Small gain theorem for systems described by quasilinear parabolic equations

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Stability of interconnection of two or several dynamical systems is a crucial property that needs to be satisfied. The small gain theorem has been recognized as an effective tool for guaranteeing stability of interconnection of dynamical systems, even for systems with time delays.
In this contribution, the small gain theorem for connection of systems described by quasilinear parabolic equations is investigated. Conditions guaranteeing Lyapunov stability for the interconnenction of two such systems are derived. This is achieved by introducing a Lyapunov function defined on a suitable Sobolev space. Attention is also paid to time-delay systems. Here, the stability of the interconnection of systems is demonstrated using a generalization of the Lyapunov-Krasovskii od Lyapunov-Razumikhin fucntionals to systems, again defined on a Sobolev space. The results are illustrated by numerical simulations.

Numerical stability of block classical Gram–Schmidt

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The block version of the classical Gram–Schmidt (BCGS) method is often employed to efficiently compute orthogonal bases for Krylov subspace methods and eigenvalue solvers, but a rigorous proof of its stability behavior has not yet been established. It is shown that the usual implementation of BCGS can lose orthogonality at a rate worse than $O(\epsilon)\kappa^2(X)$, where $\kappa(X)$ is the condition number of the input matrix X and ϵ is the unit roundoff. A useful intermediate quantity denoted as the Cholesky residual is given special attention and, along with a block generalization of the Pythagorean theorem, this quantity is used to develop more stable variants of BCGS. These variants are proven to have $O(\epsilon)\kappa^2(X)$ loss of orthogonality with relatively relaxed conditions on the intrablock orthogonalization routine satisfied by the most commonly used algorithms. A variety of numerical examples illustrate the theoretical bounds.