

# LMI-based Congestion Control Algorithms for a Delayed Network

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*In this paper buffer dynamic modeling for wireless sensor networks (WSNs) as a highly nonlinear system is accomplished in discrete time and the overall model is gained by blending subsystems obtained based on delay. Based on queue utilization and channel estimation algorithm, congestion is detected and a suitable rate is selected by an adaptive back-off interval selection. In this paper, a new approach is proposed for controller synthesis of our system based on non-quadratic Lyapunov functions, and a controller is designed to stabilize each subsystem. The controller synthesis results are expressed as a set of Linear Matrix Inequalities (LMIs). Moreover, the performance is considered and decay rate is guaranteed. Finally, a set of new LMI-based congestion control schemes (LCC) is obtained for WSNs. The closed-loop systems are globally asymptotically stable (GAS) in case of delay changes resulted from queue size changes. The simulation results using MATLAB and OPNET simulator confirm the effectiveness of our proposed schemes.*

## Article Info

### Keywords:

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## I. INTRODUCTION

Congestion control has emerged as a major issue in communication networks especially with the growing need of bandwidth, load, size and connectivity of these networks. The aforementioned fact has necessitated the design and utilization of networks including more efficient congestion control algorithms, especially for WSNs that play a dominant role in recent technologies [1]. Congestion of packets at the outgoing queues in routers causes low reliability of networks and performance degradation. Recently, a great number of schemes are proposed to address congestion control problem [2]-[15].

It should be highlighted that fading is very probable in WSNs which renders bandwidth reduction ending in queue length increase and consequently delay increase. In this case, satisfactory performance is gained only if the resultant closed-loop systems are stable, otherwise performance degradation is achieved ending in system instability. Fading

is considered in decentralized predictive congestion control (DPCC) [5] and Robust Decentralized Adaptive Nonquadratic Congestion Control Scheme (RDANQCC) [15].

Finite transmission speeds and traffic congestion renders delay in networks. So, delay is one of the most important metrics for performance evaluation in WSNs. In [8], network modeling is accomplished by a nonlinear fluid flow model where delay is considered. Afterwards, congestion control in WSNs based on sliding mode learning control is studied. In [9] the congestion control in local WSNs by the use of time-delay compensators is presented. In [10] a congestion control in WSNs based on triangle module fusion operator is addressed where the ratio of queue waiting time and queue delay is considered as the retention rate which is used to detect congestion. In [11] a congestion control algorithm in WSNs is established which copes with time-varying network parameters and large delays. In [15] a congestion controller which is robust to delay changes is designed. Also, in order to consider delay in our system, in [15], buffer dynamic modeling for WSNs is accomplished in discrete time, different subsystems are

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achieved based on delay, and the overall model is gained by blending them. It is worth mentioning that unlike [5] where the buffer occupancy error and control effort are only considered at time instances  $n$  and  $n-1$ , respectively, in [15], they are considered at different time instances.

Stability analysis and controller synthesis of systems have been comprehensively addressed during the last years. In this regard, Lyapunov-based approaches are of special concern [16]-[18]. It should

be highlighted that there have been a few attempts to imply non-quadratic Lyapunov functions for controller synthesis in WSNs in a will to reduce the conservativeness of the common quadratic Lyapunov functions. Stability analysis and controller synthesis can be accomplished using LMIs. In [12] congestion control for communication networks is studied with random parameter jumps where the controller synthesis is accomplished by LMIs. In [12], the round-trip time delay, the link capacity, and the transmission control protocol (TCP) session number are considered random finite state Markov process. In [13] the congestion control issue for linearized TCP/AQM network is addressed using LMIs where an input constraint is considered in the controller design. In [14] a distributed estimation and congestion control method is established for a sensor-network-based system where LMIs are used to formulate the controllers stabilizing the system despite observation errors in the controlled output, and minimize the communication noise effect in the observation error. In [15] RDANQCC is presented as a congestion control scheme where iterative linear matrix inequalities (ILMIs) are used to solve the control problem.

In this paper, buffer dynamic modeling for WSNs is accomplished in discrete time, different subsystems are achieved based on delay, and the overall model is gained by blending them. Also, unlike [5], the buffer occupancy error and control effort are considered at different time instances.

In this paper, a new approach is presented for controller synthesis of our proposed system based on non-quadratic Lyapunov functions. Unlike [15] where a controller is obtained to stabilize the whole system, in the current study, a controller is designed for each subsystem, and the overall controller for our proposed system is achieved by blending them. Also, unlike [15] where the results of controller synthesis are presented in terms of ILMIs, in this paper, LMIs are used for controller synthesis which are numerically feasible using commercially available software. Finally, our congestion control strategies are presented for WSNs based on these approaches, and in case of delay changes resulted from queue size changes, the closed-loop systems are GAS. Moreover, unlike [5],[15], performance is considered and decay rate is guaranteed.

The reminder of this article is as follows: First, some preliminaries are stated. Next, we address the main results, followed by our approach performance which is assessed through simulation results, and finally we conclude our study.

## II. PRELIMINARIES

In this section, the buffer occupancy changes at a node, the adaptive and predictive controller in [5], our proposed model [15] and the controller synthesis results in RDANQCC are briefly presented.

*Notations:* Through this paper  $N, \mathfrak{R}, \mathfrak{R}^n, \mathfrak{R}^{n \times m}$  denote sets of natural numbers, real numbers,  $n$  component real vectors and  $n \times m$  real matrices, respectively.

### A. Buffer occupancy changes at a node

Based on [5], buffer occupancy changes at the  $a^{\text{th}}$  node in terms of outgoing and incoming traffic is given as:

$$q_a(n+1) = \text{sat}_c [q_a(n) + Tu_a(n) - f_a(u_{a+1}(n)) + d(n)] \quad (1)$$

Where  $\text{sat}_c$  is the saturation function showing the behavior of finite-size queue,  $q_a(n)$  is the  $a^{\text{th}}$  node buffer occupancy at time instant  $n$ ,  $T$  is the measurement interval,  $u_a(n)$  is the rate of regulated incoming traffic,  $f_a(\cdot)$  is a dictated outgoing traffic by node located at the next hop which is disturbed by channel variations and  $d(n)$  is the traffic disturbance which is unknown. In order to estimate the outgoing traffic, it is essential to calculate and propagate  $u_a(n)$  as a feedback to the  $(a-1)^{\text{th}}$  node.

Considering  $q_{ad}$  as the desired buffer occupancy at the  $a^{\text{th}}$  node, the buffer occupancy error  $e_{ba}(n) = q_a(n) - q_{ad}$  is as follows:

$$e_{ba}(n+1) = \text{Sat}_c [e_{ba}(n) + Tu_a(n) - f_a(u_{a+1}(n)) + d(n)] \quad (2)$$

### B. The adaptive and predictive controller in DPCC

The adaptive and predictive controller in [5] is as follows:

$$u_a(n) = \text{Sat}_c \left( \frac{\hat{f}_a(u_{a+1}(n)) + (\kappa_{bv} - 1)e_{ba}(n)}{T} \right) \quad (3)$$

Where  $\kappa_{bv}$  is the gain parameter and  $\hat{f}_a(u_{a+1}(n))$  is the estimate of  $f_a(u_{a+1}(n))$ . The error of buffer occupancy at time instant  $n+1$  is:

$$e_{ba}(n+1) = \text{Sat}_c [k_{bv}e_{ba}(n) + \tilde{f}_a(u_{a+1}(n)) + d(n)] \quad (4)$$

Where  $\tilde{f}_a(u_{a+1}(n)) = f_a(u_{a+1}(n)) - \hat{f}_a(u_{a+1}(n))$  is the estimation error of the outgoing traffic. Also, the traffic estimate and the node actual rate are detailed in [5].

**Remark1-** Since our scheme is decentralized, model formulation and controller synthesis results are presented for each node. So, for the sake of simplicity, the index " $a$ " which is used to show the  $a^{\text{th}}$  node is omitted in our scheme.

### C. The proposed system

The proposed model is as follows [15]:

$$x(n+1) = \sum_{j=1}^r \beta_j(n)(A_j x(n) + B_j u(n))$$

$$\sum_{j=1}^r \beta_j(n) = 1, \beta_j(n) \in \{0, 1\} \quad (5)$$

Where

$x(n) = [e_b(n) \ e_b(n-1) \ \dots \ e_b(n-o) \ u(n-1) \ \dots \ u(n-p) \ s(n)]^T \in \mathfrak{R}^z$  includes buffer occupancy errors  $e_b(n-o)$  (Equ. 2), regulated incoming traffic rate (control effort)  $u(n-p)$  at different time instances and the integrator  $S(n) = \sum_{m=0}^n e_b(m) \in \mathfrak{R}$ . Also  $o, p \in N$  are the state number where  $o+1$  and  $p$  are the number of buffer occupancy error and control effort states, respectively,  $A_j \in \{A_1 \ A_2 \ \dots \ A_r\} \in \mathfrak{R}^{z \times z}$  and  $B_j \in \{B_1 \ B_2 \ \dots \ B_r\} \in \mathfrak{R}^{z \times z}$  are known constant matrices for system description showing the  $j^{\text{th}}$  subsystem,  $z \in N$  is the number of state variables,  $r \in N$  is the number of subsystems obtained due to queue size increase which renders delay increase in system and finally  $\beta_j(n)$  indicates the subsystem chosen considering delay  $(\sum_{j=1}^r \beta_j(n) = 1$  and  $\beta_j(n)$  can be either 0 or 1).

### D. The controller synthesis results in RDANQCC

The controller synthesis result in RDANQCC is as follows:

**Theorem1[15]** - System (5) with  $u(n) = kx(n)$  as the control effort is GAS if there exists a set of symmetric positive definite (PD) matrices  $P_i$  and  $P_m$  for every  $i, j, m \in L$  ( $L = \{1 \ 2 \ \dots \ r\}$ ) such that (6) is satisfied:

$$\begin{bmatrix} P_m & (A_j + B_j k)^T \\ (A_j + B_j k) & P_i^{-1} \end{bmatrix} > 0 \quad (6)$$

In the subsequent sections, controller synthesis and the simulation results will be addressed in details.

### III. MAIN RESULTS

In this paper, non-quadratic Lyapunov functions are utilized for controller synthesis of our system. Also, a controller is designed for each subsystem and afterwards, a combination of these controllers is used that guarantee system stability. The controller synthesis results are expressed as a set of LMIs which are numerically feasible with commercially available software and unlike [5], performance is considered and decay rate is guaranteed.

**Lemma 1 [19]:** If matrices  $C_m$  and  $S_m$  have appropriate dimensions and  $S_m$  is (PD), then

$$C_m^T S_m^{-1} C_m \geq C_m^T + C_m - S_m \quad (7)$$

By defining the control signal as

$$u(n) = \left[ \sum_{j=1}^r \beta_j N_j \left( \sum_{j=1}^r \beta_j P_j \right)^{-1} \right] x(n) \quad (8)$$

The following result is obtained:

**Theorem2:** System (5) with the control signal defined in (8) is GAS if

i) There exists a set of symmetric positive definite matrices  $P_j \in \mathfrak{R}^{z \times z}$  and matrices  $N_j \in \mathfrak{R}^{l \times z}$  for every

$i, j \in L$  and  $L = \{1 \ 2 \ \dots \ r\}$  such that the following LMIs (9) are satisfied:

$$(A_j P_j + B_j N_j)^T (P_i^{-1}) (A_j P_j + B_j N_j) - P_j < 0 \quad (9)$$

Or equivalently

ii) There exists a set of symmetric positive definite matrices  $P_j \in \mathfrak{R}^{z \times z}$  and the matrices  $N_j \in \mathfrak{R}^{l \times z}$  for every  $i, j \in L$  such that the following LMIs (10) are satisfied:

$$\begin{bmatrix} P_j & (A_j P_j + B_j N_j)^T \\ (A_j P_j + B_j N_j) & P_i \end{bmatrix} > 0 \quad (10)$$

Moreover, the controller gains are given by:

$$F_j = N_j P_j^{*-1} \quad (11)$$

Where the following notation is adopted for simplicity:

$$P_j^* = \left( \sum_{j=1}^r \beta_j P_j \right) \quad (12)$$

**Proof:** Consider the following Lyapunov function candidate:

$$V(x(n)) = x^T(n) P_j^{*-1} x(n) \quad (13)$$

The difference function is given as:

$$\begin{aligned} \Delta V &= V(x(n+1)) - V(x(n)) \\ &= x^T(n+1) (P_j^*(n+1))^{-1} x(n+1) - x^T(n) (P_j^*(n))^{-1} x(n) \end{aligned} \quad (14)$$

Based on system Equation (5) and the control signal (8), we have:

$$x(n+1) = \sum_{j=1}^r \beta_j(n) (A_j + B_j N_j P_j^{-1}) x(n) \quad (15)$$

Since  $\sum_{j=1}^r \beta_j(n) = 1$  and  $\beta_j \in \{0, 1\}$ , we have

$$x(n+1) = (A_j + B_j N_j P_j^{-1}) x(n) \quad (16)$$

Substituting (16) in (14) yields:

$$\Delta V = x^T(n) (A_j + B_j N_j P_j^{-1})^T (P_i^{-1}) (A_j + B_j N_j P_j^{-1}) x(n) - x^T(n) P_j^{-1} x(n)$$

$$\Delta V = x^T(n) (A_j + B_j N_j P_j^{-1})^T (P_i^{-1}) (A_j + B_j N_j P_j^{-1}) - P_j^{-1} x(n) \quad (17)$$

Pre- and post-factorization (17) by  $P_j^{-T}$  and its transpose, respectively, lead to:

$$\Delta V = x^T(n)P_j^{-T}[(A_jP_j + B_jN_j)^T(P_i^{-1}) - (A_jP_j + B_jN_j) - P_j^T P_j^{-1}P_j]P_j^{-1}x(n) \quad (18)$$

In order to prove that the proposed system (16) is GAS, it suffices to show that the following inequality is satisfied:

$$(A_jP_j + B_jN_j)^T(P_i^{-1})(A_jP_j + B_jN_j) - P_j < 0 \quad (19)$$

So, the claimed stability result of condition (i) is established.

Via the Schur complement Lemma [20], (19) can be rewritten as:

$$\begin{bmatrix} P_j & (A_jP_j + B_jN_j)^T \\ (A_jP_j + B_jN_j) & P_i \end{bmatrix} > 0 \quad (20)$$

So, condition (ii) is also satisfied and the closed-loop control system (16) is GAS. Moreover, the controller gains can be easily obtained by (11) and the proof is completed.

In the subsequent Theorem, performance is considered and decay rate is guaranteed.

**Theorem3:** The closed-loop control system (16) is GAS with decay rate  $\Phi \in \mathfrak{R}^{z \times z}$  if

- i) There exists a set of symmetric positive definite matrices  $P_j \in \mathfrak{R}^{z \times z}$  and matrices  $Q_j \in \mathfrak{R}^{z \times z}$  and positive definite full rank decay rate matrix  $\Phi \in \mathfrak{R}^{z \times z}$  and matrices  $N_j \in \mathfrak{R}^{l \times z}$  for every  $i, j \in L$  and

$L = \{1 \ 2 \ \dots \ r\}$  such that the following LMIs (21,22) are satisfied:

$$(A_jP_j + B_jN_j)^T P_i^{-1} (A_jP_j + B_jN_j) - (P_j - Q_j) < 0 \quad (21)$$

$$P_j^T \Phi^{-1} P_j - Q_j \leq 0 \quad (22)$$

Or equivalently

- ii) There exists a set of symmetric positive definite matrices  $P_j \in \mathfrak{R}^{z \times z}$  and matrices  $Q_j \in \mathfrak{R}^{z \times z}$  and positive definite full rank decay rate matrix  $\Phi \in \mathfrak{R}^{z \times z}$  and matrices  $N_j \in \mathfrak{R}^{l \times z}$  for every  $i, j \in L$  and

$L = \{1 \ 2 \ \dots \ r\}$  such that the following LMIs (23,24) are satisfied:

$$\begin{bmatrix} P_j - Q_j & (A_jP_j + B_jN_j)^T \\ (A_jP_j + B_jN_j) & P_i \end{bmatrix} > 0 \quad (23)$$

$$\begin{bmatrix} Q_j & P_j^T \\ P_j & \Phi \end{bmatrix} \geq 0 \quad (24)$$

Moreover, the control gains are given by (11) and the control signal can be rewritten as (8).

**Proof:** Consider the Lyapunov function candidate which is defined in (13),

$$V(x(n)) = x^T(n)P_j^{*-1}x(n)$$

In order to prove that the proposed system (16) is GAS, it suffices to show that the following inequality is satisfied.

So:

$$\Delta V = V(x(n+1)) - V(x(n)) < -x^T(n)[P_j^{-T}Q_jP_j^{-1}]x(n) \quad (25)$$

Substituting (16) in (25), yields:

$$\Delta V = x^T(n)[(A_j + B_jN_jP_j^{-1})^T(P_i^{-1})(A_j + B_jN_jP_j^{-1}) - P_j^{-1}]x(n) < -x^T(n)[P_j^{-T}Q_jP_j^{-1}]x(n) \quad (26)$$

Pre-and post-factorization (26) by  $P_j^{-T}$  and its transpose, respectively, lead to:

$$x^T(n)P_j^{-T}[(A_jP_j + B_jN_j)^T(P_i^{-1})(A_jP_j + B_jN_j) - P_j]P_j^{-1}x(n) < -x^T(n)[P_j^{-T}Q_jP_j^{-1}]x(n) \quad (27)$$

Which in turn implies that:

$$(A_jP_j + B_jN_j)^T(P_i^{-1})(A_jP_j + B_jN_j) - (P_j - Q_j) < 0 \quad (28)$$

Thus, the stability result (21) is established. Via the Schur complement Lemma, (28) is equal to:

$$\begin{bmatrix} P_j - Q_j & (A_jP_j + B_jN_j)^T \\ (A_jP_j + B_jN_j) & P_i \end{bmatrix} > 0 \quad (29)$$

So, the corresponding equivalent condition (23) is also satisfied.

The following inequality guarantees the performance with the decay rate  $\Phi$ :

$$P_j^{-T}Q_jP_j^{-1} \geq \Phi^{-1} \quad (30)$$

In fact, pre-and post-multiplying  $P_j^T$  and its transpose, respectively, lead to:

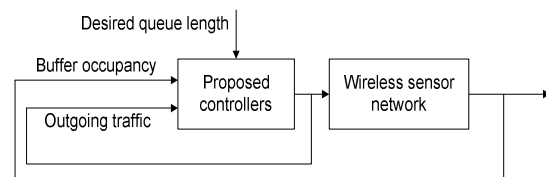
$$P_j^T \Phi^{-1} P_j - Q_j \leq 0 \quad (31)$$

Via the Schur complement Lemma, we have:

$$\begin{bmatrix} Q_j & P_j^T \\ P_j & \Phi \end{bmatrix} \geq 0 \quad (32)$$

So, conditions (22) and (24) are also established and the proof is completed.

Figure 1 depicts our proposed control scheme. Buffer occupancy and outgoing rate are used in the controller design and the resultant closed-loop systems are GAS in case of queue length changes and the consequent delay changes.



**Fig. 1.** Our proposed control scheme

#### IV. PERFORMANCE ANALYSIS

In this section, first the performance of LCC and DPCC are studied using MATLAB and outgoing flow rate variations are considered. In this case, the control effort and queue size are examined. Outgoing flow rate variations reveal MAC data rate variations which can be considered as an indication of the method performance in networks and

that it can support multiple modulation rates. Then, the mean outgoing estimation error, the sent traffic and the queue length mean error in LCC schemes, RDANQCC and DPCC are presented. Afterwards, the performance of LCC schemes, RDANQCC and DPCC is analyzed using OPNET simulator [21] and compared in case of queue size, load, delay and throughput.

**A. Performance Analysis using MATLAB**

In this subsection, first LCC performance is evaluated against DPCC. In our simulations  $o$  and  $p$  are both considered 4 and  $z$  is 9 and  $r$  are 4. It is straight forward to achieve the subsequent subsystems if delay is considered 0, 1, 2 or 3. Then, different subsystems based on delay are chosen as  $(A_1, B_1), (A_2, B_2), (A_3, B_3), (A_4, B_4)$ :

$$A_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad A_2 = \begin{bmatrix} 1 & 0 & 0 & 0 & T & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

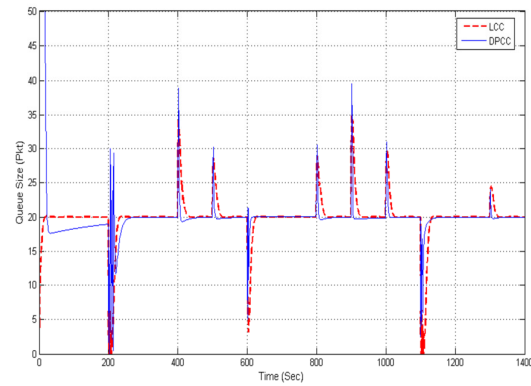
$$A_3 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & T & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad A_4 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & T & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$B_1 = [T \ 0 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0 \ 0]^T \quad B_2 = [0 \ 0 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0 \ 0]^T$$

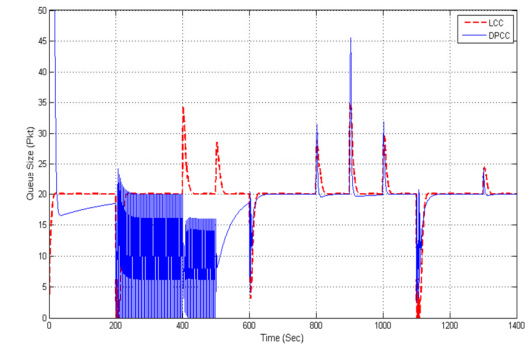
$$B_3 = [0 \ 0 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0 \ 0]^T \quad B_4 = [0 \ 0 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0 \ 0]^T$$

Also, the ideal and maximum queue lengths are considered as 20 and 50 packets, respectively, and the controller parameters are  $k_{bv} = 0.6$  and  $0.7$  and  $\lambda = 0.001$ .

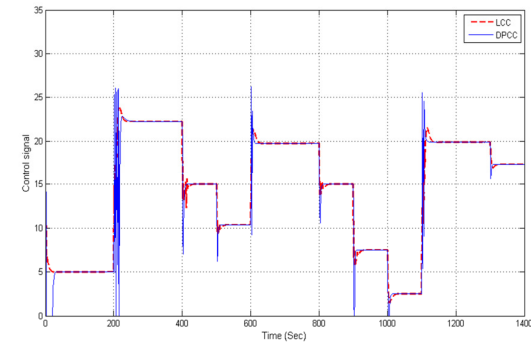
Figures 2 and 3 show the changes of queue size in DPCC and LCC considering  $k_{bv} = 0.6$  and  $k_{bv} = 0.7$ . It is clear that  $k_{bv} = 0.7$  ends in poor performance in DPCC. As stated in preliminaries subsection B,  $k_{bv}$  is a key factor in DPCC controller which directly affects the performance of system. The control effort is shown in Fig. 4. Finally, Table I compares the performance in LCC schemes, RDANQCC and DPCC.



**Fig. 2.** Queue length in LCC and DPCC with  $k_{bv} = 0.6$



**Fig. 3.** Queue length in LCC and DPCC with  $k_{bv} = 0.7$



**Fig. 4.** Control effort of LCC and DPCC

**TABLE I.** The sent traffic, mean outgoing estimation error and mean queue length error in LCC schemes, RDANQCC and DPCC

	Mean {Queue size error} <sup>2</sup>	Mean {Queue size error}	Mean {Outgoing estimation error}	Sent traffic
DPCC	53.77587	1.533551	3.2074	19920.98
RDANQCC	17.15475	1.350184	0.3790	20009.91
LCC	12.87633	1.138422	0.1733	20044.42
LCC with guaranteed decay rate	14.73304	1.235762	0.2238	20037.29

The figures and Table I demonstrate that LCC schemes and RDANQCC end in better performance comparing with DPCC which is because of using a better buffer dynamic model. It is worth mentioning that unlike DPCC, delay is



considered in our model, buffer dynamic modeling for WSNs is accomplished in discrete time, different subsystems are gained based on delay and the overall model is obtained by blending them. Also, unlike DPCC, our closed-loop systems are GAS in case of delay changes. From Table I, it can be easily found that LCC schemes have better performance comparing with RDANQCC, since in the former each subsystem is stabilized by a controller, however a single controller is used to stabilize the whole system in the latter. Also, LCC renders better performance comparing with LCC with guaranteed decay rate. So, it can be concluded that LCC outperforms LCC with guaranteed decay rate, RDANQCC and DPCC.

### B. Performance Analysis using OPNET simulator

Now in order to verify and evaluate our schemes against DPCC, OPNET simulator is used. The tree topology is used for sensor networks with clusters at leaf nodes to generate traffic. The traffic is then forwarded to base station which is at the tree root. A 2Mbps channel with shadowing and path-loss is considered. The packet size and the queue limit are considered 512 and 50 packets, respectively. All nodes generate traffic exceeding the channel capacity which renders congestion. Also, it is assumed that nodes are static and have random placement and  $k_{bv} = 0.6$  is set for DPCC.

Table II presents the simulation parameters.

**TABLE II.** The Simulation parameters

Traffic generation parameters	
On state time (sec)	Constant (100)
Off state time (sec)	Constant (0.1)
Packet generation arguments	
Inter-arrival time (sec)	Exponential (0.01)
Packet size (bytes)	Constant (512)
Traffic type of service	Best effort
Wireless LAN	
Desired queue size	20
Min Baud rate (bits/sec)	2000000

Table III presents the average delay, load, queue size and throughput in LCC schemes, RDANQCC and DPCC. It can be seen that LCC schemes and RDANQCC render better performance comparing with DPCC. Also, LCC schemes have better performance comparing with RDANQCC. Also, LCC renders better performance comparing with LCC with guaranteed decay rate. So, LCC outperforms LCC with guaranteed decay rate, RDANQCC and DPCC.

**TABLE III.** Average performance metrics in LCC schemes, RDANQCC and DPCC

	Average delay	Average load	Average queue size	Average throughput
DPCC	0.200367	76.42	7.027073	581632
RDANQCC	0.786021	58.98	19.46699	922378.2
LCC	0.788540	58.15	19.51738	939138.6
LCC with guaranteed decay rate	0.787193	58.63	19.47698	931184.5

So, the simulations using MATLAB and OPNET simulator show the superior performance of LCC comparing with LCC with guaranteed decay rate, RDANQCC and DPCC.

## V. CONCLUSION

This paper deals with buffer dynamic modeling of WSNs in discrete time considering delay.

For controller synthesis, a new approach is proposed based on non-quadratic Lyapunov functions and a controller is designed to stabilize each subsystem. Also, performance is considered and decay rate is guaranteed. Finally, a set of novel LMI-based congestion control schemes (LCC) is derived for WSNs and the resultant closed-loop systems are GAS in case of queue length changes and the consequent delay changes. The simulation results using MATLAB and OPNET simulator confirm that LCC schemes outperform RDANQCC and DPCC. Future work will include improving the proposed congestion control schemes to enlarge the stability region which renders conservativeness reduction and better performance.

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