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Multi-stage stochastic optimization of carbon risk management

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ABSTRACT

Emissions trading within the Emissions Trading Scheme of the European Union (EU ETS) strongly influences European industrial companies. The companies must choose their strategy of reduction the costs of emissions allowances as possible. The changing system's conditions and volatile prices of allowances make this decision challenging. The main aim of this study is to compare different ways of risk management: banking (i.e., buying the allowances in forward) and using derivatives: futures and options. Despite several studies devoted to the relationship between the EU ETS and companies have already been published, there is still a gap in this field. Namely, the published studies have been substantially simplified so far by ignoring the risk of driving parameters. We construct a realistic large-scale stochastic optimization model, which avoids the mentioned simplifications. We use the Markov Stochastic Dual Dynamic Programming algorithm (MSDDP) to find the optimal solution. We apply the model to the data of a real-life industrial company. We find that banking is the most costly way of risk reduction, while using derivatives is efficient in risk reduction. Surprisingly, out of the derivatives, it is always optimal to use futures and not to use options. These results are confirmed by a thorough sensitivity analysis. The preference of the futures over options is mainly due to the less price of futures in comparison to options reducing risk equivalently.

1. Introduction

Emissions trading within the European Emissions Trading System (EU ETS) is the main tool of the EU environmental policy since 2005. Rules and settings of the EU ETS are given by several EU directives (Council of European Union, 2003, 2004, 2013, 2018). This system obliges companies to use emission allowances (EUAs) to cover their CO_2 emissions, thus it is a source of additional financial risk and costs related with trading the allowances. Many studies have been already devoted to analysis of the EU ETS' impact on companies with the same results — this effect cannot be omitted and should be involved in strategic planning (Šmíd et al., 2017; Zapletal et al., 2019; Zhang & Xu, 2013). However, in our opinion, there results can only be considered a first step towards a more profound understanding of the companies' optimal behaviour in terms of emissions trading.

Based on our thorough literature review, we identified four features which in our opinion a realistic model optimizing companies' decisions on emissions trading should include:

(a) multi-period setting allowing for use of banking (transferring the allowances to following time periods);
(b) uncertain demand for products (the demand is a crucial factor for production, and thus for amount of emissions too);
(c) uncertain EUA price (since the EU ETS is a cap-and-trade system, the allowance price is essential);
(d) involving all available derivatives for allowances (futures and options).

None of the models published so far concerned with all these features together, see Table 1.

Table 1 provides a review of the available models, and the features missing in modelling therein, according to the list above. The simplifications can stem from two main reasons. First, the authors established their model under the conditions valid at the time of origin of their work, namely the rules of the EU ETS, and the state of economy. For instance, financial derivatives could not be used by companies in the beginning of the EU ETS, banking of allowances was not allowed between the first (2005–2008) and the second (2009–2012) trading phase (Zapletal & Moravcová, 2013). Moreover, during the deep economic crisis, due to very low levels of demand and EUA price and over-allocation of the system, the emissions trading was rather profitable than generating additional losses. The second reason for simplification could be that the authors may have been aware of all the mentioned factors, but they decided for simplification just to keep

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Table 1
The review on published optimization models devoted to company's behaviour regarding the emissions trading.

Authors	Omitted factors
Lemathe and Balakrishnan (2005)	a, b, c, d
Rong and Landhelma (2007)	d
Sirikitputtisak et al. (2009)	b, c, d
Mirzaesmaeeli et al. (2010)	b, c, d
Gong and Zhou (2013)	d
Tang and Song (2013)	c, d
Zhang and Xu (2013)	a, c, d
Zapletal and Šmíd (2016)	a, d
Šmíd et al. (2017)	a
Zapletal et al. (2019)	d (options)

the size of the models small enough to be still solvable, like in the case of Zapletal et al. (2019).

The goal of this paper is to explore the optimal risk management of emissions trading. Unlike Zapletal et al. (2019), we involve the possibility to use the options on EUAs into the model, we also capture the stochastic parameters in a more precise way (namely, without former discretization into scenarios), and, finally, we use the more suitable risk measure, namely nested CVaR instead of multi-period CVaR. As shown by Kozmík (2013), the nested model provides more stable (and more risk-averse) solutions. In particular, we establish a complex stochastic optimization model involving the factors (a)–(d).

The established model is too complex to be solved using the standard algorithms of stochastic programming. Therefore, we apply the Stochastic Dual Dynamic algorithm (SDDP), which is known to solve large stochastic programming models efficiently (Pereira & Pinto, 1991). To be precise, we use its modification relaxing the restricting assumption of stage-wise independence, allowing the random parameters to follow a hidden Markov models (Löhndorf & Shapiro, 2019; Philpott et al., 2013).

The proposed model is applied to a real-world Czech steel company, for which we explore the impact of the factors (a)–(d) on the costs of emissions trading and the associated financial risk. Furthermore, we construct the optimal portfolio of emission allowances and we evaluate the impact of the emissions trading on production of the modelled company. This analysis is supported by a sensitivity analysis performed for the selected parameters.

The rest of this paper is organized as follows. After this introductive section, we provide a description of the problem together with the problem definition (Section 2). Section 3 contains a description of the input data. Furthermore, we continue with the preliminary analysis (Section 4), in which we illustrate the optimal risk management on a simplified single-stage example. Section 5 discusses the solution of the multi-stage problem. Section 6 is a core part of this paper providing the results of modelling and their discussion. The last section (Section 7) provides a thorough sensitivity analysis of the results.

2. Description of the problem

A company decides on ways of covering their emissions $Y_1, \dots, Y_T$ stemming from their exogenously given production, at times $1, \dots, T$, by allowances. At each $t = 1, \dots, T$, r_t allowances are given (grandfathered) to the company for free. Further, allowances may be bought (sold) at a secondary market at any time $t = 0, \dots, T$. The allowances may be saved (banked) for future periods.

In addition to the allowances themselves (spots), at each $t = 0, \dots, T-1$, futures with maturities $t+1, t+2, \dots T$ may be bought. Moreover, at each $t = 0, \dots, T-1$, call and put options with maturities $t+1, t+2, \dots, T$, and strike prices $K_1, \dots, K_\kappa$, $L_1, \dots, L_\kappa$, respectively, may be bought. In principle, the holder of an option is not obliged to exercise it at the time of their maturity. However, as we neglect transaction costs, exercising only some options is always no better than exercising all options and selling (or buying) the spots. Thus, one can assume that all the options are exercised. The company can trade the allowances and their derivatives freely. However, it cannot take short positions and they cannot sell futures (we do not allow selling the futures to avoid speculations which are prevented by margin requirements in practice, but are neglected here).

The company is risk-averse, minimizing the discounted nested mean-CVaR dynamic risk measure, applied to the difference of the profits from the production and the costs of emissions trading.

2.1. Problem definition

As it was premised, the subject of decision is the emission trading. In particular, the decision variables at t include the amount Δs_t of the spot allowances purchased (or sold) at time t, the amounts

$$\Delta f_t = (\Delta f_t^{t+1}, \dots, \Delta f_t^T)$$

of the futures with maturities $t+1, \dots, T$ purchased at time t, and the amounts

$$\Delta\phi_t = \begin{bmatrix} \Delta\phi_t^{t+1,1} & \dots & \Delta\phi_t^{T,1} \\ \vdots & & \vdots \\ \Delta\phi_t^{t+1,\kappa} & \dots & \Delta\phi_t^{T,\kappa} \end{bmatrix}, \qquad \Delta\psi_t = \begin{bmatrix} \Delta\psi_t^{t+1,1} & \dots & \Delta\psi_t^{T,1} \\ \vdots & & \vdots \\ \Delta\psi_t^{t+1,\kappa} & \dots & \Delta\psi_t^{T,\kappa} \end{bmatrix},$$

of the traded call options and put options, respectively. In particular, $\Delta\phi_t^{\tau,i}$ stands for the number of purchased call options with maturity τ and strike price K_i, and $\Delta\psi_t^{\tau,i}$ denotes the number of sold put options with maturity τ and strike price L_i, any $0 \le t < \tau \le T$, $1 \le i \le \kappa$.

At $t = 0$, the company may buy allowances on the spot market only:

$$e_0 = s_0. \tag{1}$$

At $0 < t \le T$, the company uses free allowances r_t, spots s_t bought at t, allowances secured by the futures with maturity t (f_t^t), call options with maturity t and strike price K_i ($\phi_t^{t,i}$), and put options with maturity t and strike price L_i ($\psi_t^{t,i}$) to cover (random) CO_2 emissions at t (Y_t):

$$e_t = e_{t-1} + r_t + s_t + f_t^t + \sum_{i=1}^{\kappa} \phi_t^{t,i} + \sum_{i=1}^{\kappa} \psi_t^{t,i} - Y_t, \qquad 0 < t \le T. \tag{2}$$

All allowances have to be used at T:

$$e_T = 0. \tag{3}$$

At $t = 0$, the income of the company (z_0) is negative, consisting of (minus) costs for spots and options purchases:

$$z_0 = -P_0 \Delta s_0 - \sum_{\tau=1}^{T} \sum_{i=1}^{\kappa} B_0^{\tau,i} \Delta\phi_0^{\tau,i} - \sum_{\tau=1}^{T} \sum_{i=1}^{\kappa} C_0^{\tau,i} \Delta\psi_0^{\tau,i}, \tag{4}$$

whereas at any $0 < t \le T$, in addition, the income includes profit from production (X_t), costs of futures with maturity t, and costs of spots secured by the options:

$$\begin{aligned} z_t = X_t - P_t \Delta s_t - \sum_{i=1}^{\kappa} \min(P_t, K_i)\phi_{t-1}^{t,i} + \sum_{i=1}^{\kappa} \max(P_t, L_i)\psi_{t-1}^{t,i} - \\ - \sum_{\tau=0}^{t-1} Q_\tau^t \Delta f_\tau^t - \sum_{\tau=t+1}^{T} \sum_{i=1}^{\kappa} B_t^{\tau,i} \Delta\phi_t^{\tau-t,i} - \sum_{\tau=t+1}^{T} \sum_{i=1}^{\mu} C_t^{\tau,i} \Delta\psi_t^{\tau-t,i}. \end{aligned} \tag{5}$$

Here, $f_t = \sum_{\tau=0}^{t} \Delta f_\tau^t$, $\phi_t = \sum_{\tau=0}^{t} \Delta\phi_\tau^t$ and $\psi_t = \sum_{\tau=0}^{t} \Delta\psi_\tau^t$. Furthermore, $X_t \in \mathbb{R}_+$ is the profit from the production at time t, $P_t \in \mathbb{R}_+$ is the spot price at time t, $Q_t^\tau \in \mathbb{R}_+$ is the price of the future with maturity τ at time t, and $B_t^{\tau,i} \in \mathbb{R}_+$ and $B_t^{\tau,i} \in \mathbb{R}_+$ are the premia paid at t for the call and put option, respectively, with strike price K_i and L_i, respectively, and with maturity τ.

Finally, we assume a minimizing decision criterion (i.e., the objective function):

$$\rho(-z_0, \dots, -z_T) = \mu_1(\mu_2(\dots \mu_T(-z_0 - \varrho z_1 - \dots - \varrho^T z_T) \dots)). \tag{6}$$

where ϱ is a discount factor, and

$$\mu_t(Z) = (1-\lambda)\mathbb{E}(Z|\mathcal{F}_{t-1}) + \lambda \mathrm{CVaR}_\alpha(Z|\mathcal{F}_{t-1}), \quad 0 \le \lambda \le 1 \text{ and } 0 < \alpha < 1 \tag{7}$$

is the mean-CVaR conditional risk measure, and $\mathcal{F}_t$ is the information available at t.

To summarize, the objective function (6)) is to be minimized subject to the constraints (1)–(5), and (7).

To make orientation easier for readers, we offer a list of used mathematical symbols in Appendix A (Table 7).

3. Data

In this section, we describe the input data for the stochastic parameters of the model, i.e., the amount of CO_2 emissions Y_t, the profit of the company X_t, the price of EUA spot P_t, prices of the futures on EUA spot Q_t, and the option prices B_t. We consider 4 stages in the model where the first one corresponds to the beginning of 2018 ($t = 0$) and the last one (T) represents the end of 2020 ($t = 3$). The data on emissions trading were taken from the ICE market (theice.com), the rest of the data were provided by managers of the modelled company.

For the spot prices P and the future prices Q, we adopt the model from Zapletal et al. (2019). In particular, we assume the prices to follow the Geometrically Brownian motion. The volatility σ is estimated by Maximum Likelihood from the historical EUA prices, the mean parameter is set such the price process is a martingale (to satisfy the Efficient Market Hypothesis Cuthbertson & Nitzsche, 2005). As a result, we have

$$P_t = P_0 \exp\left\{\sum_{\tau=1}^{t} u_\tau\right\}, \qquad u_t \sim \mathcal{N}\left(-\frac{\sigma^2}{2}, \sigma^2\right), \qquad 1 \le t \le T,$$

with $\sigma = 0.439$, where $u_1, \dots, u_T$ are i.i.d. increments needed to express the Geometrical Brownian motion, and

$$Q_t^\tau = P_t \exp\{(\tau - t)(\mu + v_t^\tau)\}, \quad v_t^\tau \sim \mathcal{N}(0, \varsigma^2), \quad 1 \le t < \tau, \quad 1 \le \tau \le T, \tag{8}$$

with $\varsigma = 0.010$ and $\mu = 0.00974$. Here, Q_t^τ denotes the price of the future with maturity τ at time t following, and v_t^τ are i.i.d., independent of $u_1, \dots, u_T$. This (Geometrically Brownian) model of spot price evolution and the (cost-of-carry) model of future price evolution are adopted from Zapletal et al. (2019) together with the estimates of ς and μ.

The initial prices are equal to their historical values:

$$P_0 = 7.77, \quad Q_0^1 = 7.81, \quad Q_0^2 = 7.87, \quad Q_0^3 = 7.97.$$

The option prices $B_t^{\tau,i}$ (see the problem's definition) are computed by the Black–Scholes formula with the implied volatility (i.e., the hypothetical volatility such that the actual option prices fulfil the Black–Scholes formula). We determine the implied volatility from the historical option prices and, to have this value for an arbitrary strike price, we fit the volatility curve, i.e., the implied volatility as a function of the intrinsic value (the difference between the strike price and the market price). As a result of the estimation, we got the volatility smile, i.e., a convex (quadratic in our case) function with the minimum near to the strike price, see Fig. 1. For the notion of volatility smile, see Hull (2019). The shape of the smile function has been estimated using 110 observations of actual option prices on the ICE market. As the risk-free rate, 1.75% was taken, being equal to the repo rate in the Czech Republic in 2018. Two strike prices were considered for each type of options: $K_1 = 8$, $K_2 = 10$, $L_1 = 8$, $L_2 = 6$.

Our model of yearly profits X (in thousands of EUR) and emissions Y (in metric tonnes) is as follows

$$\begin{bmatrix} X_t \\ Y_t \end{bmatrix} = \begin{bmatrix} 37,847.73 \\ 232,561.57 \end{bmatrix} + \begin{bmatrix} 0.6981 & -0.1211 \\ 1.712 & -0.2638 \end{bmatrix} \begin{bmatrix} X_{t-1} \\ Y_{t-1} \end{bmatrix} + e_t, \quad e_t = \mathcal{N}(0, V) \tag{9}$$

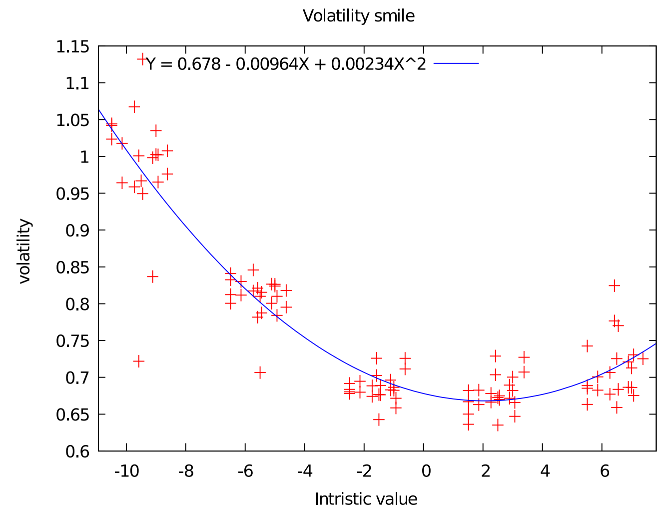


Fig. 1. Volatility smile function.

where $e_1, e_2, \dots, e_T$ are i.i.d. and V is a variance matrix defined by standard deviations 7968 and 25,499 and correlation 0.7368. As the initial values, we took

$$X_0 = 33,735, \qquad Y_0 = 246,974.$$

See Appendix B for details on construction of the model.[1]

As for the information $\mathcal{F}_t$, available at t, we assume that, at any t, the company observes the history of P, Q, X, Y up to t, and they are partially informed about X and Y one step ahead (ω is a coefficient representing the knowledge of randomness), namely that

$$e_t = f_t + g_t, \qquad f_t \sim \mathcal{N}(0, \omega V), \quad g_t \sim \mathcal{N}(0, (1-\omega)V), f_t \perp\!\!\!\perp g_t, \quad 1 \le t \le T, \tag{10}$$

for some $0 \le \omega \le 1$, and

$$\mathcal{F}_t = \sigma((P_\tau, Q_\tau, X_\tau, Y_\tau)_{\tau \le t}, f_{t+1}).$$

Finally, according to Zapletal et al. (2019), the discount factor ϱ was chosen to reflect the risk free rate, i.e., $\varrho = 0.96$.

4. Preliminary analysis

In this section, we illustrate the problem of the optimal emissions covering on a simplified single-stage example.

Say that the amount of emissions Y_1 has to be covered at $t = 1$. At $t = 0$, spots with price P_0 and futures with price Q_0 can be bought. Furthermore, call- and put options with strike prices K, L, respectively, may be bought at time zero for prices (option premia) b, c, respectively. Finally, the spots may be bought or sold for a random price P_1 at time one.

The distributions of P_1 and Y_1 are discrete. The prices b, c, P_0 and Q_0 are deterministic.

First, let us observe that buying a future at $t = 0$ is equivalent to buying the spot at $t = 0$ because the result is always having a spot at $t = 1$, either for Q_0, or for $\varrho^{-1} P_0$, where ϱ is the discount factor.[2] Thus, the cheaper option will always be used rather than the other one. Thus, we can treat both cases simultaneously, say as buying a future.

[1] We neglect the fixed costs as the income depends on these costs only up to an additive constant. Thus, once these costs are deterministic, they do not alter the optimal solutions.

[2] Here we assume that the future price is paid at $t = 1$. In practice, a part of the future price has to be paid in forward in form of a margin; however, we neglect this in this simplified setting.

We do not study all combinations of the instruments in this simplified example. Rather, we discuss using futures, call options, and put options, separately. In all these cases, the decision maker chooses the amount of the selected derivative so that the risk is minimized. As a risk measure, we use the supremum (worst case), which we denote by σ. Although it is different from the mean-CVaR, which is used in the model presented in Section 2, it is similar, as it also penalizes the worst cases. Moreover, it coincides with CVaR for the discrete distributions with the probability of the scenarios being no less than the CVaR level.

As to the first case, let us assume that x futures are bought at $t = 0$, resulting in costs

$$C_x = xQ_0 + (Y_1 - x)P_1$$

with

$$\sigma(C_x) = xQ_0 + \sup_i \sup_j \left\{(y_j - x)p_i\right\}$$

$$= xQ_0 + \sup_i \left\{(y_n - x)p_i\right\} = \begin{cases} xQ_0 + (y_n - x)p_m & x \le y_n \\ xQ_0 + (y_n - x)p_1 & x > y_n \end{cases}.$$

If we, naturally, assume that $p_1 < Q_0 < p_m$, then both the branches are minimized at $x = y_n$, giving

$$\sigma_C = \min_{x \ge 0} \sigma(C_x) = Q_0 y_n = Q_0 \sup Y_1.$$

Further, assume that the strike price K of the call option coincides with some atom p_i. If x call options are bought at $t = 0$, then the costs are

$$D_x = bx + (Y_1 - x)P_1 + x \min(P_1, K),$$

with

$$\begin{aligned}\sigma(D_x) &= bx + \sup_i \sup_j \left\{(y_j - x)p_i + x\min(p_i, K)\right\} = \\ &= bx + \sup_i \left\{(y_n - x)p_i + x\min(p_i, K)\right\} = \\ &= bx + \left(\sup_{p_i \le K} y_n p_i \vee \sup_{p_i \ge K} \left\{(y_n - x)p_i + xK\right\}\right) = bx + \left[y_n K \vee d(x)\right],\end{aligned}$$

$$d(x) = \begin{cases} (y_n - x)p_m + xK & x \le y_n \\ y_n K & x > y_n, \end{cases}$$

i.e.,

$$\sigma(D_x) = \begin{cases} bx + [y_n K \vee (y_n p_m + x(K - p_m))] & \\ = (bx + y_n K) \vee (y_n p_m + x(b + K - p_m)) & x \le y_n \\ bx + y_n K & x > y_n. \end{cases}$$

If $b + K \le p_m$ then the first branch is no less than

$$(bx + y_n K) \vee [y_n p_m + y_n(b + K - p_m)] = (bx + y_n K) \vee (y_n(b + K)) = y_n(b + K)$$

where the bound is attained by $x = y_n$. As the second branch is no less than $y_n(b + K)$, we have that

$$\sigma_D = \min_x \sigma(D_x) = (b + K) \sup Y_1.$$

If, on the other hand, $b + K > p_m$, then the first branch has a lower bound $y_n K \vee y_n p_m = y_n p_m$, attained by $x = 0$, while the second branch is no less than $y_n(b + K)$, i.e., $\sigma(D_x) \ge y_n p_m$, with the right-hand side attained by $x = 0$, giving $\sigma_D = y_n p_m$. Together, this gives

$$\sigma_D = y_n[p_m \wedge (K + b)]$$

with the optimal solution being y_n if $b + K \le p_m$, and being zero otherwise.

Finally, let us assume that x put options are bought at $t = 0$, leading to the costs

$$E_x = cx + (Y_1 + x)P_1 - x\max(P_1, K).$$

We have

$$\begin{aligned}\sigma(E_x) &= cx + \sup_i \sup_j \{(y_j + x)p_i - x\max(p_i, K)\} \\ &= cx + \sup_i \{(y_n + x)p_i - x\max(p_i, K)\} \\ &= cx + \left(\sup_{p_i \le K} \{(y_n + x)p_i - xK\} \vee \sup_{p_i \ge K} \{y_n p_i\}\right) \\ &= cx + \left([(y_n + x)p_m - xK] \vee y_n p_m\right) \\ &= \left([cx + y_n p_m + (p_m - K)x] \vee [cx + y_n p_m]\right) \ge y_n p_m\end{aligned}$$

where the bound is attained for $x = 0$, i.e., put options will no way help to reduce risk.

Clearly, buying all the spots at $t = 1$ leads to the costs $F = Y_1 P_1$ with

$$\sigma(F) = \sup P_1 \sup Y_1 = p_m y_n.$$

Summarized, if we have to choose which instrument to use, we will choose futures if

$$Q_0 < K + b, \tag{11}$$

or call options otherwise. In both cases, the number of spots secured at time zero will correspond to the worst possible need for the emissions.

Applied to the actual data we use further, our simple model suggests that the futures will be used rather than options or banking the spots because, for the call options with strike prices 8.0 and 10.0, we get $K_1 + B_0^{1,1} = 8.0 + 2.22 = 10.22$, $K_2 + B_0^{1,2} = 10 + 1.41 = 11.41$, respectively, which is both much greater than $Q_0 = 7.81$ (the future price), which itself is less than $\varrho^{-1} P_0 = 1.042 \times 7.77 = 8.09$ (the estimated spot price). The results for the other strike prices are similar.

Furthermore, if the initial future and spot prices are stochastic according to (8) (which is what we assume in the further stages), then the choice between a spot and future will depend mostly on the discount factor ρ. As it was discussed above, the spot will be chosen rather than the future (with one period maturity) if $P_0 < \varrho Q_1$, which happens if and only if $\nu > -\ln\varrho - 0.00974$ where ν is a standard normal variable. Given the discount factor $\rho = 0.96$ which we work with, the probability of this event is about 10%, and it would be 83% if $\varrho = 1$, or 15% if $\varrho = 0.98$. Thus, according the simplified model, the futures will be used much more often than (banking) the spots.

Clearly, within the general setting, the situation is more complex: the mean-CVaR is used instead of the supremum, the random variables are continuous, and, most importantly, the problem is dynamic with the nested structure. Nevertheless, as it will be shown below, the results given by the complex model are similar to this simplified example.

It should be also noted that, in financial theory, the prices of futures (forwards, more exactly) and options are tied by put–call parity, i.e., the equivalence of the forward and the portfolio containing the long position of the call and the short position of the put, so the use of forwards and options should be exchangeable. In the context of the market we examine, however, this principle can hardly be applicable because of the illiquidity of the options, sparsity of quoted strike prices and difficulty to take short positions.

5. Solution

The general decision problem defined in Section 2, as well as all its variants (see Section 6), were solved using the Markov SDDP method, introduced by Philpott et al. (2013) and thoroughly discussed by Löhndorf and Shapiro (2019). Unlike the traditional SDDP, which requires stage-wise independence, this method allows for "switching regime" stochastic model, in which the random parameters need not to be stage-wise independent, but it suffices that the tth stage parameters are conditionally independent of all the past parameters given m_t where m is a finite Markov Chain.

We give a brief description of the SDDP algorithm in order to give sufficient context for presenting our results. SDDP applies to the dynamic programming equations. During a typical iteration of the SDDP algorithm, cuts have been accumulated at each stage. These represent a piecewise linear outer approximation of the expected future cost functions, separately for each possible state. On a forward pass we sample a number of linear paths through the tree. As we solve a sequence of master programs along these forward paths, the cuts that have been accumulated so far are used to form decisions at each stage. Solutions found along a forward path in this way form a policy, which does not anticipate the future. In fact, the solutions can be found at a node on a sample path via the stage t master program, even before we sample the random parameters at stage $t+1$. The sample mean of the costs incurred along all the forward sampled paths through the tree forms an estimator of the expected cost of the current policy, which is determined by the master programs.

In the backward pass of the algorithm, we add cuts to the collection defining the current approximation of the expected future cost function at each stage. We do this by solving subproblems at the descendant nodes of each node in the linear paths from the forward pass, except in the final stage, T. The cuts collected at any node in stage t apply to all the nodes in that stage, and hence we maintain a *single set of cuts* for each stage, however, separately for each future state. This reduction is possible because of our markov property assumption.

The Markov SDDP (sometimes denoted ADDP), modifies the original procedure in a way that the dependent random parameter (P in our case) is approximated by a sparse Markov chain and, at each stage, cut collections are maintained for each possible values of the Markov chain. See Löhndorf and Shapiro (2019) for further details on Markov SDDP.

As it is clear from Section 3, random parameters P, X and Y are time dependent rather than conditionally independent, given some Markov chain. As neither X_t nor Y_t are multiplied by a decision variable in the constraints of the problem, their time-dependence may be circumvented by regarding them as decision variables, constrained by (9), the residuals e_t becoming new random parameters. However, the same procedure cannot be used for P, as it is multiplied by other decision variables in the constraints. Instead, we directly approximate P by a finite Markov process π_t with 15, 21 and 30 atoms (states) at stages 1, 2, 3, respectively. As the e_t are i.i.d. and as $Q_t^{\bullet}$ – the remaining random parameters – are functions of π_t and an i.i.d. collection $v_t^{\bullet}$, the vector of the new random parameters (i.e., $e_t, Q_t^{\bullet}, \pi_t$) is stage-wise conditionally independent given π_t, hence eligible for Markov SDDP.

To get the solutions of the model, we use our own implementation of Markov SDDP, written in `C++`, which calls `CPlex` optimizer for solving the linear subproblems. The used programming codes are available in the external repository (Šmíd, 2021). The algorithm copes with the non-linearity of CVaR by computing its subgradient rather than by a linearization of the whole problem. The solution of each problem took several hours in a regular PC with Intel Core i7 and 48 GB RAM.

6. Results

In the present section, we report the solution results for various settings of the general decision problem. In doing so, we mainly focus on actual (monetary) income rather than on the values of the decision criterion – the nested Mean-CVaR risk measure – which is difficult to interpret. The actual profits, on the other hand, are easily understandable for decision makers, e.g., managers. To obtain the actual profits, we first solved the problem (with the nested Mean-CVaR criterion) and, based on the solution, we constructed an optimal policy. Next we generated 1000 scenarios of the random parameters, this time without approximation. Then we applied the optimal policy on each scenario, and evaluated the profit. As a result, we got a random sample of 1000 values of the profit. Having a sample for each setting (variant) of the model, we could compare the results (samples) either by their mean values, but also by means of stochastic dominance (see below); this way of comparison is clearly more robust than the comparison only by means, as its avoids "victories" of risky strategies over more robust ones with only slightly less mean.

Three different settings of the model in terms of derivatives and banking were discussed. The first setting (S1) corresponded to the situation when the company can use the derivatives (futures and options) on emission allowances, and it can also transfer unused allowances between time periods (i.e., banking is allowed). The second setting (S2) does not allow for the derivatives, but it is possible to keep the allowances for future periods (banking). The last setting (S3) allows neither derivatives nor banking, i.e., all allowances held at a given period must be used till the end of this period. In fact, S3 does not provide any space for decisions on carbon management; therefore, it represents the situation when no risk management is applied at all (i.e., it is a trivial policy when the company buys the necessary amount of spots for the given year at the end of this year). The settings are summarized in Table 2.

Table 2
Explored settings and their properties.

Setting	Derivatives allowed	Banking allowed
S1	✔	✔
S2	✗	✔
S3	✗	✗

For all the three settings S1–S3, the parameters of risk aversion and awareness (see (10)) were set to moderate values $\lambda = 0.55$, $\omega = 0.5$, respectively.

First, we compared the mean profits given by the settings S1–S3. Considering the mean profit for S3 (no risk management) as the 100% benchmark, settings S1 and S2 generated the profit of 102.6% and 99.9%, respectively, on average. Thus, S1 clearly outperformed both S2 and S3 which means that the derivatives can bring savings when used in emissions trading (note that most of the profit originates from selling of the production which cannot be influenced by the decisions, while the contribution of the emissions trading is small). Not surprisingly, banking the allowances without futures (S2) brought the same profit as the setting without any carbon management (S3) (0.1% percentage point in favour of the model without banking is within the standard error). The reason for this equality is that the prices are martingales by construction, so the price of the spot at t has the same (conditional) mean as that at $t+1$.

As taking into account only mean values could be misleading because it neglects risk, we further compared the settings S1–S3 by means of stochastic dominance, which is a much stronger tool for comparison of random variables (a variable stochastically dominating another one has a greater mean, but not vice versa). In particular, we used two most commonly used versions of this notion: the first-order- and the second-order stochastic dominance.

First, we compared the settings by means of the first-order stochastic dominance (FSD). According to Post (2003), a discrete random variable *A first-order stochastically dominates* random variable B if $F_A(x) \le F_B(x)$ for all x, where F_A, F_B denote cumulative distribution functions A, B, respectively (with strict inequality at least at one x). When comparing two random samples, which is our case, this means that the ordered values of the first sample are not less than those of the second sample with at least one strict equality. When only certain percentage p of observation pairs is no less, we speak of *p% almost first-order dominance* (Leshno & Levy, 2002). When applied to the profit samples given our three settings, it appeared that there is no strict FSD relationship between the settings, however, there is 97% almost dominance of S1 over the other two settings. This superiority of S1 was also confirmed by a different measure of an imprecise stochastic dominance – the ratio between the area where FSD is violated and the total area between $F_A(x)$ and $F_B(x)$ (Leshno & Levy, 2002) – which came out as 0.0301 (for S2) and 0.0083 (for S3), respectively.

Next, we compared the settings by the less strict second-order stochastic dominance. Mathematically said, a discrete random variable *A second-order stochastically dominates* (SSD) a random variable B if $\int_{-\infty}^{x}\left(F_B(t) - F_A(t)\right)dt$ for all x, with strict inequality at least at one x. In words, this means the dominated variable involves more risk and does not have a higher mean (Post, 2003). When exploring if there is

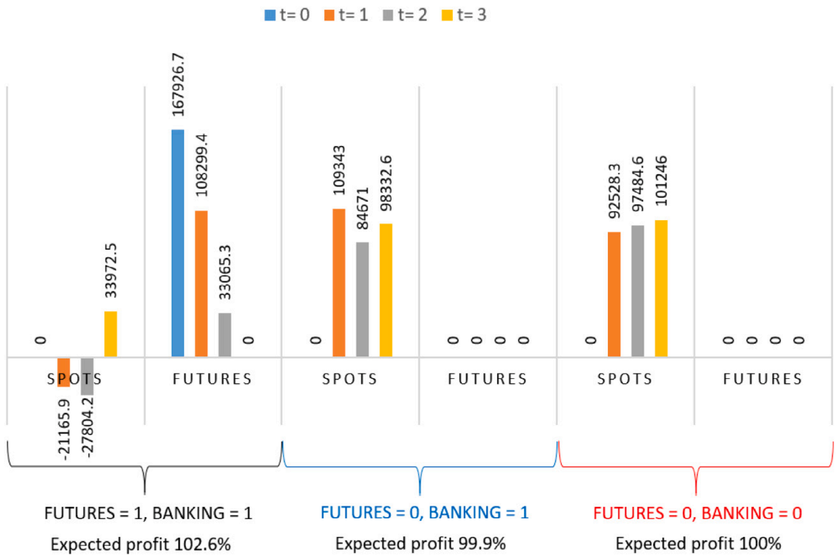


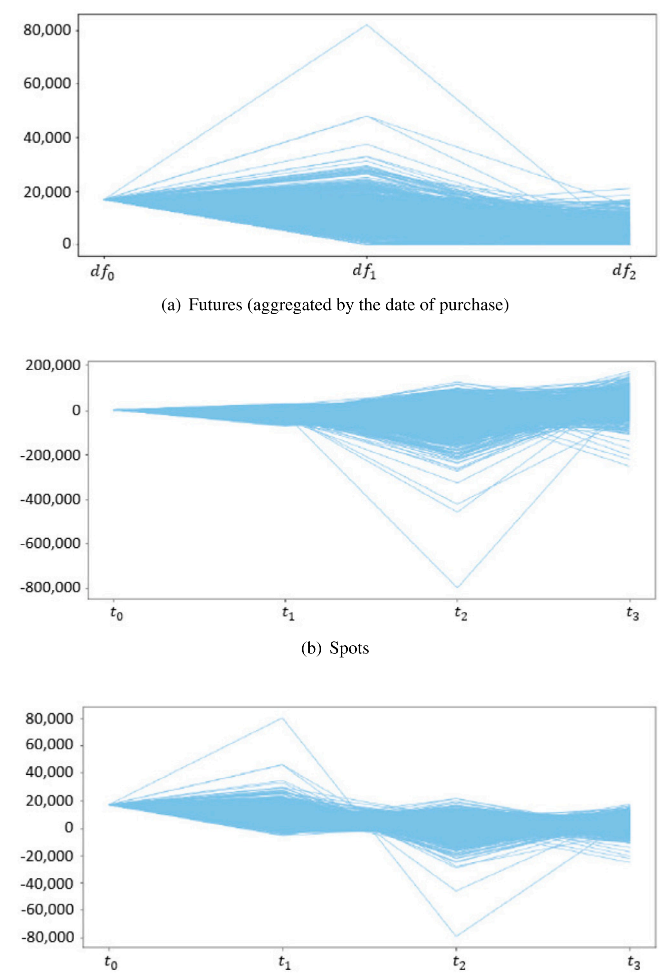
Fig. 2. Emissions management under the settings S1–S3.

a SSD between the settings, we got a positive answer: S1 second-order stochastically dominates both S2 and S3.

To summarize, the setting S1 including both the types of derivatives (futures and options) as well as banking is superior to the other two settings. In other words the carbon management including derivatives optimized using our model was clearly useful for the company.

As for the optimal mixture of the risk management tools, i.e., banking, futures, and options, the results showed that it is optimal to use only two tools, see Fig. 2, showing the average amount of individual tools in the portfolio. First, the company used futures on EUA and bought them each year. Second, the purchased allowances were used in the following years, so the company used banking of allowances. These results are fully in line with Zapletal et al. (2019). The options on EUA were not used at all, which confirmed the results of the preliminary analysis provided in Section 4. Note that a part of grandfathered allowances (approximately 20%) were sold in the second and third year; the other way around, the company bought almost 34,000 spots at the end of the last period as a possible corrective action caused by uncertainty during the last year.

Figs. 3a, 3b, and 3c show the optimal emissions management by scenarios. In couple of scenarios, the number of futures bought at the beginning of the second year deviated from the rest of the scenarios and was extremely high, see Fig. 3a. These deviations were balanced by selling spots at the beginning of the following year, see Fig. 3b. The total number of traded allowances had a slightly decreasing trend in time, see Fig. 3c. The optimal emissions covering by scenarios for settings S2 and S3 is depicted in Fig. 4. When the company could not transfer the allowances between time periods, it had to increase the number of bought spots year by year because the number of grandfathered allowances decreases each year, see Fig. 4b.



(a) Futures (aggregated by the date of purchase)

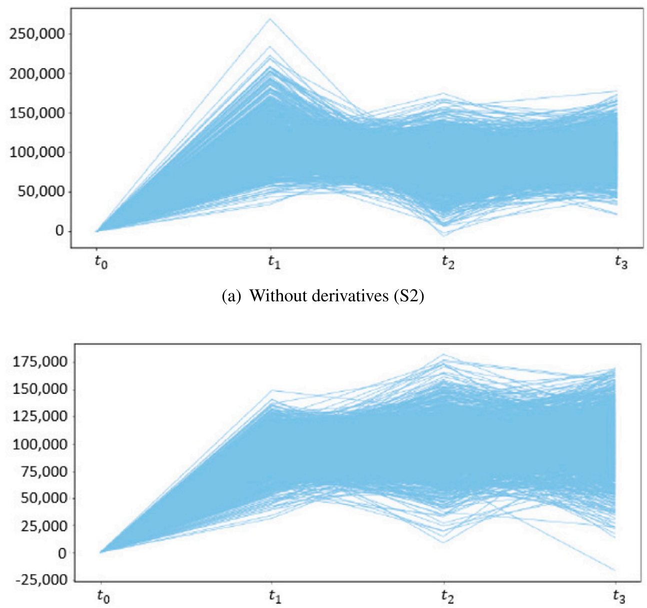
(b) Spots

(c) Spots and futures together

Fig. 3. Optimal emissions management by scenarios under S1 (the number of allowances traded in each stage).

7. Sensitivity analysis

In this section, we analyse the robustness of the model with respect to several factors. This analysis should support the reliability of the model and the obtained results and strengthen their applicability. Namely, we focus on the three following issues. First, the model was run again with various values of the input parameters λ and ω, which were originally set to the moderate values. Second, we examine whether options, which were found unfavourable under the actual market prices, can become applicable when their price decreases. Third, we tested how robust our results are with respect to the stochastic nature of the SDDP algorithm, namely the sample generated from the distribution of the random parameters.



(a) Without derivatives (S2)

(b) Without derivatives and banking (S3)

Fig. 4. Optimal emissions management by scenarios under S2 and S3 (the number of allowances traded in each stage).

7.1. Variation of λ and ω

In addition to their moderate values, we solved the problem also for more extreme values of λ and ω parameters, namely, 0.1 and 1 for λ, and 0 and 1 for ω), and all 9 resulting combinations of these parameters (together with the original moderate values). Table 3 shows the (mean) optimal profit under all these nine combinations, again under the settings S1–S3 (see their description in Table 2). For 8 out of 9 combinations of λ and ω parameters, the company generated higher profit under S1 in comparison with S3 (the values out of the standard error are written in bold). This means that it was optimal for the company to use derivatives and banking for (almost) all considered combinations of both parameters. Figs. 5–7 show optimal emissions management under all these combinations of parameters. Generally, the company used more futures with increasing risk-aversion λ. On the other hand, the more the company was informed about the future development of the random parameters (i.e., with increasing ω), the less spots had to be purchased within the corrective decision at the end of the last period. Figs. 5–7 also provide a comparison of the purchased and grandfathered allowances (the grandfathered amounts are denoted by the black, almost horizontal, lines). Unlike the results of the analysis by Zapletal et al. (2019), the company often bought futures for more than one period ahead, which is obviously a result of the fact that we used the nested risk measure here. The multi-period CVaR, used in Zapletal et al. (2019), aggregates risk in each stage separately, and, therefore, produces policies which are less risk-averse than policies produces by nested CVaR, which is used in this paper (Kozmík, 2013).

We also tested the settings S1–S3 for stochastic dominance for all 9 considered combinations of λ and ω described above. The results of the comparison of S1 with S3 differ from the baseline in some cases, see Table 5. In particular, only for 7 out of 9 combinations of λ and ω, it could be said that S1 almost first-order dominates S3, while, given $\lambda = 0.1, \omega = 0.5$ and $\lambda = 0.1, \omega = 1$, the dominance is violated in nearly half of the cases. This is also confirmed by the area criterion, see the values in parentheses. The reason for this could be that the risk-aversion level 0.1 is close to risk-neutrality, so the algorithm cares more of the expectation than of the shape of the distribution. This, however, does not explain the good dominance for $\omega = 0$. As for the SSD relationship, the results are displayed in Table 5 showing that S1 second-order stochastically dominates S3 only in 3 cases.

Table 3
Optimal profit under different levels of λ and ω and settings S1–S3 (the value of 100% corresponds with the optimum under $\lambda = 0.55$, $\omega = 0.5$ under S3)

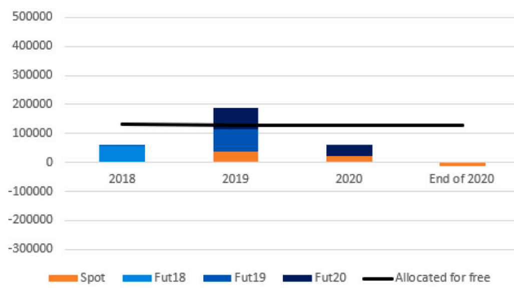
ω (→)	0		
λ (↓)	S1	S2	S3
0.1	**102.08%**	**100.88%**	99.67%
0.55	**101.82%**	99.97%	99.57%
1	**100.97%**	99.91%	100.05%
ω (→)	0.5		
λ (↓)	S1	S2	S3
0.1	100.4%	100.09%	100.46%
0.55	**102.59%**	99.86%	100%
1	**100.97%**	99.65%	99.93%
ω (→)	1		
λ (↓)	S1	S2	S3
0.1	**100.91%**	100.15%	100.49%
0.55	**101.81%**	99.65%	99.59%
1	**101.72%**	100.42%	99.96%

7.2. Variation of option prices

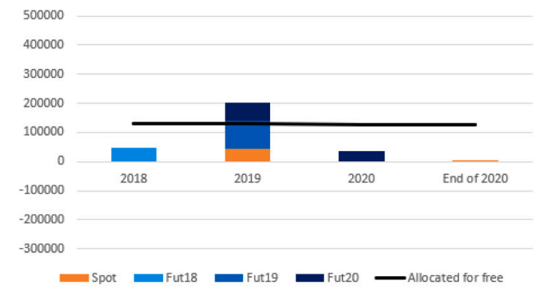
As the results of the model showed, the options on EUA were never bought by the company, which is in line with the preliminary analysis provided in Section 4. A natural question is whether they become applicable when their price decreases. To check this, we considered 3 cases with decreasing implied volatility of the spot, giving lower option prices — the case with 75%, 50% and 25% of the original implied volatility, respectively. Table 4 shows the option premia corresponding to the changed implied volatility levels, together with the results of the perturbed solutions.

For the case with 75% of the original volatility, the situation remains identical with the baseline in the sense that no options are still used. A change occurred only in two scenarios with the lowest volatility (and prices) – the company started to use all considered call options. However, maximal possible amounts of the options, many times exceeding the amounts needed for covering the emissions, are bought in order to sell the underlying spots later — this phenomenon is called arbitrage, i.e., unlimited buying of instruments, leading to unlimited profits (measured by the decision criterion). Namely, for these option prices, the arbitrage occurred in all decision stages except the first one (i.e., the one when the price of option is deterministic), and in 1% to 62% of scenarios. If the implied volatility further dropped to 25% of the original value, then the arbitrage would occur in all decision stages and more generated scenarios (from 63% to 99.9%), see Table 4 (the cases when the arbitrage occurs are written in bold). Summarized, the options would never be used for risk hedging: their realistic prices are so high that they are always outperformed by futures, and lower prices are impossible, because they would lead to arbitrage (which would push the prices back up).[3] These results are in line with the preliminary analysis provided in Section 4, and Šmíd et al. (2017).

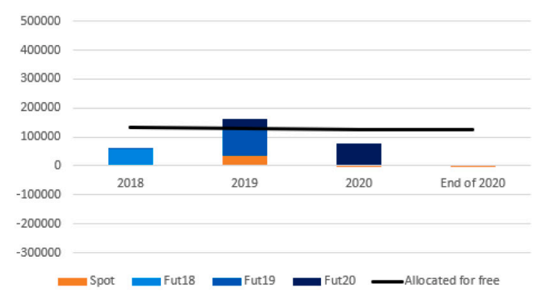
[3] In fact, the situation is slightly more complicated, as we are not speaking about the textbook arbitrage when we get a strictly positive income for zero costs, but about the *statistical arbitrage* when we get a strictly positive value of the decision criterion for zero cost. Clearly, it may happen, that the arbitrage exists given a special (unrealistic) case of a decision criterion, a good example being the sole expectation, which neglects all the risks. If, however, all the market participants had the same decision criterion, even the statistical arbitrage opportunities could not persist as they would be quickly exploited, which would move the prices to non-arbitrage levels.



(a) $\lambda = 0.1$, $\omega = 0$

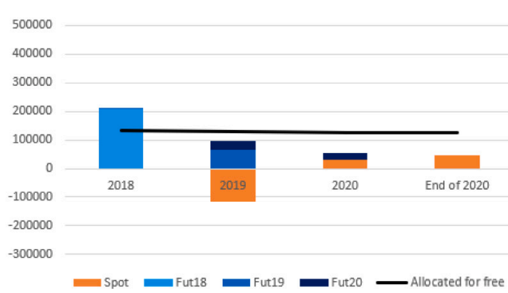


(b) $\lambda = 0.1$, $\omega = 0.5$

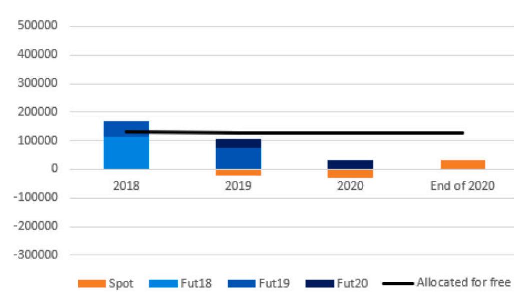


(c) $\lambda = 0.1$, $\omega = 1$

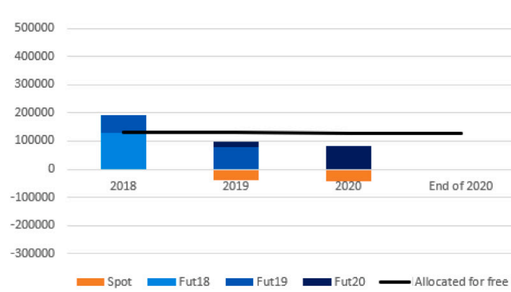
Fig. 5. Emissions management under $\lambda = 0.1$ and different values of knowledge of uncertainty ω.



(a) $\lambda = 0.55$, $\omega = 0$



(b) $\lambda = 0.55$, $\omega = 0.5$



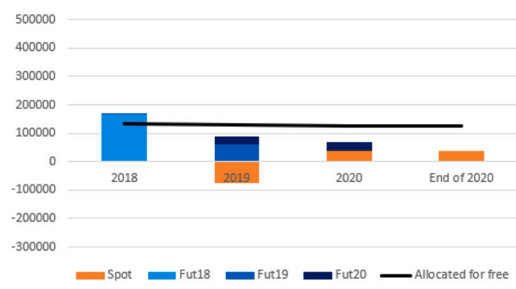
(c) $\lambda = 0.55$, $\omega = 1$

Fig. 6. Emissions management under $\lambda = 0.55$ and different values of knowledge of uncertainty ω.

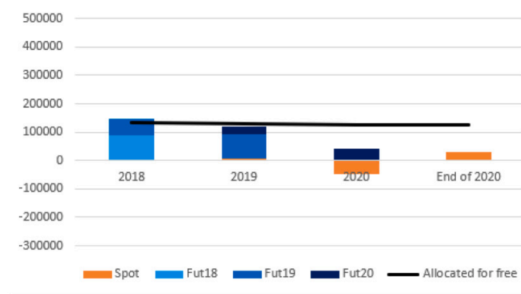
7.3. Robustness with respect to random solution

Finally, we tested to what extent our results depend on the stochasticity of the SDDP algorithm, whose solution is dependent on sampling from the distribution of the random parameters.[4] To address this issue, we ran the model with the identical settings, i.e., S1 setting (according Table 2) and $\lambda = 0.55, \omega = 0.5$, 10 times and analysed the differences between the solutions. The results are summarized in Table 6. It can be seen that the differences between the optimal values and the total profit

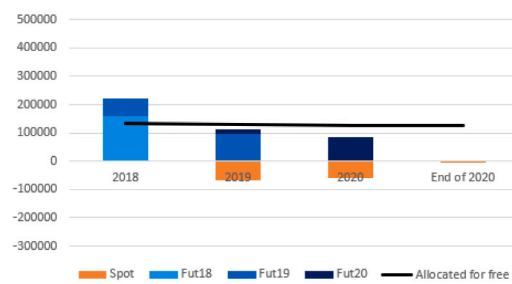
[4] Here, we do not mean the samples used for the evaluation of the profits, mentioned at the beginning of this section, but the large sample generated from the distribution of the random parameters, from which subsamples are repeatedly drawn during the solution.



(a) $\lambda = 1, \omega = 0$



(b) $\lambda = 1, \omega = 0.5$



(c) $\lambda = 1, \omega = 1$

Fig. 7. Emissions management under $\lambda = 1$ and different values of knowledge of uncertainty ω.

Table 4
Sensitivity analysis for options (Y = arbitrage occurs already at the first decision stage, N = arbitrage occurs later; percentage values show in how many scenarios the arbitrage occurs); arbitrage is denoted in bold.

Call options [EUR]	ϕ_1^1	ϕ_1^2	ϕ_2^1
1× volatility	2.023 (N)	1.409 (N)	2.868 (N, 0%)
0.75× volatility	1.519 (N)	0.895 (N)	2.192 (N, 0%)
0.5× volatility	1.004 (N)	0.412 (N)	**1.486 (N, 19.7%)**
0.25× volatility	**0.482 (Y)**	**0.053 (Y)**	**0.76 (Y, 64.2%)**
Call options [EUR]	ϕ_2^2	ϕ_3^1	ϕ_3^2
1× volatility	2.3 (N, 0%)	3.483 (N, 0%, 0%)	2.963 (N, 0%, 0%)
0.75× volatility	1.577 (N, 0%)	2.697 (N, 0%, 0%)	2.109 (N, 0%, 0%)
0.5× volatility	**0.849 (N, 43.8%)**	**1.859 (N, 14.5%, 40.6%)**	**1.218 (N, 1.1%, 61.7%)**
0.25× volatility	**0.197 (Y, 86.5%)**	**0.986 (Y, 63%, 64.9%)**	**0.359 (Y, 85.6%, 82.6%)**
Put options [EUR]	ψ_1^1	ψ_1^2	ψ_2^1
1× volatility	2.114 (N)	1.075 (N)	2.823 (N, 0%)
0.75× volatility	1.61 (N)	0.667 (N)	2.147 (N, 0%)
0.5× volatility	1.095 (N)	0.29 (N)	**1.441 (N, 12.1%)**
0.25× volatility	**0.573 (Y)**	**0.029 (Y)**	**0.715 (Y, 74.7%)**
Put options [EUR]	ψ_2^2	ψ_3^1	ψ_3^2
1× volatility	1.695 (N, 0%)	3.303 (N, 0%, 0%)	2.125 (N, 0%, 0%)
0.75× volatility	1.125 (N, 0%)	2.518 (N, 0%, 0%)	1.455 (N, 0%, 0%)
0.5× volatility	**0.561 (N, 11.1%)**	**1.68 (N, 20.4%, 43.8%)**	**0.768 (N, 26.8%, 56.1%)**
0.25× volatility	**0.097 (Y, 96.2%)**	**0.807 (Y, 77.4%, 77.3%)**	**0.16 (Y, 99.9%, 96.4%)**

of the company ranged within the standard error having low variability (variation coefficient −0.24% and 0.59%, respectively). However, the total number of used futures varied significantly (variation coefficient exceeds 20%). These results suggest that it is safe to use the SDDP algorithm, or, in other words, its user need not care about its stochasticity, because the profit achieved by the resulting policy, which is the most important quantity for the decision maker, comes out nearly the same. To explain the significant variation of the futures amounts, additional analysis would be necessary. One of the explanations could be that this phenomenon originates from situations, when the discounted spot price is close to the future price so buying any of them is nearly equivalent.

8. Conclusion

This paper investigated the impact of European emissions trading on industrial companies. Namely, we focused on the suitable risk management of emissions trading. Having learnt from existing studies, we constructed the most realistic model to date, taking both the dynamics and the stochasticity into account and involving all available derivatives on the emission allowances. Contrary to all existing models, we worked with continuous distributions rather than with scenario trees, so our model can realistically capture tale events, whose accurate modelling is indispensable for good risk management. To capture the risk aversion, we used a dynamic time-consistent risk measure — the multiperiod nested mean-CVaR.

Table 5
First-order stochastic dominance of the setting S1 over the setting S3: % of scenarios, for which the FSD holds (ratio between the area where the FSD does not hold and the total area between the CDFs); second-order stochastic dominance between the same settings (Y/N)

FSD ($\lambda \downarrow, \omega \rightarrow$)	0	0.5	1
0.1	95% (0.0196)	53% (0.5225)	54% (0.3406)
0.55	95% (0.0076)	97% (0.0301)	98% (0.0040)
1	96% (0.0258)	98% (0.0101)	97% (0.0083)
SSD ($\lambda \downarrow, \omega \rightarrow$)	0	0.5	1
0.1	N	N	N
0.55	Y	Y	N
1	N	N	Y

Table 6
Robustness of the results for 10 runs of the model (100% corresponds to the values obtained at the first run; OF = objective function (value); VC = coefficient of variation)

	1	2	3	4	5	
Optimal OF	100%	99.4%	99.7%	98.8%	99.1%	
Profit	100%	100.1%	100.1%	99.9%	99.6%	
Futures used	357896	304321	371728	313704	299610	
	6	7	8	9	10	VC
Optimal OF	98.5%	98.4%	99.4%	98.4%	99.65%	−0.24%
Profit	99.8%	99.8%	99.9%	99.3%	99.8%	0.59%
Futures used	172762	297561	276370	341957	403189	20.13%

The results showed that the company should use a combination of EUA futures and EUA spots to cover its CO_2 emissions by allowances, which confirms the conclusions made by Zapletal et al. (2019). On the other hand, options were never used at all.

To assess the impact of several parameters of the emissions trading system, we considered 3 different settings — the one where the banking and derivatives for allowances can be used, the one where the banking can be used, but without the derivatives, and the last one where neither banking, nor derivatives are available. We showed that, in terms of the average profit of the company, the EUA futures used together with banking are the best option for the company's emissions management. This combination outperformed the other two settings using the almost first-order stochastic dominance and second-order stochastic dominance for most of the combinations of input parameters. On the other hand, we showed that banking itself (used separately without the derivatives) cannot significantly reduce the costs of emissions trading, which contradicts the results by Šmíd et al. (2017). These results were also supported by the sensitivity analysis which confirmed a similar optimal policy for almost all combinations of risk-aversion and awareness parameters. Furthermore, the analysis showed that the unfavorability to use options persists even if their price decreases.

Our study provides a profound analysis of the usefulness of the EUA futures for companies, which was missing so far. The results of the empirical analysis using the data of a real-life company are even supported by the illustrative simplified theoretical analysis showing that the EUA options will never be used by companies unlike the EUA futures.

CRediT authorship contribution statement

František Zapletal: Model development, Sensitivity analysis, Communication with the managers of the modelled company, Writing, Reviewing. **Martin Šmíd:** Model development, Programming codes, Interpretation, Editing, Reviewing. **Václav Kozmík:** Model solution, Reviewing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Table 7
List of used mathematical symbols (in the same order as they appear in the text)

Notation	Description
Y_t	Amount of emissions produced by the company at time t
T	Number of considered time periods
r_t	Number of free allowances at time t
K_i	Strike price of the call option
L_i	Strike price of the put option
κ	Number of considered strike prices
s_t	Number of spots at time t
f_t^τ	Number of futures with maturity τ at time t
$\phi_t^{\tau,i}$	Number of call options with maturity τ and strike price K_i at time t
$\psi_t^{\tau,i}$	Number of put options with maturity τ and strike price L_i at time t
e_t	Remaining allowances after covering the emissions at time t
z_t	Income of the company at time t
P_t	Price of spot at time t
$B_t^{\tau,i}$	Price of the call option with maturity τ and strike price K_i at time t
$C_t^{\tau,i}$	Price of the put option with maturity τ and strike price L_i at time t
X_t	Profit of the company from production at time t
Q_t^τ	Price of the future with maturity τ at time t
ϱ	Discount factor
λ	Risk-aversion coefficient
α	Level of CVaR
$\mathcal{F}_t$	Information available at time t
σ	Standard deviation
ω	Awareness — level of knowledge of randomness

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Appendix A. List of used mathematical symbols

See Table 7.

Appendix B. Model for production income and emissions

This Appendix describes the construction of our model for yearly profits X and yearly emissions Y. For the latter, historical monthly emission amounts of the modelled company were available for years 2014–2016. As for the profits, however, only data from a single time instant in 2014 were available, i.e., no information on dynamics of the profits was at our disposal. Thus, to construct the model, we faced two challenges: to model the dynamics of the profits and to transform the monthly model into the yearly one.

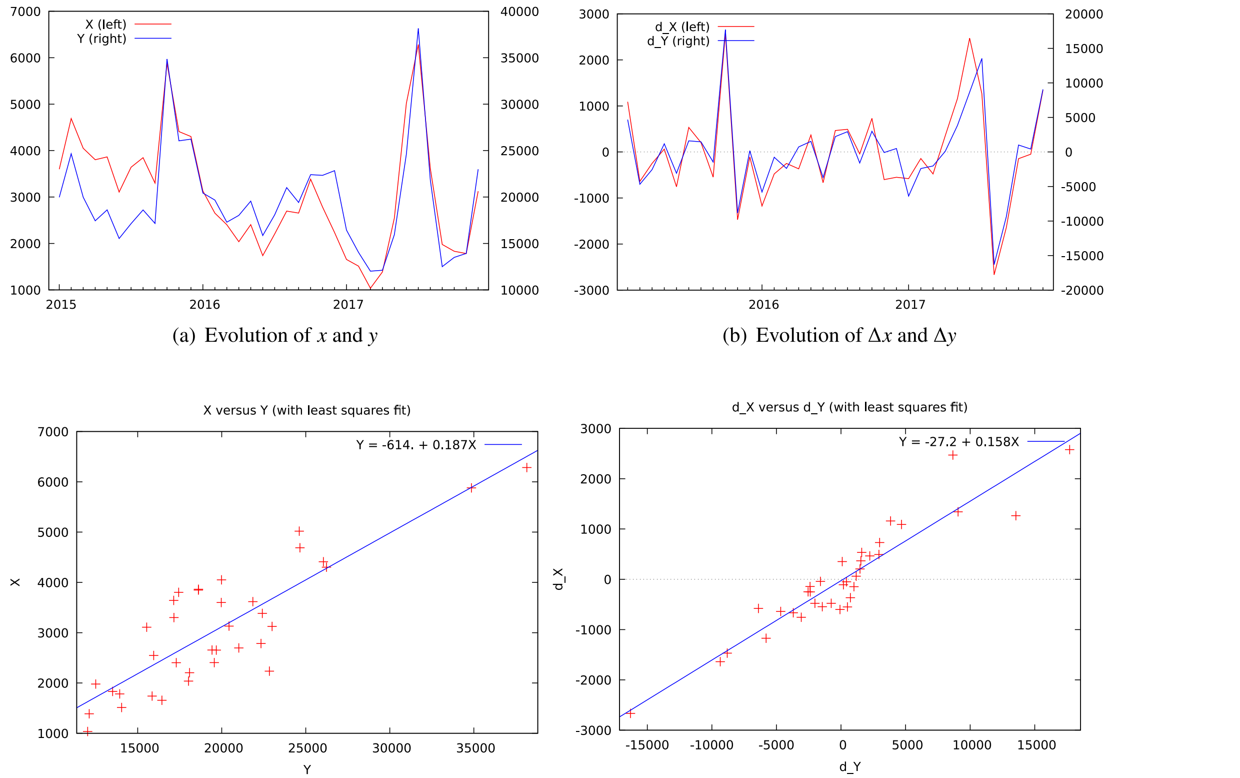
We start with the profit, computed as a difference of the selling price and the production costs.

According to an expert in steel industry, the variable costs consist mainly of three inputs: costs of coal, scrap and iron ore. The expertise showed that these three inputs represent approximately 75% of variable costs of a steel company, and each of these three inputs is equally important, i.e., represents 25% of the total variable costs. We assume that the remaining 25% of costs do not change in time (this part is assumed to be fixed). The resulting variable costs of the ith product at time t ($c_{i,t}$) can be approximated by the initial value and the input prices as:

$$c_{i,t} = c_{i,0}\left(0.25\frac{C_t^O}{C_0^O} + 0.25\frac{C_t^S}{C_0^S} + 0.25\frac{C_t^I}{C_0^I} + 0.25\right), \qquad j = O, S, I, \tag{12}$$

where 0 is the time of the last observation, C_t^j stands for the monthly observed unit price of the jth input at time t, and O, S, I stand for coal, scrap and iron ore, respectively. The historical data on prices of the inputs were taken from the (paid) Czech Steel Union's database.

As for the selling prices, according to the mentioned expert, the influence of European steel companies on the market prices is very weak, i.e., they are rather price-takers. We thus construct the selling



(a) Evolution of x and y (b) Evolution of Δx and Δy

(c) Crossplot of x and y (d) Crossplot of Δx and Δy

Fig. 8. Analysis of x and y.

prices using the monthly values of the price indices of the three main groups of steel products — semiproducts (E), flat products (F) and long products (L); the prices of each product are then constructed from its initial (known) value and the relative dynamics of the index of the group the product belongs to. In particular, in our model, the selling price of the ith product at the time t ($\pi_{i,t}$) is

$$\pi_{i,t} = \pi_{i,0} \cdot \frac{\Pi_{j_i,t}}{\Pi_{j_i,0}}, \tag{13}$$

where 0 is the time of the last observation, $j_i \in \{E, F, L\}$ is the group of the ith product, $\Pi_{j,t}$, stands for the selling price of the jth product group on the (European) market (again, the historical data on the selling prices were taken from the (paid) Czech Steel Union's database).

The resulting monthly profit (contribution margin) for the ith product at time t is then given as:

$$x_{i,t} = \pi_{i,t} - c_{i,t}, \qquad i = 1, \dots, 4. \tag{14}$$

Further, denote y_t as the monthly emissions of the company. Next we construct the joint monthly model of x and y in relation to (monthly) prices p_t of the EUA spots.

As all the correlations of any of x_t, y_t with any of p_t, p_{t-1} and correlations of any of $\Delta x_t, \Delta y_t$ with any of Δp_t, Δp_{t-1} are insignificant, we model x, y alone, independently of p. The time series plots and xy-plots of processes x_t, y_t and the processes of their first differences can be seen in Fig. 8:

It can be clearly seen that the values of x_t and y_t "go along" as well as their first differences, so it is worth to model their evolution jointly. As the ADF tests rejected unit root hypothesis for both the series, we chose VAR model with the single lag to fit their time evolution and estimated it as

$$z_t = C + Az_{t-1} + \varepsilon_t, \quad z_t = \begin{bmatrix} x_t \\ y_t \end{bmatrix}, \quad C = \begin{bmatrix} 2306.45 \\ 12852.2 \end{bmatrix},$$
$$A = \begin{bmatrix} 1.26237 & -0.158943 \\ 2.24683 & 0 \end{bmatrix} \tag{15}$$

where $\operatorname{stdev}(\varepsilon_1^1) = 809.6647$, $\operatorname{stdev}(\varepsilon_1^2) = 4882.004$ and $\operatorname{corr}(\varepsilon_1^1, \varepsilon_1^2) = 0.907$.

Finally, we derive the yearly model. Denote

$$Z_s = \begin{bmatrix} X_s \\ Y_s \end{bmatrix} \qquad X_s = \sum_{\tau=12(s-1)}^{12s} x_\tau, \quad Y_s = \sum_{\tau=12(s-1)}^{12s} y_\tau$$

the process of yearly sums.

Lemma 1.

$$Z_s | Z_{s-1}, \dots, Z_1 = \mathcal{N}\left(\mu_s, V_s\right)$$

where

$$\mu_s = D + E\mathbb{E}(z_{12(s-1)} | Z_{s-1}, \dots, Z_1),$$
$$D = \left(\sum_{k=0}^{11} (12-k)A^k\right) C, \quad E = \sum_{k=1}^{12} A^k,$$
$$V_s = E\operatorname{var}(z_{12(s-1)} | Z_{s-1}, \dots, Z_1)E' + W,$$
$$W = \sum_{i=0}^{11} F_i \operatorname{var}(\varepsilon_1) F_i', \quad F_k = \sum_{i=0}^{k} A^i.$$

Proof. For any $t > 1$ and $k > 1$, we have,

$$z_{t+1} = C + Az_t + \varepsilon_{t+1}$$

$$z_{t+2} = (I + A)C + A^2 z_t + \varepsilon_{t+2} + A\varepsilon_{t+1}$$

$$\dots$$

$$z_{t+k} = (I + \dots + A^{k-1})C + A^k z_t + \varepsilon_{t+k} + \dots + A^{k-1}\varepsilon_{t+1}.$$

Summing this, we get

$$Z_s = D + Ez_{12(s-1)} + \eta_s, \qquad \eta_s = \sum_{k=0}^{11} F_k \epsilon_{12s-k}$$

Clearly, $(Z_1, \dots, Z_s)$ is regular Gaussian, so $Z_s|Z_{s-1}, \dots, Z_1$ is Gaussian with

$$\begin{aligned}\mathbb{E}(Z_s|Z_{s-1}, \dots, Z_1) &= \mathbb{E}(D + Ez_{(s-1)12} + \eta_s|Z_{s-1}, \dots, Z_1) = \\ &= D + E\mathbb{E}(z_{(s-1)12}|Z_{s-1}, \dots, Z_1),\end{aligned}$$

$$\begin{aligned}\mathrm{var}(Z_s|Z_{s-1}, \dots, Z_1) &= \mathrm{var}(D + Ez_{(s-1)12} + \eta_s|Z_{s-1}, \dots, Z_1) = \\ &= \mathrm{var}(\eta_s) + E\mathrm{var}(z_{(s-1)12}|Z_{s-1}, \dots, Z_1)E'\end{aligned}$$

(note that $\eta_s \perp\!\!\!\perp (Z_1, \dots, Z_{s-1})$). Finally, as ε_t are i.i.d, we get $\mathrm{var}(\eta_s) = \sum_{k=0}^{11} F_k \mathrm{var}(\varepsilon_1) F_k'$.

In our case, the matrices come out as

$$D = \begin{bmatrix} 37,847.23 \\ 232,561.57 \end{bmatrix}, \qquad E = \begin{bmatrix} 8.37724 & -1.45331 \\ 20.54408 & -3.16534 \end{bmatrix},$$

$$W = w' \begin{bmatrix} 1 & 0.7369 \\ 0.7369 & 1 \end{bmatrix} w, \qquad w = \begin{bmatrix} 7,967.99 \\ 25,499.00 \end{bmatrix}.$$

The parameters of the yearly model may be computed by Lemma 1. However, the computation of the conditional expectation and the conditional variance of $z_{12(s-1)}$ is complicated (yet possible — by the formula for the conditional distribution of a Gaussian subvector). Thus, as the influence of these term is not great, we approximate $\mathbb{E}(z_{12(s-1)}|Z_{s-1}, \dots, Z_1) \doteq \mathbb{E}(\frac{Z_{s-1}}{12}|Z_{s-1}, \dots, Z_1) = \frac{Z_{s-1}}{12}$, $\mathrm{var}(z_{12(s-1)}|Z_{s-1}, \dots, Z_1) \doteq \mathrm{var}(\frac{Z_{s-1}}{12}|Z_{s-1}, \dots, Z_1) = 0$ to get

$$Z_s = D + \frac{1}{12} E Z_{s-1} + e_s, \qquad e_s \sim \mathcal{N}(0, W).$$

By imposing, we get (9). As the initial values (corresponding to the beginning of 2018), we take the linear forecast of $Z_0 = \sum_{k=1}^{12} z_{t+k}$ in the monthly model, where t corresponds to the beginning of 2017.

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