GENERALIZED PREDICTIVE CONTROL AND STATE SPACE KALMAN FILTER ESTIMATION

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Abstract: Predictive controllers based on model-based generalized Predictive algorithm gain significant and widespread application in industrial process control. Predictive control can assume all relations in a controlled system and can design so-called centralized control actions. In spite of its incontestable advantages, this control can cause occurrence of steady-state errors. It is happened not only when penalizations in design criterion of predictive control are nonzero but also e.g. when unmeasured disturbances occur. This paper deals with one possible solution based on incremental modification of state-space Predictive algorithm with state-space Kalman filter estimation. The algorithms of this combination are presented in square-root form.

Keywords: Predictive Control, State-Space Control, State Estimation, Kalman Filter.

1. INTRODUCTION

Predictive controllers based on model-based generalized Predictive algorithm (Ordys *et al.* 1993) can offer more powerful control actions than standard PID-based controllers and therefore they gain significant and widespread application in industrial process control. Their basic formulation can be adapted, without difficult modifications, directly for multi-input multi-output (MIMO) systems.

Conventional use of PID-based controllers for MIMO systems represents taking the systems as a set of single-input single-output units (setSISO) with independent (decentralized) control in their appropriate loops (Belda *et al.* 2001). Internal relations in the controlled MIMO system are assumed as outside disturbances. The use of PID based controllers may generally provide control, but it need not achieve good results, mainly in case of dynamically non-uniform systems. Moreover, it is not applicable for under or over actuated systems, where the knowledge of some model, representing system decoupling, is necessary.

On the other hand, Predictive control design, which is based on some model representation (model-based approach), can assume, just by model, most of all relations in a controlled system. The Predictive control can be applied not only for adequately actuated systems, but also for under and over actuated systems. Through the model, it can design more suitable control actions that closely correspond to actual requirements (desired values). Against independent (decentralized) PID controllers, it represents global, so-called centralized control design (Belda *et al.* 2003).

In spite of incontestable advantages of predictive control design, such control can cause, in general point of view, occurrence of steady-state errors (Belda *et al.* 2004). It is happened not only when absolute values of control actions are penalized in the criterion of predictive design, but also when unmeasured disturbances or real passive resistances – insensitivities occur. This paper deals with one possible solution based on incremental modification of state-space Predictive algorithm supported with state-space Kalman filter estimation.

The paper is organized as follows. In section two, the construction and modification of state-space model for incremental algorithm is shown. The third section deals partly with the equations of prediction and partly with predictive control design derived in square-root form. The next section, section four, concerns with the question of achievability of new state vector following from incremental model modification. For solving this question, the observer based on Kalman filter is used (Anderson *et al.* 1979). Two observer structures are considered. Design of observer gain is formulated also in square-root form. Finally, the section five concludes the paper.

2. MATHEMATICAL MODEL AND ITS MODIFICATIONS

Mathematical models represent important prior information in process of control design. Let us proceed from ordinary used mathematical model – ordinary differential equation generally of n^{th} order

$$y^{(n)} + a_{n-1}y^{(n-1)} + \cdots + a_0 y = b_0 u + \cdots + b_{n-1}u^{(n-1)} \tag{1}$$

The model of the system is represented either by single equation (1) in case of single-input single-output (SISO) systems or by set of equations for multi-input multi-output (MIMO) systems. Generally, these cases can be written in state-space form (2)

$$\begin{aligned} \dot{\mathbf{X}}(t) &= \mathbf{A}_C\,\mathbf{X}(t) + \mathbf{B}_C\,\mathbf{u}(t) \\ \mathbf{y}(t) &= \mathbf{C}_C\,\mathbf{X}(t) \end{aligned} \tag{2}$$

Due to digital realization in practice, the models are discretized to a form

$$\begin{aligned} \mathbf{X}(k+1) &= \mathbf{A}\,\mathbf{X}(k) + \mathbf{B}\,\mathbf{u}(k) \\ \mathbf{y}(k) &= \mathbf{C}\,\mathbf{X}(k) \end{aligned} \tag{3}$$

The discrete state-space model (3) is suitable form for design based on Predictive control algorithms. In order to constitute (build in) incremental character to predictive algorithms generally both for static and astatic systems, one of possibilities is to use the following simple modification of state-space model (3)

$$\begin{aligned} \mathbf{X}(k+1) &= \mathbf{A}\,\mathbf{X}(k) + \mathbf{B}\,\mathbf{u}(k) \\ \mathbf{u}(k+1) &= \qquad\qquad \mathbf{u}(k) + \Delta\mathbf{u}(k) \\ \mathbf{y}(k) &= \mathbf{C}\,\mathbf{X}(k) \end{aligned} \tag{4}$$

$$\begin{aligned} \mathbf{X}g(k+1) &= \mathbf{A}g\;\mathbf{X}g(k) + \mathbf{B}g\;\Delta\mathbf{u}(k) \\ \mathbf{y}(k) &= \mathbf{C}g\;\mathbf{X}g(k) \end{aligned}, \quad \mathbf{X}g(k+1) = \begin{bmatrix} \mathbf{X}(k+1) \\ \mathbf{u}(k) \end{bmatrix},\; \mathbf{A}g = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{0} & \mathbf{1} \end{bmatrix},\; \mathbf{B}g = \begin{bmatrix} \mathbf{0} \\ \mathbf{1} \end{bmatrix} \quad \mathbf{C}g = \begin{bmatrix} \mathbf{C} & \mathbf{0} \end{bmatrix} \tag{5}$$

State-space model (4) after condensing (5) has the same form as notation (3) (Belda 2005).

That modification can ensure consecutive change of control actions in spite of cases of static systems without possibility to measure ideal system state; (i.e. only disturbed state (output) is available). In case of astatic systems, this modification can solve model inaccuracies from reality, as well physical insensitivities (passive resistance, mechanical backlashes etc.). Finally, the incremental modification removes loss effect at penalization of control action in design criterion.

3. PREDICTIVE CONTROL DESIGN

Generalized predictive control belongs with linear quadratic control to multi-step approach (Ordys *et al.* 1993). It combines both feed-forward part and feed-back part. The feed-forward part is represented by prediction via mathematical model describing a controlled system. This part forms the dominant part of control actions. The feed-back, closed from measured outputs, compensates some inaccuracies of the model and certain bounded disturbances.

The design consists in local minimization of the criterion expressed by quadratic cost function. In it, the predictions, given by equations of prediction, are involved. The following subsections outline this approach of model-based design of control.

3.1 Equations of predictions

The prediction is fundamental part of the design. It can define the character of the algorithm (Ordys 1993, Belda 2005). May the following algorithm types are considered

- basic algorithm, generating full control actions;
- incremental algorithm, generating the increments of control actions.

Basic algorithm generates directly appropriate values of the actions, i.e. their full (absolute) values. The algorithm arises from the model (3). On the other hand, incremental algorithm generates only increments of the control actions, which are counted for absolute values applied to the system.

The prediction for both algorithms leads to the repetitive insertion of state-space formula (3) or (5). (Note: For simplicity, in further text, the symbol $(\cdot)$ will represent simultaneously both basic and incremental state model matrices, and symbol $(.)$ will mark also simultaneously either full values of control action or its increment, respectively.)

$$\begin{aligned}
\hat{\mathbf{x}}_{(\cdot)(k+1)} &= \mathbf{A}_{(\cdot)}\,\mathbf{x}_{(\cdot)(k)} + \mathbf{B}_{(\cdot)(.)}\mathbf{u}_{(k)} \\
\hat{y}_{(k+1)} &= \mathbf{C}_{(\cdot)}\mathbf{A}_{(\cdot)}\,\mathbf{x}_{(\cdot)(k)} + \mathbf{C}_{(\cdot)}\mathbf{B}_{(\cdot)(.)}\mathbf{u}_{(k)} \\
&\ \ \vdots \qquad\qquad \vdots \qquad\qquad \ddots \\
\hat{\mathbf{x}}_{(\cdot)(k+N)} &= \mathbf{A}_{(\cdot)}^{N}\,\mathbf{x}_{(\cdot)(k)} + \cdots + \mathbf{B}_{(\cdot)(.)}\mathbf{u}_{(k+N-1)} \\
\hat{y}_{(k+N)} &= \mathbf{C}_{(\cdot)}\mathbf{A}_{(\cdot)}^{N}\,\mathbf{x}_{(\cdot)(k)} + \cdots + \mathbf{C}_{(\cdot)}\mathbf{B}_{(\cdot)(.)}\mathbf{u}_{(k+N-1)}
\end{aligned} \tag{6}$$

In condensed notation, equations of prediction are

$$\hat{\mathbf{y}} = \mathbf{f} + \mathbf{G}_{(.)}\mathbf{u}\,, \quad \text{where } \mathbf{f} = \begin{bmatrix} \mathbf{C}_{(\cdot)}\mathbf{A}_{(\cdot)} \\ \vdots \\ \mathbf{C}_{(\cdot)}\mathbf{A}_{(\cdot)}^{N} \end{bmatrix}\mathbf{x}_{(\cdot)(k)},\ \mathbf{G} = \begin{bmatrix} \mathbf{C}_{(\cdot)}\mathbf{B}_{(\cdot)} & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{C}_{(\cdot)}\mathbf{A}_{(\cdot)}^{N-1}\mathbf{B}_{(\cdot)} & \cdots & \mathbf{C}_{(\cdot)}\mathbf{B}_{(\cdot)} \end{bmatrix} \tag{7}$$

3.2 Minimization of the design criterion

Design criterion is defined for certain interval of predictions (several steps to future). It includes the part of control error, in which the model of system is covered (insertion of equations of prediction (7)) and part of control actions, where the input energy (control actions) is weighted. This part redistributes control errors to individual steps of predictions and provides coupling within interval of predictions. Usual form of the criterion for predictive design is written as follows

$$J_k = \sum_{j=No+1}^{N} \left\{ \left(\mathbf{y}(k+j) - \mathbf{w}(k+j) \right)^T \mathbf{Q}_\mathbf{y} \left(\mathbf{y}(k+j) - \mathbf{w}(k+j) \right) \right\} + \sum_{j=1}^{Nu} \left\{ {}_{(.)}\mathbf{u}(k+j-1)^T \mathbf{Q}_\mathbf{u}\, {}_{(.)}\mathbf{u}(k+j-1) \right\} \quad (8)$$

The criterion is expressed in step k. N is a horizon of prediction, No is a horizon of initial insensitivity and Nu is a control horizon. $\mathbf{Q}_\mathbf{y}$ and $\mathbf{Q}_\mathbf{u}$ are output and input penalizations and $\mathbf{y}(k+j)$ and ${}_{(.)}\mathbf{u}(k+j-1)$ are output and input (full or incremental) values. The control actions are obtained by minimization of described criterion (9), which can be simply rewritten to the following matrix product

$$J_k = [(\hat{\mathbf{y}} - \mathbf{w})^T, {}_{(.)}\mathbf{u}^T] \begin{bmatrix} \mathbf{Q}_\mathrm{y} & \mathbf{0} \\ \mathbf{0} & \mathbf{Q}_\mathbf{u} \end{bmatrix} \begin{bmatrix} \mathbf{Q}_\mathrm{y} & \mathbf{0} \\ \mathbf{0} & \mathbf{Q}_\mathbf{u} \end{bmatrix} \begin{bmatrix} \hat{\mathbf{y}} - \mathbf{w} \\ {}_{(.)}\mathbf{u} \end{bmatrix} =$$

$$= \qquad \mathbf{J}^T \qquad \times \qquad \mathbf{J} \quad (9)$$

where $\hat{\mathbf{y}}$ is a vector substituted by the equation (7) (time step k+1, · · ·, k+N), $\mathbf{w}$ is a vector of desired values, corresponding to vector $\hat{\mathrm{y}}$ and ${}_{(.)}\,\mathbf{u}$ is a vector of designed future inputs, again in discrete time instants for the whole horizon (k, · · ·, N - 1). The product (9), as it is indicated, can be decomposed in so-called square roots of the criterion. From mathematical point of view, the minimization of the square root gives straightforward direction for practical use. If the square root of the criterion on the right side is selected and expression of prediction (7) is inserted in this square root, then the new criterion is given

$$\mathbf{J} = \begin{bmatrix} \mathbf{Q}_\mathrm{y} & \mathbf{0} \\ \mathbf{0} & \mathbf{Q}_\mathbf{u} \end{bmatrix} \begin{bmatrix} \hat{\mathbf{y}} - \mathbf{w} \\ {}_{(.)}\mathbf{u} \end{bmatrix} = \begin{bmatrix} \mathbf{Q}_\mathrm{y}\,\mathbf{G} \\ \mathbf{Q}_\mathbf{u} \end{bmatrix} {}_{(.)}\mathbf{u} - \begin{bmatrix} \mathbf{Q}_\mathrm{y}\,(\mathbf{w} - \mathbf{f}) \\ \mathbf{0} \end{bmatrix} \quad (10)$$

J is a column vector and its Euclidean norm equals a cost of the square root of the criterion (9). The objective is to search for such ${}_{(.)}\,\mathbf{u}$, which minimizes the square root (10); which means, that ${}_{(.)}\,\mathbf{u}$ minimizes the norm $|\mathbf{J}|$, and thus as well the criterion (9). In case of square root (10), the minimization leads to a system of algebraic equations with more rows than columns – over-determined system

$$\begin{aligned} \begin{bmatrix} \mathbf{Q}_\mathrm{y}\,\mathbf{G} \\ \mathbf{Q}_\mathbf{u} \end{bmatrix} {}_{(.)}\mathbf{u} - \begin{bmatrix} \mathbf{Q}_\mathrm{y}\,(\mathbf{w} - \mathbf{f}) \\ \mathbf{0} \end{bmatrix} &= \mathbf{0} \\ \mathbf{A} \quad \mathbf{u} - \quad \mathbf{b} \quad &= \mathbf{0} \end{aligned} \quad (11)$$

For optimization of the criterion, the orthogonal triangular decomposition (Golub *et al.* 1989) is used. It reduces excess rows of matrix **A** $[(2 \cdot N \cdot i) \times (N \cdot i)]$ and elements of vector **b** $[2 \cdot N \cdot i]$ (i is a number of inputs of controlled system) into upper triangular matrix, which makes possible to compute the control ${}_{(.)}\,\mathbf{u}$ directly by backward substitution.

Finally, let us note how to construct real control actions at incremental algorithm: after computing of vector ${}_{(.)}\,\mathbf{u}$ for whole horizon, only first control ${}_{(.)}\mathbf{u}(k)$ is used (as at basic algorithm) and for obtaining the full control actions the second line of equation (4) is used.

4. STATE-SPACE ESTIMATION

When using Predictive control in the state-space formulation and generally at the use of whichever state-space control, it is necessary to solve the question of availability of the state of the system (state vector). If it is not available and only system outputs from the measurement are known, then some state-space estimation has to be considered. Suitable, well-known solution of such estimation is the state-space observer based on Kalman filter (Anderson et al. 1979). The determination of its gain will be the main objective. It will be demonstrated in square root form – optimal way for real-time use.

Consider a linear or linearized, discrete multi-input multi-output system defined by (3) or (5). The Kalman filter is designed such that the estimate $\hat{\mathbf{X}}$ of the state $\mathbf{X}$ is 'best' in the sense of minimum of the error covariance matrix for the considered estimate – the best estimate given by the minimum variance estimates (Billings 1980). That is, the estimate $\hat{\mathbf{X}}$ is to be determined so that

$$\operatorname{trace} E\,((\mathbf{X}-\hat{\mathbf{X}})(\mathbf{X}-\hat{\mathbf{X}})^T) \;\rightarrow\; \text{minimum} \tag{12}$$

where $\operatorname{trace} E\,(\cdot)$ is conditional error variance associated with the estimate $\hat{\mathbf{X}}$ and conditional mean estimate minimizes this error variance; $\mathbf{X}$ is a random state vector, which is to be estimated from measurement – a random vector $\mathbf{Y}$ with mean value $\bar{\mathbf{y}}$ and altogether with joint covariance matrix

$$\boldsymbol{\Sigma} = \begin{bmatrix} \boldsymbol{\Sigma} yy & \boldsymbol{\Sigma} yx \\ \boldsymbol{\Sigma} xy & \boldsymbol{\Sigma} xx \end{bmatrix} \tag{13}$$

Then the conditional probability density with knowledge of $\mathbf{y}$ (measurement) can be written

$$\begin{aligned} p_{\mathbf{X}|\mathbf{Y}}(\mathbf{X}\,|\,\mathbf{Y}) &= \frac{p_{\mathbf{XY}}(\mathbf{X},\mathbf{Y})}{p_{\mathbf{Y}}(\mathbf{Y})} = \frac{1}{(2\pi)^{\frac{N+n}{2}}} \frac{e^{-\frac{1}{2}[\mathbf{X}^T-\bar{\mathbf{X}}^T \;\; \mathbf{Y}^T-\bar{\mathbf{y}}^T]\boldsymbol{\Sigma}^{-1}\left[\begin{smallmatrix}\mathbf{X}-\bar{\mathbf{X}}\\ \mathbf{Y}-\bar{\mathbf{y}}\end{smallmatrix}\right]}}{|\boldsymbol{\Sigma}|^{1/2}} \frac{(2\pi)^{n/2}\,|\boldsymbol{\Sigma}_{yy}|^{1/2}}{e^{-\frac{1}{2}(\mathbf{Y}^T-\bar{\mathbf{y}}^T)\boldsymbol{\Sigma}^{-1}yy\,(\mathbf{Y}-\bar{\mathbf{y}})}} = \\ &= \frac{1}{(2\pi)^{\frac{N}{2}}} \frac{e^{-\frac{1}{2}(\mathbf{X}^T-\hat{\mathbf{X}}^T)(\boldsymbol{\Sigma}xx-\boldsymbol{\Sigma}xy\,\boldsymbol{\Sigma}^{-1}yy\boldsymbol{\Sigma}yx)(\mathbf{X}-\hat{\mathbf{X}})}}{|\,\boldsymbol{\Sigma}xx-\boldsymbol{\Sigma}xy\,\boldsymbol{\Sigma}^{-1}yy\boldsymbol{\Sigma}yx\,|^{1/2}}, \quad \text{where } n \text{ is a number of outputs and } N \text{ is a dimension of the state} \end{aligned} \tag{14}$$

with mean
$$\hat{\mathbf{X}}(\mathbf{Y}) = E(\mathbf{X}\,|\,\mathbf{Y}) = \bar{\mathbf{X}} + \boldsymbol{\Sigma}xy\,\boldsymbol{\Sigma}^{-1}yy\,(\mathbf{Y}-\bar{\mathbf{y}}), \tag{15}$$

and covariance
$$\boldsymbol{\Sigma}(\mathbf{X}\,|\,\mathbf{Y}) = \boldsymbol{\Sigma}xx - \boldsymbol{\Sigma}xy\,\boldsymbol{\Sigma}^{-1}yy\;\boldsymbol{\Sigma}yx\,. \tag{16}$$

In the expressions (15) and (16), the marginal covariances are expressed in general

$$\begin{aligned} &\textstyle\sum xx = E((\mathbf{X}_{(k)}-\hat{\mathbf{X}}_{(k)})(\mathbf{X}_{(k)}-\hat{\mathbf{X}}_{(k)})^T) = \sum_{(k,\,k-1)} \\ &\textstyle\sum yy = E((\mathbf{y}_{(k)}-\hat{\mathbf{y}}_{(k)})(\mathbf{y}_{(k)}-\hat{\mathbf{y}}_{(k)})^T) = \mathbf{R} + \mathbf{C}\sum_{(k,\,k-1)}\mathbf{C}^T \quad , \mathbf{R} \text{ is a covariance of } \mathbf{y} \\ &\textstyle\sum xy = E((\mathbf{X}_{(k)}-\hat{\mathbf{X}}_{(k)})(\mathbf{y}_{(k)}-\hat{\mathbf{y}}_{(k)})^T) = \sum_{(k,\,k-1)}\mathbf{C}^T \\ &\textstyle\sum yx = (\sum xy)^T = \mathbf{C}^T\sum_{(k,\,k-1)} \end{aligned} \tag{17}$$

To obtain combined (time + measurement) update, the initial form of covariance is

$$\boldsymbol{\Sigma}_{k+1,k} = \begin{bmatrix} \mathbf{R}+\mathbf{C}(\mathbf{A}\boldsymbol{\Sigma}\mathbf{A}^T+\mathbf{GQG}^T)\mathbf{C}^T & \mathbf{C}(\mathbf{A}\boldsymbol{\Sigma}\mathbf{A}^T+\mathbf{GQG}^T) \\ (\mathbf{A}\boldsymbol{\Sigma}\,\mathbf{A}^T+\mathbf{GQG}^T)\mathbf{C}^T & \mathbf{A}\boldsymbol{\Sigma}\mathbf{A}^T+\mathbf{GQG}^T \end{bmatrix}_k, \quad \begin{array}{l}\mathbf{G} \text{ is a gain noise matrix} \\ \mathbf{Q} \text{ is a covariance of } \mathbf{X}\end{array} \tag{18}$$

The expression (18) can be decomposed to the product of square roots

$$\boldsymbol{\Sigma}_{k+1,k} = \begin{bmatrix} \mathbf{R}_r & \mathbf{CAS} & \mathbf{CGQ}_r \\ \mathbf{0} & \mathbf{AS} & \mathbf{GQ}_r \end{bmatrix}_k \begin{bmatrix} \mathbf{R}_r^T & \mathbf{0} \\ \mathbf{S}^T\mathbf{A}^T\mathbf{C}^T & \mathbf{S}^T\mathbf{A}^T \\ \mathbf{Q}_r^T\mathbf{G}^T\mathbf{C}^T & \mathbf{Q}_r^T\mathbf{G}^T \end{bmatrix}_k = \left(\boldsymbol{\Sigma}^{\frac{1}{2}}_{k+1,k}\right)\left(\boldsymbol{\Sigma}^{\frac{1}{2}}_{k+1,k}\right)^T \quad (19)$$

$$(\sqrt{\Sigma_{k+1}})^T = \begin{bmatrix} \mathbf{R}^T & \mathbf{0} \\ \mathbf{S}_k{}^T\mathbf{A}^T\mathbf{C}^T & \mathbf{S}_k{}^T\mathbf{A}^T \\ \mathbf{Q}^T\mathbf{G}^T\mathbf{C}^T & \mathbf{Q}^T\mathbf{G}^T \end{bmatrix} = \begin{bmatrix} (inv)^T & (\mathbf{k})^T \\ \mathbf{0} & \mathbf{S}_{k+1}{}^T \\ \mathbf{0} & \mathbf{0} \end{bmatrix}, \quad \mathbf{kfg} = ((inv)^{-1}\,\mathbf{k}\,)^T \quad (20)$$

where **R**, **Q** and **S** are square roots of **R Q** and $\Sigma xx = \mathbf{SS}^T$; **kfg** is a searched observer gain.

Finally, the equations of estimation (equations of state observer) can be written. They can be formulated either as a single equation of estimation (21) for $\hat{\mathbf{X}}_{(\cdot)\,k+1,k}$

$$\hat{\mathbf{X}}_{(\cdot)_{k+1,k}} = \mathbf{A}_{(\cdot)}\hat{\mathbf{X}}_{(\cdot)_{k,k-1}} + \mathbf{A}_{(\cdot)}\,\mathbf{kfg}\,(\mathbf{y}_k - \mathbf{C}_{(\cdot)}\hat{\mathbf{X}}_{(\cdot)_{k,k-1}}) + \mathbf{B}_{(\cdot)(.)}\mathbf{u}(k) \quad (21)$$

or as a two equations (22) and (23) used separately for $\hat{\mathbf{X}}_{(\cdot)\,k,k}$ and $\hat{\mathbf{X}}_{(\cdot)_{k+1,k}}$ (Billings 1980)

$$\hat{\mathbf{X}}_{(\cdot)_{k,k}} = \hat{\mathbf{X}}_{(\cdot)_{k,k-1}} + \mathbf{kfg}\,(\mathbf{y}_k - \mathbf{C}_{(\cdot)}\hat{\mathbf{X}}_{(\cdot)_{k,k-1}}) \quad (22)$$

$$\hat{\mathbf{X}}_{(\cdot)_{k+1,k}} = \mathbf{A}_{(\cdot)}\hat{\mathbf{X}}_{(\cdot)_{k,k}} + \mathbf{B}_{(\cdot)(.)}\mathbf{u}(k) \quad (23)$$

5. CONCLUSION

This paper deals with incremental modification of model-based control based on discrete state-space generalized predictive algorithm and briefly summarizes the solution of state-space estimation.

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