PREDICTIVE CONTROL FOR MECHATRONIC LABORATORY MODELS

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Abstract: The paper deals with the design of discrete adaptive model-based predictive control for simple mechatronic systems. Simple mechatronic systems are considered as Single-Input/Single-Output systems or possibly systems with low number of inputs and outputs. However, the methods of adaptation and model-based control are not generally limited to this condition. In the paper, a combination of on-line identification and generalized predictive control will be introduced. The identification is based on least squares. The predictive control arises from state-space formulation. This idea is applied to ARX models representing Input/Output formulation. The presented algorithms are derived in computationally suitable square-root form and their correctness is documented by tests on laboratory models.

Keywords: Mechatronic systems, Predictive control, I/O equations of predictions

1. INTRODUCTION

With steady growth of production, there is a necessity to develop different ways of the control of individual components included in the production process. The components usually combine some mechanical elements (mechanisms) and elements, mostly electrical, which power (actuators – drive units), monitor (sensors) or control (control units) the component state itself; i.e. altogether, it represents combination of mechanics and electronics, in single word – mechatronics.

The main purpose of the control often consists in fulfillment of some predetermined motion or in stabilization. The question is: ‘How to achieve such purpose?’. One of the possibilities, being at the beginning of industrial application, is approach based on model-based control strategies (Belda, 2005). These strategies offer complex solution on global level of whole controlled system and not only of its individual elements. In this paper, as a powerful way, the predictive control is presented (Ordis, 1993; Maciejowski, 2002). The main stress is laid on obtaining of suitable model and its on-line use (adaptation) within control design.

The explanation is intended for simple mechatronic systems, which are considered as Single-Input/Single-Output type or possibly the systems with low number of inputs and outputs. However, the methods of adaptation and model-based control are not generally limited to this condition.

2. MODEL DEFINITION

The model, description of a controlled system, represents very important part, which includes specifically processed information for design of control actions. The best results of control process are achieved, when the model is obtained on the basis of thoroughgoing mathematical and physical analysis. It is often difficult. Therefore, different ways, how to obtain the model describing the controlled system, are investigated.

Selection of model form is determined by used model-based control, in which the model is involved. Due to digital character of automating devices, the discrete control techniques are preferred. Therefore, the resultant models for control design are also discrete in spite of the facts that controlled system may be continuous. Discrete realization is advantageous, because naturally respects finite and predefined time for computation of control actions.

As was mentioned in introduction, the design of algorithms of predictive control will arise from state-space formulation. However, only this idea will be used as inspiration for utilization of ARX models, which are Input/Output type. Thus, let us proceed from autoregressive model with external input (ARX model) (Bobál, 2005)

$$y(k) = \sum_{i=0}^{n} b_i\, u(k-i) - \sum_{i=1}^{n} a_i\, y(k-i) + e(k)$$

where n is order of controlled system; $y(\cdot)$ and $u(\cdot)$ are values of its output and input; and $e(k)$ is error, respective, some noise of measurement of system output $y(k)$. The ARX model can be also written in the following condensed form

$$y(k) = \vartheta_k\, \mathbf{f}_k + e(k)\,, \quad \vartheta_k = [b_0\ \ b_1\ \cdots\ b_n\ \ -a_1\ \ -a_2\ \cdots\ -a_n]\,,$$
$$\mathbf{f}_k = [u(k)\ u(k-1)\cdots u(k-n)\ y(k-1)\cdots y(k-n)]^T$$

The ARX model will be used for construction of equations of predictions (subsection 4.2) and its condensed form is suitable for identification by least squares (section 3).

3. IDENTIFICATION

The sufficient and well known method of identification is method of least squares (Söderström, 1989). In this paper will be briefly summed up in square-root form (Bobál, 2005). Let us consider ARX model, where $e(k)$ represents in view of least squares model error expressed as follows:

$$e(k) = y(k) - \vartheta_k\, \mathbf{f}_k$$

On the assumption, that the parameters are close to constants or they are varied only slightly during real control process, then it is possible to write necessary number of equations with changeless vector of parameters ϑ_k

$$\mathbf{e}_k = \mathbf{y}_k - \mathbf{F}_k\, \vartheta_k^T = [\mathbf{F}_k\ \ \mathbf{y}_k\ ]\begin{bmatrix} -\vartheta_k^T \\ 1 \end{bmatrix}$$

where $\mathbf{F}_k$ is a matrix of past data, composed from data vectors $\mathbf{f}_{k-i+1}^T$, $i = 1,\cdots,2n$.

The criterion for identification is

$$J_k = \mathbf{e}_k^T \ \mathbf{e}_k \text{, i.e. alternatively } J_k = [-\vartheta_k \ \ 1] \begin{bmatrix} \mathbf{F}_k^T \\ \mathbf{y}_k^T \end{bmatrix} [\mathbf{F}_k \ \ \mathbf{y}_k] \begin{bmatrix} -\vartheta_k^T \\ 1 \end{bmatrix}$$

To minimize the criterion, it is sufficient to minimize only its square-root $\mathbf{J}$ following from

$$\min J_k = \| \mathbf{J} \|^2 = \left\| [\mathbf{F}_k \ \ \mathbf{y}_k] \begin{bmatrix} -\vartheta_k^T \\ 1 \end{bmatrix} \right\|^2$$

The computationally effective minimization is provided by orthogonal-triangular decomposition (e.g. house-holder algorithm (Golub, 1989)) which transforms extended matrix $[\mathbf{F}_k \ \ \mathbf{y}_k]$ to upper triangular matrix $\mathbf{R} = \begin{bmatrix} \mathbf{R}_{PP} & \mathbf{R}_{PR} \\ \mathbf{0} & c_l \end{bmatrix}$.

This matrix consists of sub-matrices partly corresponding to the unknown parameters ϑ_k and partly to square-root of loss of the criterion $\mathbf{c}_l$. By considering sub-matrices related to unknown parameters, the following equation is obtained

$$-\mathbf{R}_{PP}\vartheta_k^T + \mathbf{R}_{PR} = \mathbf{0}$$

from which, the parameters can be determined by backward substitution (due to triangular form of matrix $\mathbf{R}_{PP}$). This process is provided on-line in each time step with connecting refreshed data $\mathbf{f}_k$ and $y(k)$ to current triangular matrix $\mathbf{R}$, which is again restored to new upper triangular matrix $\mathbf{R}$.

4. ALGORITHM OF PREDICTIVE CONTROL

Predictive control is a multi-step approach, combining feedforward and feedback control design (Ordis, 1993). Feedforward is represented by predictions based on mathematical model. This part is dominant component of control actions. Feedback from measured outputs serves for compensation of some bounded model inaccuracies and low external disturbances.

The design consists in local minimization of quadratic criterion, in which the predictions of future outputs $y(\cdot)$ are involved. The predictions are determined from specific equations of predictions forming the basis of predictive control. The minimization is repeated in each time step.

4.1 Model reorganization

To compose equations of predictions from available ARX model (defined in sections 2 & 3), there are several possibilities how to do it. One of the possibilities is to express the equations directly from ARX model. It is possible according to (Maciejowski, 2002), however such way requires solution of Diofantic equation and storing previous values of inputs and outputs as in ‘pseudo state-space’ possibilities. These further possibilities are generally called as state-space forms with non-minimal state. They will be helpful for the use of idea of state-space formulation in predictive control, in which the forming of needed equations of predictions consists in repetitive insertion of state-space model.

The suitable model form arisen from described ARX model can be structured as follows:

$$\begin{bmatrix} y(k-n+2) \\ \vdots \\ y(k) \\ y(k+1) \end{bmatrix} = \begin{bmatrix} 0 & 1 & & \\ \vdots & & \ddots & \\ 0 & & & 1 \\ -a_n & & \cdots & -a_1 \end{bmatrix} \begin{bmatrix} y(k-n+1) \\ \vdots \\ y(k-1) \\ y(k) \end{bmatrix} + \begin{bmatrix} 0 & \cdots & 0 \\ \vdots & & \vdots \\ 0 & \cdots & 0 \\ b_n & \cdots & b_1 \end{bmatrix} \begin{bmatrix} u(k-n+1) \\ \vdots \\ u(k-1) \\ u(k) \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} y(k-n+2) \\ \vdots \\ y(k+1) \end{bmatrix} = \mathbf{A} \begin{bmatrix} y(k-n+1) \\ \vdots \\ y(k) \end{bmatrix} + \mathbf{B}_0 \begin{bmatrix} u(k-n+1) \\ \vdots \\ u(k) \end{bmatrix} \quad i.e. \quad \mathbf{X}(k+1) = \mathbf{A}\,\mathbf{X}(k) + \mathbf{B}_0\,\mathbf{u}(k)$$

$$y(k) = [0 \quad \cdots \quad 0 \;\; 1]\,\mathbf{X}(k) \qquad i.e. \quad y(k) = \mathbf{C}\,\mathbf{X}(k)$$

This state-space model is equivalent to the usual state-space model. It has two pseudo state-space matrices $\mathbf{A}$ and $\mathbf{B}_0$ with similar dimensions as in the usual model.

4.2 Equations of predictions

Principle of the equations is an expression (prediction) of future values of outputs $\mathbf{y}$ from current measured state $\mathbf{X}(k)$ (Ordis, 1993). Considering the pseudo state-space model, the equations of predictions can be composed as follows

$$\begin{aligned}
&» \hat{\mathbf{X}}_{(k+1)} = \mathbf{A}\mathbf{X}_{(k)} + \mathbf{B}_0\mathbf{u}_{(k)} \\
&\hat{\mathbf{X}}_{(k+2)} = \mathbf{A}^2\mathbf{X}_{(k)} + \mathbf{A}\mathbf{B}_0\mathbf{u}_{(k)} + \mathbf{B}_0\overline{\mathbf{u}}_{(k+1)} \\
&» \hat{\mathbf{X}}_{(k+2)} = \mathbf{A}^2\mathbf{X}_{(k)} + \underbrace{([\mathbf{A}\mathbf{B}_0 \;\; \mathbf{0}] + [\mathbf{0}\;\mathbf{B}_0])}_{\mathbf{B}_1}\,\mathbf{u}_{(k+1)} \\
&\hat{\mathbf{X}}_{(k+3)} = \mathbf{A}^3\mathbf{X}_{(k)} + \mathbf{A}\mathbf{B}_1\mathbf{u}_{(k+1)} + \mathbf{B}_0\overline{\mathbf{u}}_{(k+2)} \\
&» \hat{\mathbf{X}}_{(k+3)} = \mathbf{A}^3\mathbf{X}_{(k)} + \underbrace{([\mathbf{A}\mathbf{B}_1 \;\; \mathbf{0}] + [\mathbf{0}\;\mathbf{0}\;\mathbf{B}_0])}_{\mathbf{B}_2}\,\mathbf{u}_{(k+2)} \\
&\qquad \ddots \\
&\hat{\mathbf{X}}_{(k+N)} = \mathbf{A}^N\mathbf{X}_{(k)} + \mathbf{A}\mathbf{B}_{N-2}\mathbf{u}_{(k+N-2)} + \mathbf{B}_0\overline{\mathbf{u}}_{(k+N-1)} \\
&» \hat{\mathbf{X}}_{(k+N)} = \mathbf{A}^N\mathbf{X}_{(k)} + \underbrace{([\mathbf{A}\mathbf{B}_{N-2}\;\mathbf{0}] + [\mathbf{0}_{n,N-1}\,\mathbf{B}_0])}_{\mathbf{B}_{N-1}}\mathbf{u}_{(k+N-1)}
\end{aligned}$$

, where

$$\begin{aligned}
\overline{\mathbf{u}}_{(k+1)} &= [u_{(k-n+2)} \cdots u_{(k+1)}]^T \\
\mathbf{u}_{(k+1)} &= [u_{(k-n+1)}\; u_{(k-n+2)} \cdots u_{(k)}\; u_{(k+1)}]^T \\
&= [\mathbf{u}^T_{(k)},\, u_{(k+1)}]^T \\
&\;\;\vdots \\
\overline{\mathbf{u}}_{(k+N-1)} &= [u_{(k-n+N)} \cdots u_{(k+N-1)}]^T \\
\mathbf{u}_{(k+N-1)} &= [u_{(k-n+1)} \cdots u_{(k+N-2)}\; u_{(k+N-1)}]^T \\
&= [\mathbf{u}^T_{(k+N-2)},\, u_{(k+N-1)}]^T
\end{aligned}$$

and appropriate matrix notation is

$$\underbrace{\begin{bmatrix} \hat{y}_{(k+1)} \\ \vdots \\ \hat{y}_{(k+N)} \end{bmatrix}}_{\hat{\mathbf{y}}} = \underbrace{\begin{bmatrix} \mathbf{CA} \\ \vdots \\ \mathbf{CA}^N \end{bmatrix} \begin{bmatrix} y_{(k-n+1)} \\ \vdots \\ y_{(k)} \end{bmatrix}}_{\mathbf{f}} + \underbrace{\begin{bmatrix} \mathbf{CB}_0 & \cdots & \mathbf{0} \\ & \ddots & \vdots \\ & & \mathbf{CB}_{N-1} \end{bmatrix}}_{\overline{\mathbf{G}}} \underbrace{\begin{bmatrix} u_{(k-n+1)} \\ \vdots \\ u_{(k+N-1)} \end{bmatrix}}_{\mathbf{u}_{(k+N+1)}}$$

$$\hat{\mathbf{y}} = \mathbf{f} + \underbrace{\overline{\mathbf{G}}_{(:,1:n-1)} \begin{bmatrix} u_{(k-n+1)} \\ \vdots \\ u_{(k-1)} \end{bmatrix}}_{\bar{\mathbf{f}}} + \underbrace{\overline{\mathbf{G}}_{(:,n:N-1)}}_{\mathbf{G}} \underbrace{\begin{bmatrix} u_{(k)} \\ \vdots \\ u_{(k+N-1)} \end{bmatrix}}_{\mathbf{u}}$$

Such composed equations of predictions have the same dimension as the equations, which are based on state-space model with minimal state.

4.3 Computation of control actions

The control actions are obtained by minimization of quadratic criterion

$$J_k = \sum_{j=No+1}^{N} \left\| (\hat{y}_{(k+j)} - w_{(k+j)}) Q_y \right\|^2 + \sum_{j=1}^{Nu} \left\| u_{(k+j-1)} Q_u \right\|^2$$

where N, No and Nu are horizons; Q_y and Q_u are penalizations; and $w_{(k+j)}$ are desired values. Using effective square-root algorithm, the vector $\mathbf{u}$ is obtained. It represents control actions for whole horizon N. From it, only the first appropriate actions are really applied to the controlled system. This process is repeated in every time step.

5. TESTS WITH MODEL 'BALL ON ROD'

For real-time tests, simple laboratory model (Fig. 1) was used. From mathematical-physical analysis, this model represents system of fourth order: electrical motor is second order and the ball dynamics is expressed also by second order. The fast motor dynamics can be omitted, and only ball dynamics of second-order can be considered.

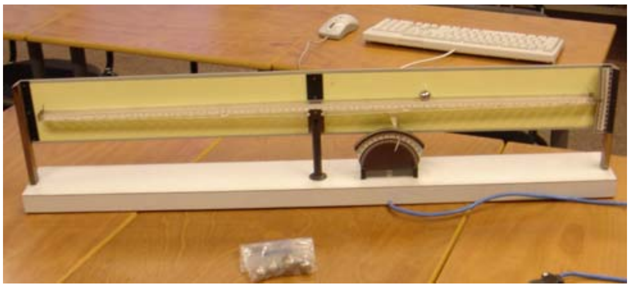

Fig. 1. Laboratory model 'ball on rod'.

The tests were realized in MATLAB-Simulink environment. The predictive controller and algorithm of identification were implemented in Simulink blocks; see schemes in Fig. 2 and in Fig. 3. The presented predictive control was partly tested in adaptive mode (Fig. 2). For comparison, the control was also tested in non-adaptive mode with constant model from mathematical-physical analysis (Fig. 3). The aim of the tests was stabilization of the ball in different positions y of desired rectangular signal w.

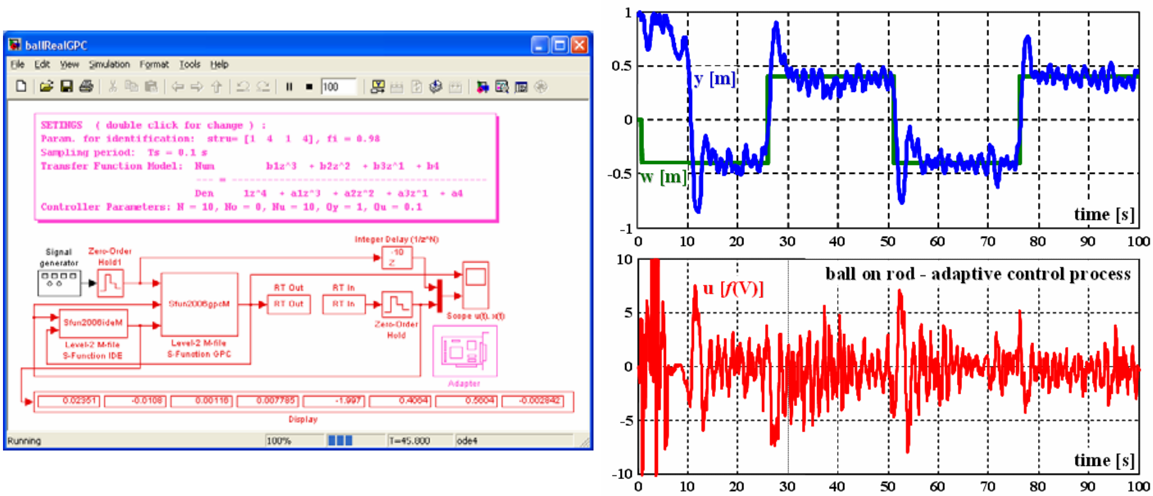

Fig. 2. Simulink scheme and time histories of adaptive predictive control.

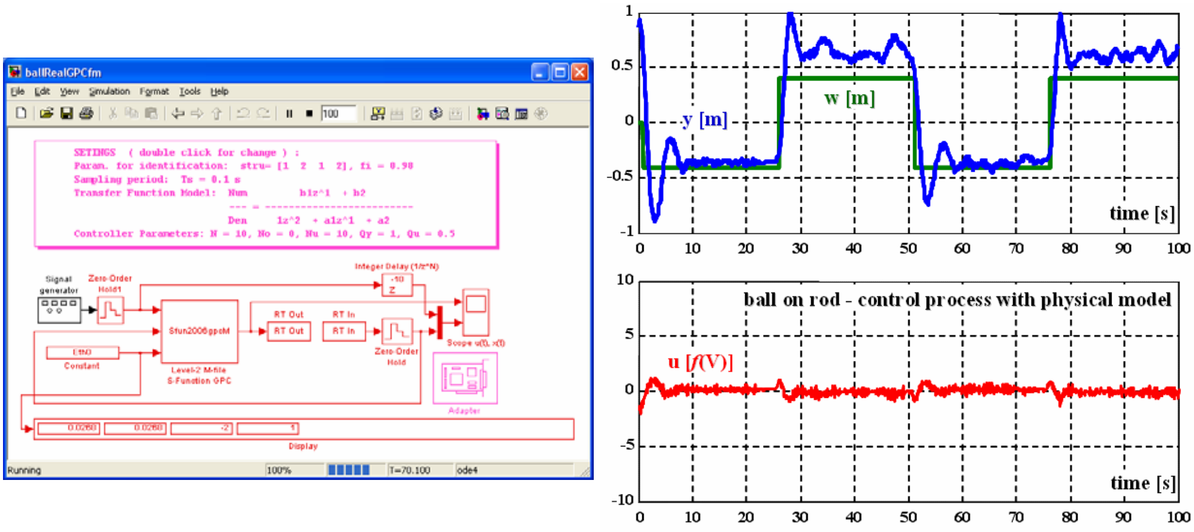

Fig. 3. Simulink scheme and time histories of predictive control with mathematical-physical model.

The control process with model identification required relatively energetic control actions for induction of the identification. It is achieved by penalization Q_u , which was smaller, than in case of using physical model. Therefore the actions were quite jittered.

The tests show successful realization of predictive control both non-adaptive and adaptive.

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