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Synchronization of a network composed of Hindmarsh-Rose neurons with stochastic disturbances ⋆

Branislav Rehák * **Volodymyr Lynnyk** **

* The Czech Academy of Sciences, Institute of Information Theory and Automation, Pod vodárenskou věží 4, 182 08 Praha 8, Czech Republic (e-mail: rehakb@utia.cas.cz.)

** The Czech Academy of Sciences, Institute of Information Theory and Automation, Pod vodárenskou věží 4, 182 08 Praha 8, Czech Republic (e-mail: voldemar@utia.cas.cz.)

Abstract: In the paper, an algorithm for synchronization of a neural network composed of interconnected Hindmarsh-Rose neurons is proposed. The interconnections of the neurons are subject to delays and an additive noise is present. The convex optimization is the tool for finding the synchronization control, the result is formulated using linear matrix inequalities. The synchronization of the recovery and adaptation variables is also guaranteed thanks to the minimum-phase property of the Hindmarsh-Rose neuron. An example illustrates the results.

Keywords: Hindmarsh Rose neuron, Multi-agent system, Stochastic retarded system

1. INTRODUCTION

1.1 Synchronization of complex networks and multi-agent systems

Complex networks or multi-agent systems were intensively investigated recently.

We can distinguish several kinds of synchronization: first, the identical synchronization where the states of all nodes (agents) composing the interconnected systems have to mutually converge each to other. Moreover, a number of other kinds of synchronization were introduced, see e.g. Čelikovský and Lynnyk (2012); Lynnyk and Čelikovský (2010); Lynnyk et al. (2019b,a); Plotnikov and Fradkov (2019b).

Let us note that the structure of the network as well as presence of different types of disturbances in the communication channels,time delays Rehák and Lynnyk (2019); Rehák and Lynnyk (2019), quantization Rehák and Lynnyk (2019) etc., is crucial and has an influence on quality and speed of the synchronization. The information transmission is often carried out through noisy channels: the received information is perturbed by adding a random part. The control design must take this into account so that the control is capable to minimize damage caused by this phenomenon. In Hu et al. (2015) a multi-agent system with noise is investigated, a LMI-based design of a synchronization control is derived. A design of a synchronization control for a stochastic system exhibiting uncertainties and jumping control signals is presented in Ren et al. (2017). Let us also mention Ma et al. (2017) where the interested reader finds information about further works concerning this area.

1.2 Synchronization of neurons

Several types of complex behavior that depends on an external input (external current of ions) can be observed in neurons. This is described e.g. in Malik and Mir (2020); Ngouonkadi et al. (2016): as long as the influx current is small, the neuron rests in a quiescent state. After increasing this current, the neuron exhibits periodic spikes: one spike per period. After increasing the current even more, a number of spikes during one period appears. After another increase, chaotic behavior takes place. A complete understanding of these phenomena has not been achieved yet. It is an intensively studied topic as it is important for recognizing causes of some health problems such as epileptic attacks or Parkinson disease. Also, it is crucial for obtaining an insight into the function of the neural system.

Several neuronal models were developed to help to understand its functions. The Hodgkin-Huxley model (HH) Hodgkin and Huxley (1952) can be mentioned first. This model is accurate as it describes the function of a neuron precisely but it is too complicated. The Fitzhugh-Nagumo (FN) FitzHugh (1961); Nagumo et al. (1962) neuronal model is simple at the cost of inaccurately describing some phenomena such as bursting. A compromise between the requirements for simplicity and accuracy is the Hindmarsh-Rose (HR) neuron Hindmarsh and Rose (1984). A detailed description of oscillatory and chaotic phenomena occurring in this neuron can be found e.g. in Malik and Mir (2020); Ngouonkadi et al. (2016). The stochastic HR neuron has been studied in e.g. Łepek and Fronczak (2018).

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Note also that synchronization problem of FN neurons with delays is investigated in Plotnikov (2015); Plotnikov et al. (2016). This is a problem closely related to the one studied here.

All the above papers describe the synchronization of a network composed of identical neurons. Contrary to this, Plotnikov and Fradkov (2019b) deals with synchronization of a heterogeneous networks of FN neurons while desynchronization of FN neurons is dealt with in Plotnikov and Fradkov (2019a) or Djeundam et al. (2018).

1.3 Purpose of this paper

Synchronization of a network composed of HR neurons subject to random disturbances and delayed communication using the exact feedback linearization is investigated. The aim of this paper is to apply the method from Rehák and Lynnyk (2019); Rehák et al. (2018) to a neuronal network composed of HR neurons. The effects of noise are studied. This leads to a the need of combining the aforementioned method with the stochastic version of the Razumikhin functional.

1.4 Notation

The notation used in this paper is introduced here.

- The Kronecker product is denoted by the symbol $\otimes$.
- The expected value of a random variable φ is denoted by $\mathbb{E}(\varphi)$.
- If A, B are matrices, then $\mathrm{diag}(A, B)$ is a block-diagonal matrix with blocks A, B on the diagonal.
- The symbol $.^T$ denotes the transposed matrix.
- The time argument t is often omitted: $f(t) = f$. However, if dependence on this time argument needs to be emphasized or the time argument is different from t, it is written in full.
- The time delay is written in the subscript: $f(t-\tau(t)) = f(t-\tau) = f_\tau(t) = f_\tau$.

2. GRAPH THEORY

The interconnection of the neurons in the network is described by means of the graph theory: the neurons are denoted by numbers $0, \ldots, N$. Define *the set of nodes* $\mathcal{V} = \{0, \ldots, N\}$ and *the set of edges* $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$ as $(i, j) \in \mathcal{E}$ if and only if there exists a connection from node i to node j: the neuron i can sends signal to neuron j directly. It is supposed that $(i, i) \notin \mathcal{E}$ for any $i \in \mathcal{V}$. Then, the graph $\mathcal{G}$ is defined as a pair $\mathcal{G} = (\mathcal{V}, \mathcal{E})$. The Laplacian matrix $\mathcal{L} \in \mathbb{R}^{(N+1)\times(N+1)}$ is defined for the graph $\mathcal{G}$ as for $i, j \in \{0, \ldots, N\}$, $i \neq j$ one defines $\mathcal{L}_{i,j} = -1$ if $(j, i) \in \mathcal{E}$, otherwise $\mathcal{L}_{i,j} = 0$. The diagonal elements are defined as $\mathcal{L}_{i,i} = -\sum_{j=1, i\neq j}^{N} \mathcal{L}_{i,j}$. Define also sets $N_i \subset \mathcal{V}$, $i = 1, \ldots, N$ as $N_i = \{j \in \mathcal{V} \mid (j, i) \in \mathcal{E}\}$. The set N_i contains agents sending information directly to the agent i.

We assume existence of (exactly) one neuron i_0 such that for any $j \in \mathcal{V}$, $i_0 \neq j$ there exists a directed path in $\mathcal{G}$ from i_0 to j but there is no such path from j to i_0. A node with this property is called the leader. Here, it is a neuron whose behavior all other neurons replicate. Without loss of generality, we suppose $i_0 = 0$. For more details, see Ni and Cheng (2010).

Define matrix $L \in \mathbb{R}^{N\times N}$ by removing the first row and column (they correspond to the leader) from the matrix $\mathcal{L}$. Then one can prove that (Song et al. (2012, 2013)) eigenvalues of L have positive real parts and also there exists a diagonal matrix $D = \mathrm{diag}(d_1, \ldots, d_N)$ with $d_i > 0$ satisfying

$$DL + L^T D > 0. \tag{1}$$

Denote $d_{\max} = \max\{d_i \mid i = 1, \ldots, N\}$ and define also *the pinning matrix* $G \in \mathbb{R}^{N\times N}$ by $G = \mathrm{diag}(g_1, \ldots, g_N)$ where $g_i = 1$ if $(0, i) \in \mathcal{E}$, otherwise $g_i = 0$. Structure of the interconnection of these neurons is described by the matrix $\bar{L} = L - G$.

3. SYNCHRONIZATION OF STOCHASTIC MULTI-AGENT SYSTEMS

Synchronization of linear multi-agent systems is thoroughly described e.g. in Hu et al. (2015).

The agents have the form

$$dx_0 = Ax_0 + \sigma_0(x_0)dw(t), \tag{2}$$

$$dx_i = (Ax_i + Bu_i)dt + \sigma_i(x_i)dw(t), \; i = 1, \ldots, N. \tag{3}$$

where $x_i \in \mathbb{R}^n$ is the state of the ith agent, u_i is its control, $\sigma_i : \mathbb{R}^n \to \mathbb{R}$ is the noise intensity function, $w(t)$ is a one-dimensional Wiener process defined on $(\Omega, \mathcal{F}, \mathcal{P})$ such that $\mathbb{E}(w(t)) = 0$, $\mathbb{E}((w(t))^2) = dt$.

This model is suitable to describe random external disturbances acting upon the whole multi-agent system Hu et al. (2015).

The goal: to find the control u_i as a function of $x_i, x_j, j \in N_i$ so that

$$\lim_{t\to\infty} \mathbb{E}(\|x_i(t) - x_0(t)\|^2) = 0. \tag{4}$$

4. THE HINDMARSH-ROSE NEURONAL MODEL

The HR neuron is defined by the following equations (see e.g. Nguyen and Hong (2013)):

$$\dot{x}_1 = ax_1^2 - x_1^3 + x_2 - x_3 + I + \bar{\sigma}(t, x_1)dw(t), \tag{5}$$

$$\dot{x}_2 = 1 + bx_1^2 - x_2, \tag{6}$$

$$\dot{x}_3 = c(x_1 + 1.56) - 0.006x_3. \tag{7}$$

The variable x_1 is the membrane potential, x_2 is the recovery variable associated with the fast current of the Na^+ and/or K^+ ions. The variable x_3 is the adaptation variable associated with the slow current of Ca^+ ions and I is the external current.

Values of the above constants are taken from in Nguyen and Hong (2013): $a = 3$, $b = -5$, $c = 0.024$ and $I = 1.25$.

To meet the requirements for the method based on the feedback linearization (see Khalil (2001)), first define the output $y = x_1$. The exact feedback linearization yields:

$$\xi_1 = x_1, \; \xi_2 = x_2, \; \xi_3 = x_3 \tag{8}$$

$$u = v - F(\xi), \tag{9}$$

$$F(\xi) = -a\xi_1^2 - \xi_1^3 + \xi_2 - \xi_3 - I. \tag{10}$$

System (5-7) is changed into:

$$\dot{\xi} = \begin{pmatrix} 0 \\ -\xi_2 \\ -0.006\xi_3 \end{pmatrix} + \begin{pmatrix} v \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \\ 1.56c \end{pmatrix}. \quad (11)$$

Variable ξ is split as follows: $\xi = (\eta, \zeta^T)^T$ where $\eta = \xi_1$, $\zeta = (\xi_2, \xi_3)^T$. The first equation is called the observable part while the remaining part is called the non-observable part. The state ζ is not observable through the output y, hence this terminology (note that in order to control the system (5-7), access of the controller to the state ζ is necessary though). One can define the zero dynamics by

$$\dot{\zeta} = \begin{pmatrix} -\zeta_1 \\ -0.006\zeta_2 \end{pmatrix}. \quad (12)$$

The HR neuronal model has an asymptotically stable zero dynamics - it is a minimum-phase system. As will be shown in the sequel, this is a crucial property for application of the synchronization algorithm from Rehák and Lynnyk (2019).

Let us consider the network composed of $N+1$ neurons denoted by $0, \ldots, N$ with the neuron 0 being the leader. If $i \in \{1, \ldots, N\}$ then the ith neuron with coupling can be described as

$$\dot{x}_{1,i} = ax_{1,i}^2 - x_{1,i}^3 + x_{2,i} - x_{3,i} + I + u_i + \sigma_i(t, x_{1,i})dw(t), \quad (13)$$

$$\dot{x}_{2,i} = 1 + bx_{1,i}^2 - x_{2,i}, \quad (14)$$

$$\dot{x}_{3,i} = c(x_{1,i} + 1.56) - 0.006x_{3,i}. \quad (15)$$

Here, the variable u_i is the input signal to be designed.

The noise intensity functions σ_i is supposed to satisfy the condition (analogous to Assumption 2 in Hu et al. (2015)):

Assumption 1. *The noise intensity functions σ_i satisfy the following Lipschitz condition: there exists a constant $\Sigma > 0$ so that*

$$\Big(\sigma_i(t, u) - \sigma_0(t, v)\Big)^2 \leq \Sigma^2 |u - v|^2 \quad (16)$$

holds for all $i = 1, \ldots, N$, $u, v \in \mathbb{R}$ and $t \geq 0$.

For simplicity, we suppose the time delay, denoted by τ, is equal for all neurons in the network, but it is not constant. On the other hand, it is bounded: there exists $\bar{\tau} > 0$ so that $\tau : [0, \infty) \to [0, \bar{\tau}]$ is a measurable function. The control u_i of the ith neuron equals to

$$u_i = -F(x_i) + k \sum_{j \in N_i} (x_{1,j;\tau} - x_{1,i;\tau}) + g_i k(x_{1,0;\tau} - x_{1,i;\tau}) \quad (17)$$

where the symbol k is called coupling gain. It is assumed the signals from the other neurons are delayed. The aim is to find the parameter k so as (4) is reached. This parameter is equal for all neurons. Note also that the control signal of the ith neuron is expressed by the formula

$$v_i = k\Big(\sum_{j \in N_i} (\xi_{1,j,\tau} - \xi_{1,i,\tau}) + g_i(\xi_{1,0,\tau} - \xi_{1,i,\tau})\Big) \quad (18)$$

after the erxact feedback linearization.

Define $\eta = (\eta_1, \ldots, \eta_N)^T$, $\zeta = (\zeta_1^T, \ldots, \zeta_N^T)^T$ and $\sigma(\eta, \zeta) = (\sigma_1(\eta_1, \zeta_1), \ldots, \sigma_N(\eta_N, \zeta_N))^T$. Then, introduce the compacted form of notation of the observable parts as

$$\dot{\eta} = k(\bar{L}\eta_\tau + G\eta_{0,\tau}) + \sigma(t, \eta)dw = kL\eta_\tau + \sigma(t, \eta)dw. \quad (19)$$

5. SYNCHRONIZATION OF THE MEMBRANE POTENTIAL IN THE STOCHASTIC NEURONAL NETWORKS

The first goal is to synchronize the observable parts of the neurons. For this purpose, we introduce the synchronization error $e_i = \eta_i - \eta_0$ for $i = 1, \ldots, N$ and also introduce $\tilde{\sigma}_i(e_i, \zeta_i) = \sigma_i(\eta_i, \zeta_i) - \sigma_0(\eta_0, \zeta_0)$.

The complete synchronization is equivalent to $e_i = 0$. This is not achievable due to the noise added to the first variable. As presented e.g. in Hu et al. (2015), this should be relaxed to (4) - this is equivalent to

$$\lim_{t \to \infty} \sum_{i=1}^{N} \mathbb{E}(\|e_i(t)\|^2) = 0. \quad (20)$$

The suitable tool to prove the main theorem is the adaptation of the Razumikhin theorem for stochastic dynamical systems (see e.g. Theorem 3.1 in Huang and Deng (2009)):

Theorem 1. *Consider system (19) and let $V : R^n \to [0, \infty)$ be a twice differentiable function satisfying $\alpha_1\|e\|^2 \leq V(e) \leq \alpha_2\|e\|^2$ for some constants $0 < \alpha_1 < \alpha_2$. Let there exist a constant $\delta > 1$ such that for all $t > 0$ the following implication hold: if for all $t' \in [t - \bar{\tau}, t]$ holds $\mathbb{E}(V(e(s)) < \delta\mathbb{E}(V(e)$ then $\mathbb{E}(\mathcal{F}(V(e)) \leq -\mathbb{E}(\|e\|^2)$. Then the solution of (19) satisfies $\lim_{t\to\infty} \mathbb{E}(\|e(t)\|^2) = 0$ for any initial condition $e(t), t \in [-\bar{\tau}, 0]$.*

Remark 1. *The proof presented in Huang and Deng (2009) is derived for general pth moments, thus for more general functions V. The formulation presented in Theorem 1 is sufficient for our purpose. For further extensions of this approach see e.g. Zhou and Luo (2018).*

Before the formulation of the main theorem of this section, choose parameters $r, \varkappa_1, \varkappa_2, \varkappa$ so that

$$r > 1 \quad (21)$$

$$L^T DL \leq \varkappa_1 D, \quad (22)$$

$$L^T L \leq \varkappa_2 D, \quad (23)$$

$$DL + L^T D > \varkappa D. \quad (24)$$

With these constants, we formulate the result:

Theorem 2. *Assume parameters $r, \varkappa_1, \varkappa_2$ satisfy inequalities (21-24). Assume there also exist positive constants Q, Z, W, R, S and also a scalar y satisfying with*

$$\Xi = \varkappa y + 2\varkappa_1\bar{\tau}(Z + W) + 2(\varkappa_2\bar{\tau} + \Sigma^2)Q + \Sigma^2 Q$$

the following LMIs hold:

$$\Xi < 0, \quad (25)$$

$$\begin{pmatrix} Z & y \\ y & Q \end{pmatrix} \geq 0, \ \begin{pmatrix} W & y \\ y & Q \end{pmatrix} \geq 0, \quad (26)$$

$$R + S < Q, \quad (27)$$

$$\begin{pmatrix} rQ & y \\ y & R \end{pmatrix} \geq 0, \ \begin{pmatrix} r\Sigma^2 Q & Q\Sigma \\ \Sigma Q & S \end{pmatrix} \geq 0, \quad (28)$$

then $\lim_{t\to\infty} \mathbb{E}(\|e\|^2) = 0$ for any initial condition $e(t), t \in [-\bar{\tau}, 0]$ and $k = yQ^{-1}$.

First, some useful notations are introduced. Then we prove three propositions. Let $P = Q^{-1}$, $k = yP$.

Proposition 1. *Let the assumptions of Theorem 2 hold. Then the following is satisfied $(L^T DL \otimes kP(R+S)Pk) \leq \varkappa_1(D \otimes (Z + W))$.*

Proof. Let us multiply (26)by $\mathrm{diag}(1, P)$ from the left and right. Then, use the Schur complement. This yields $Z \geq k^2P$ and $W \geq k^2P$ which, with help of (27), implies $Z + W \geq k^2P^2(R + S)$. This means $L^TDL \otimes k^2P^2(R + S) \leq L^TDL \otimes (Z + W)$. Using (22) finally gives the claim. □

Proposition 2. *Under assumptions of Theorem 2, the first inequality in* (28) *and* (23) *imply* $L^TL \otimes R^{-1}k^2 \leq r\varkappa_2(D \otimes P)$.

Proof. We multiply the first inequality in (28) by $\mathrm{diag}(P, 1)$ from the left and right. Then, application of the Schur complement yields $rP \geq k^2R^{-1}$. This with (23) results into $L^TL \otimes k^2R^{-1} \leq L^TL \otimes rP \leq \varkappa_2 r(D \otimes P)$. □

The proof of Theorem 2 is omitted due to space reasons.

6. SYNCHRONIZATION OF THE ADAPTATION AND RECOVERY VARIABLES

The non-observable part of the system describing the HR neuron is not controlled. However, Rehák et al. (2018) shows that these parts can be synchronized as well as the HR neuron is a minimum-phase system. The aforementioned paper presents the proof of convergence of the non-observable parts for nonlinear zero dynamics if it is exponentially stable. In the HR neuron, this is satisfied. The Theorem 5.4 in Rehák et al. (2018) useful for the proof of thsi claim, reads

Theorem 3. *Under assumptions of Lemma 2, the non-observable part (13-15) achieves synchronization.*

Proof. Eq. (12) yields that

$$\dot{\zeta}_i - \dot{\zeta}_{i_0} = \begin{pmatrix} -1 & 0 \\ 0 & -0.006 \end{pmatrix} (\zeta_i - \zeta_{i_0}) + \begin{pmatrix} -(\eta_i - \eta_{i_0}) \\ c(\eta_i - \eta_{i_0}) \end{pmatrix}. \quad (29)$$

Theorem 2 yields $\lim_{t\to\infty} \mathbb{E}\Big(\|\eta_i - \eta_{i_0}\|\Big) = 0$. This and the exponential stability of the zero dynamics imply $\lim_{t\to\infty} \mathbb{E}\Big(\|\zeta_i - \zeta_{i_0}\|\Big) = 0$.

The main result is formulated as follows:

Theorem 4. *Let assumptions of Theorem 2 and Theorem 3 hold. Let also the control input be defined as in* (17)*. Then* $\lim_{t\to\infty} \mathbb{E}(\|x_i - x_{i_0}\|) = 0$.

Proof. Theorem 3 implies that $\lim_{t\to\infty} \mathbb{E}\Big(\|\zeta_i - \zeta_{i_0}\|\Big) = 0$. This is since the exact feedback linearization converting (13-15) into (11) and the control input transformation (9) is a diffeomorphism. □

7. EXAMPLE

A network composed of 6 neurons was simulated, the neuron 0 is the leader. The parameters are as in Section 4. The interconnection topology is shown in Fig. 1.

Then, this interconnection topology implies that $D = \mathrm{diag}(0.2866, 1.007, 1.8044, 2.435, 2.689)$ which satisfies (1). Moreover, $\varkappa = 0.5$, $\varkappa_1 = 7$, $\varkappa_2 = 33$. Further, Σ was chosen as $\Sigma = 0.1$ and $r = 200$ while the maximal time delay was $0.1s$. Finally, the algorithm yields $k = -6.94$.

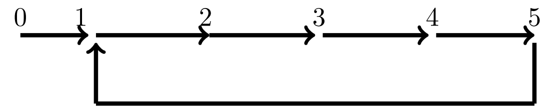

Fig. 1. Connection of neurons.

Fig. 2 illustrates the state of the leader. Note that there is no noise in the leader. The type of the curves mean: solid line: state x_1, dashed line: state x_2 and dash-dot line: state x_3. Difference $\sqrt{\sum_{i=1}^{5}(x_{3,i} - x_{3,0})}$ in Fig. 4 (note the different time range). As the synchronization is obtained through the potential only, the synchronization is fairly good in the first state, with the exception of the points with rapid changes of the potential. Synchronization of this part is improved since the nonlinearities (the function F) are exactly matched.

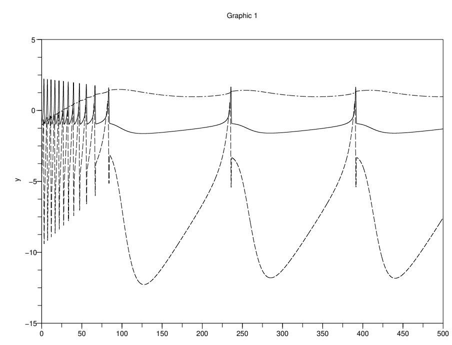

Fig. 2. State of the leader neuron

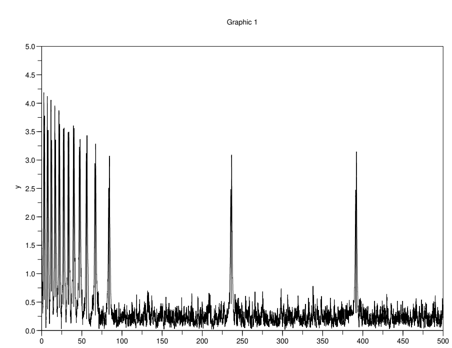

Fig. 3. Norm of synchronization error in the membrane potential

8. CONCLUSION

A synchronization algorithm for a neuronal network that is composed of Hindmarsh-Rose neurons with additive noise was derived. The observable part is synchronized by a control law that is obtained by solving a set of LMIs based on the Razumikhin theorem. The synchronization of the non-observable part results from the minimum-phase property of the Hindmarsh-Rose neurons. The algorithm is illustrated by an example.

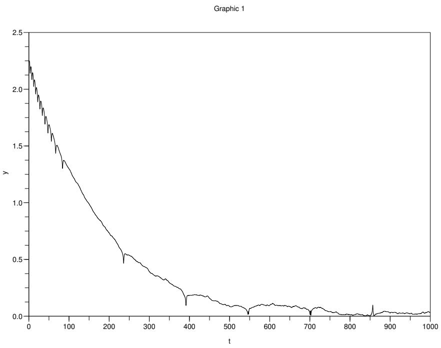

Fig. 4. Norm of the synchronization error in the adaptation variable

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