PREDICTIVE CONTROL WITH TIME-VARIANT STATE-SPACE MODELS

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Abstract: The paper is focused on Predictive control design for time-variant state-space models both SISO and MIMO type, specially on model identification/composition, further on consideration of system nonlinearities in equations of predictions and state-space estimation. The algorithms will be presented in computationally-effective square-root forms and demonstrated by several simulative examples with simple linear SISO systems and with one nonlinear system represented by MIMO model of planar redundant parallel robot.

Keywords: Predictive control, State-space estimation, Linear identification, Nonlinear prediction

1 INTRODUCTION

Predictive control based on generalized predictive algorithm [Ordys, 1993] is a powerful approach, which enables user to handle different control requirements (limits, constraints). Predictive control is a representative of model-based control. The form of model, which describes a controlled system, determines the style of control design, respectively, properties of designed control actions. However, the basic idea of predictive design is independent of specific model formulation. Due to its transparency and universality is predetermined to substitute standard PID based controllers still used by virtue of their simplicity. Moreover, the design of control actions by predictive control does not require complicated setting of parameters of the controller from end user as in PID controllers.

This paper deals with composition/identification of models of controlled systems, of which mathematical-physical expression is not available and known. Furthermore, the paper deals also with the opposite case, when the mathematical-physical model of the system is known, but it is nonlinear. In that case, one possibility, how to reflect this nonlinearity, is introduced.

The predictive control presented in the paper is based on discrete state-space models. In case of composition/identification, the state-space model is considered in transposed Frobenius' form [Šulc, 1999] obtained by transformation of autoregressive model with external input (ARX model). To use a state-space formulation of predictive algorithm, the system state is necessary to be known. When the state is not available (is not measured), then the state is estimated by discrete linear state-space observer based on Kalman filter [Anderson, 1979; Billings, 1980].

The paper is organized as follows:

The used models are defined in section two. The time arrangement of control process is discussed here. The arrangement consists in specification of available time range for support procedures, simultaneous identification and state-space estimation, model composition and mainly in specification of the time for computation of control actions in a frame of one sampling period.

In section three, the computation algorithm of Kalman gain via square-roots used in state-space observer is briefly outlined. Indicated algorithm is possible to employ either for simultaneous estimation of the model parameters and system state or only for state-space estimation. The algorithm stays the same for both cases, only models are different.

The section four deals with a predictive control design. It starts from composition of equations of predictions, which are firstly defined in usual form and then they are modified to consider nonlinearities in the model. The section ends by minimization of quadratic criterion, on which base the control actions are generated. Again the derivation is in the square-root form.

In the last two sections, there is a demonstration of simulative examples, which documents presented theory and finally, there is a conclusion, which sums up its ground.

2 MODEL DEFINITIONS, IDENTIFICATION/COMPOSITION

The model (description) of controlled system represents prior information for design of control actions. The best results of control process are achieved, when the model is obtained on the basis of thoroughgoing mathematical and physical analysis. It is often difficult. Therefore, different ways, how to obtain the model describing the controlled system, are investigated.

Important factor determining form of used models is a way of the real realization of considered model-based control, in which the models are involved. Due to digital character of automating devices, the discrete model-based control techniques are preferred. Therefore, the resultant models for control design are also discrete in spite of the facts that controlled system may be continuous. Discrete realization is advantageous, because respects naturally finite and predefined time for computation of control actions.

First of all, let us focus on time relations, respective, on time arrangement of one time period (sampling period) during the control process. These relations are presented in Figure 1.

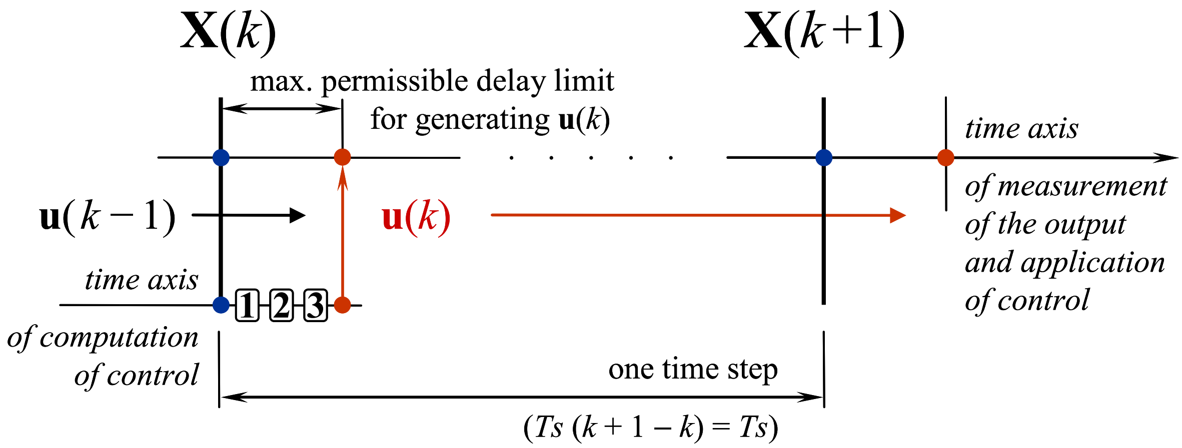


Figure 1 - Time arrangement of computation of control action by model-based approaches (1: support procedures like specific output transformations, parameter and state estimation; 2: model composition; 3: computation of real control actions)

The whole computation of control should not overrun some limit in interval of sampling, otherwise the computed control actions have no time to influence the controlled system and such design loses meaning. (Such situation should be solved as a system with delayed inputs.)

The situation illustrated in Figure 1 represents convention, in which it is supposed that controller reacts immediately, maximally with limited delay negligible in comparison with length of sampling period. On the contrary, the delay corresponding to sampling is considered in the model of controlled system.

It follows some simplification in the structure of both autoregressive model and state-space model. This convention omits the model parameters connected with direct feed-through of inputs. In the following subsections, the models will be considered generally, i.e. inclusive of these parameters. Always, with note, that proper parameter equals zero. The algorithms of estimation and generally of predictive control are universal; only, the obtained results (parameters, control actions) have to be appropriately interpreted, respectively, have to be correctly applied.

2.1 State-space model for simultaneous estimation of state and model parameters

For construction of suitable state-space model, let us proceed from simple autoregressive model with external input (ARX model), which describes relations in controlled system among its inputs and outputs.

$$y(k) = \sum_{i=0}^{n} b_i\, u(k-i) - \sum_{i=1}^{n} a_i\, y(k-i) + v(k) \tag{1}$$

where n is order of the system; $y(\cdot)$ are values of system output; $u(\cdot)$ are values of system input; and $v(k)$ is a noise of measurement of system output $y(k)$. The ARX model can be also written in the following condensed form [Havlena, 2002]

$$y(k) = \vartheta_k\, \mathbf{f}_k + v(k) \tag{2}$$

where vector ϑ_k is a vector of ARX parameters $\vartheta_k = [b_0\; b_1 \cdots b_n\; -a_1\; -a_2 \cdots -a_n] = [b_0\; \boldsymbol{\theta}_b\; \boldsymbol{\theta}_a]$ and vector $\mathbf{f}_k$ is a data vector $\mathbf{f}_k = [u(k)\; u(k-1) \cdots u(k-n)\; y(k-1) \cdots y(k-n)]^T$.

Furthermore, let us assume model of evolution of the parameters

$$\vartheta_{k+1}^T = \vartheta_k^T + \upsilon_k^T \tag{3}$$

where υ_k^T is a noise of parameters. Since the model (3) is first order, so at the same time represents directly state-space model for the parameters.

Now, let us focus on composition of state-space model of the controlled system itself. In general, the description by state-space models is not unique. One system can be described by different state-space models without loss of unique fundamental information involved in the system; i.e. thus, these state-space models generate the same system outputs for defined inputs in spite of their different internal relations. In order to obtain state-space model suitable both for parameter identification and state estimation, some deterministic formulation of such model has to be selected. Such formulation can be represented by some canonical expression (e.g. well known Frobenius' and Jordan's forms). In our case, just one of Frobenius' canonical forms – transposed Frobenius' canonical state-space model is suitable selection. Its internal structure is written as follows [Šulc, 1999].

$$\begin{aligned}
\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \\ \vdots \\ x_{n-1}(k+1) \\ x_n(k+1) \end{bmatrix} &= \begin{bmatrix} -a_1 & 1 & 0 & \cdots & 0 \\ -a_2 & 0 & 1 & & \\ \vdots & \vdots & & \ddots & \\ -a_{n-1} & 0 & & & 1 \\ -a_n & 0 & 0 & \cdots & 0 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \\ \vdots \\ x_{n-1}(k) \\ x_n(k) \end{bmatrix} + \begin{bmatrix} b_1 - a_1 b_0 \\ b_2 - a_2 b_0 \\ \vdots \\ b_{n-1} - a_{n-1} b_0 \\ b_n - a_n b_0 \end{bmatrix} u(k) + \begin{bmatrix} -a_1 \\ -a_2 \\ \vdots \\ -a_{n-1} \\ -a_n \end{bmatrix} v(k) \\
y(k) &= \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \\ \vdots \\ x_{n-1}(k) \\ x_n(k) \end{bmatrix} + b_0\, u(k) + v(k)
\end{aligned} \tag{4}$$

This structure has usual matrix notation

$$\begin{aligned}
\mathbf{X}(k+1) &= \mathbf{A}\, \mathbf{X}(k) + \mathbf{B}\, u(k) + \mathbf{G}\, v(k) \\
y(k) &= \mathbf{C}\, \mathbf{X}(k) + \mathbf{D}\, u(k) + v(k)
\end{aligned} \tag{5}$$

where unknown model parameters are included in state matrices $\mathbf{A}$, $\mathbf{B}$, $\mathbf{G}$ and $\mathbf{D}$. Thus this form represents nonlinear task of estimation, because the unknown parameters are multiplied by unknown state vector. However, for control design (section 4) with known parameters is linear and suitable.

Alternative notation with different expression of model parameters indicates possible starting point for searched solution of suitable model for simultaneous estimation [Havlena, 2002].

$$\begin{aligned} \mathbf{X}(k+1) &= [\boldsymbol{\theta}_a^T \ {}_{\mathbf{0}}^{\mathbf{I}_{n-1}}]\,\mathbf{X}(k) + [\boldsymbol{\theta}_b^T + \boldsymbol{\theta}_a^T b_0]\,u(k) + \boldsymbol{\theta}_a^T v(k) \\ y(k) &= \mathbf{C}\,\mathbf{X}(k) + b_0\,u(k) + v(k) \end{aligned} \tag{6}$$

To provide the estimation of the system state and model parameters, let us analyze both given model of parameter evolution (3) and alternative model (6) describing the system behavior (system dynamics). As was mentioned, model (3) has directly suitable state-space form and from estimation point of view is linear. This model is determined for parameters, which are either constant or slightly changeable. Furthermore, their linear character is assumed. The model (6) is linear for state $\mathbf{X}(k)$, but it is not linear simultaneously for both state $\mathbf{X}(k)$ and parameters $\vartheta_k = [b_0\ \boldsymbol{\theta}_b\ \boldsymbol{\theta}_a]$. However, due to a notation, the model (6) can be transformed to a linear relation for state and parameters according to following expressions:

$$\begin{aligned} \mathbf{X}(k+1) &= [\mathbf{0}\ \ {}_{\mathbf{0}}^{\mathbf{I}_{n-1}}]\,\mathbf{X}(k) + u(k)\boldsymbol{\theta}_b^T + \big(\underbrace{\mathbf{C}\mathbf{X}(k) + b_0 u(k) + v(k)}_{y(k)}\big)\boldsymbol{\theta}_a^T \\ y(k) &= \mathbf{C}\,\mathbf{X}(k) + b_0\,u(k) + v(k) \end{aligned}$$

$$\begin{aligned} \mathbf{X}(k+1) &= [\mathbf{0}\ \ {}_{\mathbf{0}}^{\mathbf{I}_{n-1}}]\,\mathbf{X}(k) + u(k)\boldsymbol{\theta}_b^T + y(k)\,\boldsymbol{\theta}_a^T \\ y(k) &= \mathbf{C}\,\mathbf{X}(k) + b_0\,u(k) + v(k) \end{aligned} \tag{7}$$

$$\begin{aligned} \mathbf{X}(k+1) &= \underbrace{[\mathbf{0}\ \ {}_{\mathbf{0}}^{\mathbf{I}_{n-1}}]}_{\widetilde{\mathbf{A}}}\,\mathbf{X}(k) + \underbrace{[\mathbf{0}, u(k)\mathbf{I}_n, y(k)\mathbf{I}_n]}_{\mathbf{H}}\begin{bmatrix} b_0 \\ \boldsymbol{\theta}_b^T \\ \boldsymbol{\theta}_a^T \end{bmatrix} \\ y(k) &= \mathbf{C}\,\mathbf{X}(k) + b_0\,u(k) + v(k) \end{aligned} \tag{8}$$

The transformed model (8) is already linear both for state and for parameters. In it, there is a changed state matrix $\widetilde{\mathbf{A}}$ and new matrix $\mathbf{H}$, which contains partly current control action $u(k)$ and currently measured system output $y(k)$. In view of parameters, the matrix $\mathbf{H}$ is also state matrix for parameters labeled as new states of the considered system. Assuming this interpretation, it is possible to write complete state-space model for simultaneous estimation of system state and model parameters:

$$\begin{aligned} \mathbf{X}(k+1) &= \widetilde{\mathbf{A}}\,\mathbf{X}(k) + \mathbf{H}\,\vartheta^T(k) \\ \vartheta^T(k+1) &= \mathbf{0}\,\mathbf{X}(k) + \mathbf{I}_{2n}\,\vartheta^T(k) + \upsilon^T(k) \\ \hline y(k) &= \mathbf{C}\,\mathbf{X}(k) + \mathbf{0}\,\vartheta^T(k) + \mathbf{D}\,u(k) + v(k) \end{aligned}$$

$$\begin{aligned} \begin{bmatrix} \mathbf{X}(k+1) \\ \vartheta^T(k+1) \end{bmatrix} &= \begin{bmatrix} \widetilde{\mathbf{A}} & \mathbf{H} \\ \mathbf{0} & \mathbf{I}_{2n} \end{bmatrix} \begin{bmatrix} \mathbf{X}(k) \\ \vartheta^T(k) \end{bmatrix} + \begin{bmatrix} \mathbf{I}_n & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_{2n} \end{bmatrix} \begin{bmatrix} \mathbf{0} \\ \upsilon^T(k) \end{bmatrix} \\ \hline y(k) &= [\mathbf{C}\ \ \mathbf{0}] \begin{bmatrix} \mathbf{X}(k) \\ \vartheta^T(k) \end{bmatrix} + \mathbf{D} + u(k) + v(k) \end{aligned} \tag{9}$$

i.e.

$$\begin{aligned} \mathbf{X(k+1)} &= \mathbf{A_k}\,\mathbf{X(k)} + \mathbf{G_k}\,\mathbf{w(k)} \\ y(k) &= \mathbf{C_k}\,\mathbf{X(k)} + \mathbf{D_k}\,u(k) + v(k) \end{aligned} \tag{10}$$

This model is initial form for the estimation by Kalman filter, which will be outlined in section 3.

2.2 State-space model composition on the basis of mathematical-physical analysis

Different question is, if the system e.g. mechanical system (robot etc.) was thoroughly mapped via mathematical-physical analysis. To explain this situation, let us consider e.g. model of planar parallel robot, where the mathematical-physical analysis was realized. Such system – robot structure – generally represents multi-body system with multi-inputs and multi-outputs (MIMO systems). The result of the analysis is a model in a form of system of nonlinear differential equations [Stejskal, 1996]

$$\ddot{\mathbf{y}} = \mathbf{f}(\mathbf{y}, \dot{\mathbf{y}}) + \mathbf{g}(\mathbf{y})\mathbf{u} \tag{11}$$

Its nonlinearity can consist e.g. in presence of goniometric functions of the robot geometry or in considering of passive force effects and suchlike. The system (11) can be written in state-space form (12)

$$\begin{aligned} \dot{\mathbf{X}} &= \mathbf{f}(\mathbf{X}) + \mathbf{g}(\mathbf{X})\,\mathbf{u}\,, \quad \mathbf{X} = [\mathbf{y}, \dot{\mathbf{y}}]^T \\ \mathbf{y} &= \mathbf{h}\,\mathbf{X} \end{aligned} \tag{12}$$

which should be linearized [Belda, 2005b] for state estimation and control design as follows

$$\begin{aligned} \dot{\mathbf{X}} &= \mathbf{A}(\mathbf{X})\,\mathbf{X} + \mathbf{g}(\mathbf{X})\,\mathbf{u} \\ \mathbf{y} &= \quad \mathbf{h} \quad \mathbf{X} \end{aligned} \tag{13}$$

and subsequently discretized for discrete realization

$$\begin{aligned} \mathbf{X}(k+1) &= \mathbf{A}_k\,\mathbf{X}(k) + \mathbf{B}_k\,\mathbf{u}(k) \\ \mathbf{y}(k) &= \mathbf{C}_k\,\mathbf{X}(k) \end{aligned} \tag{14}$$

This model is again initial form not only for the estimation by Kalman filter (outlined in section 3), but also for control design using generalized predictive control strategy (section 4).

3 PRINCIPLES OF ESTIMATION VIA KALMAN FILTER IN SQUARE-ROOTS (GENERAL OBSERVER WITH KALMAN GAIN)

This section provides brief outline, how to obtain optimal gain for the observer either only for single state-space estimation or for simultaneous estimation of state and model parameters. It is assumed, that the effort is not to search for steady-state solution (as e.g. solution in MATLAB), but search for variable solution corresponding to current system state and parameters.

Let us start with several assumptions. At first, let us suppose that appropriate values of system outputs $\mathbf{y}(k)$ are measured and values of control actions $\mathbf{u}(k)$ are used directly from their computation at corresponding time instant (see Figure 1).

To derive the algorithm for Kalman gain, let us precede from defined linear state-space models (10) and (14) renewed in (15) [Anderson, 1979].

$$\begin{aligned} \mathbf{X(k+1)} &= \mathbf{A_k}\ \mathbf{X(k)} + \mathbf{G_k}\,\mathbf{w(k)} & \qquad \mathbf{X}(k+1) &= \mathbf{A}_k\,\mathbf{X}(k) + \mathbf{B}_k\,\mathbf{u}(k) + \mathbf{G}_k\mathbf{w}(k) \\ y(k) &= \mathbf{C_k}\ \mathbf{X(k)} + \mathbf{D_k}\,u(k) + v(k) & \qquad \mathbf{y}(k) &= \mathbf{C}_k\,\mathbf{X}(k) + \quad \mathbf{v}(k) \end{aligned} \tag{15}$$

In the left model, the state vector $\mathbf{X(k+1)}$ includes real state of the system and its parameters. The right model assumes that parameters (elements of $\mathbf{A}_k$, $\mathbf{B}_k$, $\mathbf{G}_k$ and $\mathbf{C}_k$) are known, therefore the vector $\mathbf{X}(k+1)$ represents only real system state.

Moreover, this model is extended via noise components following from reality of measurement; i.e. the model (14) from mathematical-physical analysis was deterministic, but the right model (15) includes already stochastic components (noises).

The both models in (15) can be expressed by one general sufficient model.

$$\begin{aligned} \mathbf{X}(k+1) &= \mathbf{A}_k\,\mathbf{X}(k) + \mathbf{B}_k\,\mathbf{u}(k) + \mathbf{G}_k\,\mathbf{w}(k) \\ \mathbf{y}(k) &= \mathbf{C}_k\,\mathbf{X}(k) + \mathbf{D}_k\,\mathbf{u}(k) + \quad \mathbf{v}(k) \end{aligned} \tag{16}$$

where the matrices $\mathbf{B}_k$ and $\mathbf{D}_k$ are omitted from gain computation. They do not influence computation of the gain. Moreover, the left model (15) has no matrix $\mathbf{B}_k$ and in the both models (15), direct feed-through matrix $\mathbf{D}_k$ is assumed to be zero (see assumption of the time arrangement in section 2). $\mathbf{G}_k$ is a gain matrix of state noise; $\mathbf{w}(k)$ is a state noise and $\mathbf{v}(k)$ is output noise.

Furthermore, let us define presumptive properties of all terms in the model (16). The noises are assumed to be white, mutually independent and normally distributed $\mathcal{N}(\text{mean, covarinace})$ with zero means and known positive definite covariances

$$\begin{aligned} &\mathbf{Q}_k = E(\mathbf{w}(k)\,\mathbf{w}^T(k)),\ \mathbf{R}_k = E(\mathbf{v}(k)\,\mathbf{v}^T(k)),\ \left(\mathbf{V}_k = E(\upsilon^T(k)\,\upsilon(k))\right) \\ &p(\mathbf{w}(k+1)\,|\,\mathbf{X}(k),\mathbf{u}(k),\mathbf{y}(k),\mathbf{w}(k)) \sim \mathcal{N}(\mathbf{0},\mathbf{Q}_k) \\ &p(\mathbf{v}(k)\,|\,\mathbf{X}(k),\mathbf{u}(k)) = p(\mathbf{v}(k)) \sim \mathcal{N}(\mathbf{0},\mathbf{R}_k) \end{aligned} \tag{17}$$

where $E(a)$ is a mean of a and $p(a|b)$ means probability density of a conditioned by b.

Finally, the state-space model (16) and distributions (17) define transformation between $\mathbf{w}(k)$ and $\mathbf{X}(k+1)$ and similarly between $\mathbf{y}(k)$ and $\mathbf{v}(k)$ for given $\mathbf{X}(k)$ and $\mathbf{u}(k)$

$$p(\mathbf{X}(k+1)\,|\,\mathbf{X}(k),\mathbf{u}(k),\mathbf{y}(k),\mathbf{w}(k)) \sim \mathcal{N}(\mathbf{A}_k\mathbf{X}(k) + \mathbf{B}_k\mathbf{u}(k),\ \mathbf{G}_k\mathbf{Q}_k\mathbf{G}_k^T) \tag{18}$$

$$p(\mathbf{y}(k)\,|\,\mathbf{X}(k),\mathbf{u}(k),\mathbf{v}(k)) \sim \mathcal{N}(\mathbf{C}_k\mathbf{X}(k) + \mathbf{D}_k\mathbf{u}(k),\ \mathbf{R}_k) \tag{19}$$

The Kalman gain is designed, that estimation of $\hat{\mathbf{X}}$ was being the best in the sense of the minimum of error covariance – estimation given by minimum of variance estimates [Billings, 1980]

$$\operatorname{trace} E\,(\,(\mathbf{X}-\hat{\mathbf{X}})(\mathbf{X}-\hat{\mathbf{X}})^T\,) \ \rightarrow\ \text{minimum} \tag{20}$$

where $\operatorname{trace} E\,(\cdot)$ is conditional error variance associated with the estimate $\hat{\mathbf{X}}$ and conditional mean estimate minimizes this error variance. Finally, $\mathbf{X}$ is a random state vector. Suppose that the vector $\mathbf{X}$ is estimated from measurement – a random vector $\mathbf{Y}$ with mean value $\overline{\mathbf{y}}$.

Furthermore let us suppose, that $\mathbf{X}$ and $\mathbf{Y}$ are random vectors with mean values $\overline{\mathbf{X}}$ and $\overline{\mathbf{y}}$ and conditional probability density of normal distribution

$$\mathbf{\Sigma} = \begin{bmatrix} \mathbf{\Sigma}yy & \mathbf{\Sigma}yx \\ \mathbf{\Sigma}xy & \mathbf{\Sigma}xx \end{bmatrix} \tag{21}$$

where the covariances $\mathbf{\Sigma}$ and $\mathbf{\Sigma}yy$ have to be nonsingular and $\mathbf{\Sigma}$ is symmetric. Then the conditional probability density of vector $\mathbf{X}$ conditioned by $\mathbf{Y}$ is probability density of normal distribution

$$\begin{aligned} p_{\mathbf{X}|\mathbf{Y}}(\mathbf{X}\,|\,\mathbf{Y}) &= \frac{p_{\mathbf{XY}}(\mathbf{X},\mathbf{Y})}{p_{\mathbf{Y}}(\mathbf{Y})} = \frac{1}{(2\pi)^{\frac{N+n}{2}}} \frac{e^{-\frac{1}{2}[\mathbf{X}^T-\overline{\mathbf{X}}^T\ \ \mathbf{Y}^T-\overline{\mathbf{y}}^T]\,\mathbf{\Sigma}^{-1}\begin{bmatrix}\mathbf{X}-\overline{\mathbf{X}}\\ \mathbf{Y}-\overline{\mathbf{y}}\end{bmatrix}}}{|\mathbf{\Sigma}|^{1/2}} \frac{(2\pi)^{n/2}\ |\mathbf{\Sigma}_{yy}|^{1/2}}{e^{-\frac{1}{2}(\mathbf{Y}^T-\overline{\mathbf{y}}^T)\,\mathbf{\Sigma}^{-1}yy\,(\mathbf{Y}-\overline{\mathbf{y}})}} = \\ &= \frac{1}{(2\pi)^{\frac{N}{2}}} \frac{e^{-\frac{1}{2}(\mathbf{X}^T-\hat{\mathbf{X}}^T)\,(\mathbf{\Sigma}xx-\mathbf{\Sigma}xy\,\mathbf{\Sigma}^{-1}yy\mathbf{\Sigma}yx)\,(\mathbf{X}-\hat{\mathbf{X}})}}{|\,\mathbf{\Sigma}xx-\mathbf{\Sigma}xy\,\mathbf{\Sigma}^{-1}yy\mathbf{\Sigma}yx\,|^{1/2}}, \end{aligned} \tag{22}$$

where n is a number of outputs and N is a dimension of the state

with $\hat{\mathbf{X}}(\mathbf{Y}) = E(\mathbf{X}\,|\,\mathbf{Y}) = \overline{\mathbf{X}} + \mathbf{\Sigma}xy\,\mathbf{\Sigma}^{-1}yy\,(\mathbf{Y}-\overline{\mathbf{y}})$, and covariance $\mathbf{\Sigma}(\mathbf{X}\,|\,\mathbf{Y}) = \mathbf{\Sigma}xx - \mathbf{\Sigma}xy\,\mathbf{\Sigma}^{-1}yy\ \mathbf{\Sigma}yx$ (23)

Due to the assumption of normal distribution, only statistics (mean and covariance) of this distribution are needed to be evaluated. Now, let us directly start with outline of effective computation of gain in square-root form by analysis of marginal covariance matrices of joint covariance (21)

$$\begin{aligned}
\boldsymbol{\Sigma} xx &= E((\mathbf{X}(k)-\hat{\mathbf{X}}(k,k-1))(\mathbf{X}(k)-\hat{\mathbf{X}}(k,k-1))^T\,|\,\cdot)=\boldsymbol{\Sigma}(k,k-1)\\
\boldsymbol{\Sigma} yy &= E((\mathbf{y}(k)-\hat{\mathbf{y}}(k))(\mathbf{y}(k)-\hat{\mathbf{y}}(k))^T\,|\,\cdot)=\mathbf{R}_k+\mathbf{C}_k\,\boldsymbol{\Sigma}(k,k-1)\,\mathbf{C}_k^T\\
\boldsymbol{\Sigma} xy &= E((\mathbf{X}(k)-\hat{\mathbf{X}}(k,k-1))(\mathbf{y}(k)-\hat{\mathbf{y}}(k))^T\,|\,\cdot)=\boldsymbol{\Sigma}(k,k-1)\,\mathbf{C}_k^T\\
\boldsymbol{\Sigma} yx &= (\boldsymbol{\Sigma} xy)^T=\mathbf{C}_k\,\boldsymbol{\Sigma}(k,k-1)
\end{aligned} \tag{24}$$

$$\boldsymbol{\Sigma}_{k,k}=\begin{bmatrix}\boldsymbol{\Sigma} yy & \boldsymbol{\Sigma} yx\\ \boldsymbol{\Sigma} xy & \boldsymbol{\Sigma} xx\end{bmatrix}=\begin{bmatrix}\mathbf{R}_k+\mathbf{C}_k\,\boldsymbol{\Sigma}(k,k-1)\,\mathbf{C}_k^T & \mathbf{C}_k\,\boldsymbol{\Sigma}(k,k-1)\\ \boldsymbol{\Sigma}(k,k-1)\,\mathbf{C}_k^T & \boldsymbol{\Sigma}(k,k-1)\end{bmatrix} \tag{25}$$

To obtain directly one single update of (25) (update of measurement and time) with determination of searched Kalman gain, let us define predicted expression of joint covariance [Belda, 2005a]

$$\boldsymbol{\Sigma}_{k+1,k}=E\left(\begin{bmatrix}\mathbf{y}(k+1)-\hat{\mathbf{y}}(k+1)\\ \mathbf{X}(k+1)-\hat{\mathbf{X}}(k+1)\end{bmatrix}\begin{bmatrix}\mathbf{y}^T(k+1)-\hat{\mathbf{y}}^T(k+1) & \mathbf{X}^T(k+1)-\hat{\mathbf{X}}^T(k+1)\end{bmatrix}\right)=$$

$$=\begin{bmatrix}\mathbf{R}+\mathbf{C}(\mathbf{A}\boldsymbol{\Sigma}\mathbf{A}^T+\mathbf{GQG}^T)\mathbf{C}^T & \mathbf{C}(\mathbf{A}\boldsymbol{\Sigma}\mathbf{A}^T+\mathbf{GQG}^T)\\ (\mathbf{A}\boldsymbol{\Sigma}\,\mathbf{A}^T+\mathbf{GQG}^T)\mathbf{C}^T & \mathbf{A}\boldsymbol{\Sigma}\mathbf{A}^T+\mathbf{GQG}^T\end{bmatrix}_k \tag{26}$$

which can be decomposed as follows

$$\boldsymbol{\Sigma}_{k+1,k}=\begin{bmatrix}\mathbf{R}_r\mathbf{R}_r^T+\mathbf{C}(\mathbf{A}\,\mathbf{SS}^T\mathbf{A}^T+\mathbf{G}\,\mathbf{Q}_r\mathbf{Q}_r^T\,\mathbf{G}^T)\mathbf{C}^T & \mathbf{C}(\mathbf{A}\,\mathbf{SS}^T\mathbf{A}^T+\mathbf{G}\,\mathbf{Q}_r\mathbf{Q}_r^T\,\mathbf{G}^T)\\ (\mathbf{A}\,\mathbf{SS}^T\mathbf{A}^T+\mathbf{G}\,\mathbf{Q}_r\mathbf{Q}_r^T\,\mathbf{G}^T)\mathbf{C}^T & \mathbf{A}\,\mathbf{SS}^T\mathbf{A}^T+\mathbf{G}\,\mathbf{Q}_r\mathbf{Q}_r^T\,\mathbf{G}^T\end{bmatrix}_k=$$

$$=\begin{bmatrix}\mathbf{R}_r & \mathbf{CAS} & \mathbf{CGQ}_r\\ \mathbf{0} & \mathbf{AS} & \mathbf{GQ}_r\end{bmatrix}_k\begin{bmatrix}\mathbf{R}_r^T & \mathbf{0}\\ \mathbf{S}^T\mathbf{A}^T\mathbf{C}^T & \mathbf{S}^T\mathbf{A}^T\\ \mathbf{Q}_r^T\,\mathbf{G}^T\mathbf{C}^T & \mathbf{Q}_r^T\,\mathbf{G}^T\end{bmatrix}_k=\left(\boldsymbol{\Sigma}^{\frac{1}{2}}_{k+1,k}\right)\left(\boldsymbol{\Sigma}^{\frac{1}{2}}_{k+1,k}\right)^T \tag{27}$$

where $\mathbf{R}_r$, $\mathbf{Q}_r$ and $\mathbf{S}$ are square-roots of mutual covariances $E(\mathbf{vv}^T)$, $E(\mathbf{ww}^T)$ and $\boldsymbol{\Sigma}=\mathbf{SS}^T$. Then, by orthogonal-triangular decomposition [Golub, 1989], the upper triangular matrix is obtained, from which the Kalman gain is possible to compute

$$\mathbf{Q}^T_{ortg}\left(\boldsymbol{\Sigma}^{\frac{1}{2}}_{k+1,k}\right)^T=\mathbf{Q}^T_{ortg}\begin{bmatrix}\mathbf{R}_r^T & \mathbf{0}\\ \mathbf{S}^T\mathbf{A}^T\mathbf{C}^T & \mathbf{S}^T\mathbf{A}^T\\ \mathbf{Q}_r^T\,\mathbf{G}^T\mathbf{C}^T & \mathbf{Q}_r^T\,\mathbf{G}^T\end{bmatrix}_k=\begin{bmatrix}(inv)^T & (\mathbf{k})^T\\ \mathbf{0} & \overline{\mathbf{S}}^T\\ \mathbf{0} & \mathbf{0}\end{bmatrix}_k \tag{28}$$

where $\overline{\mathbf{S}}_k^T$ represents new estimation of square-root $\mathbf{S}^T_{k+1,k}$, which is used in next time instant. Searched Kalman gain with state-space observer is defined as follows [Anderson, 1979]

$$\mathbf{K}_k=((inv)^{-1}\,\mathbf{k}\,)^T\,|_k\quad\left(=\begin{bmatrix}\mathbf{K}_{\mathbf{X}k}\\ \mathbf{K}_{\vartheta k}\end{bmatrix},\ \ e(k)=y(k)-\mathbf{C_k}\hat{\mathbf{X}}(\mathbf{k},\mathbf{k-1})\right) \tag{29}$$

$$\hat{\mathbf{X}}(k,k)=\hat{\mathbf{X}}(k,k-1)+\mathbf{K}_k\,(\mathbf{y}(k)-\mathbf{C}_k\,\hat{\mathbf{X}}(k,k-1)),\quad \hat{\mathbf{X}}(k+1,k)=\mathbf{A}_k\hat{\mathbf{X}}(k,k)+\mathbf{B}_k\,\mathbf{u}(k)$$

(for simultaneous estimation: $\hat{\mathbf{X}}(k,k)=\hat{\mathbf{X}}(k,k-1)+\mathbf{K}_{\mathbf{X}k}e(k),\ \hat{\vartheta}(k,k)=\hat{\vartheta}(k,k-1)+\mathbf{K}_{\vartheta k}e(k)$) (30)

The indicated procedure is repeated in each time step for current state with appropriate matrices.

4 ALGORITHMS OF PREDICTIVE CONTROL

Predictive control is a multi step approach, combining feedforward and feedback control design [Ordys, 1993]. Feedforward is represented by predictions based on mathematical model. This part is dominant component of control actions. Feedback from measured outputs, serves for compensation of some bounded model inaccuracies and low external disturbances. The design consists in local minimization of quadratic criterion, in which the predictions from equations of predictions are involved. The minimization is repeated in each time step.

4.1 Equations of predictions

Equations of predictions form the basis of derivation of control actions. Their form determines character of predictive algorithms [Belda, 2005a]:

- algorithms generating full values of control
- algorithms generating increments of control (incremental algorithms).

The both types lead on repetitive insertion of appropriate model. In our case, linear discrete state-space model is considered. Let us firstly present basic algorithm and then algorithm considering nonlinearities, as mentioned in the first section.

- Basic algorithm generating full values of control

Let us consider directly already introduced linear discrete state-space model

$$\begin{aligned}\mathbf{X}(k+1) &= \mathbf{A}_k\,\mathbf{X}(k)+\mathbf{B}_k\,\mathbf{u}(k)\\ \mathbf{y}(k) &= \mathbf{C}_k\,\mathbf{X}(k)+\mathbf{D}_k\,\mathbf{u}(k) \qquad (\mathbf{D}_k=\mathbf{0})\end{aligned} \tag{31}$$

The model maps interval of one sampling period. Principle of equations of predictions is expression (prediction) of future values of outputs $\mathbf{y}$ from current measured state $\mathbf{X}(k)$ as follows

$$\begin{aligned}\mathbf{y}(k) &= \mathbf{C}\,\mathbf{X}(k)\\ \hat{\mathbf{X}}(k+1) &= \mathbf{A}\,\mathbf{X}(k)+\mathbf{B}\,\mathbf{u}(k)\\ \hat{\mathbf{y}}(k+1) &= \mathbf{CA}\,\mathbf{X}(k)+\mathbf{C}\,\mathbf{B}\,\mathbf{u}(k)\\ &\ \vdots\\ \hat{\mathbf{X}}(k+N) &= \mathbf{A}^N\,\mathbf{X}(k)+\mathbf{A}^{N-1}\mathbf{B}\,\mathbf{u}(k)+\cdots+\mathbf{B}\,\mathbf{u}(k+N-1)\\ \hat{\mathbf{y}}(k+N) &= \mathbf{CA}^N\,\mathbf{X}(k)+\mathbf{CA}^{N-1}\mathbf{B}\,\mathbf{u}(k)+\cdots+\mathbf{CB}\,\mathbf{u}(k+N-1)\end{aligned} \tag{32}$$

Equation (32) can be usefully written in matrix notation

$$\hat{\mathbf{y}}=\mathbf{f}+\mathbf{G}\,\mathbf{u},\ \ \hat{\mathbf{y}}=[\hat{\mathbf{y}}(k+1),\ \ \hat{\mathbf{y}}(k+2),\ \ \hat{\mathbf{y}}(k+3),\ \ \cdots,\ \ \hat{\mathbf{y}}(k+N)]^T$$
$$\mathbf{u}=[\mathbf{u}(k),\ \ \mathbf{u}(k+1),\ \ \mathbf{u}(k+2),\ \ \cdots,\ \ \mathbf{u}(k+N-1)]^T$$

$$\mathbf{f}=\begin{bmatrix}\mathbf{CA}\\ \mathbf{CA}^2\\ \vdots\\ \mathbf{CA}^N\end{bmatrix}\mathbf{X}(k) \qquad \mathbf{G}=\begin{bmatrix}\mathbf{C}\,\mathbf{B} & \mathbf{0} & \cdots & \mathbf{0}\\ \mathbf{CA}\,\mathbf{B} & \mathbf{C}\,\mathbf{B} & & \\ \vdots & \vdots & \ddots & \vdots\\ \mathbf{CA}^{N-1}\mathbf{B} & \mathbf{CA}^{N-2}\mathbf{B} & \cdots & \mathbf{CB}\end{bmatrix} \tag{33}$$

● Incremental algorithm with nonlinear prediction

The incremental algorithm with nonlinear prediction is developed to improve accuracy of predicted outputs $\hat{\mathbf{y}}$ by better approximation of nonlinearities of the model (e.g. model of the robot) by specific equations of predictions. The algorithm consists of two cycles as follows from Figure 2.

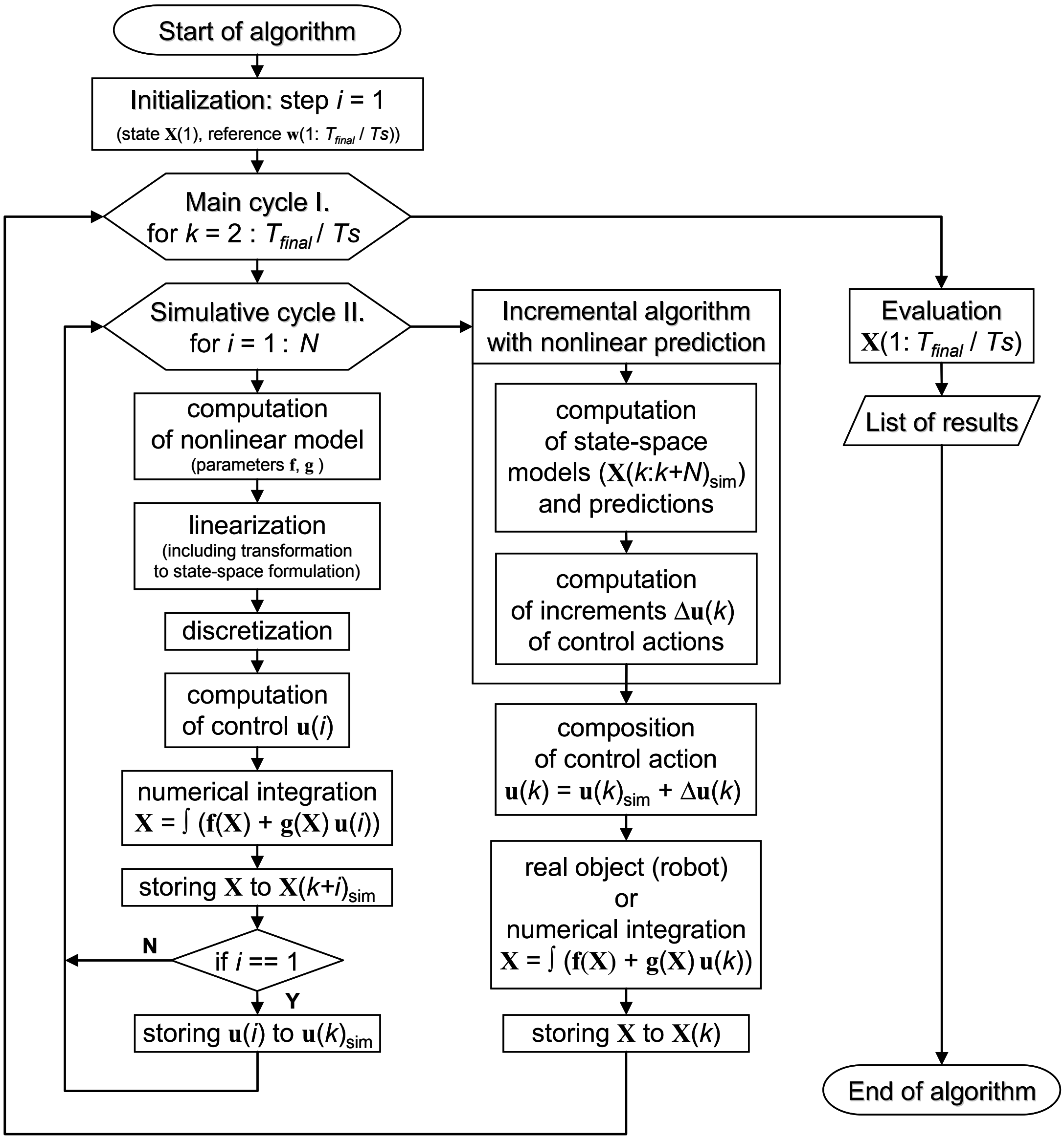


Figure 2 - Flow diagram of incremental algorithm with nonlinear prediction.

The main cycle represents usual control cycle extended by internal simulative cycle. Internal cycle composes the essential part of the control ($\mathbf{u}(k)_{sim}$) and predicts future values of the system state – $\mathbf{X}(k:k+N)_{sim}$. For these operations, the general predictive control algorithm and numerical integration of nonlinear model (function $\mathbf{f}(\mathbf{X})+\mathbf{g}(\mathbf{X})\mathbf{u}$) are used. Obtained values are stored for computations in main cycle. In it, at first, the models determined by states $\mathbf{X}(k:k+N)_{sim}$ are calculated. Then, they are used for composition of equations of predictions serving for generating of increments of control actions. Finally, the increments are added to actions from internal cycle $\mathbf{u}(k)_{sim}$. Final control actions $\mathbf{u}(k)$ are applied to controlled systems.

Now, this generally defined algorithm is prepared for mathematical formulation.

In detail, equations of predictions are specified as follows

$$
\begin{aligned}
\mathbf{X}(k+1) &= \underbrace{\mathbf{A}(\mathbf{X}(k))\,\mathbf{X}(k)+\mathbf{B}(\mathbf{X}(k))\,\mathbf{u}(k)_{sim}}_{} + \mathbf{B}(\mathbf{X}(k))\Delta\mathbf{u}(k) \\
» \mathbf{X}(k+1) &= \mathbf{X}(k+1)_{sim} + \mathbf{B}(\mathbf{X}(k))\Delta\mathbf{u}(k) \\
» \mathbf{X}(k+2) &= \mathbf{X}(k+2)_{sim} + \mathbf{A}(\mathbf{X}(k+1)_{sim})\mathbf{B}(\mathbf{X}(k))\,\Delta\mathbf{u}(k) \\
&\vdots \qquad + \mathbf{B}(\mathbf{X}(k+1)_{sim}\,\Delta\mathbf{u}(k+1) \\
» \mathbf{X}(k+N) &= \mathbf{X}(k+N)_{sim} + \mathbf{A}(\mathbf{X}(k+N-1)_{sim})\cdots\mathbf{A}(\mathbf{X}(k+1)_{sim})\mathbf{B}(\mathbf{X}(k))\,\Delta\mathbf{u}(k) \\
&\quad + \cdots + \mathbf{B}(\mathbf{X}(k+N-1)_{sim}\,\Delta\mathbf{u}(k+N-1)
\end{aligned} \tag{34}
$$

where $\mathbf{X}(k+1)_{sim}$, $\mathbf{u}(k)_{sim}$ (etc. $\mathbf{X}(k+i)_{sim}$, $\mathbf{u}(k+i-1)_{sim}, 1,\cdots,N$) is a state or control action given from pre-simulation in considered receding horizon of incremental control algorithm. The derivation is similar to previous cases – i.e. suitably repetitive substitution for predicted future states and outputs, respectively. However, the structure of equations of predictions is more complex.

$$
\begin{bmatrix} \hat{\mathbf{y}}(k+1) \\ \hat{\mathbf{y}}(k+2) \\ \vdots \\ \hat{\mathbf{y}}(k+N) \end{bmatrix} = \begin{bmatrix} \mathbf{C}\,\mathbf{X}(k+1)_{sim} \\ \mathbf{C}\,\mathbf{X}(k+2)_{sim} \\ \\ \mathbf{C}\,\mathbf{X}(k+N)_{sim} \end{bmatrix} +
$$

$$
+ \begin{bmatrix} \mathbf{CB}(\mathbf{X}(k)) & & \cdots & \mathbf{0} \\ \mathbf{CA}(\mathbf{X}(k+1)_{sim})\mathbf{B}(\mathbf{X}(k)) & \mathbf{CB}(\mathbf{X}(k+1)_{sim}) & & \vdots \\ \vdots & & \ddots & \mathbf{0} \\ \mathbf{CA}(\mathbf{X}(k+N-1)_{sim})+\cdots+\mathbf{A}(\mathbf{X}(k+1)_{sim})\mathbf{B}(\mathbf{X}(k)) & & \cdots & \mathbf{CB}(\mathbf{X}(k+N-1)_{sim}) \end{bmatrix} \times
$$

$$
\times \begin{bmatrix} \Delta\mathbf{u}(k) \\ \Delta\mathbf{u}(k+1) \\ \\ \Delta\mathbf{u}(k+N-1) \end{bmatrix} \tag{35}
$$

Corresponding matrix notation is expressed by equation (36).

$$
\begin{aligned}
\hat{\mathbf{y}} &= \hat{\mathbf{y}}_{sim} + \\
&+ \mathbf{G} \times \Delta\mathbf{u}
\end{aligned}
\quad \left|\; \begin{aligned} \hat{\mathbf{y}} &= [\hat{\mathbf{y}}(k+1),\ \hat{\mathbf{y}}(k+2),\ \cdots,\ \hat{\mathbf{y}}(k+N)]^T \\ \Delta\mathbf{u} &= [\Delta\mathbf{u}(k),\ \Delta\mathbf{u}(k+1),\ \cdots,\ \Delta\mathbf{u}(k+N-1)]^T \end{aligned} \right. \tag{36}
$$

This algorithm has integrative character. It improves the actions generated from prior states, which have been predicted in pre-simulation with linear control. The algorithm represents one possible way of nonlinear design of predictive control.

The both ways of predictions: basic and incremental with nonlinear prediction can be formally generalized to one expression

$$
\hat{\mathbf{y}} = \mathbf{f} + \mathbf{G}\mathbf{u} \tag{37}
$$

in that the vector $\mathbf{f}$ and matrix $\mathbf{G}$ are composed according to appropriate algorithm.

4.2 Minimization of quadratic criterion

To implement predictive control, the minimization can be evaluated in effective square-root form. The quadratic criterion is written in the following matrix notation

$$J_k = [(\hat{\mathbf{y}}-\mathbf{w})^T, \mathbf{u}^T] \begin{bmatrix} \mathbf{Q_y} & \mathbf{0} \\ \mathbf{0} & \mathbf{Q_u} \end{bmatrix}^T \begin{bmatrix} \mathbf{Q_y} & \mathbf{0} \\ \mathbf{0} & \mathbf{Q_u} \end{bmatrix} \begin{bmatrix} \hat{\mathbf{y}}-\mathbf{w} \\ \mathbf{u} \end{bmatrix} = \underbrace{}_{\mathbf{J}^T} \times \underbrace{}_{\mathbf{J}} \tag{38}$$

For minimization, only right side is needed. With consideration of equations of predictions, the square-root is expressed as follows

$$\mathbf{J} = \begin{bmatrix} \mathbf{Q_y} & \mathbf{0} \\ \mathbf{0} & \mathbf{Q_u} \end{bmatrix} \begin{bmatrix} \hat{\mathbf{y}}-\mathbf{w} \\ \mathbf{u} \end{bmatrix} = \begin{bmatrix} \mathbf{Q_y\,G} \\ \mathbf{Q_u} \end{bmatrix} \mathbf{u} - \begin{bmatrix} \mathbf{Q_y\,(w-f)} \\ \mathbf{0} \end{bmatrix} \tag{39}$$

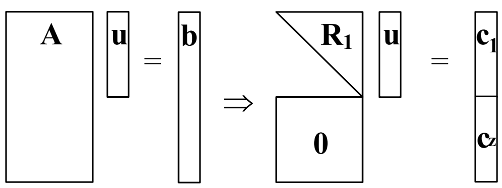
The objective is to search for such $\mathbf{u}$, which minimizes the square-root (39), which means, that control $\mathbf{u}$ minimizes the norm $|\mathbf{J}|$ of the criterion (28). In that case, the minimization leads to a system of algebraic equations

$$\begin{bmatrix} \mathbf{Q_y\,G} \\ \mathbf{Q_u} \end{bmatrix} \mathbf{u} - \begin{bmatrix} \mathbf{Q_y\,(w-f)} \\ \mathbf{0} \end{bmatrix} = \mathbf{0}$$

$$\mathbf{A} \quad \mathbf{u} - \quad \mathbf{b} \quad = \mathbf{0} \tag{40}$$

For optimization of the criterion, the orthogonal triangular decomposition is used [Lawson, 1974]. It reduces excess rows of matrix $\mathbf{A}$ and elements of vector $\mathbf{b}$ into upper triangular matrix $\mathbf{R}$ and a vector $\mathbf{c}$

$$\mathbf{A\,u} = \mathbf{b} \quad / \times \mathbf{Q}^T$$
$$\mathbf{R\,u} = \mathbf{c}$$



(41)

Vector $\mathbf{c_z}$ is a loss vector, which Euclidean norm $|\mathbf{c_z}|$ is equivalent to the value of square root $\sqrt{J}$ (i.e. $J = \mathbf{c_z}^T \mathbf{c_z}$).

To obtain vector of unknown control actions $\mathbf{u}$, only upper part of the system (41) is needed

$$\begin{aligned} \mathbf{R}_1\,\mathbf{u} &= \mathbf{c}_1 \\ \mathbf{u} &= (\mathbf{R}_1)^{-1}\,\mathbf{c}_1 \end{aligned} \tag{42}$$

Since a matrix $\mathbf{R}_1$ is upper triangle, then the vector of control $\mathbf{u}$ is given directly by backward substitution. The vector $\mathbf{u}$ represents design of control for whole horizon N, thus only first elements, corresponding to the first prediction in a considered horizon N are really used as values of control actions.

5 SIMULATIVE EXAMPLES

The aim of this section is partly to show simple application of simultaneous estimation of state and model parameters for SISO systems (Figure 3) (partly in [Belda, 2005c]), furthermore to show comparison of two presented algorithms of prediction applied to more complicated applications as parallel robots (Figure 4) and partly to show inclusion of state-space observer to control scheme for such systems, which generally represent MIMO systems (Figure 5).

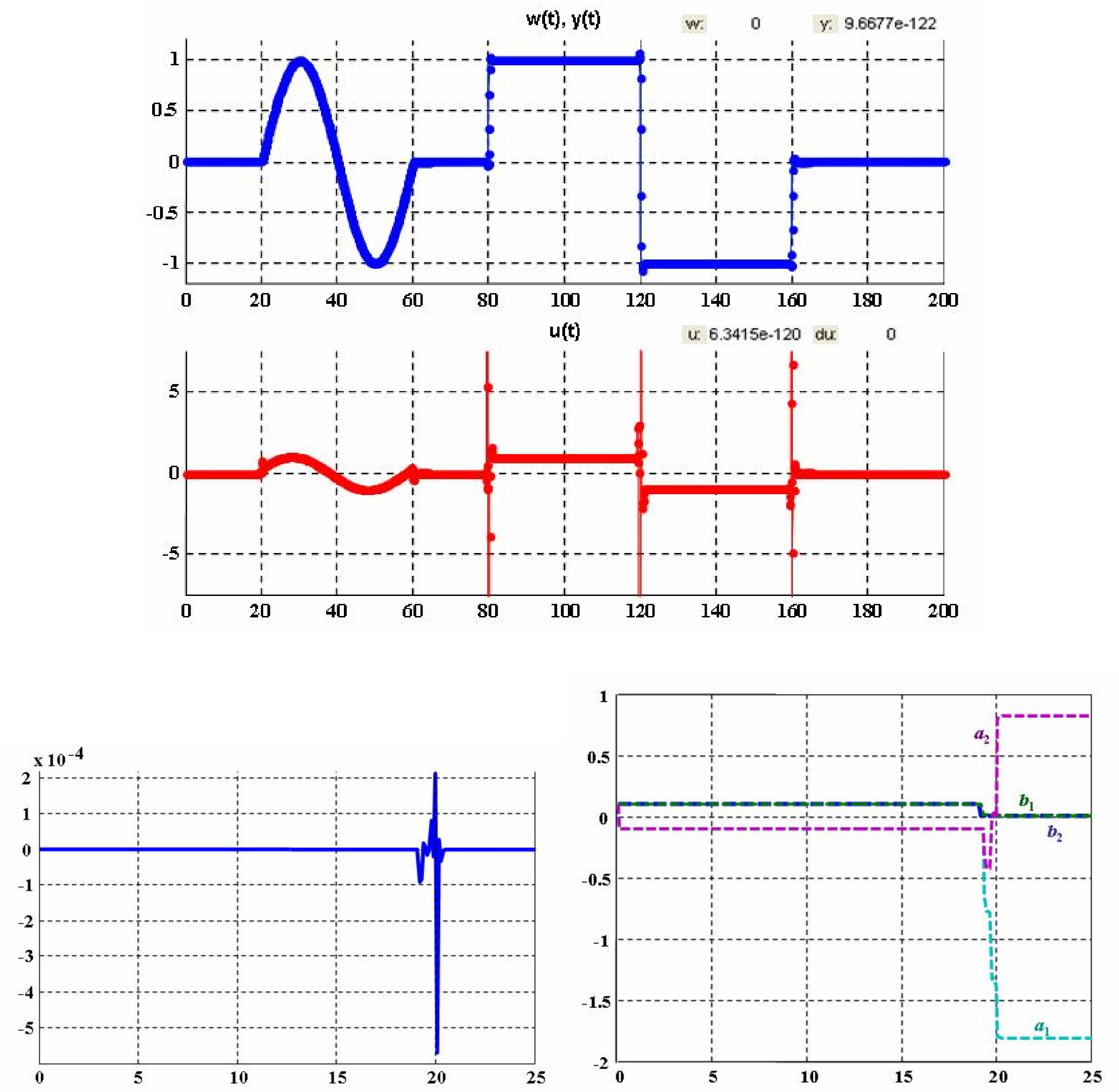


Figure 3 - Time histories of control process (up), error of state estimation (at the bottom, on the left) and parameter evolution (at the bottom, on the right)
(SISO system of second order: continuous form $\ddot{y}(t)+2\dot{y}(t)+y(t)=u(t)$,
discrete form for $Ts=0.1s$: $y(k)-1.8097y(k-1)+0.8187y(k-2)=0.0047u(k-1)+0.0044u(k-2)$)

The parameter estimation/identification begins by N time steps ahead, since predictive control generates the control actions with considering of prediction within time interval given by horizon of prediction N .

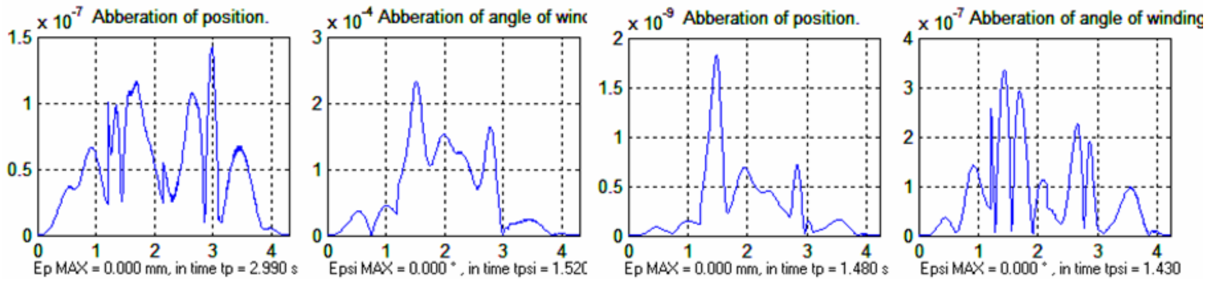


Figure 4 - Comparison of achieved errors (aberrations of position and angle of winding) partly of basic predictive algorithm (on the left) and partly incremental predictive algorithm with nonlinear prediction (on the right).

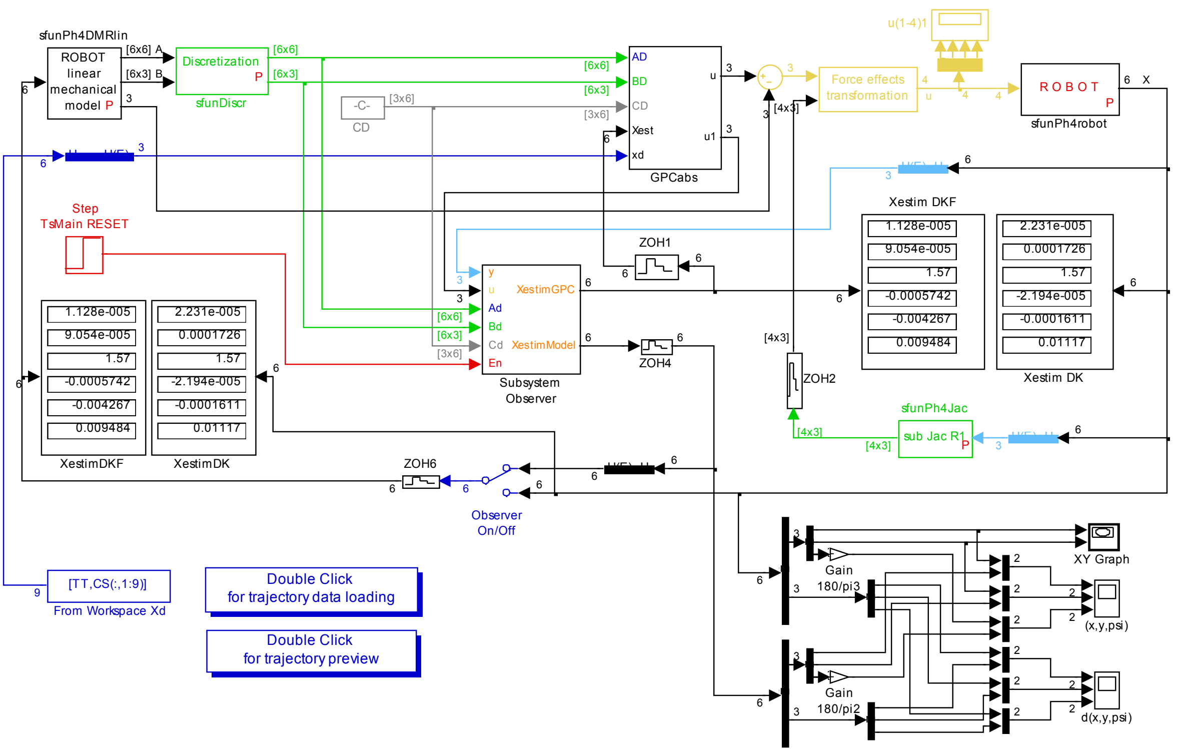


Figure 5 - Simulink simulative scheme with predictive control using estimated state of the robot (estimated state 'Xest' get off the 'Subsystem Observer' and is led to the controller 'GPCabs').

6 CONCLUSION

The paper shows possibilities, when the model parameters are unknown and system state is not available from measurement. There is a discussion also of the situation, when the model (its parameters) is known and only state is not available. As a solution for the simultaneous estimation of state and model parameters and single estimation of state, the state-space observer based on Kalman filter with effective algorithm for computation of the optimal gain is investigated. The theory is documented by several simulative experiments.

References

ANDERSON, B.D.O.; MOORE, J. B. 1979. *Optimal Filtering.* Prentice-Hall, Inc., 1979.

BELDA, K.; BÖHM, J. 2005a. Incremental State-Space Predictive Controller with State Space Kalman Estimation. In.: *Proceedings of 15th International Conference on Process Control*, Štrbské pleso, June 2005. Bratislava: Slovak University of Technology, 2005. Pp. 1-6.

BELDA, K.; BÖHM, J.; PÍŠA, P. 2005b. Predictive control for modern industrial robots, algorithms and their applications. In.: *Proc. of Int. Conf. on Informatics in Control, Automation and Robotics*, Barcelona, September 2005. Setúbal: INSTICC Press, 2005. Pp. 1-8. ISBN 972-8865-30-9.

BELDA, K. 2005c. *GPC pages (GPC Toolbox)*. Praha. <http://as.utia.cas.cz/asc/>.

BILLINGS, A. 1980. Introduction to Kalman Filters. *Int. IEE Colloquium on "Kalman Filters and Their Applications"*, 1980, pp. 1-7.

HAVLENA, V. 2002. *Odhadování a filtrace (Doplňkové skriptum).* Praha: FEL ČVUT V Praze, 2002. 74 s. ISBN 80-01-02587-X.

LAWSON, Ch. J.; HANSON, R. J. 1974. *Solving Least Square Problems.* Prentice - Hall, Inc. 1974.

GOLUB, H. G.; LOAN, Van Ch. F.1989. *Matrix computations.* The Johns Hopkins University Press, 1989.

ORDYS, A.; CLARKE, D. 1993. A state – Space Description for GPC Controllers. *INT. J. Systems SCI.*, 1993, Vol. 24, No. 9, pp. 1727-1744.

STEJSKAL, V.; VALÁŠEK, M. 1996. *Kinematics and dynamics of machinery.* New York: Marcel Dekker, Inc., 1996.

ŠULC, B. 1999. *Teorie automatického řízení.* Skriptum. Praha: Vydavatelsví ČVUT v Praze, 1999.

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