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Synchronization of a nonlinear multi-agent system with application to a network composed of Hindmarsh-Rose neurons

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Abstract— **This paper discusses synchronization of a network composed of agents that are nonlinear and minimum phase. The main tool is feedback linearization. The results are applied to the problem of synchronization of a network composed of Hindmarsh-Rose neurons. Using the developed scheme, it is demonstrated that synchronization is achieved by considering the membrane potential only, the fact that the neuron is a minimum-phase nonlinear system implies that the other states are synchronized as well. The results are illustrated by simulations.**

I. Introduction

A. Synchronization in the network

The network is a representation of the physical system or structure [5]. Complex networks (CN) have been intensively studied during the last several decades. The complex network is usually represented by the graph consisting of the nodes (vertices) interconnected by the links (edges) in a certain topology (structure). Most of the real-world and man-made systems can be described by CN, e.g. communication networks, social networks, metabolic networks, neural networks, collaboration networks, economic networks, food webs, electric power grids, etc. Commonly, the research of the complex networks was provided via classical graph theory and random graph theory introduced by [11], but these basics were extended during the last two decades. Additionally, there has been an increasing interest in the synchronization between nodes of the complex networks. Synchronization is a significant phenomenon in nature and it is generally understood as a collective state of coupled systems [4]. Indeed, it is one of the simplest types of the collective dynamics of the interconnected systems. Studies of the identical (complete) synchronization of the coupled systems were started by the papers [13], [29] and the analysis of the synchronization phenomena between interconnected chaotic systems was started thanks to the paper [28]. In case of the identical synchronization (IS), the states of the interconnected systems to be synchronized should mutually converge each to other. Besides the IS approach, many other kinds of synchronizations of the coupled chaotic systems have been introduced recently [48], [19], [20], [21], [30]. The quality and the speed of the synchronization of the coupled systems depend not only on the structure of the network but also on the presence of different types of disturbances in the communication channels, e.g. the limited-band data erase [1], time delays [37], [35], quantization [34] etc. Presence of heterogeneous (not equal throughout the network) time delays has impact on the ability to reach full synchronization as shown in [38], [37], [39], [43]. The general knowledge about different types of synchronization between coupled chaotic systems and complex networks can be found in review [3] and the books [6], [4].

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B. Synchronization of neurons

Neurons exhibit a wide range of complex behavior, this even in dependence of external input - the external current of ions. The neuron can rest in a quiescent state if the influx current is small, then it starts to exhibit periodic spikes. With increasing of this current, number of spikes during one period can increase. With further increase of this current, chaotic behavior is observed. For further details see e.g. [24], [26]. A full understanding of these phenomena is still not achieved, however, it seems to be of crucial importance for recognizing causes and proposing of treatment for various health issues like epileptic attacks, Parkinson disease etc. as well as for obtaining a thorough insight of neural system function.

To understand function of a neuron, several models were introduced. As the first one we mention the Hodgkin-Huxley model [16]. This describes the function of a neuron quite accurately but is prohibitively complicated so that its use is limited. On the other hand, the Fitzhugh-Nagumo (FN) [12], [25] neuronal model is quite simple at the cost of inaccurately illustrating some important phenomena like bursting. A compromise is the Hindmarsh-Rose (HR) neuron [15] with ability of fairly precise description of the neuron's functions without the need for extremely complicated computations when modeling a network composed of such neurons. For a thorough description of bifurcations and oscillatory and chaotic phenomena occurring in the HR neuron, see [26], [24] among others.

Synchronization of a neuronal network is a principal property for its function. Hence it is thoroughly studied. Synchronization of a neuronal network with a linear feedback is presented in [8], master-slave synchronization of two HR neurons is described in [27]. On the other hand, nonlinear coupling functions for chaotic synchonization of HR neuronal models are used in e.g. [7], [14], [10], [50], [49] investigates enhancement of synchronization by using a memristor. Synchronization of a network composed of HR neurons is studied in [2] and coupling by a magnetic flux is investigated in [22]. Adaptive synchronization allowing to adjust values of some parameters of the model is presented e.g. in [45] and of general chaotic systems in [23]. It is noteworthy that interconnection topology of the neural network has influence on the firing pattern of the neurons as pointed out e.g. in [51].

For synchronization of neurons, it is necessary to apply the methods for synchronization of nonlinear systems. Methods based on the exact feedback linearization include those presented in [41], [36], [40]. It is shown there that the complex network can be synchronized even if the nodes have a nontrivial zero dynamics, if the nodes are minimum-phase systems. This follows from the ideas stemming from the output regulation theory [31], [42] or [44]. Sometimes the neurons are assumed to exhibit time delay. This results in the fact that they are described as nonlinear time delay systems, often with polynomial nonlinearity. To deal with such systems, the sum-of-squares method can be practical [32], [33].

The inverse process to synchronization of a neural network is its desynchronization. Here, synchronization of the originally synchronized network is terminated by acting of an external force or changing some parameters. This is also a very important problem as pathologically strong synchronization is connected with some neurological diseases. Let us name few results: desynchronization of stochastic HR neuron by an external perturbation is studied in [9], [17].

All the above papers were devoted to synchronization of a network composed of identical neurons. Contrary to this, [30] deals with synchronization of a heterogeneous networks of FN neurons.

C. Purpose of this paper

Synchronization of a network composed of HR neurons using the exact feedback linearization is presented in this paper. This theory was applied to general complex networks in [41] and further extended to systems with delays in [36], [40]. As noted in these papers, the dealing with the multi-agent systems composed of nonlinear subsytems is somewhat similar to the output regulation problem of nonlinear systems [42], [31]. The method yields results better than merely approximating the nonlinearities by a linear function (modeling them as uncertainties of a linear system) as it allows to match them quite precisely. This is reflected in a larger domain of states where the algorithm yields good results. It can be seen that the HR neuron satisfies the minimum-phase requirement so the aforementioned method presented in [41] is suitable for application to the problem of synchronization of a network composed of HR neurons. Hence this paper was written to demonstrate capabilities of this method to synchronization of a practical system with a complex behavior. On the other hand, behavior of complex networks composed of neurons is still not a fully understood issue, hence it is a contribution to this area.

II. Graph theory

The interconnection of the neurons is described by a graph in the following way: assume the neurons are denoted by numbers $1,\dots,N$. Define $\mathscr{V} = \{1,\dots,N\}$ (this set is called the set of nodes) and the set of edges $\mathscr{E} \subset \mathscr{V} \times \mathscr{V}$ as $(i,j) \in \mathscr{E})$ if there exists a connection from node i to node j. Translated onto the neuronal network terminology, this means that neuron number i sends signals to neuron j. It is assumed $(i,i) \notin \mathscr{E}$ for any $i \in \mathscr{V}$ - no neuron sends signals to itself. The graph $\mathscr{G}$ is defined as a pair $\mathscr{G} = (\mathscr{V},\mathscr{E})$. For graph $\mathscr{G}$ we define the Laplacian matrix $\mathscr{L} \in \mathbb{R}^{N\times N}$ as follows: for $i,j \in \{1,\dots,N\}$, $i \neq j$ one defines $\mathscr{L}_{i,j} = -1$ if $(j,i) \in \mathscr{E}$, otherwise $\mathscr{L}_{i,j} = 0$. Moreover, $\mathscr{L}_{i,i} = -\sum_{j=1,i\neq j}^{N} \mathscr{L}_{i,j}$.

It is assumed there exists one neuron i_0 such that for any $j \in \mathscr{V}$, $i_0 \neq j$ there exists a path in $\mathscr{G}$ from i_0 to j but there is no path from j to i_0. Such a node is called the leader. In our framework, it is a neuron whose behavior should all other neurons mimic.

Define also matrix $\bar{L} \in R^{(N-1)\times(N-1)}$ by removing the i_0th row and column from the matrix $\mathscr{L}$. Paper [46], [47] prove that there matrix $\bar{L}$ has eigenvalues with positive real part and there exists a diagonal matrix $D = \mathrm{diag}(d_1,\dots,d_N)$ with $d_i > 0$ so that $D\bar{L} + \bar{L}^T D > 0$. For the space reasons, we will mostly assume that matrix L has simple real eigenvalues. The proof of the case when eigenvalues are real but with multiplicity greater of 1 is technically more demanding. Here, procedure from [52] can be used. The results obtained by this procedure are the same as mentioned here. The case of complex conjugated eigenvalues is mentioned later in the text.

III. Exact feedback linearization

A very useful tool for control of nonlinear systems is the so-called exact feedback linearization. It has became a classical tool for nonlinear control design so far and is thoroughly described in [18]. Consider the system

$$\dot{x} = f(x) + g(x)u,\ x(0) = x_0,\ y = h(x) \tag{1}$$

where $f : \mathbb{R}^n \to \mathbb{R}^n$, $g : \mathbb{R}^n \to \mathbb{R}$ are sufficiently smooth (this requirement will be precised later) functions satisfying $f(0) = 0$, $h(0) = 0$.

Lie derivative of function h along the vector field f is defined as

$$L_f(h)(x) = (\frac{\partial h}{\partial x_1}(x),\dots,\frac{\partial h}{\partial x_n}(x)).f(x). \tag{2}$$

Moreover, the kth-order Lie derivative is defined as follows: $L_f^1 h = L_f h$, if $k > 1$ and $L_f^{k-1}h$ is defined, then $L_f^k h = L_f(L_f^{k-1}h)$.

We assume system (1) has relative degree r. This means, there exists an integer $r \leq n$ such that

1) $L_g L_f^{r-1} h(x)(0) \neq 0$,
2) for all $j = 1,\dots,r-2$ holds $L_g L_f^j h(x)(0) = 0$.

Define the transformation $\xi = \mathscr{T}(x)$ as follows:

$$\begin{aligned}
\xi_1 =& h(x),\\
\xi_2 =& L_f h(x),\\
&\vdots\\
\xi_r =& L_f^{(r-1)} h(x)
\end{aligned}$$

and for $j=r+1,\dots,n$ we define

$$\xi_j = x_j. \tag{3}$$

Moreover, define also functions F, G by $F(\xi) = L_f^{(r)} h(\mathscr{T}^{-1}(x))$, $G(\xi) = L_g L_f^{(r-1)}(\mathscr{T}^{-1}(x))$. Let also $\eta = (\xi_1,\dots,\xi_r)^T$, $\zeta = (\xi_{r+1}, \dot{\xi}_n)^T$ and $\varphi(\eta,\zeta) = (f_{r+1}(\mathscr{T}^{-1}(x)),\dots,f_n(\mathscr{T}^{-1}(x)))$.

In the transformed coordinates, system (1) attains the form

$$\begin{aligned} y =& \xi_1, \\ \dot{\xi}_1 =& \xi_2, \\ & \vdots \\ \dot{\xi}_{r-1} =& \xi_r, \\ \dot{\xi}_r =& F(\xi)+G(\xi)u, \\ \dot{\zeta} =& \varphi(\eta,\zeta). \end{aligned}$$

Defining

$$v = F(\xi)+G(\xi)u, \tag{4}$$

the transformed system can be written as

$$\dot{\eta} = \begin{pmatrix} 0 & 1 & 0 & \dots & 0 & 0 \\ 0 & 0 & 1 & \dots & 0 & 0 \\ & \vdots & & & & \vdots \\ 0 & 0 & 0 & \dots & 0 & 1 \\ 0 & 0 & 0 & \dots & 0 & 0 \end{pmatrix} \eta + \begin{pmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix} v, \tag{5}$$

$$\dot{\zeta} = \varphi(\eta,\zeta), \tag{6}$$

$$y = \xi_1. \tag{7}$$

The system (5) will be called *the observable part* while system (6) is the *unobservable part*.

Note that even a more general form with function φ depending on the control input u can be defined but for our purpose, this formulation is sufficient.

IV. Neuronal model

As explained in [27], the HR neuron is described by the following nonlinear system:

$$\dot{x}_1 = ax_1^2 - x_1^3 + x_2 - x_3 + I, \tag{8}$$

$$\dot{x}_2 = 1 + bx_1^2 - x_2, \tag{9}$$

$$\dot{x}_3 = c(x_1+1.56) - 0.006x_3. \tag{10}$$

The constants attain the following values:

a	3
b	-5
c	-0.024
I	1.25

TABLE I

Values of parameters of the HR neuronal model

The meaning of the variables is as follows: x_1 is the membrane potential, x_2 stands for the recovery variable associated with the fast current of the Na^+ and/or K^+ ions, x_3 is the adaptation variable associated with the slow current of Ca^+ ions, finally I is the external current.

It is noteworthy that behavior of the HR neuron varies significantly in dependence on the external current. For example, for $I > 3.25$, one can observe a periodic spiking. Chaotic behavior is exhibited for $I \in (2.75, 3.25)$. For $I < 2.75$, periodic bursting is observed, however, for $I < 1.14$, the quiescent state appears.

As the coupling is done through the variable x_1, we set $y = x_1$. Then proceeding as in the previous section, we define $\xi_i = x_i$, $F(\xi_1,\xi_2,\xi_3) = -a\xi_1^2 - \xi_1^3 + \xi_2 - \xi_3$. Moreover, $\eta = \xi_1$, $\zeta = (\xi_2,\xi_3)^T$. The zero dynamics reads

$$\widetilde{\varphi}(\zeta) = \begin{pmatrix} -\zeta_1 \\ -0.006\zeta_2 \end{pmatrix}. \tag{11}$$

As one can see from (11), the system has exponentially stable zero dynamics (it is a *exponentially minimum-phase system*; for further details, see [18]). Hence the synchronization algorithm described in [41] can be used.

The problem solved here is more general than synchronization of a pair of neurons as mentioned in [27] as not only a pair of neurons but a connected neuronal network is synchronized.

The ith neuron with coupling is described as

$$\dot{x}_{1,i} = ax_{1,i}^2 - x_{1,i}^3 + x_{2,i} - x_{3,i} + I + u_i, \tag{12}$$

$$\dot{x}_{2,i} = 1 + bx_{1,i}^2 - x_{2,i}, \tag{13}$$

$$\dot{x}_{3,i} = c(x_{1,i}+1.56) - 0.006x_{3,i} \tag{14}$$

where u_i is the input signal guaranteeing synchronization. It is assumed to be in form

$$u_i = -F(x_i) + k\sum_{j\in N_i}(x_{1,j}-x_{1,i}). \tag{15}$$

with coupling gain k. Here, the coupling gain k which must be designed comes into play. It describes the strength of coupling between neuronal pairs. It is assumed to be constant throughout the network. If we denote $\eta_i = \xi_{1,i}$ then control of the ith neuron after the transformation (4) turns into

$$v_i = k\sum_{j\in N_i}(\eta_j - \eta_i).$$

Then, the observable part of the ith neuron in the neuronal model after feedback linearization reads

$$\dot{\eta}_i = k\sum_{i\in N_i}(\eta_j - \eta_i). \tag{16}$$

To write it compactly, for the observable parts holds (with $\eta = (\eta_1,\dots,\eta_N)^T$):

$$\dot{\eta} = kL\eta. \tag{17}$$

V. Synchronization of a network composed of HR neurons

Obviously, the problem of synchronization of a network composed of neurons (12-14) boils down to the problem of synchronization of a network of a multi-agent systems where the agents admit exact feedback linearization in form (5,6).

This problem has already been solved, method developed in [41] will be used here.

Lemma V.1. *The system* (17) *is stabilized for* $k < 0$.

As mentioned above, matrix L has eigenvalues with positive real part. Let matrices D, T (T nonsingular) satisfy $L = T^{-1}DT$ and D is the real Jordan canonical form of L. Then, with $\eta' = T\eta$, system (17) is converted into $\dot{\eta}' = kD\eta'$, hence the result.

Denote $\delta = \frac{1}{2}(d_N - d_1)$, $\bar{d} = \frac{1}{2}(d_1 + d_N)$.

The unobservable part is not controlled. However, as shown in [41], since it is asymptotically stable, its synchronization is achieved. In the aforementioned paper, the proof of convergence is formulated for nonlinear functions φ. However, as in the case of the HR neuron, this function is linear, the result holds globally and is simplified. The direct counterpart of Theorem 5.4 in [41] is

Lemma V.2. *Under assumption of Lemma V.1, the unbservable part of the neurons described by (12-14) is synchronized.*

For the sake of completeness, a sketch of the proof is presented here. First, let $\zeta_i = (\xi_{2,i}, \xi_{3,i})^T$. From (11) follows that

$$\dot{\zeta}_i - \dot{\zeta}_{i_0} = \begin{pmatrix} -1 & 0 \\ 0 & -0.006 \end{pmatrix} (\zeta_i - \zeta_{i_0}) + \begin{pmatrix} -(\eta_i - \eta_{i_0}) \\ c(\eta_i - \eta_{i_0}) \end{pmatrix}. \quad (18)$$

From Lemma V.1 follows that $\lim_{t\to\infty} \|\eta_i - \eta_{i_0}\| = 0$. This and exponential stability of the zero dynamics imply also $\lim_{t\to\infty} \|\zeta_i - \zeta_{i_0}\| = 0$.

Thus, we have

Theorem V.3. *Under assumptions of the Lemma V.1 and V.2, if the control input is defined as*

$$u_i = -F(x_i) + k \sum_{j\in N_i} (x_{1,j} - x_{1,i}).$$

then $\lim_{t\to\infty} \|x_i - x_{i_0}\| = 0$.

This is a direct consequence of the previous lemmata and the fact that the exact feedback linearization together with the control input transformation (4) is a diffeomorphism.

VI. Simulations

A network of 6 neurons was used for simulations with neuron 1 being the leader. All parameters were as in Table I. The interconnection of the neurons is depicted in Fig. 1.

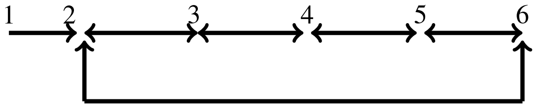

Fig. 1. Connection of neurons.

The corresponding matrix L has eigenvalues 4.1149, 3.6180, 0.1392, 1.3820, 1.7459. Hence $\bar{d} = 2.2$ and $\delta = 2$.

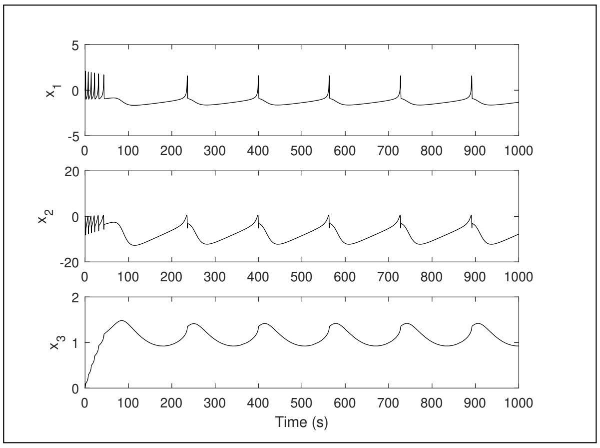

Fig. 2. Master neuron

Use $k = 4.8932$ as this coupling gain guarantees stability of (17). The state trajectories of the master neuron on a large time scale are shown in Fig. 2.

Comparison of the corresponding variables is shown in Figs 3 - 5. The solid line shows the corresponding state of the neuron 4 while the dotted line depicts the leader's state. One can see that the spikes are not precisely tracked, however, the times of the spikes and their overall shape is well copied.

Norm of errors $e_1 = \left(\sum_{i=2}^{6} \|x_{1,i} - x_{1,1}\|^2\right)^{\frac{1}{2}}$, $e_2 = \left(\sum_{i=2}^{6} \|x_{2,i} - x_{2,1}\|^2\right)^{\frac{1}{2}}$ and $e_3 = \left(\sum_{i=2}^{6} \|x_{3,i} - x_{3,1}\|^2\right)^{\frac{1}{2}}$ is shown in Fig. 6. One can see that, apart from the points where the spike occurs, the state is well synchronized.

VII. Conclusions

Synchronization of a neuronal network composed of Hindmarsh-Rose neurons was considered in this paper. To synchronize the neurons, the method based on the feedback linearization was used. It is shown that all three components of the neuron model are synchronized.

In future, robust synchronization of the neuronal network as well as the case of delayed coupling will be investigated.

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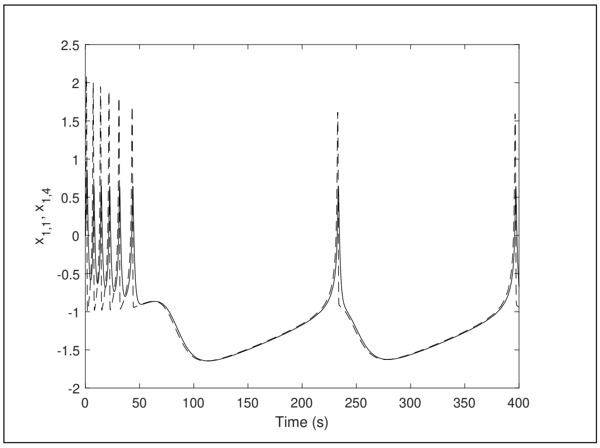

Fig. 3. Comparison of $x_{1,1}$ and $x_{1,4}$

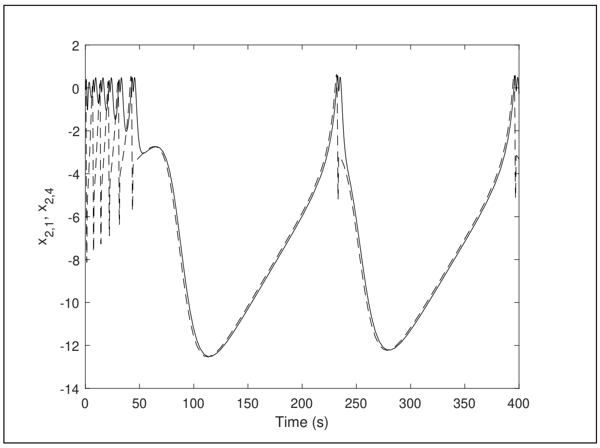

Fig. 4. Comparison of $x_{2,1}$ and $x_{2,4}$

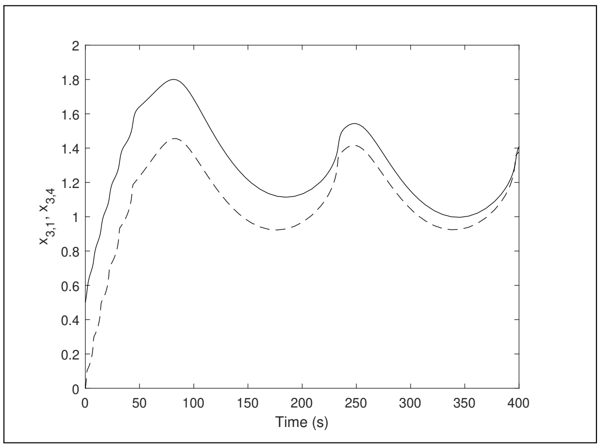

Fig. 5. Comparison of $x_{3,1}$ and $x_{3,4}$

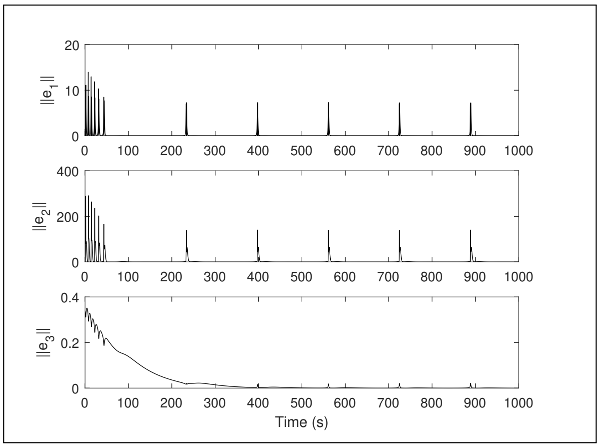

Fig. 6. Norm of errors

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