Transfer Learning of Mixture Texture Models

Michal Haindl[1,2](✉) and Vojtěch Havlíček[1]

[1] The Institute of Information Theory and Automation of the Czech Academy of Sciences, Prague, Czechia
{haindl,havlicek}@utia.cz

[2] Faculty of Management, University of Economics, Jindřichův Hradec, Czechia

Abstract. A transfer learning approach for multidimensional parametric mixture random field-based textural representation is introduced. The proposed transfer learning approach allows alleviating the multidimensional mixture models requirement for sufficiently large, but not always available, learning data sets. These compound random field models consist of an underlying structure model that controls transitions between several sub-models, each of them has different characteristics. The structure model proposed is a two-dimensional probabilistic mixture model, either of the Bernoulli or Gaussian mixture type. Local textures are modeled using the fully multispectral three-dimensional Gaussian mixture sub-models. Both presented compound random field models allow the reproduction of, compresses, edits, and enlarges a given measured color, multispectral, or bidirectional texture function (BTF) texture so that ideally, both measured and synthetic textures are visually indiscernible.

Keywords: Texture · Texture modeling · Transfer learning · Three-dimensional Gaussian mixture · Compound random field model · Bidirectional texture function

1 Introduction

Realistic, visually convincing, and physically correct virtual models require precise object shapes and their surfaces covered with nature-like surface material textures to present realism in virtual scenes. The principal objective of any modeling texture approach is to reproduce and enlarge a given measured material texture so that ideally, both measured natural and modeled synthetic texture will be mutually visually indiscernible. This aim is not easy to reach due to the enormous variability of the natural material's appearance. The surface material semblance dramatically changes with illumination and viewing variations, among others, and we cannot even measure them in their full complexity. The most advanced current texture representation is the seven-dimensional Bidirectional Texture Function (BTF) [11]. Although BTF texture data can be measured, this task is expensive and requires a very demanding measurement setup. Additionally, measured BTF data are nearly always too limited to estimate reliably

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complex seven-dimensional BTF models, inevitably leading to some simplifying factorization [11], such as the presented compound random field models which use a set of three-dimensional factor models for the estimation of the complex overall BTF material model [11]. Very often not enough data is available such that the multidimensional mixture model can be trained [4–6,16,17]. Transfer learning (or domain adaptation) [21–24] partially alleviates the problem of lack of learning data by transferring knowledge learned from other similar learning tasks. Compound random field models (CRF) consist of several sub-models with different characteristics along with an underlying structure model that controls transitions between these sub-models [19]. Several image restoration [2,3,19,20], segmentation [25], or modeling [8,9,12] applications already benefitted of the compound Markov field models. However, these models always require demanding numerical solutions with all their well-known drawbacks. Our exceptional CMRF [8] model allows analytical synthesis at the cost of a slightly compromised compression rate. The transfer learning is illustrated on the three-dimensional Gaussian mixture model (3DGMM) but the same conclusion also holds for other atypical multidimensional mixture models such as 3D Bernoulli distribution mixture model [17] or 3D discrete distribution mixture model [17].

We propose two textural models - CRF$^{BM-3DGMM}$, CRF$^{GM-3DGMM}$, based on complex spatial probabilistic mixture models. These models have both the two-dimensional control field model and the three-dimensional local, regional models, either Gaussian mixture or probabilistic Bernoulli models. They differ only in dimensionality. While the principal control field is a simpler two-dimensional field, the local, regional models are much more demanding than three-dimensional. The three-dimensional random fields require much larger learning set to reliable estimate or their parameters, but unfortunately, their learning sets are also much smaller than the learning control fields. Thus we often face the situation when there are not enough data to use such models.

2 Compound Random Field Texture Models

Let us denote a multiindex $r = (r_1, r_2)$, $r \in I$, where I is a discrete two-dimensional rectangular lattice and r_1 is the row and r_2 the column index, respectively. $X_r \in \mathcal{K} = \{1, 2, \ldots, K\}$ is a random variable with natural number value (a positive integer), Y_r is multispectral pixel at location r and $Y_{r,j} \in \mathcal{R}$ is its j-th spectral plane component. Both random fields (X, Y) are indexed on the same lattice I. Let us assume that each multispectral or BTF observed texture $\tilde{Y}$ (composed of d spectral planes) can be modelled by a compound random field model, where the principal random field X controls switching to a regional local model $Y = \bigcup_{i=1}^{K} {}^{i}Y$. Single K regional sub-models ${}^{i}Y$ are defined on their corresponding lattice subsets ${}^{i}I$, ${}^{i}I \cap {}^{j}I = \emptyset \quad \forall i \neq j$ and they are of the same random field (RF) type. The sub-models differ only in their contextual support sets ${}^{i}I_r$ and the corresponding parameters sets ${}^{i}\theta$. The CRF model has posterior probability $P(X, Y \,|\, \tilde{Y}) = P(Y \,|\, X, \tilde{Y})\, P(X \,|\, \tilde{Y})$ and the corresponding optimal MAP solution is:

$$(\hat{X},\hat{Y}) = \arg \max_{X\in\Omega_X, Y\in\Omega_Y} P(Y \mid X,\tilde{Y})\, P(X \mid \tilde{Y}) \ ,$$

where Ω_X, Ω_Y are corresponding configuration spaces for random fields (X,Y). To avoid an iterative MCMC MAP solution, we propose the following two step approximation [8]:

$$(\breve{X}) = \arg \max_{X\in\Omega_X} P(X \mid \tilde{Y}) \ , \tag{1}$$

$$(\breve{Y}) = \arg \max_{Y\in\Omega_Y} P(Y \mid \breve{X},\tilde{Y}) \ . \tag{2}$$

This simplifying approximation significantly reduces the BTF-CRF$^{BM-3DGMM}$, BTF-CRF$^{GM-3DGMM}$ learning set requirements because it allows to estimate the principal switching random field X (1) and regional sub-models ${}^{i}Y$ (2) independently.

3 Principal Switching Model

The principal part (X) of the BTF compound random models (BTF-CMRF) is assumed to to be independent on illumination and observation angles, i.e., it is identical for all possible combinations $\phi_i, \phi_v, \theta_i, \theta_v$ azimuthal and elevation illumination/viewing angles, respectively. This assumption does not compromise the resulting BTF space quality, because it influences only a material texture macro-structure, which is independent of these angles for static BTF textures.

The control RF $(P(X \mid \tilde{Y}))$ is supposed to be represented by a two-dimensional random filed model. Such model can be a non-parametric random field [8,14,15] or some parametric random field hierarchical two-scale Potts model [9], Potts-Voronoi Markov random field model [18], and Gaussian or Bernoulli distribution mixture model [13], respectively. Mixture models are appropriate for regular or near-regular textures such as textile materials presented in this article. The mixture distribution $P(X_{\{r\}})$ has the form:

$$P(X_{\{r\}}) = \sum_{m\in\mathcal{M}} P(X_{\{r\}} \mid m)\, p(m) = \sum_{m\in\mathcal{M}} \prod_{s\in I_r} p_s(X_s \mid m)\, p(m) \tag{3}$$

where $X_{\{r\}} \in \mathcal{K}^{\eta}$, $\mathcal{M} = \{1,2,\dots,M\}$, $I_r \subset I$ is an index set, $\eta = cardinality\{I_r\}$, and $p(m)$ are probability weights $\sum_{m\in\mathcal{M}} p(m) = 1$. The maximum-likelihood parameter estimates $p(m)$ (probability weights), μ_{ms}, σ_{ms} (Gaussian mixture component means and standard deviation), $\theta_{m,s}$ (Bernoulli mixture component parameters) are computed using the EM algorithm [1,5] $p_s^{(t+1)}(. \mid m)$ and

$$q^{(t)}(m \mid X_{\{r\}}) = \frac{p^{(t)}(m)\, P^{(t)}(X_{\{r\}} \mid m)}{\sum_{j\in\mathcal{M}} p^{(t)}(j) P^{(t)}(X_{\{r\}} \mid j)} \ , \tag{4}$$

$$p^{(t+1)}(m) = \frac{1}{|\mathcal{S}|} \sum_{X_{\{r\}}\in\mathcal{S}} q^{(t)}(m \mid \ X_{\{r\}}) \ . \tag{5}$$

3.1 Principal Field Synthesis

We can assume without loss of generality at a given position r of the contextual neighborhood I_r to have some part of the pixel-wise synthesized control field $X_{\{r\}}$ already specified. If $X_{\{\rho\}}$ is a sub-vector of all of $X_{\{r\}}$ pixels previously specified within this window and $I_\rho \subset I_r$ the corresponding index subset, then the statistical properties of the remaining unspecified variables are fully described by the corresponding conditional distribution:

$$p_{n\,|\,\rho}(X_n\,|\,X_{\{\rho\}}) = \sum_{m=1}^{M} W_m(X_{\{\rho\}})\, p_n(X_n\,|\,m) \ , \tag{6}$$

where $W_m(X_{\{\{\rho\}})$ are the a posteriori component weights corresponding to the given sub-vector $X_{\{\rho\}}$:

$$W_m(X_{\{\rho\}}) = \frac{p(m) P_\rho(X_{\{\rho\}}\,|\,m)}{\sum_{j=1}^{M} p(j) P_\rho(X_{\{\rho\}}\,|\,j)} \ , \tag{7}$$

$$P_\rho(X_{\{\rho\}}\,|\,m) = \prod_{n\in\rho} p_n(X_n\,|\,m) \ .$$

X_n can be randomly generated by the conditional distribution $p_{n\,|\,\rho}(X_n\,|\,X_{\{\rho\}})$ whereby Eq. (6) can be applied to all the unspecified variables $n = \eta -$ card$\{\rho\}$ given a fixed position of the control field. Each newly generated X_n is used to upgrade the conditional weights $W_m(X_{\{\rho\}})$.

3.2 Bernoulli Distribution Mixture Model

We assume that the control field distinguishes between K sub-models and the distribution $P(X_{\{r\}})$ to be a multivariable Bernoulli mixture (BM), The control field is further decomposed into separate binary bit planes of binary variables $\xi \in \mathcal{B}, \mathcal{B} = \{0, 1\}$ and these planes are separately modeled and can be estimated from a much smaller learning texture than a multi-level discrete mixture model. We further suppose that a bit factor of a control field can be fully characterised by a marginal probability distribution of binary levels on pixels within the scope of a window centered around the location r and specified by the index set $I_r \subset I$, i. e., $X_{\{r\}} \in \mathcal{B}^\eta$ and $P(X_{\{r\}})$ is the corresponding marginal distribution of $P(X\,|\,\tilde{Y})$. The component distributions $P(\cdot\,|\,m)$ are factorisable, and multivariable Bernoulli:

$$P(X_{\{r\}}\,|\,m) = \prod_{s\in I_r} \theta_{m,s}^{X_s} (1-\theta_{m,s})^{1-X_s} \qquad X_s \in X_{\{r\}} \ . \tag{8}$$

The mixture model parameters (8) include component weights $p(m)$ and the univariate discrete distributions of binary levels. They are defined by one parameter $\theta_{m,s}$ as a vector of probabilities:

$$p_s(\cdot\,|\,m) = (\theta_{m,s}, 1-\theta_{m,s}) \ . \tag{9}$$

The EM solution is (4), (5) and

$$p_s^{(t+1)}(\xi \,|\, m) = \frac{1}{|\mathcal{S}|\, p^{(t+1)}(m)} \sum_{X_{\{r\}} \in \mathcal{S}} \delta(\xi, X_s)\, q^{(t)}(m \,|\, X_{\{r\}}), \quad \xi \in \mathcal{B} \ . \tag{10}$$

The total number of mixture (3), (9) parameters is thus $M(1+\eta)$ – confined to the appropriate norming conditions. The advantage of the multivariable Bernoulli model (9) is a simple switch-over to any marginal distribution by deleting superfluous terms in the products $P(X_{\{r\}} \,|\, m)$.

3.3 Gaussian Mixture Model

The principal (control) random field is discrete, but a continuous RF can alternatively model it if we map single indices into continuous random variables with uniformly separated mean values and small variance. The continuous synthetic results are subsequently inversely mapped back into a corresponding synthetic discrete control field. We assume the joint probability distribution $P(X_{\{r\}})$, $X_{\{r\}} \in \mathcal{K}^\eta$ in the form of a two-dimensional normal mixture and the mixture components are defined as products of univariate Gaussian densities

$$P(X_{\{r\}} \,|\, \mu_m, \sigma_m) = \prod_{s \in I_{\{r\}}} p_s(X_s \,|\, \mu_{ms}, \sigma_{ms}) \ , \tag{11}$$

$$p_s(X_s \,|\, \mu_{ms}, \sigma_{ms}) = \frac{1}{\sqrt{2\pi}\sigma_{ms}} \exp\left\{ -\frac{(X_s - \mu_{ms})^2}{2\sigma_{ms}^2} \right\} \ ,$$

i. e., the components are multivariate Gaussian densities with diagonal covariance matrices. The maximum-likelihood estimates of the parameters $p(m), \mu_{ms}, \sigma_{ms}$ can be computed by EM algorithm [1,5]. Anew we use a data set $\mathcal{S}$ obtained by pixel-wise shifting the observation window within the original texture image $\mathcal{S} = \{X_{\{r\}}^{(1)}, \ldots, X_{\{r\}}^{(K)}\}$, $X_{\{r\}}^{(k)} \subset X$. The corresponding log-likelihood function is maximized by the EM algorithm $(m \in \mathcal{M}, n \in \mathcal{N}, X_{\{r\}} \in \mathcal{S})$ and the iterations are (4), (5) and

$$\mu_{m,n}^{(t+1)} = \frac{1}{\sum_{X_{\{r\}} \in \mathcal{S}} q^{(t)}(m \,|\, X_{\{r\}})} \sum_{X_{\{r\}} \in \mathcal{S}} X_n \, q(m \,|\, X_{\{r\}}) \ , \tag{12}$$

$$(\sigma_{m,n}^{(t+1)})^2 = -(\mu_{m,n}^{(t+1)})^2 + \frac{\sum_{X_{\{r\}} \in \mathcal{S}} X_n^2 \, q^{(t)}(m \,|\, X_{\{r\}})}{\sum_{X_{\{r\}} \in \mathcal{S}} q(m|X_{\{r\}})} \ . \tag{13}$$

Details and examples about both principal random field models are illustrated in [13]. These BTF principal models usually do not suffer from lack of learning data, because there is one common principal field for thousands of measured combinations of illumination and observation angles. However, in the rare case of insufficient data, the transfer learning from the subsequent section can be applied without any change.

3.4 Constant Principal Model

The simplest principal model is a constant field which contains only one model BTF-CMRF$^{c...}$ $P(X \mid \tilde{Y}) = 1$. Then there is no need to use the MAP approximation (1), (2), and the compound Markov model simplifies into a single random field BTF-MRF model, and this model can be any of the local MRF or mixture models. To simplify further exposition and better illustrate the achieved results (Fig. 2) on larger images, we will further assume the constant principal field.

4 Local Spatial 3D Gaussian Mixture Model

A homogeneous static texture image Y is assumed to be defined on a finite rectangular $N_1 \times N_2 \times d$ lattice I, $r = (r_1, r_2, r_3) \in I$ denotes a pixel multiindex with the row, columns and spectral indices, respectively. Let us suppose that Y represents a realization of a random vector with a probability distribution $P(Y)$. The statistical properties of interior pixels of the moving window on Y are translation invariant due to assumed textural homogeneity. They can be represented by a joint probability distribution and the properties of the texture can be fully characterized by statistical dependencies on a sub-field, i. e., by a marginal probability distribution of spectral levels on pixels within the scope of a window centered around the location r and specified by the index set:

$$I_r = \{r + s : |r_1 - s_1| \leq \alpha \wedge |r_2 - s_2| \leq \beta\} \subset I \ .$$

The index set I_r depends on modeled visual data and can have any other than this rectangular shape. $Y_{\{r\}}$ denotes the corresponding matrix containing all Y_s in some fixed order arrangement such that $s \in I_r$, $Y_{\{r\}} = [Y_s \ \ \forall\, s \in I_r]$, $Y_{\{r\}} \subset Y$, $\eta = \text{cardinality}\{I_r\}$ and $P(Y_{\{r\}})$ is the corresponding marginal distribution of $P(Y)$. If we assume the joint probability distribution $P(Y_{\{r\}})$, in the form of a normal mixture

$$\begin{aligned} P(Y_{\{r\}}) &= \sum_{m \in \mathcal{M}} p(m)\, P(Y_{\{r\}} \mid \mu_m, \Sigma_m) \qquad Y_{\{r\}} \subset Y \ , \\ &= \sum_{m \in \mathcal{M}} p(m) \prod_{s \in I_r} p_s(Y_s \mid \mu_{m,s}, \Sigma_{m,s}) \end{aligned} \tag{14}$$

where $Y_{\{r\}} \in \Re^{d \times \eta}$ is $d \times \eta$ matrix, μ_m is $d \times \eta$ mean matrix, Σ_m is $d \times d \times \eta$ a covariance tensor, and $p(m)$ are probability weights and the mixture components are defined as products of multivariate Gaussian densities

$$P(Y_{\{r\}} \mid \mu_m, \Sigma_m) = \prod_{s \in I_{\{r\}}} p_s(Y_s \mid \mu_{ms}, \Sigma_{ms}) \ , \tag{15}$$

$$p_s(Y_s \mid \mu_{ms}, \Sigma_{ms}) = \frac{|\Sigma_{m,s}|^{-\frac{1}{2}}}{(2\pi)^{\frac{d}{2}}} \exp\left\{ -\frac{1}{2}(Y_r - \mu_{m,s})^T \Sigma_{m,s}^{-1} (Y_r - \mu_{m,s}) \right\} \ , \tag{16}$$

i. e., the components are multivariate Gaussian densities with covariance matrices (23). The underlying structural model of conditional independence is estimated from a data set $\mathcal{S}$ obtained by the step-wise shifting of the contextual window I_r within the original texture image, i. e., for each location r one realization of $Y_{\{r\}}$.

$$\mathcal{S} = \{Y_{\{r\}}\ \forall\, r \in I,\ I_r \subset I\} \qquad Y_{\{r\}} \in \Re^{d\times\eta} \ . \tag{17}$$

4.1 Parameter Estimation Using Transfer Learning

Local i-th texture region (not necessarily continuous) is represented by the 3D Gaussian mixture random (3DGMM) field model [7,10]. This model can be analytically estimated as well as synthetically enlarged to any required size if there is enough learning data, what is its typical application problem. The unknown parameters of the approximating the mixture can be estimated using the iterative EM algorithm [1]. To obtain robust parameter estimates for the unusually large 3DGMM models, we need an extensive learning set. The lower frequencies are in the BTF texture the larger cardinality η of $Y_{\{r\}} \in \mathcal{K}^\eta$, is needed. The lack of learning data is bypassed other similar texture ${}^1Y$ with approximately similar frequencies as the target texture ${}^2Y$. We assume the same GMM model structure for both textures, i.e., ${}^1m = {}^2m$. However, both textures can differ in their spectral information. The similar first texture ${}^1Y$ is used to learn the initial estimates for the target texture ${}^2Y$ with the exception of the component mean vectors:

$$ {}^2p^{(0)}(m) = {}^1p^{(t+1)}(m) \qquad \forall m \ , \tag{18}$$

$$ {}^2\Sigma^{(0)}_{m,s} = {}^1\Sigma^{(t+1)}_{m,s} \qquad \forall m, s \ . \tag{19}$$

The parameter transfer learning algorithm can be summarized:

1. EM estimation (20)–(23) of GMM parameters from ${}^1Y$,
2. EM estimation (20)–(23) of GMM parameters from ${}^2Y$ using the EM initialization (18),(19).

In order to estimate the unknown distributions $p_s(\cdot\,|\,m)$ and the component weights $p(m)$ we maximize the likelihood function corresponding to the training set (17):

$$L = \frac{1}{|\mathcal{S}|} \sum_{{}^iY_{\{r\}} \in \mathcal{S}} \log \left[\sum_{m\in\mathcal{M}} P({}^iY_{\{r\}}\,|\,\mu_m, \Sigma_m)\, p(m) \right] \ .$$

The likelihood is maximized using the iterative EM algorithm (with non-diagonal covariance matrices):
E:

$$q^{(t)}(m|\,{}^iY_{\{r\}}) = \frac{P^{(t)}({}^iY_{\{r\}}\,|\,\mu_m, \Sigma_m)\, p^{(t)}(m)}{\sum_{j\in\mathcal{M}} P^{(t)}({}^iY_{\{r\}}\,|\,\mu_j, \Sigma_j)\, p^{(t)}(j)} \ , \tag{20}$$

M:

$$p^{(t+1)}(m) = \frac{1}{|\mathcal{S}|} \sum_{^iY_{\{r\}} \in \mathcal{S}} q^{(t)}(m \,|\, {}^iY_{\{r\}}) \ , \tag{21}$$

$$\mu_{m,s}^{(t+1)} = \frac{1}{\sum_{^iY_{\{r\}} \in \mathcal{S}} q^{(t)}(m \,|\, {}^iY_{\{r\}})} \sum_{^iY_{\{r\}} \in \mathcal{S}} {}^iY_s \, q^{(t)}(m \,|\, {}^iY_{\{r\}}) \ . \tag{22}$$

The $M\eta$ covariance matrices are:

$$\begin{aligned}
\Sigma_{m,s}^{(t+1)} &= \frac{\sum_{^iY_{\{r\}} \in \mathcal{S}, {}^IY_s \in {}^IY_{\{r\}}} q^{(t)}(m \,|\, {}^iY_{\{r\}})}{\sum_{^iY_r \in \mathcal{S}} q^{(t)}(m \,|\, {}^iY_{\{r\}})} ({}^iY_s - \mu_{m,s}^{(t+1)})({}^iY_s - \mu_{m,s}^{(t+1)})^T \\
&= \frac{\sum_{^iY_{\{r\}} \in \mathcal{S}, {}^iY_s \in {}^iY_{\{r\}}} q^{(t)}(m \,|\, {}^iY_{\{r\}}) \, {}^iY_s^i Y_s^T}{\sum_{^iY_r \in \mathcal{S}} q^{(t)}(m \,|\, {}^iY_{\{r\}})} \\
&\quad - \frac{p^{(t+1)}(m) \, |\mathcal{S}| \, \mu_{m,s}^{(t+1)} \left(\mu_{m,s}^{(t+1)}\right)^T}{\sum_{^iY_r \in \mathcal{S}} q^{(t)}(m \,|\, {}^iY_{\{r\}})} \ .
\end{aligned} \tag{23}$$

The iteration process is stopped when the criterion increments are sufficiently small. The EM algorithm iteration scheme has the monotonic property: $L^{(t+1)} \geq L^{(t)}, \ t = 0, 1, 2, \ldots$ which implies the convergence of the sequence $\{L^{(t)}\}_0^\infty$ to a stationary point of the EM algorithm (local maximum or a saddle point of L).

4.2 Texture Synthesis

The advantage of a mixture model is its simple synthesis based on the marginals:

$$p_{n\,|\,\rho}(Y_n \,|\, Y_{\{\rho\}}) = \sum_{m=1}^{M} W_m(Y_{\{\rho\}}) \, p_n(Y_n \,|\, m) \ , \tag{24}$$

where $W_m(Y_{\{\{\rho\}})$ are the a posteriori component weights corresponding to the given submatrix $Y_{\{\rho\}} \subset Y_{\{r\}}$:

$$W_m(Y_{\{\rho\}}) = \frac{p(m) P_\rho(Y_{\{\rho\}} \,|\, m)}{\sum_{j=1}^{M} p(j) P_\rho(Y_{\{\rho\}} \,|\, j)} \ , \tag{25}$$

$$P_\rho(Y_{\{\rho\}} \,|\, m) = \prod_{n \in \rho} p_n(Y_n \,|\, m) \ . \tag{26}$$

The unknown multivariate vector-levels Y_n can be synthesized by random sampling from the conditional density (24) or the mixture RF can be approximated using the GMM mixture prediction [17].

5 Experiments

The proposed compound random field models ($BTF - CRF^{BM-3GMM}$, $BTF - CRF^{GM-3GMM}$) are well convenient for near-regular textures such as textile materials. Textures with the near-regular structure are difficult for Markov random field type of textural models [8,11], which are better suited for the random type of materials. The dimension of the estimated control field model distribution is not too high ($\eta \approx 10^1 - 10^2$) and the number of the training data vectors is relatively large ($|\mathcal{S}| \approx 10^4 - 10^5$). However, the window size should always be kept reasonably small and the sample size as large as possible. We have used a regular left-to-right and top-to-down shifting of the generating window in our experiments. Figure 1 illustrates the synthesis of Cloth35 texture control field using the two-dimensional Bernoulli mixture model ($BTF - CRF^{BM-3DGMM}$). Figure 2 shows four textile BTF material measurements (only one measurement with perpendicular viewing and illumination angle from the whole sets of 6561 measurements per material). These samples were synthesized using the $BTF - CRF^{c-3DGMM}$ models either with the transfer learning from similar textile textures or without. If we are comparing both synthesis variants, the usefulness of the additional information obtained from the transfer learning is pronounced. The transfer learning cost doubles the learning time because, in our experiments, we double (2×512^2) the overall learning set. Experiments with a chain of two similar textures suggest no further noticeable improvement. Significantly different textures might even decrease the performance (negative transfer).

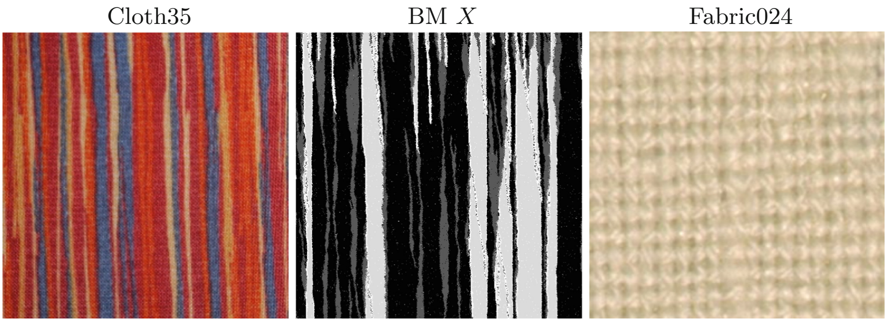

Fig. 1. Cloth measurements (left), its synthesized control field using the BM model with $K = 4$ (middle), and Fabric024 texture used in the transfer learning (Fig. 2 first row).

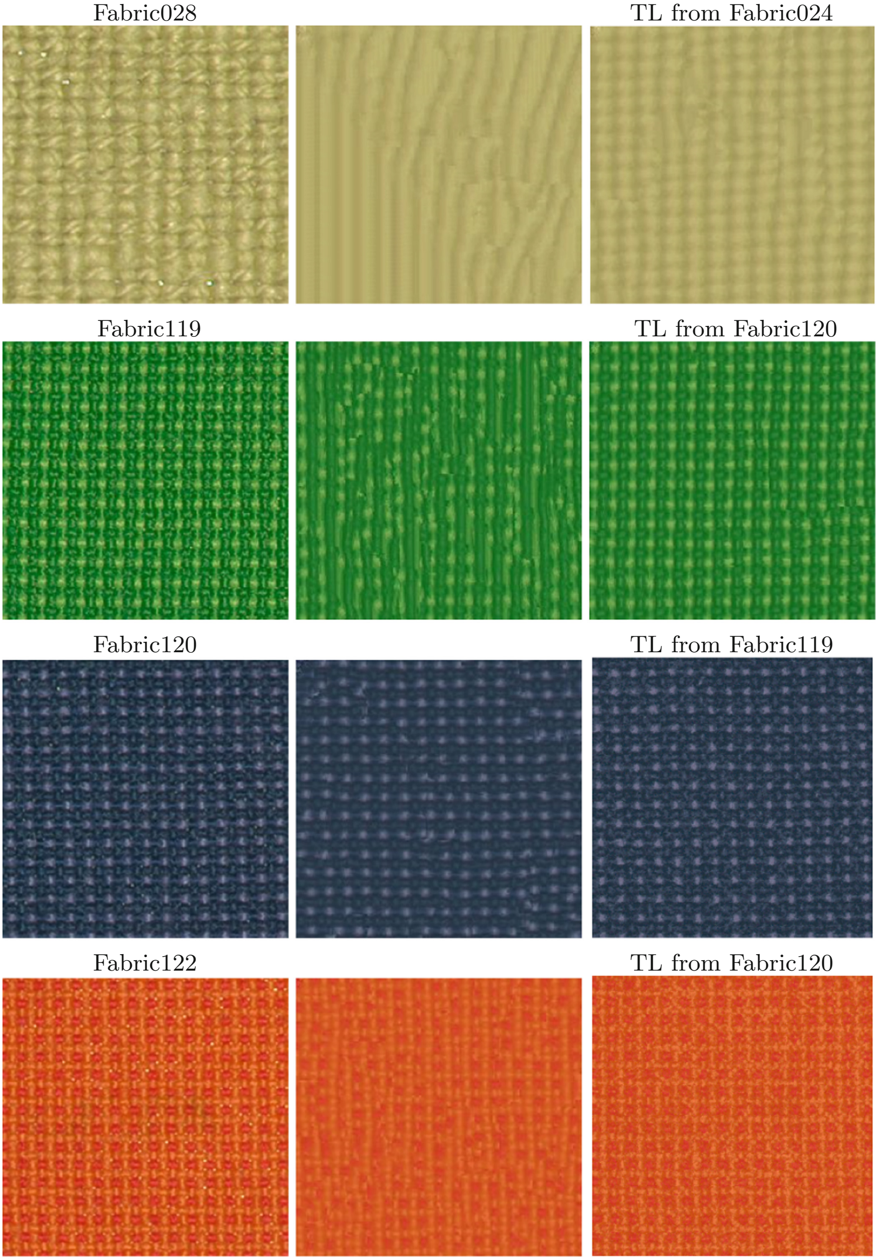

Fig. 2. Fabric measurements (left column), synthesis without transfer learning (TL, middle column), and with the transfer learning (right column) using the $3DGMM$ model.

6 Conclusion

Both presented BTF-CRF ($BTF-CRF^{GM-3DGMM}$, $CRF^{BM-3DGMM}$ methods with the local random field models learned using the supported information from similar textures through transfer learning suggest improved visual achievement on selected real-world measured textile materials. The models do not compromise spectral correlation; thus, they reliably model and enlarge motley textures. Both methods can be smoothly generalized for color, hyperspectral or BTF texture editing by learning some local models from alternative target materials. The proposed transfer learning approach allows alleviating typical multidimensional mixture models drawback - their requirement of sufficiently large learning data that are not always available.

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