit_t = 1) = \frac{e^c}{1+e^c} = \tau$. Exact finite sample critical values for the likelihood ratio test are obtained from Monte Carlo simulations, as suggested by Berkowitz et al. (2011).

The relative performance of benchmark models in the portfolio VaR application is tested using the expected tick loss for pairwise model comparison (Giacomini and Komunjer, 2005; Clements et al., 2008). The loss function is defined as:

$$\boldsymbol{\mathcal{L}}_{\tau,m} = E\left[\left(\tau - I\left\{e_{t+1}^m < 0\right\}\right) e_{t+1}^m\right],$$

where $I\{\cdot\}$ is the indicator function and $e_{t+1}^m = r_{P,t+1} - Q_{r_{P,t+1}}^m(\tau)$ and $Q_{r_{P,t+1}}^m(\tau)$ is the $m$'th model quantile forecast. The forecasting accuracy of the two models is assessed using the Diebold and Mariano (1995) test. The null hypothesis of the test that expected losses of two models are equal, i.e., $H_0 : \boldsymbol{\mathcal{L}}_{\tau,1} = \boldsymbol{\mathcal{L}}_{\tau,2}$, is tested against the general alternative.

*4.3.3. Economic evaluation*

In the economic evaluation, we study portfolio VaR forecasts using the modified Markowitz (1952) approach. Compared to Markowitz (1952), we concentrate on the relationship between the return and the VaR instead of the original risk-return trade-off.[9] To overcome the difficulties of specifying a proper model for returns and covariance/correlation matrices, we use their ex post realizations, i.e., for day $T$, we use the returns realized in day $T$, and the realized covariance/correlation matrix in day $T$.

In general, the efficient frontier of the optimal portfolio can be constructed in two equivalent ways: expected portfolio return is maximized for various levels of portfolio VaR; portfolio VaR is minimized for various levels of expected portfolio return. In both approaches, asset weights, $w_{t+1} = (w_{1,t+1}, \dots, w_{N,t+1})^\top$, maximizing the utility of risk-averse investors can be found by solving the following problem:

$$\begin{aligned} \min_{w_{t+1}} \quad & w_{t+1}^\top \widehat{\Xi}_{t+1|t} w_{t+1} \\ \text{s.t.} \quad & l^\top w_{t+1} = 1 \\ & w_{t+1}^\top \ge 0 \\ & w_{t+1}^\top \widehat{\mu}_{t+1} = \mu_{P,t+1}, \end{aligned} \tag{21}$$

where $w_{t+1}$ is the $N \times 1$ vector of asset weights, $l$ denotes a $N \times 1$ vector of ones, $\widehat{\mu}_{t+1}$ is a vector of ex post returns, $\mu_{P,t+1}$ stands for portfolio return and $\widehat{\Xi}_{t+1|t} = diag\left(\widehat{\%VaR}_{t+1|t}\right)\widehat{\Omega}_{t+1} diag\left(\widehat{\%VaR}_{t+1|t}\right)$ represents a correlated VaR covariance matrix, where $\widehat{\%VaR}_{t+1|t}$ is the $N \times 1$ vector of the univariate %VaR forecast and $\widehat{\Omega}_{t+1}$ is the correlation matrix obtained from the realized covariance matrix estimate. Once we solve the optimization problem for different levels of risk, we construct an efficient frontier. In the Markowitz-type portfolio optimization exercise, we do not allow short-selling to meet restrictions imposed mainly by regulators on certain types of investors (pension funds, etc.).

The second economic evaluation criterion we use is the global minimum VaR portfolio. The basic problem of the GMVaRP is similar to that of Markowitz (1952), and there are only two differences in the setup. The first is the existence of a closed-form solution. As a consequence, we are not restricting asset weights, i.e., we allow short-selling because the global minimum of

[9] Note that if we assume that the quantiles of returns are standard normally distributed and we use standard cut-off points, i.e., −1.645 for the 5% quantile, both approaches are equivalent.

the optimization problem might require negative weights for some assets. The second difference is the absence of a targeted portfolio return. Therefore, in some cases, we might obtain negative portfolio returns for the asset weights, minimizing the overall risk of the portfolio. The GMVaRP optimization problem can be written as:

$$\min_{w_{t+1}} \quad w_{t+1}^{\top} \widehat{\Xi}_{t+1|t} w_{t+1} \tag{22}$$

$$\text{s.t.} \quad l^{\top} w_{t+1} = 1.$$

Kempf and Memmel (2006) show that the analytical solution of the problem is:

$$w_{t+1}^{GMVaR} = \frac{\widehat{\Xi}_{t+1|t}^{-1} l}{l^{\top} \widehat{\Xi}_{t+1|t}^{-1} l}, \tag{23}$$

and the portfolio VaR corresponding to calculated asset weights is obtained as follows:

$$\%VaR_{t+1}^{GMVaR} = w_{t+1}^{GMVaR^{\top}} \widehat{\Xi}_{t+1|t} w_{t+1}^{GMVaR}.$$

## 5. Results: the role of unobserved heterogeneity in the tails

We now turn to applications of the proposed models on empirical data. First, we describe the in-sample fit of the PQR-RV, PQR-RSV, and PQR-BPV model specifications. Second, we present results for our out-of-sample VaR forecasting exercise. Third, we complement our statistical evaluation by computing a simple portfolio allocation exercise where we study global minimum VaR portfolios and Markowitz-like relationships between VaR and portfolio return.

Our empirical application is carried out using 29 U.S. stocks[10] that are traded on the National Association of Securities Dealers Automated Quotations (NASDAQ) and New York Stock Exchange (NYSE). These stocks have been chosen according to their market capitalization and liquidity. The sample we study spans from July 1, 2005 to December 31, 2015, and we consider trades executed within U.S. trading hours (9:30–16:00 EST). To ensure sufficient liquidity and eliminate possible bias, we exclude weekends and holidays, e.g., Christmas, New Year's Day, Thanksgiving Day, Independence Day etc. In total, our final dataset consists of 2613 trading days. Basic descriptive statistics for the data are in Table A.1 in Online Appendix A.

For estimation and forecasting purposes, we use a rolling window with a fixed length of 1000 observations.[11] Hence, our model is always calibrated using a four-year history. Our analysis is restricted to 5 min of intraday log returns that are used for computation of the daily returns and realized measures.

All the results in this section were obtained using nonpenalized fixed effects panel quantile regression. In the Online Appendix B., we also present estimation results when we consider the penalty, $\lambda = 1$, which serves as a robustness check. Moreover, in Online Appendix D., we present the results of a small Monte Carlo experiment.

### 5.1. In-sample analysis

Estimation results of the in-sample analysis are detailed in Table 1. In addition, to obtain a better view of the dynamics, we show the results of the PQR-RV, PQR-RSV, and PQR-BPV graphically in Figs. 1–3, respectively.

The results in Table 1, Panel A, show that the parameters of the PQR-RV model specification are significantly different from zero for all quantiles except the median. Moreover, the signs of the estimated parameters correspond to our expectations – coefficients at lower (upper) quantiles are negative (positive). Note that these values can be interpreted directly as semi-parametric estimates of VaR. Our model therefore shows that the RiskMetrics VaR[12] overestimates both left and right tails of the returns distributions. Furthermore, the insignificant parameter estimate at the median confirms the stylized fact about the randomness/unpredictability of the daily returns (Cont, 2001).

In Table 1, Panel A, we can also see that the absolute values of the parameter estimates are not symmetric around the median, which highlights the relative importance of the realized volatility for the estimation of the lower quantiles of returns. We arrive at a similar conclusion when looking at Fig. 1, which displays the PQR-RV estimates together with their corresponding 95% confidence intervals and the individual UQR-RV parameter estimates. Importantly, Fig. 1 displays that once we control for

[10] Apple Inc. (AAPL), Amazon.com, Inc. (AMZN), Bank of America Corporation (BAC), Comcast Corporation (CMCSA), Cisco Systems, Inc. (CSCO), Chevron Corporation (CVX), Citigroup Inc. (C), The Walt Disney Company (DIS), General Electric Company (GE), The Home Depot, Inc.(HD), International Business Machines Corporation (IBM), Intel Corporation (INTC), Johnson & Johnson (JNJ), JPMorgan Chase & Co. (JPM), The Coca-Cola Company (KO), McDonald's Corporation (MCD), Merck & Co., Inc. (MRK), Microsoft Corporation (MSFT), Oracle Corporation (ORCL), PepsiCo, Inc. (PEP), Pfizer Inc. (PFE), The Procter & Gamble Company (PG), QUALCOMM Incorporated (QCOM), Schlumberger Limited (SLB), AT&T Inc. (T), Verizon Communications Inc. (VZ), Wells Fargo & Company (WFC), Walmart, Inc. (WMT), and Exxon Mobil Corporation (XOM).

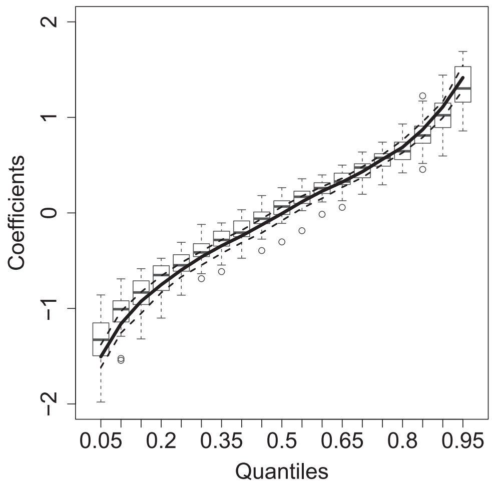
[11] We have tried different lengths of rolling windows, and the qualitative results of our analysis remain unchanged. These results are available from the authors upon request.

[12] The quantiles of the standard normal distribution rescaled by volatility.

**Table 1**
Coefficient estimates of the panel quantile regressions.

| $\tau$ | 5% | 10% | 25% | 50% | 75% | 90% | 95% |
|---|---|---|---|---|---|---|---|
| Panel A | | | | | | | |
| $\widehat{\beta}_{RV^{1/2}}$ | −1.50 | −1.16 | −0.60 | −0.01 | 0.56 | 1.11 | 1.42 |
| | (−23.50) | (−20.62) | (−15.65) | (−0.20) | (20.37) | (24.84) | (20.70) |
| Panel B | | | | | | | |
| $\widehat{\beta}_{RS^{+1/2}}$ | −0.97 | −0.75 | −0.44 | −0.16 | 0.18 | 0.41 | 0.53 |
| | (−12.74) | (−11.98) | (−8.31) | (−2.73) | (2.69) | (4.55) | (4.51) |
| $\widehat{\beta}_{RS^{-1/2}}$ | −1.18 | −0.90 | −0.41 | 0.14 | 0.62 | 1.14 | 1.49 |
| | (−11.72) | (−14.05) | (−9.90) | (2.70) | (9.17) | (13.66) | (10.39) |
| Panel C | | | | | | | |
| $\widehat{\beta}_{BPV^{1/2}}$ | −1.55 | −1.18 | −0.62 | 0.00 | 0.59 | 1.15 | 1.44 |
| | (−19.5) | (−18.15) | (−16.27) | (−0.13) | (23.84) | (23.22) | (25.72) |
| $\widehat{\beta}_{JV^{1/2}}$ | −0.25 | −0.21 | −0.14 | −0.03 | 0.06 | 0.21 | 0.44 |
| | (−3.24) | (−3.54) | (−3.39) | (−0.58) | (1.11) | (1.90) | (2.56) |

Note: The table displays the coefficient estimates, with bootstrapped t-statistics in parentheses. Panels A, B, and C display results for PQR-RV, PQR-RSV, and PQR-BPV models respectively. Individual fixed effects $\alpha_i(\tau)$ are not reported for brevity but are available from the authors upon request.



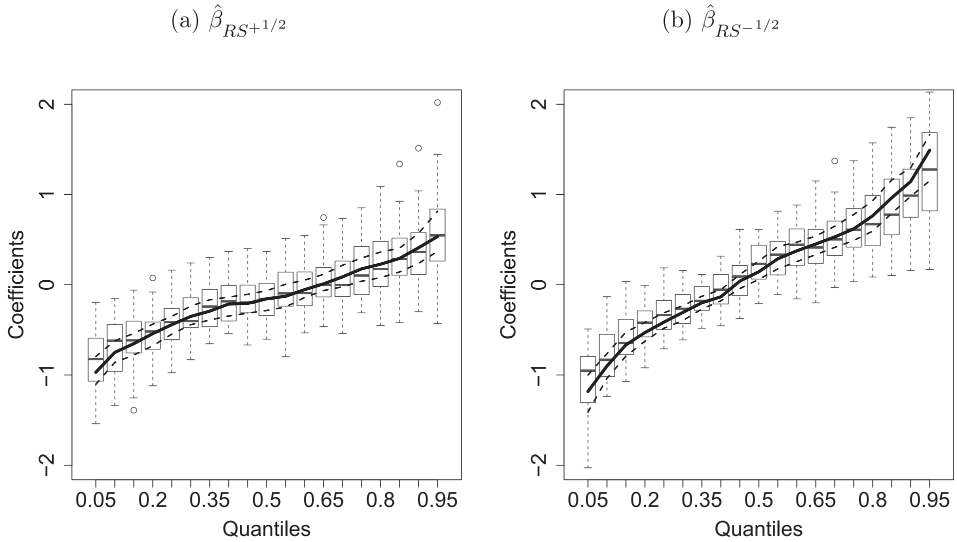

Note: Parameter estimates with corresponding 95% confidence intervals from the PQR-RV specification are plotted using solid and dashed lines, respectively. Individual UQR-RV estimates are plotted in boxplots.

**Fig. 1.** PQR-RV parameter estimates.

unobserved heterogeneity using the PQR-RV, past volatility has a larger influence on both the lower and the upper quantiles of returns than the majority of the individual UQR-RV. This phenomenon is highlighted in far quantiles, e.g., the coefficient of PQR-RV in the 5% quantile is −1.50, whereas the median of the individual UQR-RV coefficient is −1.33 (mean −1.36) or the 95% quantile PQR-RV coefficient is 1.42 and the median of the individual UQR-RV is only 1.30 (mean 1.31). This finding constitutes an important result, as we document unobserved heterogeneity in the far quantiles that needs to be controlled.

Coefficients from the second model specification (PQR-RSV), where realized variance is decomposed into realized downside ($RS^-$) and upside ($RS^+$) semivariance, are significantly different from zero for all considered quantiles (Table 1, Panel B). The magnitude of the coefficients driving the impact of both variables is the highest at far quantiles, showing the strongest impact of both negative and positive semivariance on the tails of the returns distributions. However, the influence of $RS^-$ is far more important in the upper quantiles where it dominates $RS^+$. In contrast, in the lower quantiles, the values of the parameters are close to each other, and therefore, we cannot draw a similar conclusion as in the upper quantiles. Median performance is slightly different from the PQR-RV case.

We can see that median coefficients for both $RS^-$ and $RS^+$ displayed in Table 1, Panel B, are statistically significant, and in the case that the magnitudes of $RS^-$ and $RS^+$ are equal, they sum to −0.02, which translates into loss in 50% of cases. However,




Note: For both realized upside and downside semivariance parameters, estimates with corresponding 95% confidence intervals are plotted by solid and dashed lines, respectively. Individual UQR-RSV estimates are plotted in boxplots.

**Fig. 2.** PQR-RSV parameter estimates.

as theory and stylized facts about financial time series suggest, the influence of negative returns and subsequently negative semivariances should be greater than the effect of positive ones. Therefore, one cannot draw straightforward conclusions about the sign and magnitude of the median return.

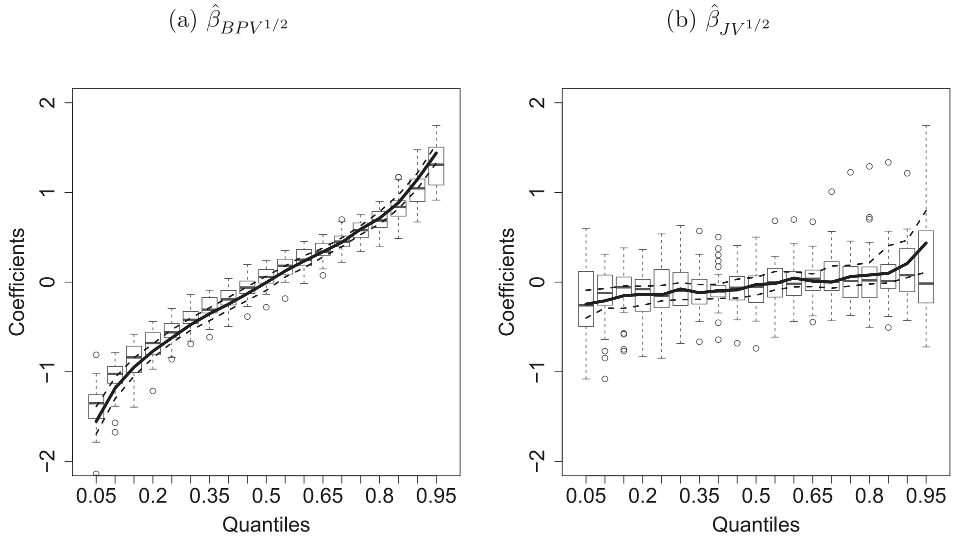
The careful reader might also notice that the median coefficient of $\widehat{\beta}_{RS^{+1/2}}$ is negative and the opposite is true for $\widehat{\beta}_{RS^{-1/2}}$. The explanation of this feature relies on the short- and long-term mean-reversion nature of the returns and the fact that we use lagged values of realized semivariances as regressors. As a result, the negative return at day $t-1$ will cause $RS^-_{t-1} > RS^+_{t-1}$ and the prediction of the median quantile for day $t$ will be positive because $\widehat{\beta}_{RS^{-1/2}}$ is positive and vice versa for positive return and subsequent $RS^-_{t-1} < RS^+_{t-1}$. This behavior leads to mean reversion.

Similar to the PQR-RV specification, we can see in Fig. 2 that controlling for unobserved heterogeneity among financial assets is important because the influence of both downside and upside semivariance is greater in the lower quantiles than in individual UQR-RSV. For example, the 5% quantile coefficients obtained by PQR-RSV are −0.97 and −1.18 for $RS^+$ and $RS^-$, respectively; however, the median values of individual UQR-RSV are −0.82 (mean −0.84) for $RS^+$ and −0.95 (mean −1.10) for $RS^-$. Moreover, in the upper quantiles of negative semivariance (Fig. 2b), PQR-RSV coefficients differ substantially from individual UQR-RSV (95% quantile $\widehat{\beta}_{RS^{-1/2}}$ coefficient of 1.49 as compared to the individual UQR-RSV median/mean coefficient of 1.28/1.27); however, the opposite is true for $RS^+$ (95% quantile $\widehat{\beta}_{RS^{+1/2}}$ coefficient of 0.54 as compared to the individual UQR-RSV median/mean coefficient of 0.55/0.55). These findings support our previous conclusion that $RS^-$ influences upper quantiles of returns more than $RS^+$.

Finally, Table 1, Panel C, reveals interesting results about parameter estimates of the PQR-BPV model specification, where the bi-power variation and jump component are used to drive the quantiles of returns. We can infer that jumps have a significant impact on both far upper and lower quantiles of financial returns. To be precise, the magnitude of the jump coefficient $\widehat{\beta}_{JV^{1/2}}$ is the highest for the 95% quantile, with a value of 0.44. For the remaining quantiles above median, jumps are not statistically significant, and therefore, the influence of the quadratic variation reduces to integrated variance, which is represented by bi-power variation. We observe the opposite situation for the quantiles below median, where the $\widehat{\beta}_{JV^{1/2}}$ coefficients are always significant.

Fig. 3 helps us to graphically confirm the results of our previous analysis. If we compare Figs. 3a to 1, we obtain an almost identical picture. Moreover, in Fig. 3b, we can see that from the 45%–85% quantiles, confidence intervals of the jump component become wider and include zero. Once we combine these two findings, we can state that for these quantiles, quadratic variation reduces to integrated variance. In contrast, none of the confidence intervals of the 5%–40% quantiles contains zero, which highlights the relative importance of the jump component in modeling lower quantiles of the financial returns.

Overall, the results of the in-sample analysis show the asymmetric impact of the regressors on the quantiles of financial returns. This impact is higher in the quantiles below median. We have also found evidence for positive/negative news asymmetry. This asymmetry is the highest in the 95% quantile (0.53 coefficient of $RS^+$ vs. 1.49 of $RS^-$), while the 5% quantile shows




Note: For both realized bipower variation and jump component parameters, estimates with corresponding 95% confidence intervals are plotted with solid and dashed lines, respectively. Individual UQR-BPV estimates are plotted in boxplots.

**Fig. 3.** PQR-BPV parameter estimates.

little asymmetry (−0.97 in case of $RS^{+}$ vs. −1.18 for $RS^{-}$). In addition, we show the importance of jumps below the median and far above the median quantiles. Importantly, we document unobserved heterogeneity in the far quantiles. We have also tested all three models (PQR-RV, PQR-RSV, and PQR-BPV) for correct dynamic specification, and we find that none of them is systematically misspecified.

### 5.2. Out-of-sample performance

We now turn to the results of our out-of-sample forecasting exercises. We analyze the VaR performance of an equally-weighted portfolio of the 29 stocks described earlier. The results of our analysis are presented in the following way: first, we comment on the absolute performance of the PQR models; second, the absolute performance of the benchmark models is discussed; third, we concentrate on the most interesting relative performance comparison of the PQR models with respect to the benchmark models. All results are summarized in Table 2. For illustrative purposes, we present a graphical comparison of the 5% portfolio VaR in the Online Appendix A (Fig. A.1), together with the individual PQR-RV 5% VaR of representative firms[13] from seven main market sectors defined in accordance with the Global Industry Classification Standard[14](Fig. A.2).

The unconditional coverage, $\hat{\tau}$, shown in Panel A.1 and Panel A.2 of Table 2 reveals that almost all models in almost all quantiles underestimate risk. Specifically, the values of unconditional coverage are higher than corresponding quantiles $\tau$, with few exceptions (median quantiles, 5% quantile of portfolio UQR, and 90% quantile of the PQR-RSV). We must also stress here that the deviation from nominal quantile rates is generally lower than 1% leading to correct unconditional coverage.

In the median quantiles, we can see that all the models overestimate risk. Moreover, the deviations from the nominal quantiles are higher compared to off-median quantiles. We attribute this finding to the nature of financial time series, especially to stylized fact about the unpredictability of the returns. More importantly, this result corresponds to our motivation of explaining quantiles of the cross-section of market returns instead of expected value. This result is in line with our previous result that median estimates are not statistically significant.

If we concentrate on the correct dynamic specification of the models represented by the CAViaR test (i.e., the second and third lines of Panels A.1 and A.2), we see that all the models in all quantiles are dynamically correctly specified except for the median of RiskMetrics. In this case, we strongly reject the null hypothesis of proper dynamic specification given a $p$-value $<$ 0.01. We attribute the poor median RiskMetrics performance to the construction of equation (19) where the cut-off point at the

[13] We choose firms with the highest market capitalization.
[14] Details about the classification can be found here: https://www.msci.com/gics.

**Table 2**
Out-of-sample performance of various specifications of the panel quantile regression model for returns.

| | | PQR-RV | | | | | PQR-RSV | | | | | PQR-BPV | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Panel A.1 | $\tau$ | 5% | 10% | 50% | 90% | 95% | 5% | 10% | 50% | 90% | 95% | 5% | 10% | 50% | 90% | 95% |
| | $\widehat{\tau}$ | 0.06 | 0.11 | 0.47 | 0.90 | 0.96 | 0.06 | 0.11 | 0.47 | 0.90 | 0.96 | 0.06 | 0.11 | 0.47 | 0.90 | 0.96 |
| | DQ | 8.92 | 3.37 | 10.16 | 6.94 | 5.69 | 8.18 | 3.34 | 10.13 | 1.48 | 9.15 | 7.96 | 4.63 | 10.16 | 5.30 | 6.21 |
| | $p$-val | 0.18 | 0.76 | 0.12 | 0.33 | 0.46 | 0.23 | 0.77 | 0.12 | 0.96 | 0.17 | 0.24 | 0.59 | 0.12 | 0.51 | 0.40 |

| | | RiskMetrics | | | | | UQR | | | | | Portfolio UQR | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Panel A.2 | $\tau$ | 5% | 10% | 50% | 90% | 95% | 5% | 10% | 50% | 90% | 95% | 5% | 10% | 50% | 90% | 95% |
| | $\widehat{\tau}$ | 0.06 | 0.09 | 0.45 | 0.92 | 0.96 | 0.06 | 0.11 | 0.47 | 0.90 | 0.96 | 0.04 | 0.10 | 0.49 | 0.91 | 0.96 |
| | DQ | 9.65 | 3.1 | 20.60$^{\ddagger}$ | 9.45 | 10.90 | 8.32 | 3.04 | 9.07 | 7.17 | 6.80 | 9.43 | 5.99 | 3.27 | 4.51 | 3.24 |
| | $p$-val | 0.14 | 0.80 | 0.00 | 0.15 | 0.09 | 0.22 | 0.80 | 0.17 | 0.31 | 0.34 | 0.15 | 0.43 | 0.77 | 0.61 | 0.78 |

| | | Benchmark | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | RiskMetrics | | | | | UQR | | | | | Portfolio UQR | | | | |
| Panel B | $\tau$ | 5% | 10% | 50% | 90% | 95% | 5% | 10% | 50% | 90% | 95% | 5% | 10% | 50% | 90% | 95% |
| PQR-RV | DM | −2.43* | −2.26* | −3.35* | −2.13* | −1.94* | 0.13 | −1.73* | 1.35 | 0.80 | −0.36 | −0.73 | −1.60 | −0.31 | −2.05* | −2.26* |
| | $p$-val | 0.01 | 0.01 | 0.00 | 0.02 | 0.03 | 0.55 | 0.04 | 0.91 | 0.79 | 0.36 | 0.23 | 0.06 | 0.38 | 0.02 | 0.01 |
| PQR-RSV | DM | −2.37* | −2.25* | −3.56* | −2.37* | −2.24* | 1.24 | −1.57 | −0.44 | −1.27 | −2.02* | −0.56 | −1.58 | −0.76 | −2.92* | −3.10* |
| | $p$-val | 0.01 | 0.01 | 0.00 | 0.01 | 0.01 | 0.89 | 0.06 | 0.33 | 0.10 | 0.02 | 0.29 | 0.06 | 0.22 | 0.00 | 0.00 |
| PQR-BPV | DM | −2.54* | −2.42* | −3.30* | −2.06* | −1.85* | −1.80* | −1.89* | 1.42 | 0.84 | 0.70 | −1.19 | −1.89* | −0.29 | −1.98* | −1.71* |
| | $p$-val | 0.01 | 0.01 | 0.00 | 0.02 | 0.03 | 0.04 | 0.03 | 0.92 | 0.80 | 0.76 | 0.12 | 0.03 | 0.38 | 0.02 | 0.04 |

Note: The table displays absolute and relative performance of PQR models for returns with RV, RSV, and BPV as regressors and benchmark models. Panel A.1 reports the absolute performance of PQR models, and Panel A.2 reports the absolute performance of benchmark models. For each model and quantile $\tau$, unconditional coverage ($\widehat{\tau}$), the value of the CAViaR test for a correct dynamic specification (DQ) with a corresponding Monte Carlo-based $p$-value are displayed. Models that are not correctly dynamically specified are denoted by ‡. Panel B reports the relative performance of panel quantile regression models for returns. For each specification and quantile $\tau$, we report Diebold-Mariano test statistics for pairwise comparison with benchmark models (DM) with corresponding p-values. Significantly more accurate forecasts with respect to benchmark models at the 5% significance level are denoted by *. A full matrix of pairwise comparisons is available from the authors upon request.

**Table 3**
Global minimum VaR portfolio.

| $\tau$ | 5% | 10% | 50% | 90% | 95% |
|---|---|---|---|---|---|
| PQR-RV | 11.76* | 8.69* | 0.02 | 9.46* | 12.37* |
| UQR | 11.85 | 8.79 | 0.01* | 9.52 | 12.43 |
| RiskMetrics | 12.77 | 9.95 | NaN | 9.95 | 12.77 |

Note: The table displays the absolute percentage values of the global minimum VaR portfolio for a given quantile $\tau$. We use * to denote the best model for a given quantile.

50% quantile, $\gamma_{50\%}$, is 0.[15]

The relative performance of the PQR models is summarized in Panel B.[16] The results of our analysis indicate good relative performance of the PQR models. All three PQR model specifications significantly outperform RiskMetrics in all quantiles. Moreover, all PQR specifications consistently outperform portfolio UQR in the upper quantiles and UQR in several quantiles, i.e., the PQR-RV model outperforms individual UQR estimates in the 10% quantile, while the performance of PQR-RSV is significantly better in the 95% quantile, and PQR-BPV delivers significantly more accurate forecasts in the 5% and 10% quantiles. If we concentrate on the full pairwise comparison, the most important is the performance of the UQR as the main competitor of the PQR specifications. In all the quantiles, UQR is not able to outperform any of the PQR specifications. This fact highlights the importance of controlling for unobserved heterogeneity among assets. Moving from the comparison of the PQR and UQR models to the relative performance of the portfolio UQR, we can see that it outperforms RiskMetrics only at the 5% and 10% quantiles. In contrast, UQR, similar to PQR, outperforms RiskMetrics in all quantiles. These results reveal the importance of the asset-specific contribution to overall future portfolio risk, as the approach of aggregating data first and modeling them second is not able to capture the dynamics that create variation in the distribution of future portfolio returns.

### *5.3. Economic evaluation*

We would like to see whether statistical gains also translate to economic value. We concentrate on comparing three models – PQR-RV, UQR, and RiskMetrics – and we refrain from presenting results for PQR-RSV and PQR-BPV for the sake of brevity. The construction of portfolio UQR rules out economic evaluation in our set-up because asset weights will be set before applying the quantile regression, and therefore, the results will be driven by the covariance structure only.

We begin our description of the results using the global minimum VaR portfolio followed by Markowitz-like optimization where we show the VaR-return relationship. In both approaches, we use annualized portfolio returns[17] and annualized portfolio VaR[18] of the whole out-of-sample period. In the GMVaRP comparison, we focus on both left and right tails, together with the median because we do not set any constraints regarding asset weights; according to equation (23), GMVaRP has a closed form solution. In contrast, the Markowitz-like optimization is purely numeric and does not offer a closed-form solution. Therefore, we restrict our analysis to long-only positions. As a result, we concentrate on the left tail of the return distribution only, which shows a potential loss for the investor.

The results of the GMVaRP analysis are displayed in Table 3. The PQR-RV model performs best in all quantiles except for the median, where UQR has the lowest VaR. The RiskMetrics model finished last, and for the median quantile, we were not able to calculate the value of GMVaRP due to the problem of singularity of the correlated VaR matrix.[19] Concentrating on the 5% VaR, for a commercial bank, the direct benefit of using PQR is a reduction in the required reserves by almost 0.1% compared to UQR, i.e., a bank needs to put a lower amount of money aside to cover potential losses.

Efficient frontiers for the VaR-return trade-off are plotted in Fig. 4a for the 5% quantile and Fig. 4b for the 10% quantile. In both quantiles, the PQR-RV model has the best performance. Similar to the GMVaRP analysis, UQR has the second-best performance, while the RiskMetrics model has the worst VaR-return trade-off. In Fig. 4b, we can also see that the benefits from using PQR are greater for lower values of VaR. Overall, the panel quantile regression model for returns generates better economic performance than the remaining benchmark models.

## 6. Results: common risk factors in the tails

We select the VIX index as an exogenous factor that has the potential to drive the tails of the return distributions. The VIX is often used as the measure of the ex ante/anticipated uncertainty, and it complements the realized volatility we use in our

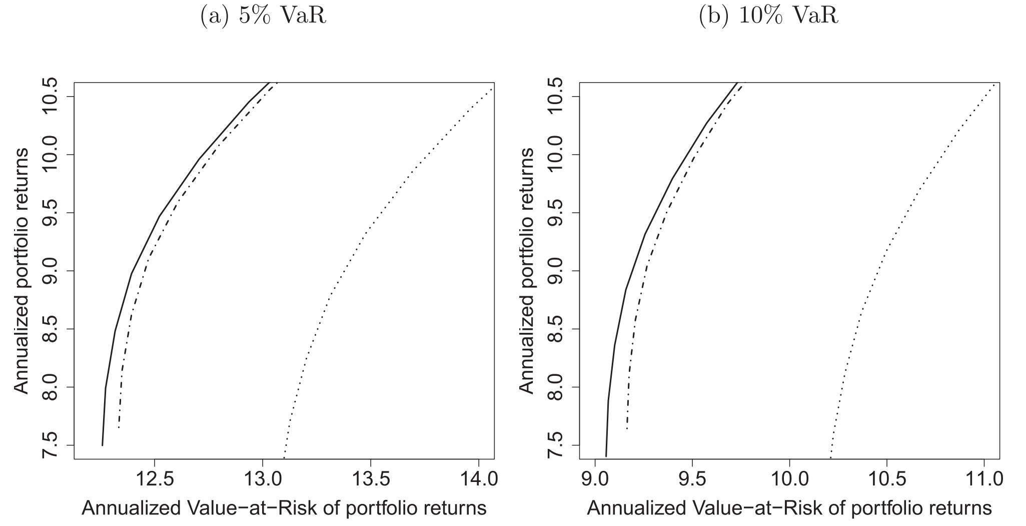
[15] The median of the standard normal distribution is 0.

[16] For brevity, we report only pairwise comparisons against benchmark models in Table 2. The full matrix of pairwise comparisons is available from the authors upon request.

[17] $\left(\prod_{t=1}^{T}(1+r_t)\right)^{\frac{250}{T}}$

[18] $\sqrt{250}\frac{\sum_{t=1}^{T}\%VaR_t}{T}$

[19] If we set the cut-off point in equation (19) to zero, we obtain a singular matrix of zeros that is not invertible.




Note: The figure shows the values of portfolio VaR and returns. PQR-RV model is plotted by solid line, UQR-RV model is plotted by dotted-dashed line, and RiskMetrics model is plotted by dotted line.

**Fig. 4.** VaR-return efficient frontiers.

previous analysis. It is also referred to as the "fear gauge" since it measures the expectations about the 30-day volatility using the weighted and aggregated prices of the call and put options with various strike prices on the S&P 500 Index. The value of the index is calculated according to the VIX methodology[20] as:

$$\sigma_{VIX}^2 = \frac{2}{T}\sum_i \frac{\Delta K_i}{K_i^2} e^{RT} Q(K_i) - \frac{1}{T}\left(\frac{F}{K_0} - 1\right)^2, \tag{24}$$

where $T$ is the time to expiration; $F$ is the forward index level derived from index option prices; $K_0$ is the first strike below the forward index level $F$; $K_i$ is the strike price of the out-of-the-money option (call if $K_i > K_0$, put if $K_i < K_0$); $\Delta K_i$ is the interval between strike prices; $R$ is the risk-free interest rate to expiration; and $Q(K_i)$ is the midpoint of the bid-ask spread for each option with strike $K_i$.

The value of the VIX represents the annual percentage volatility and is reported by the CBOE as:

$$VIX = 100\sigma_{VIX}. \tag{25}$$

The daily counterpart of the annual option implied volatility measure is constructed by dividing the VIX by $\sqrt{250}$. We further divide the daily VIX by 100 to scale it to units of realized volatility, i.e., $VIX_{daily} = \frac{VIX_{annual}}{\sqrt{250}}\frac{1}{100}$. The historical data can be freely downloaded from the Federal Reserve Bank of Saint Louis. Moreover, since the VIX was launched in 1993, it effectively covers the sample period we use in our empirical analysis. In the empirical application, we estimate panel quantile regression models containing the VIX as defined in equations (10)–(12).

Table 4 documents the stability of the relative influence of the VIX on the quantiles of financial returns. The VIX coefficient estimates are of the same magnitude for all realized measures model specifications. Although market participants perceive the VIX as the so-called "fear" index, our analysis reveals a higher relative influence in the upper quantiles than in the lower quantiles, e.g., in the RV + VIX model specification, a 0.44 coefficient estimate for the 95% quantile vs. −0.32 for the 5% quantile. Moreover, when we compare the results in Table 4 to Table 1 (PQR models with and without the VIX), we can see that the VIX index reduces the relative influence of the realized measures more in the upper quantiles than in lower quantiles. In the RV + VIX and BPV + VIX specifications, the coefficients are reduced by 0.17 and 0.20, respectively, in the 5% quantile, while in the 95% quantile, the reduction is 0.30 in both specifications. In the RSV + VIX specification, the total reduction in the coefficients is higher than in the previous two cases (0.28 and 0.40 in the 5% and 95% quantiles, respectively). The influence reduction of the positive semivariance is higher than that of the negative semivariance in the 5% quantile and vice versa in the 95% quantile, where the positive semivariance is almost not reduced and the influence of the negative semivariance is lowered by 0.39. Figs. 5–7 also support our findings graphically; in all three figures, the patterns of the $\hat{\gamma}_{VIX}$ coefficient estimates are almost identical.

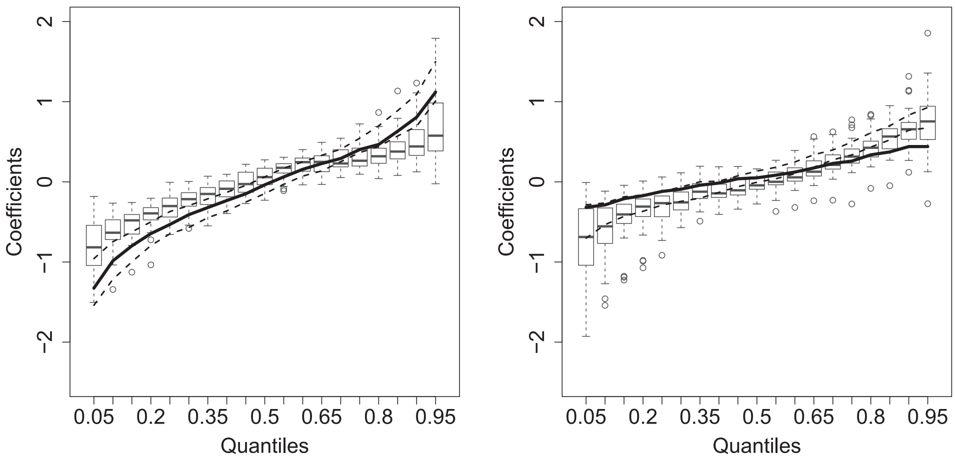
[20] Full details on the VIX calculation can be found at http://www.cboe.com/micro/vix/vixwhite.pdf.

**Table 4**
Coefficient estimates of the panel quantile regressions.

| $\tau$ | 5% | 10% | 25% | 50% | 75% | 90% | 95% |
|---|---|---|---|---|---|---|---|
| Panel A | | | | | | | |
| $\widehat{\beta}_{RV^{1/2}}$ | −1.33 | −0.99 | −0.53 | −0.04 | 0.4 | 0.81 | 1.12 |
| | (−10.87) | (−9.65) | (−8.77) | (−0.83) | (11.00) | (7.76) | (9.09) |
| $\widehat{\gamma}_{VIX}$ | −0.32 | −0.29 | −0.12 | 0.05 | 0.26 | 0.44 | 0.44 |
| | (−4.39) | (−4.49) | (−3.45) | (1.90) | (9.58) | (6.30) | (5.38) |
| Panel B | | | | | | | |
| $\widehat{\beta}_{RS^{+1/2}}$ | −0.78 | −0.65 | −0.40 | −0.18 | 0.08 | 0.28 | 0.52 |
| | (−9.46) | (−5.91) | (−6.12) | (−3.07) | (1.22) | (3.50) | (4.45) |
| $\widehat{\beta}_{RS^{-1/2}}$ | −1.09 | −0.75 | −0.35 | 0.11 | 0.49 | 0.89 | 1.10 |
| | (−7.76) | (−10.54) | (−6.90) | (2.00) | (6.32) | (5.63) | (5.83) |
| $\widehat{\gamma}_{VIX}$ | −0.33 | −0.28 | −0.12 | 0.06 | 0.26 | 0.41 | 0.42 |
| | (−4.63) | (−4.58) | (−3.35) | (1.99) | (9.28) | (5.73) | (4.70) |
| Panel C | | | | | | | |
| $\widehat{\beta}_{BPV^{1/2}}$ | −1.35 | −0.99 | −0.54 | −0.03 | 0.42 | 0.82 | 1.14 |
| | (−9.88) | (−9.50) | (−7.94) | (−0.65) | (9.42) | (7.68) | (10.34) |
| $\widehat{\beta}_{JV^{1/2}}$ | −0.21 | −0.14 | −0.14 | −0.04 | 0.07 | 0.17 | 0.44 |
| | (−2.76) | (−2.43) | (−2.79) | (−0.77) | (1.03) | (1.39) | (2.58) |
| $\widehat{\gamma}_{VIX}$ | −0.33 | −0.30 | −0.12 | 0.04 | 0.26 | 0.45 | 0.44 |
| | (−4.10) | (−4.90) | (−3.57) | (1.87) | (8.35) | (6.87) | (5.70) |

Note: The table displays the coefficient estimates, with bootstrapped t-statistics in parentheses. Panels A, B, and C display results for PQR-RV-VIX, PQR-RSV-VIX, and PQR-BPV-VIX models respectively. Individual fixed effects $\alpha_i(\tau)$ are not reported for brevity but are available from the authors upon request.

(a) $\hat{\beta}_{RV^{1/2}}$ (b) $\hat{\gamma}_{VIX}$

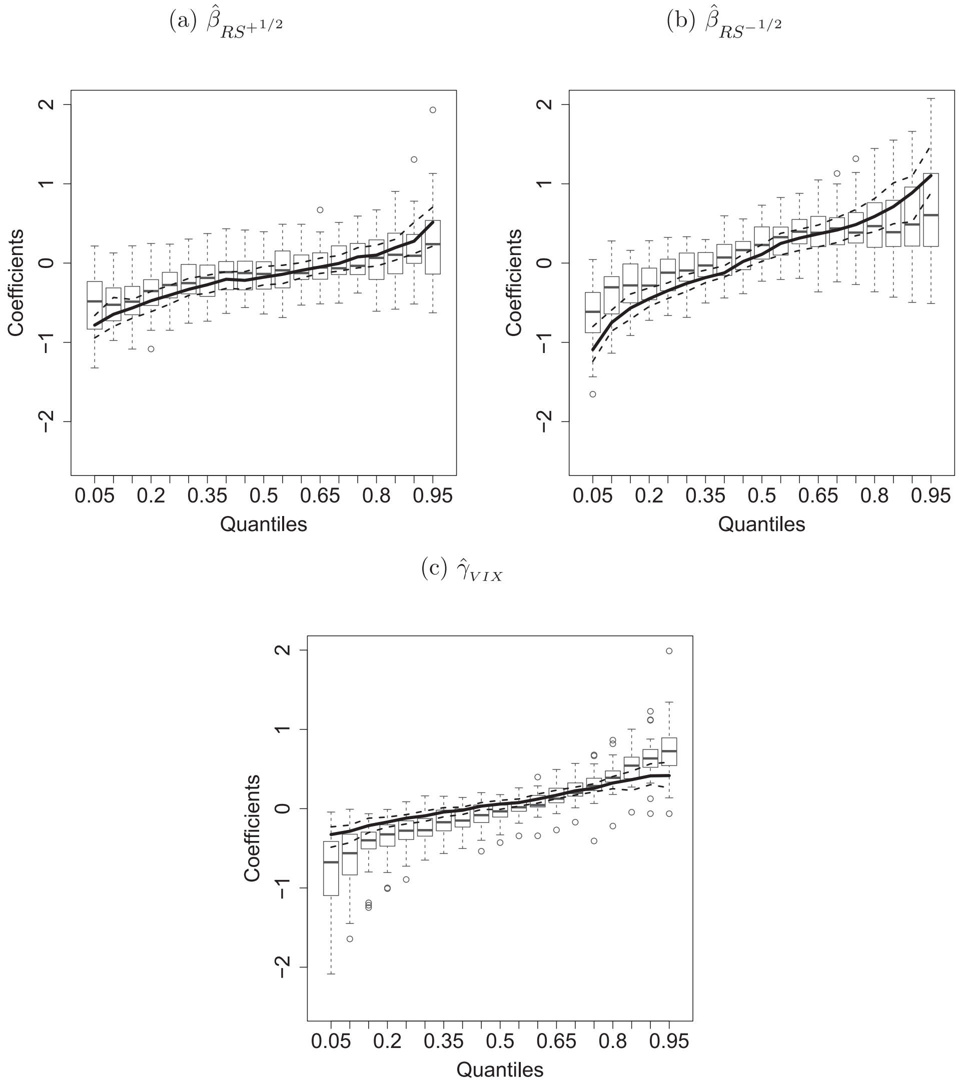



Note: For both the realized volatility and VIX index parameters, estimates with corresponding 95% confidence intervals are plotted by solid and dashed lines, respectively. Individual UQR-RV-VIX estimates are plotted in boxplots.

**Fig. 5.** PQR-RV-VIX parameter estimates.

Overall, we conclude that the VIX includes important partial information about risk that is not fully captured by any of the realized measures, and the expectations about the future risk affect the higher quantiles more than the lower quantiles. Controlling for the unobserved heterogeneity and idiosyncratic volatility, the VIX proves to be a strong common factor driving the tails of the return distributions.

The out-of-sample performance of the models containing the VIX is summarized in Table 5. In Panel A.1 and Panel A.2, there is only one change in absolute performance compared to models without the VIX: the 5% quantile of the PQR-RV-VIX specification is not dynamically correctly specified. All the remaining results for absolute performance qualitatively match the results of the




Note: For all realized upside semivariance, downside semivariance and VIX index parameter estimates with corresponding 95% confidence intervals are plotted using solid and dashed lines, respectively. Individual UQR- RSV-VIX estimates are plotted in boxplots.

**Fig. 6.** PQR-RSV-VIX parameter estimates.

models without the VIX presented in Table 2, i.e., unconditional coverage $\hat{\tau}$ is close to the nominal quantile rates, and all the models in all quantiles are dynamically correctly specified except for the median of the RiskMetrics model.

In the relative performance comparison presented in Panel B of Table 5, all the PQR model specifications significantly outperform the RiskMetrics model in all quantiles and portfolio UQR in the upper quantiles, similar to models without the VIX (see Panel B, Table 2). We can also see that PQR-RSV-VIX specification outperforms UQR in the 95% quantile and PQR-BPV-VIX outperforms the portfolio UQR in the 10% quantile. In contrast to the results without the VIX, the UQR significantly outperforms both the PQR-RV-VIX and PQR-BPV-VIX specifications in the median quantile.

### *6.1. Economic evaluation*

Furthermore, we report the results from the economic evaluation of the models. Table 6 and Fig. 8 reveal qualitatively similar patterns to models without the VIX (Table 3 and Fig. 4), whereas the PQR-RV-VIX specification achieves the lowest values of VaR in all quantiles. It also provides us with the best VaR-return trade-off.

In the comparison of models with and without exogenous factors, we can see the direct economic benefits of using the VIX. In the GMVaRP comparison (Table 3 vs. Table 6), the reductions in the VaR are 0.34, 0.173, 0.014, 0.422, and 0.609 percentage points

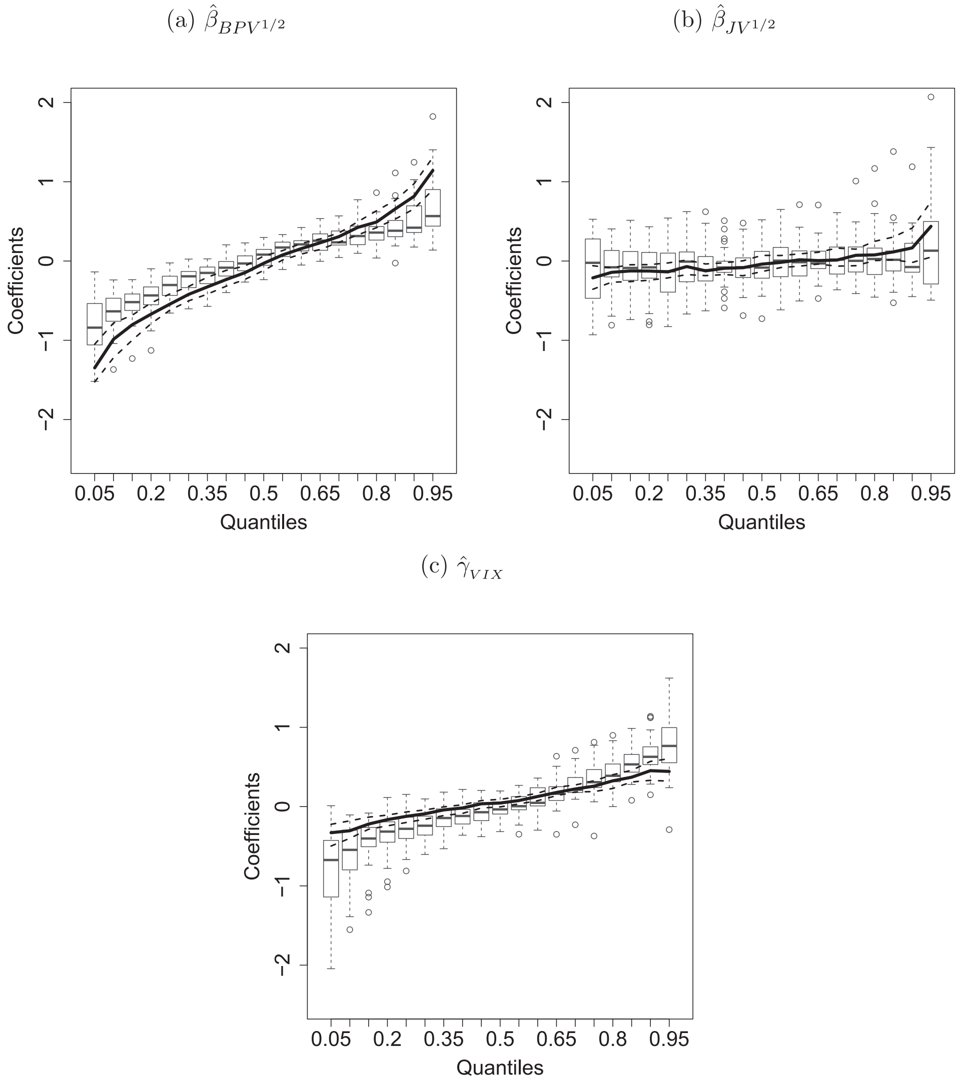
**Table 5**
Out-of-sample performance of various specifications of the panel quantile regression model for returns with the VIX.

| | | PQR-RV-VIX | | | | | PQR-RSV-VIX | | | | | PQR-BPV-VIX | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Panel A.1 | $\tau$ | 5% | 10% | 50% | 90% | 95% | 5% | 10% | 50% | 90% | 95% | 5% | 10% | 50% | 90% | 95% |
| | $\widehat{\tau}$ | 0.06 | 0.11 | 0.46 | 0.90 | 0.96 | 0.06 | 0.11 | 0.46 | 0.90 | 0.96 | 0.06 | 0.11 | 0.46 | 0.90 | 0.96 |
| | DQ | 12.93‡ | 3.08 | 11.83 | 3.48 | 5.23 | 12.31 | 3.08 | 11.43 | 2.06 | 5.23 | 11.03 | 2.47 | 11.83 | 3.97 | 5.88 |
| | *p*-val | 0.04 | 0.80 | 0.07 | 0.75 | 0.52 | 0.06 | 0.80 | 0.08 | 0.91 | 0.52 | 0.09 | 0.87 | 0.07 | 0.68 | 0.44 |

| | | RiskMetrics | | | | | UQR | | | | | Portfolio UQR | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Panel A.2 | $\tau$ | 5% | 10% | 50% | 90% | 95% | 5% | 10% | 50% | 90% | 95% | 5% | 10% | 50% | 90% | 95% |
| | $\widehat{\tau}$ | 0.06 | 0.09 | 0.45 | 0.92 | 0.96 | 0.04 | 0.10 | 0.49 | 0.91 | 0.96 | 0.06 | 0.09 | 0.45 | 0.92 | 0.96 |
| | DQ | 9.65 | 3.10 | 20.60‡ | 9.45 | 10.90 | 8.32 | 3.04 | 9.07 | 7.17 | 6.80 | 9.43 | 5.99 | 3.27 | 4.51 | 3.24 |
| | *p*-val | 0.14 | 0.80 | 0.00 | 0.15 | 0.09 | 0.22 | 0.80 | 0.17 | 0.31 | 0.34 | 0.15 | 0.43 | 0.77 | 0.61 | 0.78 |

| | | Benchmark | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | RiskMetrics | | | | | UQR | | | | | Portfolio UQR | | | | |
| Panel B | $\tau$ | 5% | 10% | 50% | 90% | 95% | 5% | 10% | 50% | 90% | 95% | 5% | 10% | 50% | 90% | 95% |
| PQR-RV-VIX | DM | −2.32* | −2.34* | −2.45* | −2.83* | −2.37* | 1.46 | −0.46 | 1.87# | −1.98* | −1.43 | −0.31 | −1.62 | 0.05 | −4.57* | −3.05* |
| | *p*-val | 0.01 | 0.01 | 0.01 | 0.00 | 0.01 | 0.93 | 0.32 | 0.97 | 0.02 | 0.08 | 0.38 | 0.05 | 0.52 | 0.00 | 0.00 |
| PQR-RSV-VIX | DM | −2.28* | −2.32* | −2.25* | −2.94* | −2.63* | 1.84# | −0.36 | 1.34 | −2.19* | −2.19* | −0.21 | −1.57 | −0.04 | −4.68* | −3.50* |
| | *p*-val | 0.01 | 0.01 | 0.01 | 0.00 | 0.00 | 0.97 | 0.36 | 0.91 | 0.01 | 0.01 | 0.42 | 0.06 | 0.48 | 0.00 | 0.00 |
| PQR-BPV-VIX | DM | −2.44* | −2.46* | −2.43* | −2.76* | −2.35* | 0.06 | −0.89 | 1.89# | −1.79* | −1.29 | −0.70 | −1.92* | 0.06 | −4.36* | −2.86* |
| | *p*-val | 0.01 | 0.01 | 0.01 | 0.00 | 0.01 | 0.52 | 0.19 | 0.97 | 0.04 | 0.10 | 0.24 | 0.03 | 0.52 | 0.00 | 0.00 |

Note: The table displays the absolute and relative performance of PQR models for returns with RV, RSV, and BPV as regressors combined with VIX and benchmark models. Panel A.1 reports the absolute performance of PQR models, Panel A.2 reports the absolute performance of benchmark models. For each model and quantile $\tau$, unconditional coverage ($\widehat{\tau}$), the value of the CAViaR test for a correct dynamic specification (DQ) with a corresponding Monte Carlo-based $p$-value are displayed. Models that are not correctly dynamically specified are denoted by ‡. Panel B reports the relative performance of panel quantile regression models for returns. For each specification and quantile $\tau$, we report Diebold-Mariano test statistics for pairwise comparison with benchmark models (DM) with corresponding p-values. Significantly more/less accurate forecasts with respect to benchmark models at the 5% significance level are denoted by */#. A full matrix of pairwise comparisons is available from the authors upon request.




Note: For all realized bipower variations, the jump component and VIX index parameter estimates with corresponding 95% confidence intervals are plotted using solid and dashed lines, respectively. Individual UQR-BPV-VIX estimates are plotted in boxplots.

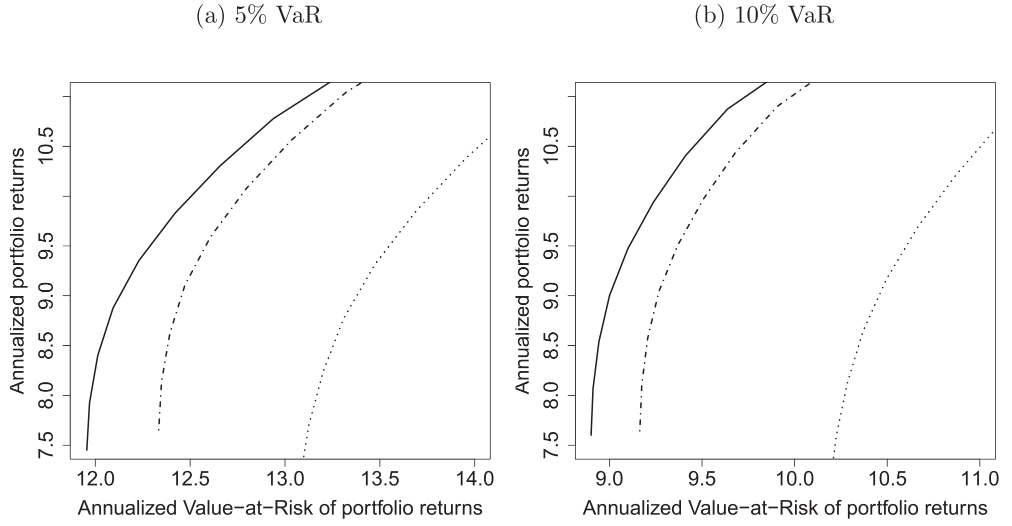
**Fig. 7.** PQR-BPV-VIX parameter estimates.

**Table 6**
Global minimum VaR portfolio: PQR with the VIX.

| $\tau$ | 5% | 10% | 50% | 90% | 95% |
|---|---|---|---|---|---|
| PQR-RV-VIX | 11.42* | 8.51* | 0.007* | 9.04* | 11.76* |
| UQR-RV | 11.85 | 8.79 | 0.011 | 9.52 | 12.43 |
| RiskMetrics | 12.77 | 9.95 | NaN | 9.95 | 12.77 |

Note: The table displays the absolute percentage values of the global minimum VaR portfolio for a given quantile $\tau$. We use * to denote the best model for a given quantile.

or 2.89%, 1.99%, 68.54%, 4.46%, and 4.92% for the 5%, 10%, 50%, 90%, and 95% quantiles, respectively. In the visual comparison, the efficient frontiers of the model with VIX shift to the right; thus, they have a superior VaR-return trade-off. Generally, we document the direct economic benefits of the ex ante volatility measure.




Note: The figure shows the values of portfolio VaR and returns. PQR-RV-VIX model is plotted by solid line, UQR-RV model is plotted by dotted-dashed line, and RiskMetrics model is plotted by dotted line.

**Fig. 8.** VaR-return efficient frontiers: PQR with the VIX.

**Table 7**
S&P 500 Index data: Coefficient estimates of the panel quantile regressions.

| $\tau$ | 5% | 10% | 25% | 50% | 75% | 90% | 95% |
|---|---|---|---|---|---|---|---|
| Panel A | | | | | | | |
| $\widehat{\beta}_{RV^{1/2}}$ | −1.51 | −1.15 | −0.58 | −0.01 | 0.55 | 1.10 | 1.45 |
| | (−96.03) | (−94.56) | (−78.79) | (−1.73) | (60.04) | (90.56) | (88.03) |
| Panel B | | | | | | | |
| $\widehat{\beta}_{RV^{1/2}}$ | −1.17 | −0.88 | −0.46 | −0.04 | 0.38 | 0.81 | 1.11 |
| | (−42.81) | (−43.89) | (−38.05) | (−4.50) | (37.12) | (39.23) | (40.67) |
| $\widehat{\gamma}_{VIX}$ | −0.67 | −0.54 | −0.26 | 0.06 | 0.36 | 0.57 | 0.63 |
| | (−21.62) | (−21.90) | (−20.25) | (7.23) | (27.85) | (26.06) | (25.04) |

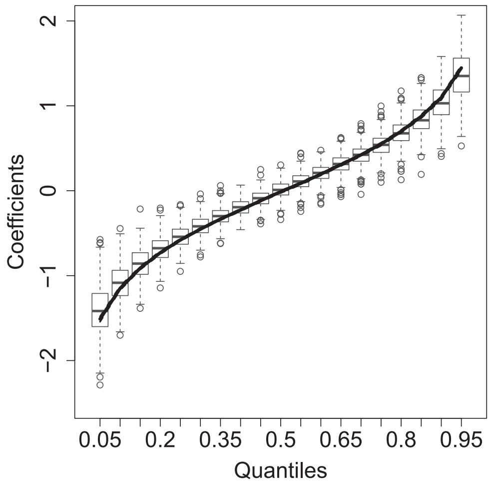
Note: The table displays coefficient estimates with bootstrapped t-statistics in parentheses. Panels A, and B display results for PQR-RV, and PQR-RV-VIX models respectively. Individual fixed effects $\alpha_i(\tau)$ are not reported for brevity but are available from the authors upon request.

### *6.2. Robustness check: portfolio of S&P 500 constituents*

To see how robust the findings are for the portfolio with a large number of stocks, we apply our methodology to the constituents of the S&P 500 Index. Since the composition of the S&P 500 Index vary substantially over time, we study stocks that are liquid enough,[21] have a full history, and are included in the index at least once during the period from July 1, 2005 to December 31, 2015. Similar to the previous analysis, we consider trades executed within U.S. trading hours, and we explicitly exclude weekends and holidays. In total, our dataset consists of the trades on 496 stocks over 2613 trading days.

In our analysis, we concentrate on two panel quantile regression model for financial returns specifications: PQR-RV and PQR-RV-VIX. The results are summarized in Table 7, Fig. 9, and Fig. 10, and they reveal interesting findings about the role of ex ante volatility. In the PQR-RV specification, the coefficient estimates for $\widehat{\beta}_{RV^{1/2}}$ are almost identical to those obtained using our portfolio of 29 stocks (Table 1). In contrast, in the PQR-RV-VIX case, both $\widehat{\beta}_{RV^{1/2}}$ and $\widehat{\gamma}_{VIX}$ differ substantially for the 29 and 496 stocks (Tables 4 and 7). Specifically, the relative influence of the realized volatility is lower as we move from 29 to 496 stocks in the left tail (e.g., in the 5% quantile coefficient estimates are −1.33 and −1.17 for 29 and 496 stocks respectively), while it remains at the same level in the right tail. In contrast, the $\widehat{\gamma}_{VIX}$ coefficient of the large portfolio almost doubled in all below-median quantiles and increased by 50% in the above-median quantiles. Moreover, the relative influence of the VIX index becomes almost symmetric, while in the 29-stock portfolio, the upper quantiles are influenced more than the lower quantiles. Overall, the results of our analysis suggest that anticipation of future market volatility translates directly into the conditional

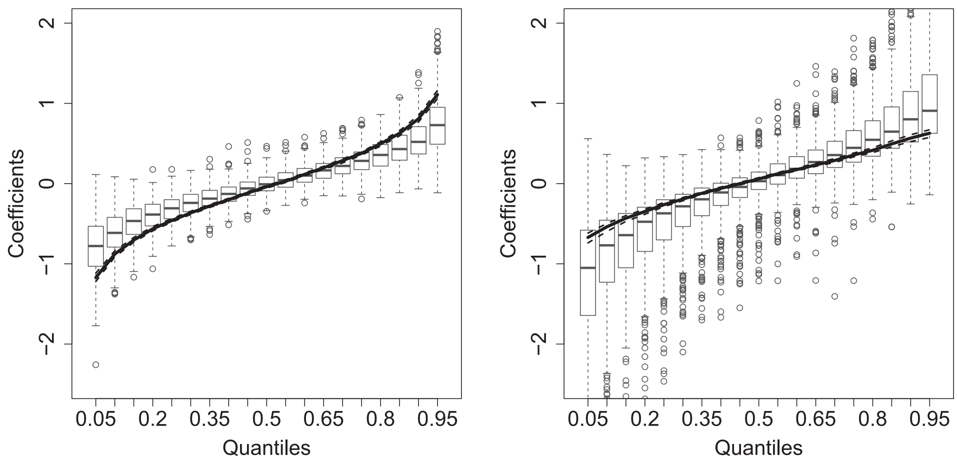
[21] At least five active trading hours during the day.




Note: Parameter estimates with corresponding 95% confidence intervals from the PQR-RV specification are plotted using solid and dashed lines, respectively. Individual UQR-RV estimates are plotted in boxplots.

**Fig. 9.** S&P 500 Index constituents: PQR-RV parameter estimates.

(a) $\hat{\beta}_{RV^{1/2}}$ (b) $\hat{\gamma}_{VIX}$




Note: For both the realized volatility and VIX parameters, estimates with corresponding 95% confidence intervals are plotted using solid and dashed lines, respectively. Individual UQR-RV-VIX estimates are plotted in boxplots.

**Fig. 10.** S&P 500 Index constituents: PQR-RV-VIX parameter estimates.

distribution of financial returns of the firms listed on the NASDAQ and NYSE.

## 7. Conclusion

In this paper, we propose a model to measure common risks in tails of financial returns distributions. We employ a series of panel quantile regressions together with non-parametric measures of quadratic return variation and ex ante measures of market volatility as the common factor to model conditional quantiles of financial asset return series. For estimation purposes, we use the fixed effects estimator introduced in Koenker (2004). The resulting panel quantile regression model for financial returns inherits all the favorable properties offered by panel data and quantile regression. A key advantage of our methodology is the ability to control for otherwise unobserved heterogeneity among financial assets, such that it is possible to disentangle overall market risk into its systemic and idiosyncratic components. Another advantage is the dimensionality reduction because the number of estimated parameters is always less than or equal to $K + N$, where $K$ is the number of regressors and $N$ the number of assets. Last but not least, to the best of our knowledge, this is one of the first applications of panel quantile regression using a dataset in which $T >> N$. As a result, we obtain estimates of quantile-specific individual fixed effects that account for unobserved heterogeneity and represent the idiosyncratic part of market risk. Moreover, these estimates translate into better forecasting performance of our model compared to traditional benchmarks. Overall, we test the accuracy and performance of our panel quantile regression model for financial returns in a simple portfolio VaR forecasting exercise using highly liquid stock market data. In our empirical application, the in-sample model fit highlights the importance of different components of quadratic variation and an ex ante volatility measure for various quantiles of return series. An out-of-sample statistical comparison shows the superiority of our approach. Better statistical performance moreover translates directly into economic gains, as shown by the global minimum VaR portfolio set-up and efficient frontiers of the VaR-return trade-off.

Our results make the model attractive not only from an academic but also from a practical point of view. In particular, it is highly attractive for portfolio and risk managers because of its ability to handle high-dimensional problems. More importantly, it can be easily used to obtain VaR measures of portfolios consisting of a high number of assets.

## Appendix A. Supplementary data

Supplementary data to this article can be found online at https://doi.org/10.1016/j.finmar.2020.100562.

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