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Nonmonotonic-Based Congestion Control Schemes for a Delayed Nonlinear Network

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Abstract

In this paper, buffer dynamic modeling for wireless sensor networks as a highly nonlinear system is accomplished in discrete time, different subsystems are achieved based on delay, and the overall model is gained by blending them. According to nonlinear dynamics point of view, considering delay in the analysis of congestion control schemes is of paramount importance. In this paper, an adaptive back-off interval selection works with the proposed robust controller. Based on queue utilization and channel estimation algorithm, congestion is detected and a suitable rate is selected by adaptive back-off interval selection. An augmented form of our proposed system is utilized for controller synthesis. A new approach is proposed for controller synthesis based on non-quadratic and common quadratic Lyapunov candidates where the former is generalized to be more relaxed. Also, the monotonicity requirement of Lyapunov's theorem is relaxed. The closed-loop systems are globally asymptotically stable in case of delay changes resulted from queue size changes. Extended simulation results confirm the effectiveness of our proposed schemes.

Keywords Wireless sensor networks (WSNs) · Congestion control · Controller synthesis · Non-quadratic Lyapunov stability · Linear matrix inequality (LMI) · Globally asymptotically stable (GAS)

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1 Introduction

A WSN is a sensor node collection which is distributed and organized in a network to monitor different environmental or physical conditions as pressure, vibration, sound, temperature and motion to estimate the area or the monitored system state. There exist several unique characteristics in WSNs which make them distinguished from traditional networks. First, there exist resource limitations in WSN nodes in communication bandwidth, memory, energy source and processing power. Second, multi-hop tree topology is used where the sink is located at the tree's root. Third, sensor nodes have peculiar traffic characteristics [25].

The complex nature of networks ends in a variety of research issues. Acoustic source localization in WSNs is studied in [23], secure data collection using chaotic compressed sensing in WSNs is presented in [17], distributed parameter estimation for univariate generalized distribution in sensor networks is addressed in [16], and the issue of resource allocation and cooperative spectrum sensing optimization in cognitive radio sensor networks with multi-channels is established in [32]. Also, special attention has been paid to the effect of noise on WSNs [5, 11, 33], bias-constrained optimal fusion filtering considering correlated noise in WSNs is studied in [33], soft fault detection for interval type-2 fuzzy-model-based stochastic systems in WSNs is presented in [11] and H∞ filtering for switched stochastic delayed systems with fading measurements in sensor networks is addressed in [5].

Congestion is a critical issue in wireless networks. It occurs when either input load exceeds available capacity which ends in buffer overflow in node, or wireless channel is commonly used by multiple nodes which ends in collision, or when link bandwidth is reduced due to fading channels [30]. Due to the dominant role of WSNs in recent technologies, it is necessary to design more efficient congestion control algorithms. Recently, a large and growing number of schemes are presented to address congestion control problem in WSNs [2, 4, 6, 8, 9, 13–15, 18, 19, 21, 22, 24, 26, 28–30].

It should be highlighted that fading is quite probable in WSNs. It renders bandwidth reduction and ends in queue length increase and consequently delay increase. In this case, satisfactory performance is gained only if the resultant closed-loop system is stable, otherwise, performance degradation is achieved which ends in system instability. Fading is considered in decentralized predictive congestion control (DPCC) [30], robust decentralized adaptive non-quadratic congestion control scheme (RDANQCC) [9] and decentralized non-common quadratic Lyapunov-based congestion control (DNCQLCC) [8].

Finite transmission speeds and traffic congestion renders delay in networks. Based on nonlinear dynamics point of view, considering delay in congestion control schemes is of paramount importance. In this regard, congestion controllers which are robust to delay changes are designed in [8, 9]. Also, in order to consider delay in our system, buffer dynamic modeling for WSNs is accomplished in discrete time, different subsystems are achieved based on delay, and the overall model is gained by blending them [8, 9]. It is worth mentioning that unlike

[30] where the buffer occupancy error and the control effort are only considered at time instances n and $n-1$, respectively, they are considered at different time instances in [8, 9].

Stability analysis and controller synthesis of systems have been comprehensively addressed during the last years. In this regard, Lyapunov-based approaches are of special concern [1, 10, 27]. In order to achieve less conservative stability conditions, the monotonicity requirement of Lyapunov's theorem is relaxed to enlarge the class of functions which can provide stability [7]. Unlike monotonic Lyapunov theorems that require Lyapunov function decrease in every single step, in nonmonotonic point of view, the Lyapunov function decreases every few steps; although, it can be locally increased. So, the Lyapunov function is not necessarily descent in every step and the enlargement of stability region is obtained which renders conservativeness reduction and better performance. It is worth mentioning that unlike [7] where a fixed Lyapunov function is considered for all subsystems, different Lyapunov functions are utilized for different subsystems in our proposed approach.

In this paper, an augmented form of our proposed system is utilized for controller synthesis to reduce conservativeness. The controller synthesis is accomplished based on non-quadratic Lyapunov candidates. The proposed scheme with non-quadratic Lyapunov candidate is generalized to be more relaxed, and the enlargement of stability region is obtained which renders conservativeness reduction. Also, the nonmonotonic point of view is generalized and used in this paper. In this regard, different Lyapunov functions are utilized for different subsystems. In this paper, a controller is designed in each approach and for each subsystem, and the overall controller for our proposed system is achieved by blending them. However, a controller is obtained to stabilize the whole system in [9]. Also, unlike [9] where the controller synthesis results are expressed in terms of ILMIs, in this paper, they are expressed in terms of LMIs which are numerically feasible using commercially available software. Finally, a set of novel congestion control schemes is proposed for WSNs where the resultant closed-loop systems are GAS in case of queue length changes and the consequent delay changes.

The reminder of this article is as follows: First, some preliminaries are stated. Next, we address the main results, followed by our approach performance which is assessed through simulation results, and finally we conclude our study.

2 Preliminaries

In this section, the buffer occupancy changes at a node, the adaptive and predictive controller in [30], our proposed model [9], the controller synthesis results in RDANQCC and DNCQLCC and the comparison of DPCC [30], RDANQCC [9], DNCQLCC [8] and the proposed schemes are briefly presented.

Notations: Through this paper $N, \mathfrak{R}, \mathfrak{R}^n, \mathfrak{R}^{n\times m}$ denote sets of natural numbers, real numbers, n component real vectors and $n \times m$ real matrices, respectively.

2.1 Buffer occupancy changes at a node

Based on [30], buffer occupancy changes at the ith node in terms of outgoing and incoming traffic are given as:

$$q_i(n+1) = sat_c[q_i(n) + Tu_i(n) - f_i(u_{i+1}(n)) + d(n)] \tag{1}$$

where sat_c is the saturation function showing the behavior of finite-size queue, $q_i(n)$ is the ith node buffer occupancy at time instant n, T is the measurement interval, $u_i(n)$ is the rate of regulated incoming traffic, $f_i(\cdot)$ is a dictated outgoing traffic by node located at the next hop which is disturbed by channel variations and $d(n)$ is the traffic disturbance which is unknown. In order to estimate the outgoing traffic, it is essential to calculate and propagate $u_i(n)$ as a feedback to the $(i-1)$th node.

Considering q_{id} as the desired buffer occupancy at the ith node, the buffer occupancy error $e_{bi}(n) = q_i(n) - q_{id}$ is as follows:

$$e_{bi}(n+1) = Sat_c[e_{bi}(n) + Tu_i(n) - f_i(u_{i+1}(n)) + d(n)] \tag{2}$$

2.2 The adaptive and predictive controller in DPCC

If the outgoing traffic $f_i(.)$ is unknown, the traffic rate input is as follows [30]:

$$u_i(n) = Sat_p\left(\frac{\hat{f}_i(u_{i+1}(n)) + (\kappa_{bv} - 1)\, e_{bi}(n)}{T}\right) \tag{3}$$

where κ_{bv} is the gain parameter and $\hat{f}_i(u_{i+1}(n))$ is the estimate of $f_i(u_{i+1}(n))$. The error of buffer occupancy at time instant $n+1$ is:

$$e_{bi}(n+1) = Sat_p[k_{bv} e_{bi}(n) + \tilde{f}_i(u_{i+1}(n)) + d(n)] \tag{4}$$

where $\tilde{f}_i(u_{i+1}(n)) = f_i(u_{i+1}(n)) - \hat{f}_i(u_{i+1}(n))$ is the estimation error of the outgoing traffic.

If there is no estimation error, the traffic estimate is defined as [30]:

$$\hat{f}_i(u_{i+1}(n)) = \hat{\theta}_i(n) f_i(n-1) \tag{5}$$

where $\hat{\theta}_i(n)$ is the actual vector of traffic parameter, $\hat{f}_i(u_{i+1}(n))$ is the estimation of unknown outgoing traffic and $f_i(n-1)$ is the past value of outgoing traffic. If parameter θ_i is updated as:

$$\hat{\theta}_i(n+1) = \hat{\theta}_i(n) + \lambda u_i(n) e_{fi}(n+1) \tag{6}$$

provided $\lambda \left\| u_i(n) \right\|^2 < 1$ and $\kappa_{fv}\ \max < 1/\sqrt{\delta}$ where λ is the adaptation gain, $\kappa_{fv}\ \max$ the maximum singular value of κ_{fv}, $\delta = 1/[1 - \lambda \left\| u_i(n) \right\|^2]$ and $e_{fi}(n) = f_i(n) - \hat{f}_i(n)$, then the mean estimation error of θ_i along with queue utilization mean error converges to zero asymptotically [30].

Remark 1 It is important to note that (3) ignores fading channels and detects congestion just by monitoring the buffer occupancy. Since the transmitted packets are not decoded in fading channels, they will be dropped at the receiver and end in retransmission increase. So, in order to mitigate congestion due to fading channels, the rate from (3) has to be decreased when the transmission power calculated by the DPC scheme [31] exceeds the maximum threshold. This is accomplished by an adaptive technique which selects virtual rates and back-off intervals for a given node. Back-off interval selection for nodes plays an important role to decide which node can access the channel. The back-off interval at the ith node is defined as BO_i, and it is denoted that $\mathrm{VR}_i = 1/\mathrm{BO}_i$, where VR_i is the ith node virtual rate. The ith node actual rate is as follows:

$$R_i(t) = \frac{B(t) \cdot \mathrm{VR}_i(t)}{\sum_{l \in S_i} \mathrm{VR}_l(t)} = \frac{B(t) \cdot \mathrm{VR}_i(t)}{TV\,R_i(t)} \tag{7}$$

where $B(t)$ is channel bandwidth, VR_i is the ith node virtual rate and $\mathrm{TVR}_i(t)$ is the sum of all virtual rates of all neighbors S_i [30].

Thereafter, (7) is differentiated and transformed to discrete-time domain using Euler's formula. The back-off interval for each node and the estimation of variation of back-off intervals of flows at the neighboring nodes from time instant n to $n+1$ is obtained based on the adaptive back-off interval selection established in [30].

In this regard, the back-off interval of the node at time instant $n+1$ is as follows:

$$\mathrm{BO}_i(n+1) = {}^{1}\!/_{v_i(n)} = {}^{1}\!/\mathrm{VR}_i(n+1) \tag{8}$$

where $v_i(n)$ is selected as [30]:

$$v_i(n) = \frac{f_i(n) - R_i(n)\hat{\alpha}_i(n) - \kappa_v e_i(n)}{\xi_i(n)} \tag{9}$$

where $\xi_i(n)$ is the ratio between the actual and used virtual rate at time instant n,k_v is the feedback gain parameter, $e_i(n) = R_i(n) - f_i(n)$ is the throughput error and the variable $\hat{\alpha}_i(n)$ is the estimate of variation of back-off intervals of flows at the neighboring nodes from time instant n to $n+1$ which is updated as [30]:

$$\hat{\alpha}_i(n+1) = \hat{\alpha}_i(n) + \sigma R_i(n) e_i(n+1) \tag{10}$$

provided

$$1)\sigma \left\|R_i(n)\right\|^2 < 1 \quad \text{and} \quad 2)k_{v\max} < 1/\sqrt{\delta} \tag{11}$$

where $k_{v\max}$ is the maximum singular value of k_v, σ is the adaptation gain and $\delta = {}^{1}\!\big/\left(1 - \sigma\left\|R_i(n)\right\|^2\right)$

Remark 2 Since our scheme is decentralized, model formulation and controller synthesis results are presented for each node. So, for the sake of simplicity, the index "i" which is used to show the ith node is omitted in our scheme.

2.3 The proposed system

The proposed model is as follows [9]:

$$\begin{aligned} x(n+1) &= \sum_{j=1}^{r} \beta_j(n)(A_j x(n) + B_j u(n)) \\ \sum_{j=1}^{r} \beta_j(n) &= 1, \beta_j(n) \in \{0, 1\} \end{aligned} \tag{12}$$

where $x(n) = \left[e_b(n)\ e_b(n-1)\ \ldots\ e_b(n-o)\ u(n-1)\ \ldots\ u(n-p)\ s(n) \right]^{\mathrm{T}} \in \mathfrak{R}^z$ includes buffer occupancy errors $e_b(n-o)$(pkts) (Eq. 2), regulated incoming traffic rate (control effort) $u(n-p)$ at different time instances(pkt/sec) and the integrator $S(n) = \sum_{m=0}^{n} e_b(m) \in \mathfrak{R}$(pkt). Also, $o, p \in N$ are the number of states where $o+1$ and p are the number of buffer occupancy error and control effort states, respectively, $A_j \in \left\{ A_1\ A_2\ \ldots\ A_r \right\} \in \mathfrak{R}^{z \times z}$ and $B_j \in \left\{ B_1\ B_2\ \ldots\ B_r \right\} \in \mathfrak{R}^z$ are known constant matrices for system description showing the jth subsystem, $z \in N$ is the number of state variables, $r \in N$ is the number of subsystems obtained due to queue size increase which renders delay increase in system and finally, $\beta_j(n)$ indicates which subsystem is chosen based on delay ($\sum_{j=1}^{r} \beta_j(n) = 1$ and $\beta_j(n)$ can be either 0 or 1).

2.4 The controller synthesis results in RDANQCC and DNCQLCC

The controller synthesis results in RDANQCC and DNCQLCC are as follows:

Theorem 1 [30] *System* (12) *with* $u(n) = kx(n)$ *as the control effort is GAS if there exists a set of symmetric positive definite (PD) matrices* P_i *and* P_m *for every* $i, j, m \in L$ *(*$L = \left\{ 1\ 2\ \ldots\ r \right\}$*) such that (13) is satisfied:*

$$\begin{bmatrix} P_m & (A_j + B_j k)^{\mathrm{T}} \\ (A_j + B_j k) & P_i^{-1} \end{bmatrix} > 0 \tag{13}$$

Remark 3 The control problem in DPCC [30] and RDANQCC [9] is solved offline; however, outgoing estimation is accomplished online.

Remark 4 Although calculating k in Theorem 1 is complex, but it is once done offline and a fixed vector k is achieved, afterward our fixed proposed controller gain k is applied to WSN, and it is not changing in WSN when the parameters of WSN are changing. So, in case of applying our controller in WSN, there is no complexity and using a fixed k is neither time consuming nor needs much processing and congestion control is easily accomplished in WSN.

Theorem 2 [8] *System* (12) *with* $u(n) = \sum_{j=1}^{r} \beta_j N_j (\sum_{i=1}^{r} \beta_i G_i)^{-1} x(n)$ *as the control effort is GAS if there exists a set of symmetric positive definite (PD) matrices* $P_j \in \Re^{z \times z}$ *and* $G_i \in \Re^{z \times z}$ *and* $N_j \in \Re^{1 \times z}$ *for every* $i, j, m, c \in L$ *(*$L = \{ 1 \ 2 \ \ldots \ r \}$*) such that* (14) *is satisfied:*

$$\begin{bmatrix} P_j & (A_j G_i + B_j N_j)^{\mathrm{T}} \\ (A_j G_i + B_j N_j) & G_m + G_m^{\mathrm{T}} - P_c \end{bmatrix} > 0 \tag{14}$$

and the controller gains are $F_{ji} = N_j G_i^{*-1}$ *where* $G_i^* = \sum_{i=1}^{r} \beta_i G_i$.

2.5 Comparison of DPCC [30], RDANQCC [9], DNCQLCC [8] and the proposed schemes

- Unlike [2, 4, 6, 13–15, 18, 19, 21, 22, 24, 26, 28, 29], congestion resulted from fading channels in dynamic environments is addressed in DPCC [30], RDANQCC [9], DNCQLCC [8] and this paper.
- In DPCC [30] and our previous schemes [8, 9] and this paper, an adaptive back-off interval selection works with the proposed robust controller. Based on queue utilization and channel estimation algorithm, congestion is detected and a suitable rate is selected by adaptive back-off interval selection.
- Unlike DPCC [30] and previous schemes [2, 4, 6, 13–15, 18, 19, 21, 22, 24, 26, 28, 29], robustness to delay changes resulted from queue size changes is considered in [8, 9] and this paper. Since our model includes delay, it is more realistic comparing with [30] and unlike [30], controller synthesis is presented in case of delayed systems.
- Unlike DPCC [30] and [2, 4, 6, 13–15, 18, 19, 21, 22, 24, 26, 28, 29], buffer dynamic modeling for WSNs is accomplished in discrete time, different subsystems are achieved based on delay, and the overall model is gained by blending them in [8, 9] and this paper.
- Unlike DPCC [30] where the buffer occupancy error and the control effort are only considered at time instances n and $n-1$, respectively, they are considered at different time instances in [8, 9] and this paper.
- The gain parameter k_{bv} is an important factor in the design of controller in DPCC [30], and DPCC is highly dependent on it. However, unlike [30], our controllers in [8, 9] and this paper are not dependent on k_{bv}.
- Unlike [9] where a controller is obtained to stabilize the whole system, in [8] and this paper, in each approach and for each subsystem, a controller is designed and the overall controller for our proposed system is achieved by blending them.
- Unlike [9] where the results of controller synthesis are presented in terms of ILMIs, in [8] and this paper, LMIs are used for controller synthesis which are numerically feasible using commercially available software.

- Unlike DPCC [30], previous schemes [2, 4, 6, 13–15, 18, 19, 21, 22, 24, 26, 28, 29] and our previous schemes [8, 9], in this paper, an augmented form of our proposed system is utilized for controller synthesis.
- Unlike DPCC [30], previous schemes [2, 4, 6, 13–15, 18, 19, 21, 22, 24, 26, 28, 29], and our previous schemes [8, 9], in this paper, in order to achieve less conservative stability conditions, the Lyapunov theorem monotonicity requirement is relaxed to enlarge the class of functions which can provide stability [7].
- The controller synthesis is presented based on different Lyapunov candidates.
- The control signals in DPCC [30], our previous schemes [8, 9] and this paper are different from each other.
- Unlike DPCC [30] and previous schemes [2, 4, 6, 13–15, 18, 19, 21, 22, 24, 26, 28, 29], our congestion control strategies in [8, 9] and this paper are presented for WSNs and in case of delay changes resulted from queue size changes, the closed-loop systems are GAS.

3 Main result

In this section, an augmented form of our proposed system is utilized for controller synthesis to reduce conservativeness. Afterward, a new approach is proposed for controller synthesis of our WSN system, and the resultant congestion controller based on augmented decentralized non-common quadratic Lyapunov-based congestion control (ADNCQLCC) is achieved. Thereafter, for computation cost reduction, a novel approach is presented for controller synthesis of our WSN system, and a congestion controller based on augmented decentralized common quadratic Lyapunov-based congestion control (ADCQLCC) is gained. Finally, the proposed scheme with non-quadratic Lyapunov candidate is generalized to be more relaxed and the enlargement of stability region is obtained which renders conservativeness reduction, and a congestion controller based on generalized augmented decentralized non-common quadratic Lyapunov-based congestion control (GADNCQLCC) is presented. In this paper, the monotonicity requirement of Lyapunov's theorem is relaxed which renders enlargement of stability region and performance improvement. In each approach, a controller is designed for each subsystem, and the overall controller for our proposed system is achieved by blending them. The controller synthesis results are presented in terms of LMIs. Finally, a set of novel congestion control schemes is proposed for WSNs where the resultant closed-loop systems are GAS in case of queue length changes and the consequent delay changes.

Lemma 1 [20]: *If matrices C_m and S_m have appropriate dimensions and S_m is (PD), then*

$$C_m^{\mathrm{T}} S_m^{-1} C_m \geq C_m^{\mathrm{T}} + C_m - S_m \tag{15}$$

By defining the control effort as:

$$u(n) = \left[\sum_{j=1}^{r} \beta_j N_j\right] x(n) \tag{16}$$

One has the following result:

Theorem 3 *System* (12) *with the control effort* (16) *is GAS, if*

There exists a set of symmetric (PD) matrices $P_j \in \Re^{z\times z}$ *and* $\bar{P}_j \in \Re^{z\times z}$ *and matrices* $G_j \in \Re^{z\times z}$ *and* $F_j \in \Re^{1\times z}$ *for every* $i,j \in L$ ($L = \{1\ 2\ \ldots\ r\}$) *such that the following LMIs (17, 18) are satisfied:*

$$\begin{bmatrix} \bar{P}_j & (A_jG_j + B_jF_j)^{\mathrm{T}} \\ (A_jG_j + B_jF_j) & G_i^{\mathrm{T}} + G_i - P_i \end{bmatrix} > 0 \tag{17}$$

$$\begin{bmatrix} P_j & (A_jG_j + B_jF_j)^{\mathrm{T}} \\ (A_jG_j + B_jF_j) & G_i^{\mathrm{T}} + G_i - \bar{P}_i \end{bmatrix} > 0 \tag{18}$$

moreover, the controller gains are given by:

$$N_j = F_jG_j^{-1} \tag{19}$$

Proof First the existence of G_j^{-1} should be checked. If (Eq. 18) is hold true, we have:

$$\begin{bmatrix} 0 & I \end{bmatrix} \begin{bmatrix} P_i & (A_iG_i + B_iF_i)^{\mathrm{T}} \\ (A_iG_i + B_iF_i) & G_j^{\mathrm{T}} + G_j - \bar{P}_j \end{bmatrix} \begin{bmatrix} 0 \\ I \end{bmatrix} > 0 \quad i,j \in L$$

So $G_j^{\mathrm{T}} + G_j - \bar{P}_j > 0$ which can be written as $G_j^{\mathrm{T}} + G_j > \bar{P}_j$. Since $\bar{P}_j \in \Re^{z\times z}$ is considered symmetric (PD) matrix, it can be concluded that $G_j^{\mathrm{T}} + G_j > 0$.

If G_j is not invertible, there exists a non-zero $\Pi_j \in \Re^{1\times z}$ where $G_j\Pi_j = 0$ (Π_j is in G_j null space). Since $G_j^{\mathrm{T}} + G_j > 0$, so $\Pi_j^{\mathrm{T}}(G_j^{\mathrm{T}} + G_j)\Pi_j > 0$,which can be written as $\Pi_j^{\mathrm{T}}G_j^{\mathrm{T}}\Pi_j + \Pi_j^{\mathrm{T}}G_j\Pi_j > 0$ or $(G_j\Pi_j)^{\mathrm{T}}\Pi_j + \Pi_j^{\mathrm{T}}(G_j\Pi_j) > 0$, since $G_j\Pi_j = 0$, we have $0 + 0 > 0$ (incorrect), so there exists no Π_j where $G_j\Pi_j = 0$, and so G_j is invertible.

Based on (12) and the control effort (16) and defining $A_c(n) \triangleq \sum_{j=1}^{r} \beta_j(n)\,(A_j + B_jN_j)$, we have:

$$\begin{bmatrix} x(n+2) \\ x(n+1) \end{bmatrix} = \begin{bmatrix} 0 & A_c(n+1)A_c(n) \\ I & 0 \end{bmatrix} \begin{bmatrix} x(n+1) \\ x(n) \end{bmatrix} \tag{20}$$

(20) is an augmented form of (12) where both $x(n+1)$ and $x(n)$ are considered and $x(n+2)$ is presented based on $x(n)$. It is worth mentioning that our controller

synthesis results are presented based on (20) where (12) can be easily obtained. Consider the following Lyapunov candidate,

$$V(x(n)) = \begin{bmatrix} x(n+1) \\ x(n) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} P(n+1) & 0 \\ 0 & P(n) \end{bmatrix} \begin{bmatrix} x(n+1) \\ x(n) \end{bmatrix} \tag{21}$$

where

$$P(n) = \tilde{G}^{\mathrm{T}}(n)\tilde{P}(n)\tilde{G}(n) \tag{22}$$

and we have:

$$\tilde{G}(n) = \left(\sum_{j=1}^{r} \beta_j(n) G_j \right)^{-1} \tag{23}$$

$$\tilde{P}(n) = \sum_{j=1}^{r} (\beta_j(n) P_j) \tag{24}$$

The difference function is given as:

$$\begin{aligned} \Delta V &= V(x(n+1)) - V(x(n)) \\ &= \begin{bmatrix} x(n+2) \\ x(n+1) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} P(n+2) & 0 \\ 0 & P(n+1) \end{bmatrix} \begin{bmatrix} x(n+2) \\ x(n+1) \end{bmatrix} - \begin{bmatrix} x(n+1) \\ x(n) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} P(n+1) & 0 \\ 0 & P(n) \end{bmatrix} \begin{bmatrix} x(n+1) \\ x(n) \end{bmatrix} \end{aligned} \tag{25}$$

which in turn implies that:

$$\begin{aligned} \Delta V = \begin{bmatrix} x(n+1) \\ x(n) \end{bmatrix}^{\mathrm{T}} \Bigg(& \begin{bmatrix} 0 & A_c(n+1)A_c(n) \\ I & 0 \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} P(n+2) & 0 \\ 0 & P(n+1) \end{bmatrix} \begin{bmatrix} 0 & A_c(n+1)A_c(n) \\ I & 0 \end{bmatrix} \\ & - \begin{bmatrix} P(n+1) & 0 \\ 0 & P(n) \end{bmatrix} \Bigg) \begin{bmatrix} x(n+1) \\ x(n) \end{bmatrix} \end{aligned} \tag{26}$$

and can be rewritten as:

$$\Delta V = \begin{bmatrix} x(n+1) \\ x(n) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} 0 & 0 \\ 0 & A_c^{\mathrm{T}}(n)A_c^{\mathrm{T}}(n+1)P(n+2)A_c(n+1)A_c(n) - P(n) \end{bmatrix} \begin{bmatrix} x(n+1) \\ x(n) \end{bmatrix} \tag{27}$$

Considering $x(n+1) = A_c(n)\,x(n)$, (27) can be written as:

$$\Delta V = x^{\mathrm{T}}(n) \begin{bmatrix} A_c^{\mathrm{T}}(n) & I \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & A_c^{\mathrm{T}}(n)A_c^{\mathrm{T}}(n+1)P(n+2)A_c(n+1)A_c(n) - P(n) \end{bmatrix} \begin{bmatrix} A_c(n) \\ I \end{bmatrix} x(n) \tag{28}$$

In order to prove that the proposed system is GAS, it is sufficient to have

$$A_c^{\mathrm{T}}(n)A_c^{\mathrm{T}}(n+1)P(n+2)A_c(n+1)A_c(n) - P(n) < 0 \tag{29}$$

Substituting (22) in (29) yields:

$$A_c^{\mathrm{T}}(n)A_c^{\mathrm{T}}(n+1)\tilde{G}^{\mathrm{T}}(n+2)\tilde{P}(n+2)\tilde{G}(n+2)A_c(n+1)A_c(n) - \tilde{G}^{\mathrm{T}}(n)\tilde{P}(n)\tilde{G}(n) < 0 \tag{30}$$

Since the controller synthesis results achieved from (30) are not LMI conditions, the terms $\pm\, A_c^{\mathrm{T}}(n)\tilde{G}^{\mathrm{T}}(n+1)\hat{P}(n+1)\tilde{G}(n+1)A_c(n)$ with $\hat{P}(n+1) = \sum_{j=1}^{r}(\beta_j(n)\bar{P}_j)$ are added to (30), and we have:

$$\begin{aligned}&\left\{A_c^{\mathrm{T}}(n)A_c^{\mathrm{T}}(n+1)\tilde{G}^{\mathrm{T}}(n+2)\tilde{P}(n+2)\tilde{G}(n+2)A_c(n+1)A_c(n) - A_c^{\mathrm{T}}(n)\tilde{G}^{\mathrm{T}}(n+1)\hat{P}(n+1)\tilde{G}(n+1)A_c(n)\right\}\\ &\quad + \left\{A_c^{\mathrm{T}}(n)\tilde{G}^{\mathrm{T}}(n+1)\hat{P}(n+1)\tilde{G}(n+1)A_c(n) - \tilde{G}^{\mathrm{T}}(n)\tilde{P}(n)\tilde{G}(n)\right\} < 0\end{aligned} \tag{31}$$

So, if the following inequalities are satisfied, our proposed system is GAS:

$$\begin{aligned}&A_c^{\mathrm{T}}(n)A_c^{\mathrm{T}}(n+1)\tilde{G}^{\mathrm{T}}(n+2)\tilde{P}(n+2)\tilde{G}(n+2)A_c(n+1)A_c(n)\\ &\quad - A_c^{\mathrm{T}}(n)\tilde{G}^{\mathrm{T}}(n+1)\hat{P}(n+1)\tilde{G}(n+1)A_c(n) < 0\end{aligned} \tag{32}$$

$$A_c^{\mathrm{T}}(n)\tilde{G}^{\mathrm{T}}(n+1)\hat{P}(n+1)\tilde{G}(n+1)A_c(n) - \tilde{G}^{\mathrm{T}}(n)\tilde{P}(n)\tilde{G}(n) < 0 \tag{33}$$

(32) can be written as:

$$A_c^{\mathrm{T}}(n)\left\{A_c^{\mathrm{T}}(n+1)\tilde{G}^{\mathrm{T}}(n+2)\tilde{P}(n+2)\tilde{G}(n+2)A_c(n+1) - \tilde{G}^{\mathrm{T}}(n+1)\hat{P}(n+1)\tilde{G}(n+1)\right\}A_c(n) < 0 \tag{34}$$

If the following inequality is satisfied, it can be concluded that (34) is also satisfied:

$$A_c^{\mathrm{T}}(n+1)\tilde{G}^{\mathrm{T}}(n+2)\tilde{P}(n+2)\tilde{G}(n+2)A_c(n+1) - \tilde{G}^{\mathrm{T}}(n+1)\hat{P}(n+1)\tilde{G}(n+1) < 0 \tag{35}$$

Pre- and post-multiplying (35) by $\tilde{G}^{-1}(n+1)$ and its transpose, respectively, lead to:

$$\tilde{G}^{-T}(n+1)A_c^{\mathrm{T}}(n+1)\tilde{G}^{\mathrm{T}}(n+2)\tilde{P}(n+2)\tilde{G}(n+2)A_c(n+1)\tilde{G}^{-1}(n+1) - \hat{P}(n+1) < 0 \tag{36}$$

Since $A_c(n) \triangleq \sum_{j=1}^{r}\beta_j(n)\,(A_j + B_jN_j)$ and $\beta_j(n) \in \{0, 1\}$, we have:

$$\begin{aligned}A_c(n+1)\tilde{G}^{-1}(n+1) &= \left(\sum_{j=1}^{r}\beta_j(n+1)(A_j + B_jN_j)\right)\left(\sum_{j=1}^{r}\beta_j(n+1)G_j\right)\\ &= \sum_{j=1}^{r}\beta_j(n+1)(A_jG_j + B_jN_jG_j)\end{aligned} \tag{37}$$

Considering (19), (37) can be rewritten as:

$$= \sum_{j=1}^{r} \beta_j(n+1)(A_j G_j + B_j F_j) \tag{38}$$

and we have:

$$\sum_{j=1}^{r} \beta_j(n+1)(A_j G_j + B_j F_j) = \tilde{\Lambda}_j(n+1) \tag{39}$$

So (36) can be rewritten as:

$$\tilde{\Lambda}_j^{\mathrm{T}}(n+1)\,\tilde{G}^{\mathrm{T}}(n+2)\tilde{P}(n+2)\tilde{G}(n+2)\tilde{\Lambda}_j(n+1) - \hat{P}(n+1) < 0 \tag{40}$$

Via the Schur complement Lemma [3] (40) can be rewritten as:

$$\begin{bmatrix} \hat{P}(n+1) & \tilde{\Lambda}_j^{\mathrm{T}}(n+1) \\ \tilde{\Lambda}_j(n+1) & (\tilde{G}^{\mathrm{T}}(n+2)\tilde{P}(n+2)\tilde{G}(n+2))^{-1} \end{bmatrix} > 0 \tag{41}$$

Via the matrix inversion lemma, we have:

$$(\tilde{G}^{\mathrm{T}}(n+2)\tilde{P}(n+2)\tilde{G}(n+2))^{-1} = \tilde{G}^{-1}(n+2)\tilde{P}^{-1}(n+2)\tilde{G}^{-T}(n+2)$$

Then, it follows from Lemma 1 that:

$$\tilde{G}^{-1}(n+2)\tilde{P}^{-1}(n+2)\tilde{G}^{-T}(n+2) \geq \tilde{G}^{-T}(n+2) + \tilde{G}^{-1}(n+2) - \tilde{P}(n+2)$$

So, (41) can be expressed as:

$$\begin{bmatrix} \hat{P}(n+1) & \tilde{\Lambda}_j^{\mathrm{T}}(n+1) \\ \tilde{\Lambda}_j(n+1) & \tilde{G}^{-T}(n+2) + \tilde{G}^{-1}(n+2) - \tilde{P}(n+2) \end{bmatrix} > 0 \tag{42}$$

Considering (39), (23) and (24), and the fact that $\beta_j(n) \in \{0, 1\}$, we have:

$$\sum_{j=1}^{r} \sum_{i=1}^{r} \beta_j(n+2)\beta_i(n+1) \begin{bmatrix} \bar{P}_j & (A_j G_j + B_j F_j)^{\mathrm{T}} \\ (A_j G_j + B_j F_j) & G_i^{\mathrm{T}} + G_i - P_i \end{bmatrix} > 0 \tag{43}$$

and since

$$\sum_{j=1}^{r} \beta_j(n) = 1, \beta_j(n) \in \{0, 1\}$$

If the following inequality is satisfied, (42) is also satisfied

$$\begin{bmatrix} \bar{P}_j & (A_j G_j + B_j F_j)^{\mathrm{T}} \\ (A_j G_j + B_j F_j) & G_i^{\mathrm{T}} + G_i - P_i \end{bmatrix} > 0 \tag{44}$$

So, the claimed GAS result of condition (17) is established.

Since $\beta_j(n)$ shows which subsystem is chosen based on delay in our system (12), so the fact that $\beta_j(n)$ can be either 0 or 1 does not affect conservativeness.

Following the procedure above for (33), we have:

$$\begin{bmatrix} P_j & (A_j G_j + B_j F_j)^{\mathrm{T}} \\ (A_j G_j + B_j F_j) & G_i^{\mathrm{T}} + G_i - \bar{P}_i \end{bmatrix} \tag{45}$$

So, the claimed GAS result of condition (18) and the claim of Theorem 3 are established and the proof is completed. Also, the resultant congestion controller based on augmented decentralized non-common quadratic Lyapunov-based congestion control (ADNCQLCC) is achieved based on (19). By this way, the non-quadratic Lyapunov candidate decreases every two steps; although, it can be locally increased.

Remark 5 In case $P(n) = P$(for computation cost reduction), system (12) with the control effort (16) is GAS, if

There exists a symmetric (PD) matrix $P \in \mathfrak{R}^{z\times z}$ and $\bar{P}_j \in \mathfrak{R}^{z\times z}$ and matrices $G_j \in \mathfrak{R}^{z\times z}$ and $F_j \in \mathfrak{R}^{1\times z}$ for every $i,j \in L$ ($L = \left\{ 1 \ 2 \ \ldots \ r \right\}$) such that the following LMIs (46, 47) are satisfied:

$$\begin{bmatrix} P & (A_j G_j + B_j F_j)^{\mathrm{T}} \\ (A_j G_j + B_j F_j) & G_i^{\mathrm{T}} + G_i - \bar{P}_i \end{bmatrix} > 0 \tag{46}$$

$$\begin{bmatrix} \bar{P}_j & (A_j G_j + B_j F_j)^{\mathrm{T}} \\ (A_j G_j + B_j F_j) & G_i^{\mathrm{T}} + G_i - P \end{bmatrix} > 0 \tag{47}$$

Moreover, the controller gains are given by (19) and the congestion controller based on augmented decentralized common quadratic Lyapunov-based congestion control (ADCQLCC) is gained.

It is worth mentioning that by setting $P(n) = P$, the common quadratic Lyapunov candidate is recovered. The LMIs obtained in Theorem 3 satisfy the results of the common quadratic case. Common quadratic Lyapunov functions tend to be conservative and even might not exist for many complex highly nonlinear systems; however, non-quadratic Lyapunov functions are less conservative, but their computation cost is much higher. Next, Theorem 3 is generalized to m-step differences where m shows the number of steps the Lyapunov candidate is decreasing and m > 2.

Corollary 1 *System (12) with the control effort (16) is GAS, ifThere exists a set of symmetric (PD) matrices* $P_j \in \mathfrak{R}^{z\times z}$ *and* $\bar{P}_{j_k} \in \mathfrak{R}^{z\times z}$ *and matrices* $G_j \in \mathfrak{R}^{z\times z}$ *and* $F_j \in \mathfrak{R}^{1\times z}$ *for every* $i,j \in \left\{ 1 \ 2 \ \ldots \ r \right\}$ *and* $k \in \{3, 4, \ \ldots, \ m-1\}$, *such that the following LMIs are satisfied:*

$$\begin{bmatrix} \bar{P}_{j_2} & (A_jG_j + B_jF_j)^{\mathrm{T}} \\ (A_jG_j + B_jF_j) & G_i^{\mathrm{T}} + G_i - P_i \end{bmatrix} > 0$$
$$\begin{bmatrix} \bar{P}_{j_k} & (A_jG_j + B_jF_j)^{\mathrm{T}} \\ (A_jG_j + B_jF_j) & G_i^{\mathrm{T}} + G_i - \bar{P}_{i_{k-1}} \end{bmatrix} > 0, \quad k = 3, 4, \dots m-1 \tag{48}$$
$$\begin{bmatrix} P_j & (A_jG_j + B_jF_j)^{\mathrm{T}} \\ (A_jG_j + B_jF_j) & G_i^{\mathrm{T}} + G_i - \bar{P}_{j_{m-1}} \end{bmatrix} > 0$$

moreover, same as Theorem 3, the controller gains are given by:

$$N_j = F_jG_j^{-1} \tag{49}$$

Proof Similar to the proof of Theorem 3, by considering (50) as an augmented form of (12) and defining (51) as a Lyapunov candidate, the result directly follows from Corollary 1.

$$\begin{bmatrix} x(n+m) \\ \vdots \\ x(n+3) \\ x(n+2) \\ x(n+1) \end{bmatrix} = \begin{bmatrix} 0_{n\times n} & 0_{n\times n} & 0_{n\times n} & 0_{n\times n} & A_c(n+m-1)\dots A_c(n+2)A_c(n+1)A_c(n) \\ I_{n\times n} & 0_{n\times n} & 0_{n\times n} & 0_{n\times n} & 0_{n\times n} \\ \vdots & \ddots & \vdots & \vdots & 0_{n\times n} \\ 0_{n\times n} & \cdots & I_{n\times n} & 0_{n\times n} & 0_{n\times n} \\ 0_{n\times n} & \cdots & 0_{n\times n} & I_{n\times n} & 0_{n\times n} \end{bmatrix} \begin{bmatrix} x(n+m-1) \\ \vdots \\ x(n+2) \\ x(n+1) \\ x(n) \end{bmatrix} \tag{50}$$

$$V(x(n)) = \begin{bmatrix} x(n+m-1) \\ \vdots \\ x(n+2) \\ x(n+1) \\ x(n) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} P(n+1) & \cdots & 0_{n\times n} & 0_{n\times n} & 0_{n\times n} \\ \vdots & \ddots & \vdots & \vdots & \vdots \\ 0_{n\times n} & \cdots & P(n+2) & 0_{n\times n} & 0_{n\times n} \\ 0_{n\times n} & \cdots & 0_{n\times n} & P(n+1) & 0_{n\times n} \\ 0_{n\times n} & \cdots & 0_{n\times n} & 0_{n\times n} & P(n) \end{bmatrix} \begin{bmatrix} x(n+m-1) \\ \vdots \\ x(n+2) \\ x(n+1) \\ x(n) \end{bmatrix} \tag{51}$$

So, the proposed scheme with non-quadratic Lyapunov candidate is generalized to be more relaxed and the enlargement of stability region is obtained which renders conservativeness reduction. In this case, a congestion controller based on generalized augmented decentralized non-common quadratic Lyapunov-based congestion control (GADNCQLCC) is obtained.

It should be emphasized that better performance is gained since the Lyapunov candidate decreases every few steps; although it can be locally increased. So, enlargement of stability region is obtained which renders conservativeness reduction. Also, increasing "*m*" ends in enlargement of stability region and fully reflects the important role of "*m*" in reducing the conservatism. However, it ends in more LMIs and the computation cost is increased.

Remark 6 In this paper, conservativeness reduction is obtained since the monotonicity requirement of Lyapunov's theorem is relaxed to enlarge the class of functions which can provide stability, an augmented form of our proposed system is utilized for controller synthesis to reduce conservativeness, the controller synthesis is accomplished based on non-quadratic Lyapunov candidates and the proposed scheme with non-quadratic Lyapunov candidate is generalized to be more relaxed.

4 Performance analysis

In this section, first the performance of ADNCQLCC and DPCC is studied using MATLAB and outgoing flow rate variations are considered. In this case, the control effort and queue size are examined.

Variations in outgoing flow rate reveal variations in MAC data rate which can be considered as an indication of the method performance in networks and that it can support multiple modulation rates. Then, the mean outgoing estimation error, the sent traffic and the mean queue length error in ADNCQLCC, ADCQLCC, GADNC-QLCC ($m = 3, 6$), RDANQCC, DNCQLCC and DPCC are presented. Afterward, the performance of our schemes and DPCC is analyzed using OPNET simulator [12] and is compared in case of queue size, load, delay and throughput.

4.1 Performance analysis using MATLAB

In this subsection, first ADNCQLCC performance is evaluated against DPCC. In our simulations o, p, z and r are considered 2, 5, 9 and 4, respectively. It is straight forward to achieve the subsequent subsystems if delay is considered 0, 1, 2 or 3. Then, different subsystems based on delay are chosen as (A_1, B_1), (A_2, B_2), (A_3, B_3), (A_4, B_4):

$$A_1 = \begin{bmatrix} 1&0&0&0&0&0&0&0&0 \\ 1&0&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0&0 \\ 0&0&0&0&1&0&0&0&0 \\ 0&0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&0&1&0&0 \\ 1&0&0&0&0&0&0&0&1 \end{bmatrix} \quad A_2 = \begin{bmatrix} 1&0&0&T&0&0&0&0&0 \\ 1&0&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0&0 \\ 0&0&0&0&1&0&0&0&0 \\ 0&0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&0&1&0&0 \\ 1&0&0&0&0&0&0&0&1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 1&0&0&0&T&0&0&0&0 \\ 1&0&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0&0 \\ 0&0&0&0&1&0&0&0&0 \\ 0&0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&0&1&0&0 \\ 1&0&0&0&0&0&0&0&1 \end{bmatrix} \quad A_4 = \begin{bmatrix} 1&0&0&0&0&T&0&0&0 \\ 1&0&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0&0 \\ 0&0&0&0&1&0&0&0&0 \\ 0&0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&0&1&0&0 \\ 1&0&0&0&0&0&0&0&1 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} T&0&0&1&0&0&0&0&0 \end{bmatrix} \quad B_2 = \begin{bmatrix} 0&0&0&1&0&0&0&0&0 \end{bmatrix}$$

$$B_3 = \begin{bmatrix} 0&0&0&1&0&0&0&0&0 \end{bmatrix} \quad B_4 = \begin{bmatrix} 0&0&0&1&0&0&0&0&0 \end{bmatrix}$$

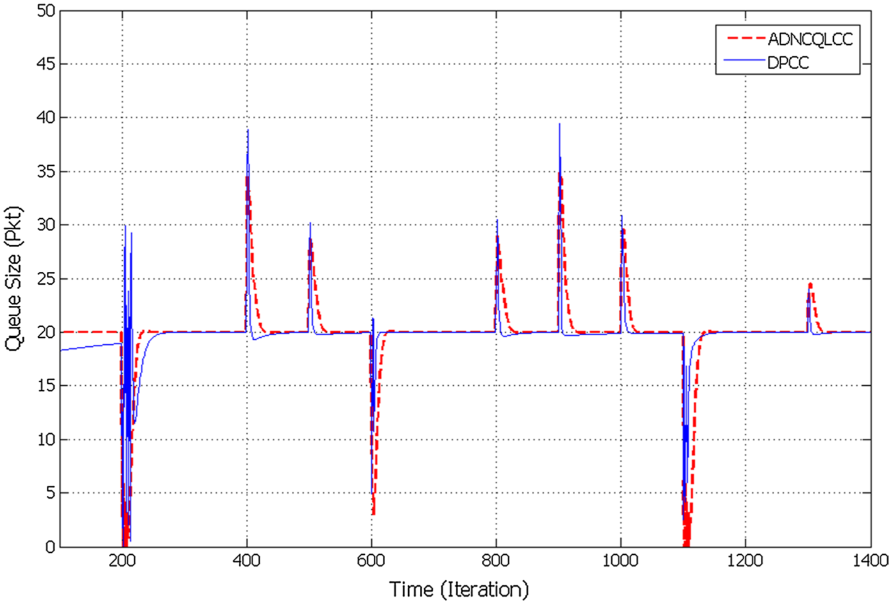

Fig. 1 Queue length in ADNCQLCC and DPCC with $k_{bv} = 0.6$

Also, the ideal and maximum queue lengths are considered 20 and 50 packets, respectively, and the controller parameters are $k_{bv} = 0.6$ and 0.7 and $\lambda = 0.001$.

Figures 1 and 2 show the changes of queue size in DPCC and ADNCQLCC considering $k_{bv} = 0.6$ and $k_{bv} = 0.7$. It is clear that $k_{bv} = 0.7$ ends in poor performance in DPCC. As stated in Sect. 2, k_{bv} is a key factor in DPCC controller which directly affects the performance of system. The control effort is shown in Fig. 3. Finally, Table 1 compares the performance in ADNCQLCC, ADCQLCC, GADNCQLCC ($m=3$, 6), RDANQCC, DNCQLCC and DPCC.

The figures and Table 1 demonstrate that ADNCQLCC, ADCQLCC, GADNCQLCC ($m=3$, 6), RDANQCC and DNCQLCC end in better performance comparing with DPCC which is because of using a better buffer dynamic model. It is worth mentioning that unlike DPCC, delay is considered in our model, buffer dynamic modeling for WSNs is accomplished in discrete time, different subsystems are gained based on delay, and the overall model is obtained by blending them. Also, unlike DPCC, our closed-loop systems are GAS in case of delay changes. From Table 1, it can be easily found that DNCQLCC, ADNCQLCC, ADCQLCC and GADNCQLCC ($m=3$, 6) have better performance comparing with RDANQCC, since in the former each subsystem is stabilized by a controller; however, a single controller is used to stabilize the whole system in the latter. ADNCQLCC renders better results in comparison with ADCQLCC, since non-quadratic Lyapunov candidates are less conservative in comparison with common quadratic Lyapunov candidates. GADNCQLCC ($m=3$, 6) outperforms ADNCQLCC and ADCQLCC, since

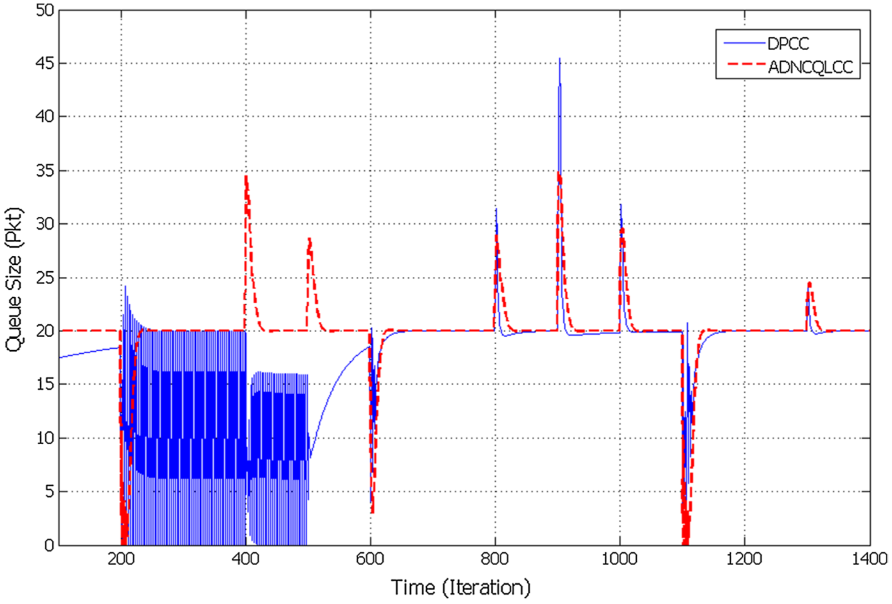

Fig. 2 Queue length in ADNCQLCC and DPCC with $k_{bv} = 0.7$

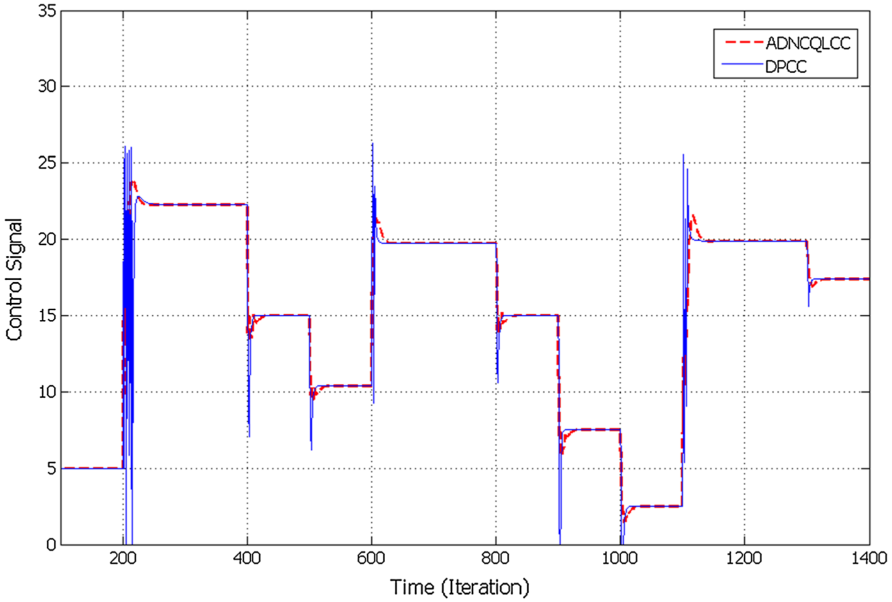

Fig. 3 Control effort of ADNCQLCC and DPCC

Table 1 The sent traffic, mean outgoing estimation error and mean queue length error in ADNCQLCC, ADCQLCC, GADNCQLCC ($m=3$, 6), RDANQCC, DNCQLCC and DPCC

	Mean error {*Queue size* }^{2}	*Mean* {|*Queue size error*|}	*Mean* {|*Outgoing estimation error*|}	*Sent traffic*
DPCC	53.77587	1.533551	3.2074	19920.98
RDANQCC	17.15475	1.350184	0.3790	20009.91
DNCQLCC	12.98235	1.156574	0.1883	20042.52
ADCQLCC	11.21213	0.09078	0.1524	20082.24
ADNCQLCC	10.12781	0.08521	0.1212	20095.19
GADNCQLCC *(m=3)*	10.02349	0.083426	0.1023	20134.84
GADNCQLCC *(m=6)*	9.92302	0.077603	0.0931	20283.15

GADNCQLCC is obtained by generalizing ADNCQLCC and making ADNCQLCC more relaxed. So, enlargement of stability region is obtained which renders conservativeness reduction.

The enlargement of the stability region is very obvious as "m" increases, which fully reflects the important role of "m" in reducing the conservatism. Also, better results are obtained in GADNCQLCC ($m=6$) comparing with GADNCQLCC ($m=3$). So, it can be concluded that GADNCQLCC (m = 6) outperforms GADNCQLCC ($m=3$), ADNCQLCC, ADCQLCC, DNCQLCC, RDANQCC and DPCC.

4.2 Performance analysis using OPNET simulator

Now in order to verify and evaluate our schemes, OPNET simulator is used. The tree topology is used for sensor networks with clusters at leaf nodes to generate traffic. The traffic is then forwarded to base station which is at the tree's root. A 2Mbps channel with shadowing and path-loss is considered. The packet size and the queue limit are considered 512 and 50 packets, respectively. All nodes generate traffic exceeding the channel capacity which renders congestion. Also, it is assumed that nodes are static and have random placement and $k_{bv}=0.6$ is set for DPCC. Table 2 presents the simulation parameters.

Table 3 presents the average delay, load, queue size and throughput in ADNCQLCC, ADCQLCC, GADNCQLCC ($m=3$, 6), DNCQLCC, RDANQCC and DPCC. It can be seen that ADNCQLCC, ADCQLCC, GADNCQLCC ($m=3$, 6), DNCQLCC and RDANQCC render better performance comparing with DPCC. Also, ADNCQLCC, ADCQLCC and GADNCQLCC ($m=3$, 6) have better performance comparing with DNCQLCC and RDANQCC. Also, ADNCQLCC renders better performance comparing with ADCQLCC. GADNCQLCC ($m=3$, 6) outperforms ADNCQLCC and ADCQLCC, and GADNCQLCC ($m=6$) outperforms GADNCQLCC ($m=3$). So, it can be concluded that GADNCQLCC ($m=6$) outperforms GADNCQLCC ($m=3$), ADNCQLCC, ADCQLCC, DNCQLCC, RDANQCC and DPCC.

Table 2 The simulation parameters

Traffic generation parameters	
On state time (sec)	Constant (100)
Off state time (sec)	Constant (0.1)
Packet generation arguments	
Inter-arrival time (s)	Exponential (0.01)
Packet size (bytes)	Constant (512)
Traffic type of service	Best effort
Wireless LAN	
Desired queue size	20
Min Baud rate (bits/s)	2000000

Table 3 Average performance metrics in ADNCQLCC, ADCQLCC, GADNCQLCC ($m=3$, 6), DNC-QLCC, RDANQCC and DPCC

	Average delay(sec)	Average load(packet)	Average queue size(packet)	Average throughput(packet/iteration)
DPCC	0.200367	76.42	7.027073	581632
RDANQCC	0.786021	58.98	19.46699	922378.2
DNCQLCC	0.787832	58.23	19.49914	935167.8
ADCQLCC	0.789094	57.79	19.53591	9487345.5
ADNCQLCC	0.794828	56.34	19.71699	965476.4
GADNCQLCC ($m=3$)	0.802237	56.08	19.75319	9788169.5
GADNCQLCC ($m=6$)	0.814119	55.69	19.89792	1010649.2

So, the simulations using MATLAB and OPNET simulator show the superior performance of GADNCQLCC ($m=6$) comparing with GADNCQLCC ($m=3$), ADNCQLCC, ADCQLCC, DNCQLCC, RDANQCC and DPCC.

5 Conclusion

This paper deals with buffer dynamic modeling of WSNs in discrete time considering delay. Afterward, an augmented form of our proposed system is utilized for controller synthesis. In this paper, a new approach is proposed which implies non-quadratic Lyapunov candidates for controller synthesis in WSNs, and the resultant congestion controller based on augmented decentralized non-common quadratic Lyapunov-based congestion control (ADNCQLCC) is achieved. Afterward, for computation cost reduction, a new approach is presented for controller synthesis of our WSN system and a congestion controller based on augmented decentralized common quadratic Lyapunov-based congestion control (ADCQLCC) is gained. Finally, (ADNCQLCC) is generalized to m-step differences and a more relaxed approach (GADNCQLCC) is proposed. In each approach, a controller is designed for each subsystem, and the overall controller for our proposed system is achieved by blending them. The controller synthesis results are presented in terms of LMIs. Finally, a set of novel congestion control schemes is proposed for WSNs where the resultant closed-loop systems are GAS in case of queue length changes and the consequent delay changes. The simulation results using MATLAB and OPNET simulator confirm that GADNCQLCC outperforms ADNCQLCC, ADCQLCC, DNCQLCC, RDANQCC and DPCC.

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