Ambiguity effect: decision-making influenced by lack of information

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***Abstract*—Quite often, the best human decision-makers outperform computer-aided decision systems. It is not only because humans can take into account faint pieces of information that cannot be formalized but also that they occasionally behave intuitively, which can hardly be incorporated into a formal optimization criterion. Therefore, mathematicians enhance their decision models to make their behavior similar to that of human decision-makers. They fit decision models up with different parameters controlling the optimality of the considered decision. From this point of view, the simplest and perhaps the most popular is the Hurwitz coefficient of pessimism controlling whether the decision process tends to expect more the best or the worst outcome. In this paper, we design a model with a parameter controlling the strength of ambiguity aversion of the resulting decision process.**

***Index Terms*—Decision making, ambiguity, belief function, subjective characteristics.**

I. Introduction

Human decision-making depends on many factors, some of which are derived from the subjective attitude of decision-makers, like, e.g., their attitude to risk and their pessimism/optimism. These characteristics may significantly influence the results of a decision process. It is known that, in general, the most careful decision need not be the best one. It makes the simulation of the human way of decision-making by a machine very difficult. In [1], the authors experimentally compared what they called conscious and unconscious thoughts in decision-making. The latter one was enforced by a limited time available. Surprisingly, experiments have shown that unconscious thinkers made the best decisions. In addition, unconscious thought seems to lead to a better organization and polarization of the thinker's memory. Similar results, in the case of the creative idea selection problem, were achieved in [2], where the authors distinguished intuitive and deliberative decision making.

To simulate the human way of decision-making, mathematicians fit some of their models up with different parameters controlling the optimality of the considered decision. From this point of view, the simplest, and maybe also the most famous is the Hurwitz coefficient of pessimism [3], [4] controlling whether the decision process tends to expect more the best or the worst outcome. To control the risk aversion of decision models, the employed optimality criterion may be based not only on the expected value of the yield but also on its standard variation.

Starting with [5], in the last decades, psychologists have devoted their attention also to another subjective attitude influencing the human decision. It comes into consideration in case of a lack of information. We have in mind the *ambiguity aversion*. One can hardly incorporate this attitude into probabilistic models because this theory does not have the proper means to describe ambiguity. This can be seen, for example, from the fact that the Bayesian approach uses uniform priors in case that all the considered situations are equally probable, as well as in situations when there is no information about their distribution.

In the second half of the last century, the belief function theory of evidence [6], [7] was designed. As we will see in the next section, having all the power of probability theory, belief functions can describe and distinguish the situations under risk and the situations under vague information. The basic ideas on how to apply this theory to the decision were laid by Gilboa and Schmeidler [8], and other authors like Strat [9].

Ambiguity and its effect on decision-making are well-known, established topics in economics [10], [11]. It may seem that, nowadays, there is plenty of data to support business decision-making and that there is no place for ambiguity. Nevertheless, some situations have never occurred before. Moreover, even if data are available, it may not be the question of their amount, but the question of trust [12].

Human decision makers distinguish from each other not only by their intuition by also by their subjective risk and ambiguity attitude. Therefore, when constructing a mathematical model simulating the behavior of human decision-makers, one has to introduce a parameter allowing the adaptation of the model. The model should be able to simulate persons avoiding ambiguity, as well as those who are irrelevant to ambiguity, or even those who are seeking ambiguity.

The main contribution of the paper is to present a decision method, the behavior of which depends on a coefficient of ambiguity aversion. If no available information is ambiguous, then the method corresponds to a usual optimal probabilistic decision-making. The more ambiguous information is available, the more influenced is the resulting decision by the coefficient of ambiguity. Moreover, the user can adapt the result of the decision process to a type of ambiguity aversion discussed below.

As explained below, the method is based on the idea that humans with ambiguity aversion expect a lower yield than that corresponding to any (reasonably selected) probability distribution. It looks like that their subjective probabilities do not sum up to one, and therefore the expected yield must be reduced accordingly. As shown in [13], the method thus simulates the behavior of most of the experimental persons described in the above-cited Ellesberg's seminal paper. However, in the same way, it can easily imitate the behavior of a decision-maker seeking ambiguity, which occurred in our experiments more often than we expected.

The rest of the paper is organized as follows. Based on a simple example guiding the reader through the whole paper, the next section introduces the basic notions and notation from belief function theory. The main contribution is explained in Section III, where a coefficient of ambiguity and its role in a decision model is explained.

II. Belief function notions and notation

Let us explain the basic notion of belief functions and the corresponding notation. We will do it with an example inspired by the current pandemic situation. Trying to specify a model, one can hardly gain reliable estimates of the probabilities about the future development. What is the probability that a required amount of doses of a given vaccine will be available within the next ten days? What is the probability that a specific mutation of a virus will be detected in a given region? What is the probability that a given vaccine will be authorized by EMA by the end of this month? These and many similar questions should be answered to set up a probabilistic model.

Using belief function models, we can do with rough estimates of the limits of considered probabilities. To simplify the situation, assume that the future development will follow one of the mutually exclusive scenarios A, B, C, and denote by D that none of A, B, C realizes. Thus, we are sure (with probability 1) that one and only one of the four considered scenarios comes true. Let scenarios A, B, and C be such that one of them realizes with high probability (90 %). Further, assume that B materializes with at least 40 % probability. Assume, it is difficult to say whether A is more probable than C, or vice versa, but we guess that one of them comes true with the probability not lower than 20 %. All this knowledge should be encoded using the tools of the belief function apparatus. In probability theory, we would have to define a probability measure for which the specified knowledge is not satisfactory. Let us do it in belief function theory. For this, let us start with the notion, which is perhaps the simplest one for the reader familiar with probability theory. It is the notion of a *basic probability assignment*.

Formally, basic probability assignment (BPA) is a mapping[1] $m : 2^{\Omega} \rightarrow \{0, 1\}$, for which $m(\emptyset) = 0$, and $\sum_{a \subseteq \Omega} m(a) = 1$. Ω (often called a *frame of discernment*) represents the possible situations. In our example $\Omega = \{A, B, C, D\}$. Values of m encode what we are sure about. We are sure, that probability of B is (at least) 40 %, so we assign $m(\{B\}) = 0.4$. Analogously, assigning $m(\{A, C\}) = 0.2$, we encode the knowledge that one from the couple A, C comes true with the probability of at least 20 %. Not knowing which of them, we place the probability on the corresponding pair of scenarios. The last piece of knowledge to be encoded is that one of the scenarios A, B, C realizes with probability 90 %. Realize that we have already encoded that B (belonging to the considered triplet) occurs with probability 0.4, and that the couple A, C (also belonging to the considered triplet) occurs with probability 0.2. Thus, from this, we already assured that the probability of a triplet A, B, C is at least $40 + 20 = 60$ %. To guarantee that a scenario from A, B, C realizes with the probability at least 0.9, we have to assign $m(\{A, B, C\}) = 0.9 - 0.4 - 0.2 = 0.3$. In this way, we exploited all the given knowledge. To assure that we do not add any misleading knowledge, and to meet the requirement that values of the assignment m sum up to one, we eventually assign $m(\{\Omega\}) = 0.1$. Thus, we have

$m(\{B\}) = 0.4$	B will be available with 40 %,
$m(\{A, C\}) = 0.2$	A or C with probability 20 %,
$m(\{A, B, C\}) = 0.3$	with 90 % it will not be D,
$m(\{\Omega\}) = 0.1$	for sure, one of them occurs.

Repeat that the specified values of BPA m sum up to one for the given subsets of Ω, which means that for all other subsets (not specified above), their basic assignment is zero.

Apart from BPA, the same information can be expressed by the corresponding *belief* or *plausibility* functions (both of which are also defined on the power set 2^{Ω}):

$$Bel_m(\mathbf{a}) = \sum_{\mathbf{b} \subseteq \Omega:\, \mathbf{b} \subseteq \mathbf{a}} m(\mathbf{b}), \tag{1}$$

$$Pl_m(\mathbf{a}) = \sum_{\mathbf{b} \subseteq \Omega:\, \mathbf{b} \cap \mathbf{a} \neq \emptyset} m(\mathbf{b}), \tag{2}$$

Let us point out that, whenever one of these functions is given, it is always possible to reconstruct the corresponding BPA m. For example, denoting $\neg\mathbf{a} = \Omega \setminus \mathbf{a}$:

$$Pl_m(\mathbf{a}) = 1 - Bel_m(\neg\mathbf{a}),$$

$$m(\mathbf{a}) = \sum_{\mathbf{b} \subseteq \mathbf{a}} (-1)^{|\mathbf{a} \setminus \mathbf{b}|} Bel_m(\mathbf{b}).$$

In this paper, we do not need too many notions from belief function theory, but it may help the reader better understand the model introduced in this paper when realizing that each BPA m specifies a set of probability measures

[1] Note that 2^{Ω} denotes the powerset of Ω, i.e. the set of all subsets of Ω.

defined on Ω. Denote the set of all probability measures on Ω by $\mathcal{P}_\Omega$. Then

$$\mathcal{P}_m = \left\{ P \in \mathcal{P}_\Omega : \sum_{x \in \mathbf{a}} P(x) \geq Bel_m(\mathbf{a}) \ \forall \mathbf{a} \subseteq \Omega \right\}, \quad (3)$$

is called a *credal set* of BPA m. It is not difficult to show that for any probability measure $P \in \mathcal{P}_m$

$$Bel_m(\mathbf{a}) \leq P(\mathbf{a}) \leq Pl_m(\mathbf{a}),$$

for any event $\mathbf{a} \subseteq \Omega$, which explains a possible interpretation of belief functions [14]. For event $\mathbf{a}$, we are sure that it will come true with the probability at least $Bel_m(\mathbf{a})$, and on contrary, that its negation $\neg\mathbf{a}$ will come true with the probability at least $Bel_m(\neg\mathbf{a}) = (1 - Pl_m(\mathbf{a}))$. This is why some authors interpret $Bel_m(\mathbf{a})$ as a *lower probability* and $Pl_m(\mathbf{a})$ as an *upper probability* of $\mathbf{a}$. From this, the reader can see that a basic assignment is a more general notion than a probability distribution as it encodes a whole class of probability distributions.

The credal set interpretation of a belief function may also better clarify why we assigned $m(\{A, B, C\}) = 0.3$ in our example. It guarantees that $Bel_m(\{A, B, C\}) = 0.9$, and $Pl_m(\{A, B, C\}) = 1$. Simultaneously, $Pl_m(\Omega \setminus \{A, B, C\}) = Pl_m(\{D\}) = 0.1$ – see Table I.

Realize that any probability distribution can also be specified by a BPA, the credal set of which contains only one probability distribution. It occurs if and only if the corresponding BPA assigns positive values only to singletons (one-element sets). Such a BPA is called *Bayesian*.

For various reasons, one may want to replace a BPA just by one probability distribution – its probabilistic representative. There are many such transformations proposed in the literature [15]–[17] nevertheless, for the sake of simplicity, we will do with that, which was strongly advocated as a basis for decision-making by Philippe Smets [18], [19]. For BPA m, the probability distribution[2]

$$\pi_m(\mathbf{a}) = \sum_{a \in \mathbf{a}} \sum_{\mathbf{b} \subseteq \Omega : a \in \mathbf{b}} \frac{m(\mathbf{b})}{|\mathbf{b}|}, \quad (4)$$

is called a *pignistic transform* of m.

III. Coefficient of Ambiguity Aversion

A situation in which the probability is unknown, or when it is not well specified, is called *ambiguous* in this paper. Let x be an element of Ω. In our example, it means that x is one of the considered scenarios. If for all $P \in \mathcal{P}_m$, $P(x) = p$, then we are sure that the probability of x equals p, and there is no ambiguity about x. The greater the difference

$$\max_{P \in \mathcal{P}_m} \{P(x)\} - \min_{P \in \mathcal{P}_m} \{P(x)\} = Pl_m(\{x\}) - Bel_m(\{x\}),$$

the more ambiguous is x. This difference is used to measure the amount of ambiguity connected with the state x by Srivastava [20]. As we will see later, we use a slightly different measure of ambiguity, but the difference is almost negligible.

[2]Recall that $|\mathbf{b}|$ denote the cardinality of set $\mathbf{b}$.

TABLE I
Uncertainty of the Scenarios

Scenarios	Set functions				
	π_m	Bel_m	Pl_m	$r_{m,0.2}$	$r_{m,0.4}$
A	0.225	0	0.6	0.18	0.135
B	0.525	0.4	0.8	0.5	0.475
C	0.225	0	0.6	0.18	0.135
D	0.025	0	0.1	0.02	0.015
A, B	0.75	0.4	1	0.68	0.61
A, C	0.45	0.2	0.6	0.4	0.35
A, D	0.25	0	0.6	0.2	0.015
B, C	0.75	0.4	1	0.68	0.61
B, D	0.55	0.4	0.8	0.52	0.49
C, D	0.25	0	0.6	0.2	0.15
A, B, C	0.975	0.9	1	0.96	0.945
A, B, D	0.775	0.4	1	0.7	0.625
A, C, D	0.475	0.2	0.6	0.42	0.365
B, C, D	0.775	0.4	1	0.7	0.625
A, B, C, D	1	1	1	1	1

In his seminal paper [5], Ellsberg showed that most of human decision-makers try to avoid situations burdened with ambiguity. In our experiments [13], the participants had to decide how much was their maximum bet they were willing to pay to take part in two lotteries. In both the considered lotteries, balls of six colors were placed in the drawing urn. It appeared that, on average, the experimental persons were willing to pay by about 28 % more to take part in the lottery, in which they knew that all colors were equally distributed in the urn, in comparison with the lottery when the distribution of colors in the drawing urn was unknown. This observation suggests the basic principle of the proposed model: *The lack of knowledge may psychologically decrease the subjective chance of success. This attitude is subjective and differs from person to person, depending on the intensity of ambiguity aversion of experimental persons.*

Let us get back to the example considered in this paper. As said above, Smets [19] proposed to base the decision on the pignistic transform, the values of which are computed using Formula (4) (see the second column of Table I). We propose to first reduce the values of the pignistic transform due to the above expressed basic principle. The magnitude of the reduction should be affected by two parameters: the strength of subjective aversion to ambiguity and the amount of ambiguity connected with the specific state.

The strength of subjective ambiguity aversion is expressed by the coefficient of ambiguity aversion α. $\alpha = 0$ corresponds to decision-makers, who are neutral to ambiguity. The higher $\alpha \in (0, 1]$, the greater ambiguity aversion is manifested by the resulting process. As it is known from literature [21], [22], though rarely, there are situations in which some people manifest a positive attitude to ambiguity; they are seeking ambiguity. Thus, like there are people with a positive attitude to risk, so-called *risk-*

takers, there are also *ambiguity-seekers*. In the described model, this fact is reflected by the possibility of choosing α negative. In our experiments [13] we came about (very rare) situations when $\alpha < -2$.

The amount of ambiguity connected with state $x \in \Omega$ is encoded in the respective BPA. As already said above, Srivastava [20] measures this strength by the difference $Pl_m(\{x\}) - Bel_m(\{x\})$. We use for this purpose the difference $\pi_m(x) - Bel_m(\{x\})$ [23]. Namely, as described in the next section, we propose to base the decision-making on computing the *reduced expected reward*, the computation of which is based on the set function

$$r_{m,\alpha}(\mathbf{a}) = (1-\alpha)\pi_m(\mathbf{a}) + \alpha Bel_m(\mathbf{a}), \tag{5}$$

called *reduced capacity function*. Notice that, for positive α, $r_{m,\alpha}(\mathbf{a}) = \pi(\mathbf{a})$ if and only if $Bel_m(\mathbf{a}) = \pi(\mathbf{a})$, which means, in our model, that there is no ambiguity connected with the state $\mathbf{a}$. The greater the difference $\pi(\mathbf{a}) - Bel_m(\mathbf{a})$, the more ambiguous is the information about the state $\mathbf{a}$, and the greater reduction is realized when switching from π to $r_{m,\alpha}$.

Functions π to $r_{m,\alpha}$, corresponding to BPA m from our example for two different α, are in the last two columns of Table I.

IV. Decision-Making with the Reduced Capacity

The reduced capacity function is a convex combination of a probability transform of the considered BPA and the corresponding belief function. In the previous section, we considered only a pignistic transform of BPA. Let us say that similar results can also be achieved using other probability transforms of BPAs. The interested reader is referred to [13], [17] where the decision models based on five other probabilistic transforms were discussed. The presented results showed that there are no strong reasons to prefer one of the considered transforms to others. Another conclusion says that in practical situations corresponding to psychological tests described in the literature, all the decision models evince the same results regardless of the used probability transform.

For positive α, $r_{m,\alpha} \le \pi_m$. It corresponds to what was said above: the ambiguity may subjectively decrease the chances of success. Notice also that the reduction does not depend only on coefficient α, but also on the difference between π_m and Bel_m. In comparison with a probability measure, set function $r_{m,\alpha}$ is neither normalized nor additive. It is only superadditive in the sense that for $\alpha \in [0,1]$, and disjoint subsets $\mathbf{a}$ and $\mathbf{b}$ of Ω

$$r_{m,\alpha}(\mathbf{a} \cup \mathbf{b}) \ge r_{m,\alpha}(\mathbf{a}) + r_{m,\alpha}(\mathbf{b}).$$

Such functions are called capacities in mathematics, and a lot of literature was written on their application to decision. Let us only note that the seminal paper by D. Choquet [24], in which the so-called *Choquet integral* was introduced, has more than five thousand citations.

TABLE II
Decision Table

Expected Gain	Scenarios			
	A	***B***	***C***	***D***
Action I	110	30	5	0
Action II	100	10	70	10
Action III	21	40	60	21
Action IV	32	32	32	160

Considering a nonnegative real function $g : \Omega \longrightarrow \mathbb{R}^+$, denote $g(\Omega) = \{g(x) : x \in \Omega\}$ the set of values of function g. For a probability measure π on Ω, the well-known expected value of g with respect to probability measure π is defined by the sum

$$\sum_{x\in\Omega} g(x)\pi(x) = \sum_{c\in g(\Omega)} c\ \pi(\{x : g(x) = c\}),$$

where the equality holds due to the additivity of a probability measure π.

As an analogue of the expected value of a nonnegative function g for nonadditive measures, Choquet proposed to use the value computed in the following way. Order set $g(\Omega) = \{c_1, c_2, \ldots, c_k\}$ in the way that $0 \le c_1 < c_2 < \ldots < c_k$. Set $c_0 = 0$. Then, the Choquet integral of g with respect to capacity function r is defined

$$\mathbb{C}_r(g) = \sum_{i=1}^{k} (c_i - c_{i-1})\ r(\{x : g(x) \ge c_i\}). \tag{6}$$

Thus in this paper, we accept the decision for which the value defined by Formula (6) achieves its extreme. If g expresses gains, then we seek the maximum of its value. If g measures losses, then, naturally, we look for its minimum.

Returning to the considered pandemic example, we assume that the enterprise management has to make a decision on which of the four possible actions should be realized.

The revenue of the individual actions depends on which of the four considered scenarios comes true. Let the relationship between the actions and scenarios be as expressed in a decision table (see Table II), where the values correspond to the expected gain (for the sake of simplicity expressed using monetary values). For example, under scenario A, Action I produces the gain of \$ 110 thousand, whereas Action III results with only \$ 21 thousand. Thus, if we knew that none of the scenarios A, B, C comes true, we should undertake Action IV because it brings the highest revenue of \$ 160 thousand. Not knowing anything more than what is in Tables I and II, the managers should compute the Choquet integral for all four actions.

For Action I, four values of gain functions must be considered $g(\Omega) = \{0, 5, 30, 110\}$. Using Formula (6), we get for $r_{m,0.2}$

$$\begin{aligned} 0 \cdot r_{m,0.2}(\{A,B,C,D\}) + 5 \cdot r_{m,0.2}(\{A,B,C\}) \\ +25 \cdot r_{m,0.2}(\{A,B\}) + 80 \cdot r_{m,0.2}(\{A\}) \\ = 0 \cdot 1 + 5 \cdot 0.96 + 25 \cdot 0.68 + 80 \cdot 0.18 = 36.2. \end{aligned}$$

TABLE III
VALUES OF CHOQUET INTEGRALS

Choquette Integral	Coefficient of Ambiguity			
	0.2	**0.4**	**0.7**	**-1.0**
Action I	36.2	30.78	22.64	**68.75**
Action II	**39.4**	35.05	28.53	65.5
Action III	37.52	**35.29**	31.94	50.9
Action IV	34.56	33.92	**32.96**	38.4

Analogously, for Action II we consider $g(\Omega) = \{10, 70, 100\}$, and the corresponding Choquet integral equals

$$10 \cdot r_{m,0.2}(\{A,B,C,D\}) + 60 \cdot r_{m,0.2}(\{A,C\}) + 30 \cdot r_{m,0.2}(\{A\}) = 10 \cdot 1 + 60 \cdot 0.4 + 30 \cdot 0.18 = 39.4.$$

Values of Choquet integrals for gain functions corresponding to Action I – Action IV, for four different reduced gain functions (we consider four different coefficients of ambiguity α) are in Table III. From this, one can see that Action I is recommendable for a decision-maker seeking ambiguity. The other actions may be recommended to decision-makers with different strengths of ambiguity aversion.

V. CONCLUSIONS

The goal of the paper is twofold. It offers the managers an alternative decision-supporting tool adapting the optimality criterion to conform with their subjective feelings. It should be stressed that we do not suggest a new approach guaranteeing to achieve an optimal solution. On the contrary, we start with the assumption that the notion of optimality is subjective; it is based on probabilities (usually subjective) and other subjective parameters like the introduced coefficient of ambiguity.

The other message conveyed by the paper claims that the decision achieved with the help of Gilboa and Schmeidler's approach (described in [8]) is not the only one that may be considered optimal because, as said above, the optimality is subjective. It depends on the attitude of decision-makers to risk, ambiguity, and perhaps some other personal characteristics.

To show that the behavior of the presented model is similar to that of human decision-makers, we tested the behavior of colleagues and students in simple decision situations. Almost two hundred persons took part in experiments, in which the participants had a chance to win money in lotteries if they were willing to pay some amount of money as a participation fee. In each lottery, the participants should determine the maximum amount of money they were willing to pay to be allowed to participate. In each lottery, they received different (incomplete) information about the content of the urn. For example, there was a situation when the participants learned that there were eight balls in the drawing urn, and one and only one of them was red. The participants knew nothing more than that balls of six colors were used in the experiments. So, in this situation, the urn could contain balls of only two colors (one red and seven, say, blue), as well as balls of all six colors. Among the lotteries, there were also situations corresponding to Ellsberg's experiments [5]. In each lottery, they could win CZK 100 if they correctly predicted the color of a ball drawn from a drawing urn.

In the experiments mentioned above, we tested, among others, the hypothesis that the coefficient of ambiguity for a person does not vary too much in time. We can hardly make definite conclusions from our experiments, but only less than one-half of participants manifested such a stable behavior. Thus, based on the results described in [13], we tend to conclude that the strength of ambiguity aversion (reflected in a personal coefficient of ambiguity α) varies in dependence on other volatile factors like the type of a decision task and the current mood of a decision-maker. Nevertheless, regarding the described model and the considered coefficient of ambiguity α, we can say that in practice, all values from the interval $[-3, 1]$ appear possible. In agreement with other authors [21], the positive attitude to ambiguity (i.e., negative α) is rare, as well as the strong ambiguity aversion modeled by $\alpha = 1$. On average, the subjective coefficient of ambiguity fluctuated around $\alpha = 0.28$ (for details, the reader is referred to [13]).

Regarding what was said above, the orientation of future research suggests itself. The studied models should cover another parameter(s), which opens a way to study the mutual dependence of two (or even several) subjective characteristics influencing the behavior of human decision-makers. The long-term goal is to design models simulating the behavior of human decision-makers. Such human-like decision models may be applicable not only in business decision-making (micro-economy) but perhaps mainly in other fields like in predicting consumer (or competitor) decision-making. Simply, it may apply to problems where the subjective attitude of humans may play its role.

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