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# Flexible negotiation process to adhere to human preferences; a case of work equipment risk assessment

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**Abstract -** Making structured and reliable decisions on relevant business problems often requires expert assistance. In decision making practice, experts are frequently required to pairwise compare elements to support the decision made. This paper proposes a user-friendly negotiation procedure to establish an effective feedback relation with experts to globally increase the consistency of their pairwise comparisons judgments, where necessary. To this aim, we develop a flexible tool, which makes use of an algebraic consistency-improving algorithm and a sensitivity analysis technique to identify which judgments contribute most to inconsistency. The framework pursues friendliness for the involved decision makers, as they are asked to reconsider only a few *a priori* judgments, instead of rethinking the entire set of previously elicited comparisons. A real-world case study on risk assessment in industry is implemented to demonstrate the practical applicability of the proposed approach.

## 1 Introduction and state of the art

Decision making permeates business dynamics and, in general, human life. In real-world optimization processes, variables are usually difficult to quantify, especially in complex decision-making problems [1, 2]. Thus, decisions may be not simple, being directly or indirectly connected to qualitative, subjective, intangible elements so that, in practice, individuals and companies resort to Multi-Criteria Decision-Making (MCDM) methods to achieve effective solutions [3] for their problems.

Various MCDM methods rely on pairwise comparisons elicited by one expert or a panel of decision makers [4]. Judgments provided by experts are collected in so-called Pairwise Comparison Matrices (PCMs). For instance, the Analytic Hierarchy Process (AHP) [5], one of the most common MCDM methods, gets priorities from pairwise comparisons of elements via the Perron eigenvector of PCMs. To make decisions reliable and appropriate, pairwise comparisons are required to be acceptably consistent. However, due to cognitive limitations, humans are not fully consistent when eliciting judgments. It goes without saying that the higher the number of elements to be pairwise compared, the lower the probability for consistency conditions to be met. Moreover, currently, since information gathered from big databases and the Internet is also susceptible to being handled through pairwise comparisons, the number of elements taken into account can be huge (see [6, 7, 8], among others). On the whole, one of the problems limiting the applicability of pairwise comparisons to large-scale decision problems is the so-called curse of dimensionality. This means that many comparisons have to be issued or built from a body of information. As a result, PCMs related to real decision-making problems may suffer lack of consistency, and this may have a negative impact on the quality of the final decisions. Consistency is crucial in decision-making, since it would be unwise to rely on randomly elicited judgments. When the consistency of a PCM is not satisfactory, it is necessary to improve it [9].

Suitable mechanisms aimed at improving consistency can be implemented and adapted to the problem under analysis [10], but they require the alteration of one or more judgments previously elicited by decision-makers. In any case, one has to consider the fundamental importance of sharing any change with the decision makers, since they may not agree with some changes. In other words, it is necessary to carry out a negotiation process capable to balance mathematical consistency enforcement with the issued judgments reflecting the (expert) reality. It is clear as decision-making itself is a complex process for various reasons. Among them, the approach proposed in the present article deals with subjectivity and negotiation. The ability to make a rational use of subjective perceptions of experts is indeed crucial and, in this context, fruitful feedback exchanges [11] ensure the facilitation of discussion and deliberation even among group members to share opinions and eventually agree on a final decision [12].

The main objective of this contribution consists in developing a framework that, using a sensitivity analysis of an input inconsistent PCM, highlights which judgment(s) previously expressed by the expert should be preferably modified to increase consistency, while keeping the matrix within an acceptable threshold. This way, parsed suggestions for changes are presented in turn, what is more friendly than going back to the expert for an entire judgment reconsideration. After a concise setting of the mathematical problem, we present the negotiation process and apply it to solve a real industrial problem on risk assessment for core work equipment.

# 2 The proposed approach

## 2.1 Problem setting

In AHP, pairwise comparisons are formally collected in a PCM by asking to express (typically by using a 9-point scale [5]) judgments of preference between pairs of elements. For a set of $n$ elements, a PCM is an $n \times n$ matrix, $A = (a_{ij})$, whose (positive) entries have to adhere to the properties of homogeneity ($a_{ii} = 1$) and reciprocity ($a_{ji} = 1/a_{ij}$), $i, j = 1, \dots, n$. The problem for matrix $A$ consists in producing a set of numerical values $w_1, ..., w_n$ expressing the importance of the $n$ elements under analysis. If the expert gives his/her judgments in a completely consistent way, the relations between weights $w_i$ and numerical values translating judgments $a_{ij}$ are simply given by $w_i/w_j = a_{ij} (i, j = 1, ..., n)$, and the PCM $A$ is said to be consistent. For a consistent PCM, the Perron eigenvector (normalized to one) gives the vector of weights [13] of the analysed decision-making elements.

However, considering the limits of human reasoning, a certain degree of inconsistency is always expected. For a non consistent PCM, one has to solve the eigenvector problem:

$$A\mathbf{w} = \lambda_{max}\mathbf{w}, \tag{1}$$

where $\lambda_{max}$ is the principal eigenvalue of matrix $A$, providing the vector of weights, $\mathbf{w}$. The so-called Consistency Index

$$CI = \frac{\lambda_{max} - n}{n - 1}, \tag{2}$$

is used as a common measure of inconsistency and is used to calculate the Consistency Ratio:

$$CR = \frac{CI}{RI}. \tag{3}$$

Average consistencies ($RI$ values) of randomly generated matrices are given in [14]. Consistency is accepted if $CR < 0.1$, new judgments should be elicited again otherwise.

As outlined above, the literature offers various methodologies aimed at improving consistency, mainly based on iterative optimisation techniques. We herein use a direct, algebraic method, the linearisation technique [15], to get the (fully) consistent matrix $A^c$ closest to a non-consistent PCM $A$. A simple formula provides it [16] in terms of an algebraic projection $p_n$ of the (entry-wise) logarithm of $A$, $L(A)$:

$$p_n(L(A)) = \frac{1}{n}[(L(A)U_n) - (L(A)U_n)^T], \tag{4}$$

where $U_n = \mathbf{1}_n\mathbf{1}_n^T$ and $\mathbf{1}_n$ is the unit column vector. The (entry-wise) exponential of this projection provides the sought matrix.

We underline as the closest consistent matrix is a synthetic result that may not properly reflect the original opinions provided by the involved expert(s). This is the reason why the process of feedback exchange is fundamental at any time, to assure that the obtained results are not far from the expert reality. In other terms, the closest consistent matrix has to be evaluated by the expert, who will have to adjust its entries where necessary, and this process will be iterated until producing a final matrix with acceptable consistency, representing a reasonable compromise between synthetic consistency and expert judgment. A basic procedure that presents the entire fully consistent matrix is provided in [17]. Next, we formally develop a more friendly process that simplifies this modifying procedure.

## 2.2 Process description

Figure 1 presents a scheme of the conciliation process between consistency and expert judgments. In this flowchart, trapezoids represent inputs, rectangles correspond to procedures, rhombuses stand for if-else decisions and rounded figures for outputs.

Detailed explanations are now provided.

- Obtaining pairwise comparisons: the elicitation of pairwise comparison judgments (upper left trapezoid) represents the input of the process and initiates the diagram.
- Drawing up the input matrix: judgments of preference are collected into the PCM by using, for instance, the scale proposed in [5] (first upper left rectangle in Figure 1).
- Checking consistency: this step (second rectangle) uses equations (1), (2) and (3) to calculate $CR$.
- Establishing consistency acceptability: in this case (lower leg of the rhombus), the PCM is validated; otherwise, a negotiation process will be undertaken with the decision-maker, including possible modifications to get a consistency ratio within the acceptable consistency threshold (rhombus's leg 'No'). This process is described next.

The negotiation process is represented in Figure 2: for a non acceptably consistent PCM (upper rhombus's 'No' leg), this process consists of two main steps.

- calculation of sensitivity and closest fully consistent matrix. It includes two calculations:
  - obtaining the closest fully consistent matrix $A^c$ by applying the linearisation process, equation (4);
  - calculating sensitivity values for the entries of $A$ as described in the next subsection. Note that just the $M = \frac{n\times(n-1)}{2}$ values over the main diagonal are considered.
- Implementation of the iterative procedure to get the adjusted matrix, represented in Figure 3 and shortly described here.
  - The process starts by ranking the entries of $A$ according to their impact on consistency.
  - Next, an adjusted matrix $B$ is built, all whose elements but one correspond to the input matrix, this last element being replaced by the element of $A^c$ occupying the first position in the sensitivity ranking.

- It follows by calculating consistency again (central rhombus in Figure 3); if matrix $B$ is consistent the process stops ('Yes' leg of that rhombus). If matrix $B$ continues to be inconsistent, the iteration consists in changing the element corresponding to the element occupying the second position in the sensitivity ranking with the corresponding value in the closest consistent matrix.
- These previous steps, together with the updating of a control parameter, $M$, are repeated until consistency is assured. Matrix $B$ is the output of the process along with parameter $M$, which is an indicator of the magnitude of the modification made.

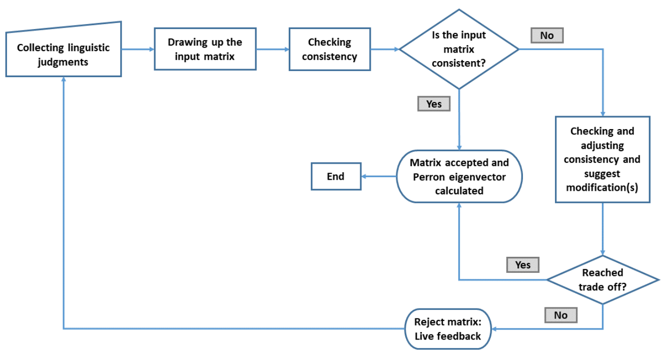


Figure 1. Conciliation of consistency and expert judgment

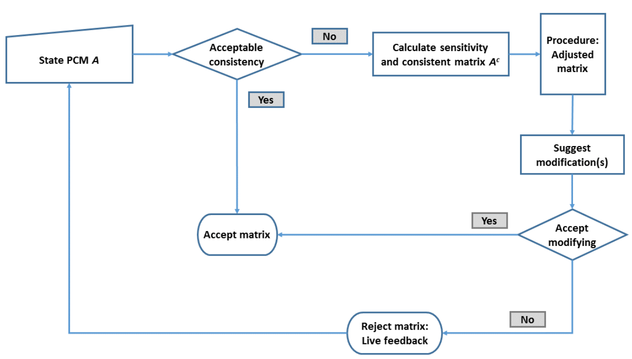


Figure 2. Process of "checking and adjusting consistency"

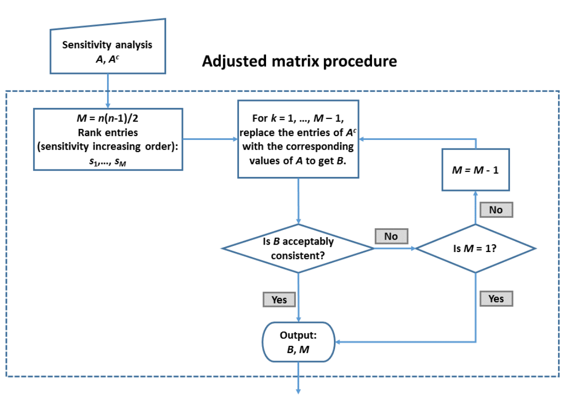


Figure 3. Adjusted matrix iterative procedure

The process to achieve the adjusted matrix has been designed to make the adjusted matrix as adherent as possible to the input matrix, by changing as fewer as possible evaluations previously given by the decision makers. The modified judgments will be eventually proposed to the expert, who will be invited to agree with the final evaluations. In case of disagreement ('No' leg of lower rhombus in Figure 2), he/she will be asked to elicit new evaluations, so that a new matrix may be drawn up. At the point in which a decision maker has associated a final adjusted consistent matrix, the process goes back to the central rounded box of Figure 2, and then through the leg 'Yes' of the lower right rhombus in Figure 1 leads to its rounded central box, where the matrix is accepted and the process stops after calculating the sought vector of priorities.

## 2.3 Sensitivity analysis

The sensitivity method applied to rank the most "influencing" judgments is presented next. Starting from an $n \times n$ PCM $A$, the method consists in calculating a second matrix $D$ giving the partial derivatives of $\lambda_{max}$ with respect to the entries of $A$, thus identifying which entries are more sensible to increase consistency. These partial derivatives are given by the following formula (Section 1.1, [18]):

$$D = \mathbf{w}\mathbf{v}^T - A^2 \circ \mathbf{v}\mathbf{w}^T, \tag{5}$$

where: $\mathbf{w}$ represents the Perron eigenvector, associated with the value of $\lambda_{max}$; $\mathbf{v}$ represents the left Perron eigenvector of $A$, that is the (right) Perron eigenvector of the transpose of $A$, also associated with $\lambda_{max}$, and normalized such that $\mathbf{v}\mathbf{w}^T = 1$; $\circ$ is the Hadamard (entry-wise) product. The Hadamard product operates on identically-shaped matrices and produces a third matrix of the same dimensions, whose elements $(i, j)$ correspond to the product of elements $(i, j)$ of the original two matrices.

The values corresponding to the partial derivatives allow to rank the corresponding entries of matrix $A$ and then know which one has higher influence on consistency. The case study proposed in the next section demonstrates the applicability of the presented algorithm and its effectiveness to solve real problems in industry.

# 3 Case Study

The present case study is focused on the topic of work equipment risk assessment for a manufacturing company operating in the food industrial sector and located in the South of Italy.

Compliance of work equipment with safety requirements is a crucial aspect that must be verified according to the existing standards. In addition, the employer is required by law to carry out an exhaustive assessment aimed at formally and quantitatively evaluating all the potential risks for employees working with the equipment. Risk assessment has to thoroughly analyse, for each working station, dangerous situations potentially occurring during tasks execution. It is also necessary to highlight which workers may be critically exposed to risks in the area where a given equipment operates.

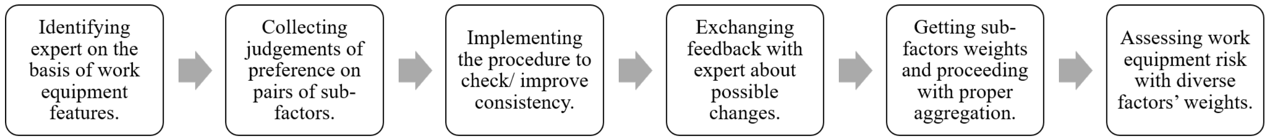


Figure 4. Process for establishing different degrees of importance for main factors in work equipment risk assessment

With this regard, the company analysed in the present case study applies a technique aligned with the standard [19] (and subsequent amendments and additions) as a risk evaluation procedure. Three main factors are considered to quantitatively evaluate the global risk associated to each work equipment: material ($M$), environment ($E$), organisation ($O$). The first factor refers to the risk related to physical work equipment and its use by workers. The second factor considers the specific and potentially risky features of the workplace where equipment (and consequentially workers) operate. Lastly, the third factor is connected to the ability of personnel to master risk occurrence, especially in terms of expertise and work organisation. All these aspects are important for risk assessment purposes, but it is clear that they may have different influence on that process. This is the reason why the management of the company has decided to upgrade the process of risk assessment for some of its core system. The objective of the proposed case study consists in supporting the company by calculating the degrees of importance associated to the three main risk factors (as shown in Figure 4) on the basis of judgments of preference provided by a professional on work equipment maintenance. In this way, the risk assessment will be more precise, by considering the diverse contribution of the three factors instead of assuming them as equally weighted. The global risk is eventually sorted into proper classes (low, medium, high risk) aimed at highlighting priorities of intervention in preventing and/or reducing risk associated to work equipment.

Each risk factor is split into various sub-factors. In Table 1 we give and synthetically describe those most significant sub-factors among ones considered of interest for the company.

TABLE 1. Factors and sub-factor for risk assessment

| **Factor** | **Sub-factor** | **Description** |
|---|---|---|
| $M$ | $M_1$ | Dangerous events and potential injuries. |
| | $M_2$ | Frequency and duration of exposition. |
| $E$ | $E_1$ | Workplace physical location. |
| | $E_2$ | Lighting, microclimate, noise. |
| $O$ | $O_1$ | Scarce personnel qualification. |
| | $O_2$ | Poor workflow organisation. |

The sub-factors have been pairwise compared by the involved expert by using the Saaty scale [5], and the judgments of preference are collected in a PCM, $A$ (Table 2).

TABLE 2. Input PCM, $A$

| $A$ | $M_1$ | $M_2$ | $E_1$ | $E_2$ | $O_1$ | $O_2$ |
|---|---|---|---|---|---|---|
| $M_1$ | 1.00 | 1.00 | 1.00 | 2.00 | 3.00 | 5.00 |
| $M_2$ | 1.00 | 1.00 | 3.00 | 4.00 | 4.00 | 4.00 |
| $E_1$ | 1.00 | 0.33 | 1.00 | 0.50 | 1.00 | 1.00 |
| $E_2$ | 0.50 | 0.25 | 2.00 | 1.00 | 0.33 | 0.33 |
| $O_1$ | 0.33 | 0.25 | 1.00 | 3.00 | 1.00 | 2.00 |
| $O_2$ | 0.20 | 0.25 | 1.00 | 3.00 | 0.50 | 1.00 |

Matrix $A$ (with $\lambda_{max} = 6.7596$) is not acceptably consistent since $CR = 12.15\% > 10\%$; so, some judgments need adjustment. The matrix of partial derivatives of $\lambda_{max}$ with respect to $A$'s entries, $D$, is calculated by (5) and provided in Table 3.

TABLE 3. Matrix $D$ of partial derivatives of $\lambda_{max}$

| $D$ | $M_1$ | $M_2$ | $E_1$ | $E_2$ | $O_1$ | $O_2$ |
|---|---|---|---|---|---|---|
| $M_1$ | 0.00 | −0.10 | 0.34 | 0.33 | −0.46 | −1.22 |
| $M_2$ | 0.10 | 0.00 | 0.09 | 0.10 | −0.50 | −0.13 |
| $E_1$ | −0.34 | −0.01 | 0.00 | 0.21 | −0.07 | 0.03 |
| $E_2$ | −0.08 | −0.01 | −0.82 | 0.00 | 0.07 | 0.13 |
| $O_1$ | 0.05 | 0.03 | 0.07 | −0.67 | 0.00 | −0.26 |
| $O_2$ | 0.05 | 0.01 | −0.03 | −1.13 | 0.06 | 0.00 |

Pairwise comparison corresponding to entry $(M_1, O_2)$ influences consistency most, having associated the highest absolute value, followed by comparison $(O_2, E_2)$; then $(E_2, E_1)$, etc. Thus, consistency can be improved by approaching the related comparison values towards the corresponding values of the closest consistent matrix $A^c$. This matrix, calculated by (4), is not herein presented for the sake of space. By just exchanging $(M_1, O_2)$ and $(O_2, E_2)$ for the corresponding values in $A^c$, an acceptably consistent ($CR = 7.78\%$) matrix $B$ (Table 4) is obtained. The process of negotiation started by suggesting to replace his/her initial evaluations 5.00 for $(M_1, O_2)$ and 3.00 for $(O_2, E_2)$ with the values 2.71 and 1.18, respectively, obtained in $A^c$. The expert approved these modifications, considering that they do not significantly change the practical meaning of the original assessment.

TABLE 4. Adjusted matrix $B$ to be shared with the expert

| $B$ | $M_1$ | $M_2$ | $E_1$ | $E_2$ | $O_1$ | $O_2$ | **Weights** |
|---|---|---|---|---|---|---|---|
| $M_1$ | 1.00 | 1.00 | 1.00 | 2.00 | 3.00 | 2.71 | 23.34% |
| $M_2$ | 1.00 | 1.00 | 3.00 | 4.00 | 4.00 | 4.00 | 33.61% |
| $E_1$ | 1.00 | 0.33 | 1.00 | 0.50 | 1.00 | 1.00 | 11.20% |
| $E_2$ | 0.50 | 0.25 | 2.00 | 1.00 | 0.33 | 0.85 | 9.87% |
| $O_1$ | 0.33 | 0.25 | 1.00 | 3.00 | 1.00 | 2.00 | 13.49% |
| $O_2$ | 0.37 | 0.25 | 1.00 | 1.18 | 0.50 | 1.00 | 8.49% |

The last column of Table 4 shows the sought vector of weights for the sub-factors, which we aggregate to get main factors' weights: $w_M = 56.95\%$, $w_E = 21.07\%$ and $w_O = 21.98\%$.

This final outcome provides the company with various degrees of importance associated to the risk factors used for work equipment risk assessment, while adhering as much as possible to the preferences issued by a stakeholder with proven experience in the field. According to the expert's opinions, the "material" factor ($M$) mostly influences the risk assessment process for work equipment, followed by the "organization" ($O$) and "environment" ($E$) factors, the last two ones having associated approximately the same importance. Aspects related to work equipment management, above all including safety and security, have hence to be controlled with particular attention to pursue effective risk minimization.

# 4 Conclusions

This article proposes an iterative procedure to carry out an effective negotiation with experts involved in complex business decision making processes. The objective consists in calculating priorities of elements (expressing their mutual importance) by balancing the mathematical need of consistency of judgments with adherence to reality. The proposed procedure is based on a sensitivity analysis signalling those comparisons most "influencing" to be used as drivers for consistency improvement. As a result, the decision maker will not be asked to elicit a new whole set of judgments, but just to reconsider a few preferences to quickly reach the necessary consensus. The approach has been applied to an industrial case study aimed at supporting the process of risk assessment for work equipment management in a real Italian company. An effective feedback relation has been established with the expert towards an easy calculation of priorities for the main elements of analysis. The negotiation process can be applied to solve generic problems in any business field, what confirms the flexibility of the proposed approach. Future development of this research may regard the integration of the negotiation process within work equipment maintenance to optimise organisational aspects also in terms of scheduling.

# 5 Aknowledgment

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# References

[1] J. Francés-Chust, B. M. Brentan, S. Carpitella, J. Izquierdo, and I. Montalvo, “Optimal placement of pressure sensors using fuzzy dematel-based sensor influence,” *Water*, vol. 12, no. 2, p. 493, 2020.

[2] S. Carpitella, S. J. Ocaña-Levario, J. Benítez, A. Certa, and J. Izquierdo, “A hybrid multi-criteria approach to gpr image mining applied to water supply system maintenance,” *Journal of Applied Geophysics*, vol. 159, pp. 754–764, 2018.

[3] A. Ishizaka and S. Siraj, “Are multi-criteria decision-making tools useful? an experimental comparative study of three methods,” *European Journal of Operational Research*, vol. 264, no. 2, pp. 462–471, 2018.

[4] S. Safarzadeh, S. Khansefid, and M. Rasti-Barzoki, “A group multi-criteria decision-making based on best-worst method,” *Computers & Industrial Engineering*, vol. 126, pp. 111–121, 2018.

[5] T. L. Saaty, “A scaling method for priorities in hierarchical structures,” *Journal of mathematical psychology*, vol. 15, no. 3, pp. 234–281, 1977.

[6] S. Bozóki, L. Csató, and J. Temesi, “An application of incomplete pairwise comparison matrices for ranking top tennis players,” *European Journal of Operational Research*, vol. 248, no. 1, pp. 211–218, 2016.

[7] W. Gu, M. Basu, Z. Chao, and L. Wei, “A unified framework for credit evaluation for internet finance companies: Multi-criteria analysis through ahp and dea,” *International Journal of Information Technology & Decision Making*, vol. 16, no. 03, pp. 597–624, 2017.

[8] J. C. L. Lopez, J. J. S. Noriega, and D. A. G. Chavira, “A multi-criteria approach to rank the municipalities of the states of mexico by its marginalization level: the case of jalisco,” *International Journal of Information Technology & Decision Making*, vol. 16, no. 02, pp. 473–513, 2017.

[9] J. S. Finan and W. J. Hurley, “The analytic hierarchy process: does adjusting a pairwise comparison matrix to improve the consistency ratio help?,” *Computers & operations research*, vol. 24, no. 8, pp. 749–755, 1997.

[10] J. Franek and A. Kresta, “Judgment scales and consistency measure in ahp,” *Procedia Economics and Finance*, vol. 12, pp. 164–173, 2014.

[11] Z.-p. Tian, R.-x. Nie, J.-q. Wang, and H.-y. Zhang, “A two-fold feedback mechanism to support consensus-reaching in social network group decision-making,” *Knowledge-Based Systems*, vol. 162, pp. 74–91, 2018.

[12] I. J. Pérez, F. J. Cabrerizo, S. Alonso, Y. Dong, F. Chiclana, and E. Herrera-Viedma, “On dynamic consensus processes in group decision making problems,” *Information Sciences*, vol. 459, pp. 20–35, 2018.

[13] T. L. Saaty, “Decision-making with the ahp: Why is the principal eigenvector necessary,” *European journal of operational research*, vol. 145, no. 1, pp. 85–91, 2003.

[14] T. L. Saaty, *Fundamentals of decision making and priority theory with the analytic hierarchy process*, vol. 6. RWS publications, 2000.

[15] J. Benítez, X. Delgado-Galván, J. Izquierdo, and R. Pérez-García, “Achieving matrix consistency in ahp through linearization,” *Applied Mathematical Modelling*, vol. 35, no. 9, pp. 4449–4457, 2011.

[16] J. Benítez, J. Izquierdo, R. Pérez-García, and E. Ramos-Martínez, “A simple formula to find the closest consistent matrix to a reciprocal matrix,” *Applied Mathematical Modelling*, vol. 38, no. 15-16, pp. 3968–3974, 2014.

[17] J. Benítez, X. Delgado-Galván, J. Gutiérrez, and J. Izquierdo, “Balancing consistency and expert judgment in ahp,” *Mathematical and Computer Modelling*, vol. 54, no. 7-8, pp. 1785–1790, 2011.

[18] G. W. Stewart, *Matrix Algorithms: Volume II: Eigensystems*. SIAM, 2001.

[19] M. del Lavoro e delle Politiche Sociali, “Testo unico sulla salute e sicurezza sul lavoro,” *D.lgs. 9 aprile 2008 n. 81, Testo coordinato con il D.Lgs. 3 agosto 2009, n. 106*.