Output-Feedback Model Predictive Control Using Set of State Estimates

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Abstract. The paper deals with an algorithm of output-feedback model predictive control (MPC) where the required point state estimate is selected from the set of possible estimates. The involved state estimator is based on an approximate uniform Bayesian filter. In the paper, there are compared conservative mean and progressive composite state estimates. The proposed method is illustrated by the motion control of a specific robotic system.

Keywords: Output-feedback control · Model predictive control · State estimation · Bayesian methods · Robotic system · Bounded disturbances

1 Introduction

The output-feedback model predictive control (MPC) is popular as the states of the involved state space model are often unmeasurable in the praxis [2]. In such case, the control performance depends on the quality of the state estimates. This quality is usually influenced by uncertainties that are related to a model inaccuracy and to unmeasured noises. The statistics of these uncertainties are rarely known. In many practical applications, they are only known to be bounded, and any additional information about their nature and properties is unavailable [10]. Therefore, the output-feedback MPC, that considers a bounded uncertainty, is one of the recent research concerns.

The estimation techniques to cope with bounded disturbances are based either on stochastic or set-membership approach. Set-membership algorithms provide state estimates that are confined in constrained sets such as boxes [20], zonotopes [23], ellipsoids [16] or their combination [26]. Stochastic state estimation is based on particle filtering [22] or can resemble Kalman filter with data and time update steps [8]. Stochastic and set-membership paradigms are merged in [6].

Set-membership state estimation has been used e.g. in [21], [5] while in [27], a specific robust Kalman filter has been used. Recently, a tube-based robust MPC scheme was proposed where the states are bounded by the tubes whose center is the state of the nominal system [11,15,19,25]. The paper [17] combines set-membership estimation with prediction tubes.

In our research, we focus on the output-feedback MPC intended for industrial stationary robots-manipulators, specifically parallel kinematic machine (PKM) [18] where

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the system outputs correspond to the Cartesian coordinates and angular position. The unmeasured states consist of the relevant velocities. In this setting, measurements are often influenced by physically bounded uncertainties.

We propose an algorithm of output-feedback MPC for discrete-time systems influenced by bounded state and output disturbances. The required estimates are provided by the Bayesian state estimator presented in [8]. The control aim is to follow a given reference trajectory. The paper builds on the previous authors works presented in [12,13], and proposes a novel way how to choose a point estimate from the admissible set of state estimates. In [12], the center of a set estimate was chosen as a point estimate in each simulation step. In [13], the choice of the point estimate was included into the optimization step. Here, we will choose it in advance based on the previous state evolution.

The paper is organised as follows. This section ends with a summary of the notation used. Section 2 introduces a linear state space model with uniform disturbances including its approximate Bayesian estimation. In Sect. 3, two algorithms of output-feedback MPC using the above mentioned model are explained. Section 4 presents experiments with a model of the parallel kinematic machine where the proposed control scheme is applied to the reference tracking. Section 5 concludes the paper.

Notation. Matrices are in capital letters (e.g. A), vectors and scalars are in lowercase letters (e.g. b). A_{ij} is the element of a matrix A on i-th row and j-th column. A_i denotes the i-th row of A. Column vectors are considered, where z_t denotes the value of a vector variable z at a discrete-time instant $t \in \{1, \cdots, \overline{t}\}$; $z_{t;i}$ is the i-th entry of z_t; $\underline{z}$ and $\overline{z}$ are lower and upper bounds on z, respectively. $\hat{z}$ denotes the estimate of z. The symbol $f(\cdot|\cdot)$ denotes a conditional probability density function (pdf); names of arguments distinguish respective pdfs; no formal distinction is made between a random variable, its realisation and an argument of the pdf. $\mathcal{U}_z(\underline{z}, \overline{z})$ denotes a multivariate uniform distribution of z, $\underline{z} \leq z \leq \overline{z}$, inequalities are meant entrywise; $\|.\|_2^2$ means the squared Euclidean norm.

2 Bayesian State Estimation of LSU Model

A linear state space model with uniform disturbances (LSU model) is defined as

$$x_t = \underbrace{A_t\, x_{t-1} + B_t\, u_{t-1}}_{\tilde{x}_t} + \nu_t,\ \nu_t \sim \mathcal{U}_\nu(-\rho, \rho) \tag{1}$$

$$y_t = \underbrace{Cx_t}_{\tilde{y}_t} + n_t, \qquad n_t \sim \mathcal{U}_n(-r, r) \tag{2}$$

where A_t, B_t are time varying model matrices; $C = [\mathrm{I}\ \ 0]$; $\tilde{x}_t$ and $\tilde{y}_t$ correspond to the nominal values of x_t and y_t, respectively; ν_t and n_t are independent and identically distributed (i.i.d.) state and observation disturbances. They are uniformly distributed within an orthotope with known bounds ρ and r, respectively.

In the Bayesian filtering framework [9], a controlled system is described by the following pdfs:

$$\text{time evolution model:} \quad f\left(x_t | x_{t-1}, u_{t-1}\right) \tag{3}$$

$$\text{observation model:} \quad f\left(y_t | x_t\right) \tag{4}$$

$$\text{prior pdf:} \quad f\left(x_0\right) \tag{5}$$

Bayesian state estimation (filtering) consists in the evolution of the posterior pdf $f(x_t|d(t))$ where $d(t)$ is a sequence of observed data records $d_t = (y_t, u_t)$, $d_0 \equiv u_0$. The evolution of posterior pdf $f(x_t|d(t))$ is described by a two-steps recursion that starts from the prior pdf $f(x_0|u_0) \equiv f(x_0)$ (5): (i) *time update* that uses theoretical knowledge described by model (3) and reflects the evolution $x_{t-1} \rightarrow x_t$; it provides prediction $f(x_t|d(t-1))$, and (ii) *data update* that uses theoretical knowledge described by model (4) and incorporates information about data d_t; it provides The LSU model (1), (2) can be equivalently described, using pdf notation (3)–(5), as follows

$$f(x_t|u_{t-1}, x_{t-1}) = \mathcal{U}_x(\tilde{x}_t - \rho, \tilde{x}_t + \rho) \tag{6}$$

$$f(y_t|x_t) = \mathcal{U}_y(\tilde{y}_t - r, \tilde{y}_t + r) \tag{7}$$

$$f(x_0) = \mathcal{U}_x(\underline{x}_0, \overline{x}_0) \tag{8}$$

The exact solution of the Bayesian filtering of LSU model (6), (7) leads to a very complex form of posterior pdf. Recently, an approximate Bayesian state estimation was proposed by one of authors [7]. It provides the evolution of the uniformly distributed posterior pdf $f(x_t|d(t))$ as follows.

Time Update – time update starts at $t = 1$ with $\underline{m}_0 = \underline{x}_0$, $\overline{m}_0 = \overline{x}_0$ and holds

$$f(x_t|d(t-1)) \approx \prod_{i=1}^{\ell} \mathcal{U}_{x_{t;\,i}}(\underline{m}_{t;\,i} - \rho_i, \overline{m}_{t;\,i} + \rho_i) = \mathcal{U}_{x_t}(\underline{m}_t - \rho, \overline{m}_t + \rho), \tag{9}$$

where $\underline{m}_t = [\,\underline{m}_{t;\,1}, \ldots, \underline{m}_{t;\,\ell}\,]^T$, $\overline{m}_t = [\,\overline{m}_{t;\,1}, \ldots, \overline{m}_{t;\,\ell}\,]^T$, ℓ is the size of x,

$$\underline{m}_{t;\,i} = \sum_{j=1}^{\ell} \min(A_{ij}\,\underline{x}_{t-1;\,j} + B_i\,u_{t-1}, A_{ij}\,\overline{x}_{t-1;\,j} + B_i\,u_{t-1}), \tag{10}$$

$$\overline{m}_{t;\,i} = \sum_{j=1}^{\ell} \max(A_{ij}\,\underline{x}_{t-1;\,j} + B_i\,u_{t-1}, A_{ij}\,\overline{x}_{t-1;\,j} + B_i\,u_{t-1}).$$

Data Update – in data update, the observation y_t (7) is processed by the Bayes rule together with the prior (9) from the time update as $y_t - r \leq Cx_t \leq y_t + r$. The resulting uniform pdf posses a support in the form of polytope. It is approximated by a uniform pdf with an orthotopic support

$$f(x_t|d(t)) \approx \mathcal{U}_{x_t}(\underline{x}_t, \overline{x}_t). \tag{11}$$

This approximation is based on a minimising of Kullback-Leibler divergence of two pdfs [7]. The result of (11) says that the estimate $\hat{x}_t$ belong to a set

$$\hat{x}_t \in \langle \underline{x}_t, \overline{x}_t \rangle \tag{12}$$

where all points have the same probability.

For the intended task of the output-feedback MPC, we need a state point estimate. In [12], this point estimate corresponded to the center of a set estimate (12) at each control design step. In [13], we did not choose the particular point estimate to be used in the control design but consider the whole set (12). In each time step, the optimization run several times for a chosen sequence of points from this set. Then, the point connected with a minimal cost was chosen for the control input computation.

Now, we will come back to the concept of a priori chosen estimate. Contrary to the paper [12], the choice will not be fixed to the center of the set estimate but will depend on the shape of reference trajectory and on the previous state evolution. Details concerning the mentioned choice are presented in Sect. 4.

3 Control Design

To design an optimal control action, MPC employs predictions of expected future outputs of controlled system represented by a state space model. The equations of predictions are composed using current state estimate in nominal parts of model (1) and (2). For simplicity, we omit here the time indices, i.e., $A_t \to A$ and $B_t \to B$, as for one optimisation step, we will consider the matrices to be constant within a prediction horizon N that is for control horizon as well.

Prediction Equations – *positional control algorithm* [4,12]:

$$\hat{Y}_{t+1} = \left[\hat{y}_{t+1}^T, \cdots, \hat{y}_{t+N}^T\right]^T = F_1 \hat{x}_t + G_1 U_t,$$
$$U_t = \left[u_t^T, \cdots, u_{t+N-1}^T\right]^T \tag{13}$$
$$F_1 = \begin{bmatrix} CA \\ \vdots \\ CA^{N-1} \\ CA^N \end{bmatrix}, \quad G_1 = \begin{bmatrix} CB & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ CA^{N-2}B & \cdots & CB & 0 \\ CA^{N-1}B & \cdots & CAB & CB \end{bmatrix}.$$

To achieve integral property in the design, the nominal parts of model (1) and (2) are rewritten in incremental forms as follows [13]

$$\begin{aligned} \Delta\hat{x}_{t+1} &= \hat{x}_{t+1} - \hat{x}_t = A\, \Delta\hat{x}_t + B\, \Delta u_t \\ \Delta\hat{y}_{t+1} &= \hat{y}_{t+1} - y_t = C\, \Delta\hat{x}_{t+1}. \end{aligned} \tag{14}$$

Prediction Equations – *incremental control algorithm*: the equations are composed analogically to the positional algorithm but using the model (14):

$$\Delta\hat{x}_{t+j} = A^j\, \Delta\hat{x}_t + \sum_{i=1}^{j} A^{i-1} B \Delta u_{t+j-i} \tag{15}$$

$$\Delta\hat{y}_{t+j} = CA^j\, \Delta\hat{x}_t + \sum_{i=1}^{j} CA^{i-1} B \Delta u_{t+j-i} \tag{16}$$

The evolution of the full-value predictions of the system outputs $\hat{y}$ is

$$\hat{y}_{t+j} = y_t + \sum_{i=1}^{j} \Delta\hat{y}_{t+i} \tag{17}$$

The relevant matrix notation of (17) is as follows

$$\hat{Y}_{t+1} = [\hat{y}^T_{t+1} \cdots \hat{y}^T_{t+N}]^T = F_{\mathrm{I}}\, y_t + F_2\, \Delta\hat{x}_t + G_2\, \Delta U_t, \tag{18}$$

$$\Delta U_t = \left[\Delta u^T_t,\ \cdots, \Delta u^T_{t+N-1}\right]^T \tag{19}$$

$$F_{\mathrm{I}} = [I\ \cdots I]^T,\ \ F_2 = \begin{bmatrix} CA \\ \vdots \\ \sum\limits_{i=1}^{N} CA^i \end{bmatrix},\ \ G_2 = \begin{bmatrix} CB & \cdots & 0 \\ \vdots & \ddots & \vdots \\ \sum\limits_{i=1}^{N} CA^{i-1}B & \cdots & CB \end{bmatrix}$$

The behaviour of a control process is influenced by the choice of the cost function. MPC involves a quadratic cost function that balances control errors, i.e. differences between predicted outputs and reference values, against amount of input energy given by control vector (for the the positional algorithm) or control increments (for the incremental algorithm).

Cost Function: has the form

$$J_t = (\hat{Y}_{t+1} - W_{t+1})^T Q^T_{YW} Q_{YW} (\hat{Y}_{t+1} - W_{t+1}) + \mathbb{U}^T_t Q^T_{\mathbb{U}} Q_{\mathbb{U}} \mathbb{U}_t \tag{20}$$

where $\mathbb{U}_t = U_t$ for the *positional algorithm* [12] and $\mathbb{U}_t = \Delta U_t$ for the *incremental algorithm* [13]; $W_{t+1} = \left[w^T_{t+1},\ \cdots, w^T_{t+N}\right]^T$ represents a sequence of references.

Optimality Criterion: is defined as follows

$$\min_{\mathbb{U}_t}\ J_t\,(\hat{Y}_{t+1}, W_{t+1}, \mathbb{U}_t),\ \mathbb{U}_t \in \{U_t, \Delta U_t\} \tag{21}$$

s. t. state space model (1), (2) (or (15), (16))

state estimate $\hat{x}_t$ (12)

where $\hat{Y}_{t+1}$ are prediction Eq. (13) or (18), respectively. The involved cost function J_t (20) is rewritten into the square-root form

$$J_t = \mathbb{J}^T_t \mathbb{J}_t \tag{22}$$

For the *Positional algorithm*, the square-root $\mathbb{J}_t$ of J_t (22) is

$$\mathbb{J}_t = \begin{bmatrix} Q_{YW} & 0 \\ 0 & Q_U \end{bmatrix} \begin{bmatrix} \hat{Y}_{t+1} - W_{t+1} \\ U_t \end{bmatrix} = \begin{bmatrix} Q_{YW}(\,G_1\, U_t - Z_{pos}) \\ Q_U\, U_t \end{bmatrix} \tag{23}$$

where $\quad Z_{pos} = W_{t+1} - F_1\, \hat{x}_t.$

For the *Incremental algorithm*, the square-root $\mathbb{J}_t$ of J_t (22) is

$$\mathbb{J}_t = \begin{bmatrix} Q_{YW} & 0 \\ 0 & Q_{\Delta U} \end{bmatrix} \begin{bmatrix} \hat{Y}_{t+1} - W_{t+1} \\ \Delta U_t \end{bmatrix} = \begin{bmatrix} Q_{YW}\,(G_2\,\Delta U_t - \; Z_{inc}) \\ Q_{\Delta U}\,\Delta U_t \end{bmatrix} \tag{24}$$

where $Z_{inc} = W_{t+1} - F_I\,y_t - F_2\,\Delta\hat{x}_t$ and Q_{YW}, $Q_{\Delta U}$ and Q_U are penalisation matrices defined as follows

$$Q_\diamond^T Q_\diamond = \begin{bmatrix} Q_*^T Q_* & & 0 \\ & \ddots & \\ 0 & & Q_*^T Q_* \end{bmatrix} \left| \begin{array}{l} \text{subscripts } \diamond, * \ : \\ \diamond \in \{YW,\ \Delta U,\ U\} \\ * \in \{yw,\ \Delta u,\ u\} \end{array} \right. \tag{25}$$

Optimization: consist in the minimisation of the cost function. Considering the square-root $\mathbb{J}_t$ (23) or (24), the minimisation, as a specific solution of least-squares problem, leads to the following algebraic equations for (23) [12]:

$$\underbrace{\begin{bmatrix} Q_{YW}\,G_1 & Q_{YW}\,Z_{pos} \\ Q_U & 0 \end{bmatrix}}_{\mathcal{A} \qquad\qquad b} \begin{bmatrix} U_t \\ -I \end{bmatrix} = 0 \tag{26}$$

and for (24) [13]:

$$\underbrace{\begin{bmatrix} Q_{YW}\,G_2 & Q_{YW}\,Z_{inc} \\ Q_{\Delta U} & 0 \end{bmatrix}}_{\mathcal{A} \qquad\qquad b} \begin{bmatrix} \Delta U_t \\ -I \end{bmatrix} = 0 \tag{27}$$

The over-determined system (26) or (27), respectively, can be transformed by orthogonal-triangular decomposition [14] so that matrix $[\,\mathcal{A}\ \ b\,]$ is transformed into upper triangle matrix R_1, and solved for unknown $\mathbb{U}_t \in \{U_t, \Delta U_t\}$. This transformation is indicated by the following equation diagram

$$\begin{bmatrix} \mathcal{A} & b \end{bmatrix} \begin{bmatrix} \mathbb{U}_t \\ \text{-I} \end{bmatrix} = 0 \;\Rightarrow\; \begin{bmatrix} R_1 & c_1 \\ 0 & c_z \end{bmatrix} \begin{bmatrix} \mathbb{U}_t \\ \text{-I} \end{bmatrix} = 0 \tag{28}$$

where the vector c_z represents a loss vector. Its Euclidean norm $||c_z||_2$ corresponds to the square-root of the minimum of cost function (20), i.e., $J_t = c_z^T c_z$. Note that for control, only the first elements corresponding to u_t or Δu_t are used from computed vector $\mathbb{U}_t$ (13) or (19), respectively.

In the previous paper of authors, [12], the point estimate corresponding to the centre of (12) was used in (28). The paper [13] extended these result both by using the incremental algorithm (24) and by considering the set estimate (12) without choice a particular point estimate. The transformation into (28) was performed successively for preselected points from the whole set. Subsequently, the realisation with the minimal value of $||c_z||_2$ was chosen as the result.

Here, we aim to optimize the choice of the relevant point estimate before optimization step to avoid the multiple optimisation run. For details on this choice see Sect. 4.3.

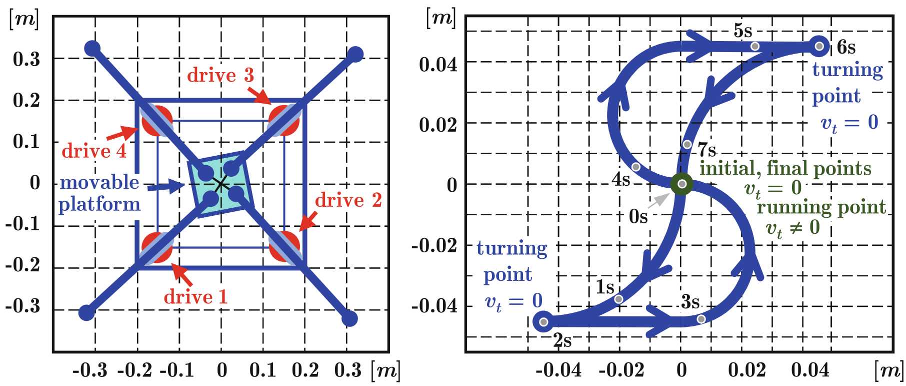

Fig. 1. Considered robot 'Moving Slide' and used testing trajectory [13].

Table 1. Overview of performed experiments.

Exp.	Control algorithm	State estimate	Fig. No	Mean E_t (31)	Max. E_t (31) (t)
1	Positional (23)	Central point (30)	Fig. 2	0.8977 mm	2.0958 mm (3.24 s)
2	Positional (23)	Set (12) [13]	Fig. 3, 4	0.9053 mm	2.3157 mm (4.70 s)
3	Positional (23)	Best point, Sect. 4.3	Fig. 5	0.9192 mm	2.2842 mm (3.24 s)
4	Incremental (24)	Central point (30)	Fig. 6	0.8248 mm	1.9193 mm (3.81 s)
5	Incremental (24)	Set (12) [13]	Fig. 7, 8	0.8317 mm	1.9069 mm (3.81 s)
6	Incremental (24)	Best point, Sect. 4.3	Fig. 9	0.8231 mm	1.8836 mm (3.81 s)

4 Experiments

4.1 Robot Model

To illustrate the proposed algorithm, we use the redundant planar parallel robot-manipulator [1]. It has a four-dimensional input u (four torques) and a three-dimensional output y (Cartesian positions of tool center point (TCP): x_{TCP} and y_{TCP}; and rotation angle φ_{TCP} of robot movable platform around the vertical axis), see Fig. 1. The dynamics of the robot can be described by a set of non-linear differential equations representing equations of motion. They are composed using Lagrange equations [3]

$$\ddot{y} = \mathsf{f}(\dot{y}, y) + \mathsf{g}(y)\, u \tag{29}$$

where $y = [\ x_{TCP},\ y_{TCP},\ \varphi_{TCP}]^T$. The corresponding non-linear continuous-time state-space model can be transformed into the linear-like continuous-time state-space model by a special decomposition [24]. Then, using standard time discretisation and considering additive bounded disturbances, the LSU model (1), (2) is obtained [13] where the system state $x_t = [y_t^T, \dot{y}_t^T]^T$.

4.2 Experiment Setup

The controlled system is represented by the robot model (29) with an additive uniformly distributed output noise. The set state estimates (12) are obtained using the linearised robot model (1), (2) as described in Sect. 2. The noise bounds are set as follows: $\rho = 10^{-6}[m, m, rad, m\,s^{-1}, m\,s^{-1}, rad\,s^{-1}]^T$, $r = 10^{-3}[m, m, rad]^T$. The control parameters in (20) are set as follows: $N = 10$; $Q_{yw} = I$, $Q_u = 10^{-2}\,I$, $Q_{\Delta u} = 2.5 \cdot 10^{-2}\,I$, where I is the identity matrix of the appropriate order. The reference trajectory to be followed is depicted in Fig. 1.

The central point estimate, i.e. corresponding to the mean value of the set estimate (12), has the form [12]

$$\hat{x}_t = \frac{\underline{x}_t + \overline{x}_t}{2}. \tag{30}$$

The control error, i.e. a difference between the reference and a measured output, is defined as follows

$$E_t = \sqrt{\sum_{i=1}^{2}(y_i - w_i)^2} = \sqrt{\sum_{i=1}^{2} e_i^2}. \tag{31}$$

4.3 Search for the Best Point Estimate

Running the series of experiments based on the combination of boundary state values for corresponding couples: [position y_i + velocity $\dot{y}_i]\!]_{i=1,2,3;}$, we have prove experimentally our initial hypothesis that the control results depend both on the choice of particular point from the set estimate (12) and on the previous state evolution that is related to the shape of the reference trajectory.

4.4 Results and Discussion

We have run series of experiments comparing the performance of the positional (23) and the incremental (24) output-feedback MPC algorithms with the following variants concerning the state point estimate: (i) a conservative choice (30) as presented in our previous paper [12], (ii) multiple optimisation using the scheme (28) with selection of points from (12) where the realization with a minimal loss c_z provides the required control input presented in our previous paper [13], (iii) offline setting of the "best" point state estimate as described in Sect. 4.3. A summary of the experiments is presented in Table 1.

In [13], the choice of the point estimate is performed in every optimisation step withing prediction horizon N. This strategy results in a discontinuous switching between subsequent point choices in control process, see the second halves of the control input courses in Fig. 3.

The proposed method of the "best" point state estimates applied to the incremental control algorithm (experiment 5) , delivers the least control error.

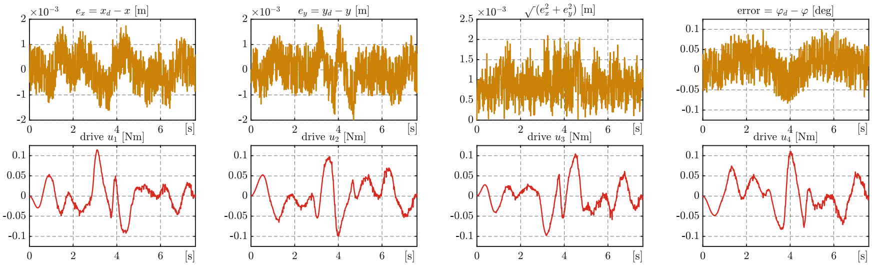

Fig. 2. Positional algorithm with mean value.

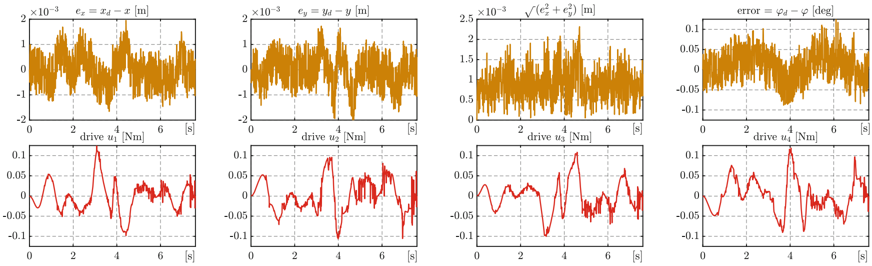

Fig. 3. Positional algorithm with selection according to loss value $c(z)$.

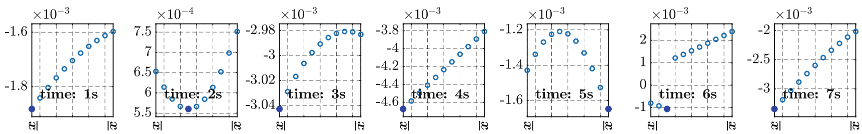

Fig. 4. Positional algorithm selected illustrative courses of loss values $c(z)$.

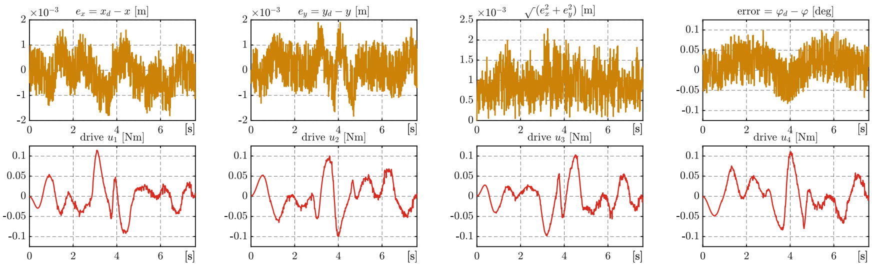

Fig. 5. Positional algorithm with selection based on prior information (1, 11, 6).

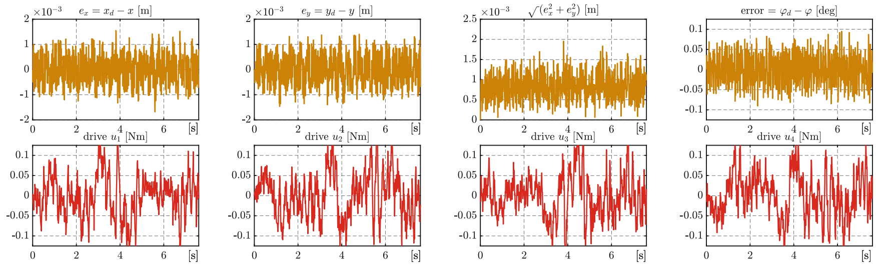

Fig. 6. Incremental algorithm with mean value.

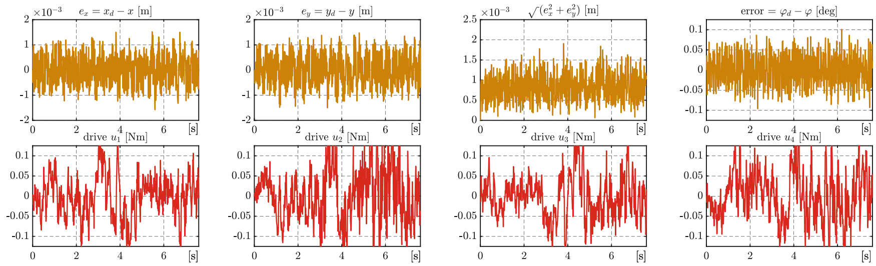

Fig. 7. Incremental algorithm with selection according to loss value $c(z)$.

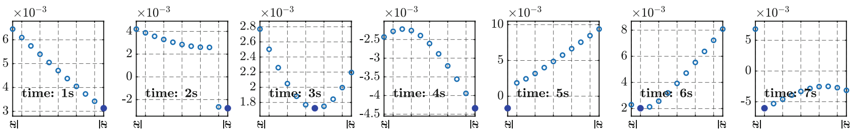

Fig. 8. Incremental algorithm selected illustrative courses of loss values $c(z)$.

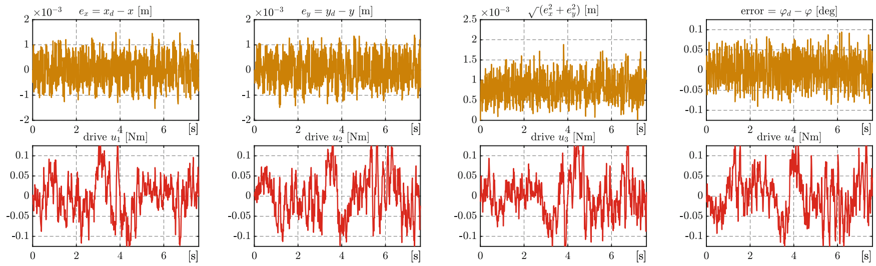

Fig. 9. Incremental algorithm with selection based on prior information (1, 11, 6).

5 Conclusion

The paper presents a method for the choice of the point state estimate used in the output-feedback MPC design. The results are compared with the previous authors results presented in [13] and [12]. In this paper, we proved experimentally our initial hypothesis that the control results depend both on the choice of particular point from the set estimate (12) and on the previous state evolution that is related to the shape of the reference trajectory.

Future work will focus on a theoretical justification of our hypothesis. We will aim to propose the optimal choice of point estimate based on available physical prior information about system motion behavior and on a physical substance of individual state variables e.g. position and corresponding velocity.

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