FULLY PROBABILISTIC CONTROL DESIGN FOR GAUSSIAN STOCHASTIC SYSTEMS

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Abstract: Control of stochastic systems is generally formulated as a minimization of expected value of a suitably chosen loss function of system inputs, outputs and desired behavior with respect to feedback control strategies. The standard strategies (e.g. Linear Quadratic Gaussian control) choose control actions that make the closed-loop behavior as close as possible to desired one using expected and desired output values. More general approach is to consider complex information on stochastic system behavior by complex probabilistic description. On this approach, fully probabilistic design is based. It uses probabilistic description for characterization of closed-loop of stochastic system and its desired behavior. This paper points out the basic principles of fully probabilistic design and its practical application to the control of Gaussian stochastic systems.

Keywords: Stochastic control, fully probabilistic control design, LQ Gaussian control

1 INTRODUCTION

The most general formulation of stochastic control problem is based on minimization of expected value of a suitably chosen loss function. The loss function is defined as function of system inputs, outputs and desired behavior with respect to feedback control strategies. Control strategy has to be chosen in correspondence to the purpose of control. Standard strategies select control actions that make the closed-loop behavior as close as possible to desired one by minimization of loss function including differences between actual and desired system outputs. One example of standard strategy is LQG control employing linear system model, quadratic criterion, Gaussian noise [Bobál et al., 2005]. More general strategy called fully probabilistic control design [Kárný, 1996] is presented here. It considers more complex information on stochastic system behavior by probabilistic description of the closed-loop system.

In fully probabilistic design all aspects of the closed-loop (Figure 1), including expected and desired inputs and outputs, are defined as probability density functions. Consequently, the probabilistic design may use more of available information contrary to standard control design, which may have a lack of representative parameters or interpretations for such information.

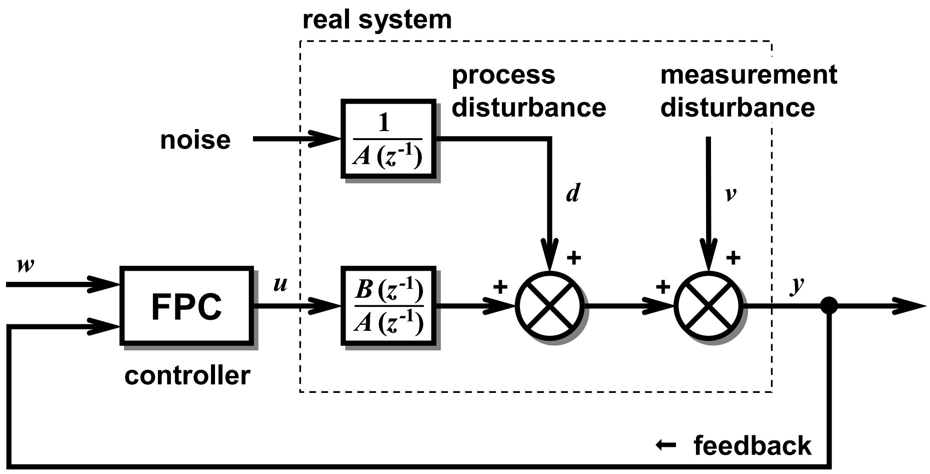


Figure 1 - Closed-loop control circuit with fully probabilistic controller and stochastic system

This paper aims to introduce probabilistic design as promising control strategy in view of its possibilities in control applications. At the beginning, the basic principles of fully probabilistic control design will be outlined for stochastic systems with general probability distribution. Then, for real control design, the models of stochastic systems will be described. At the end, the practical implementation of proposed design will be explained and several examples of practical application of the fully probabilistic control design will be demonstrated.

2 PRINCIPLES OF FULLY PROBABILISTIC CONTROL DESIGN

In general, the fully probabilistic control design determines admissible control strategy, which forces the joint distribution of all closed-loop variables as close as possible to the desired (ideal) distribution. To measure level of proximity of these distributions, the Kullback-Leibler divergence (*KL-divergence*) $\mathcal{D}(f \,\|\, {}^I f)$ is used [Kárný, 1996; Kárný et al., 2006] as follows

$$\mathcal{D}(f \,\|\, {}^I f) \equiv \int f(X) \ln\left(\frac{f(X)}{{}^I f(X)}\right) dX \tag{1}$$

where the pair of probability density functions (*pdf*s) f and ${}^I f$ – representatives of distribution is considered to be acting on their domains i.e. on a set of all values X^*. The *KL-divergence* from control point of view represents loss function or optimality criterion. By its minimization, the optimal control law is obtained. The following lines outline the minimization process. Due to necessity to consider time for computation of control law, the discrete design is considered.

Let us start from explanation of pair of *pdf*s mentioned above. In control design, they represent joint *pdf*s of real and ideal closed-loop behavior. Let us make an assumption that succeeding system state $\mathbf{x}_j$ arises from previous system state $\mathbf{x}_{j-1}$ and system input u_{j-1} only and is not depending on past system states and system inputs.

$$\begin{gathered} f(\mathbf{x}_j \mid \mathbf{x}_{j-1}, u_{j-1}, \cdots, \mathbf{x}_0, u_0) = f(\mathbf{x}_j \mid \mathbf{x}_{j-1}, u_{j-1}), \quad f(u_j \mid \mathbf{x}_j, u_{j-1}, \cdots, \mathbf{x}_0, u_0) = f(u_j \mid \mathbf{x}_j) \\ {}^I f(\mathbf{x}_j \mid \mathbf{x}_{j-1}, u_{j-1}, \cdots, \mathbf{x}_0, u_0) = {}^I f(\mathbf{x}_j \mid \mathbf{x}_{j-1}, u_{j-1}), \quad {}^I f(u_j \mid \mathbf{x}_j, u_{j-1}, \cdots, \mathbf{x}_0, u_0) = {}^I f(u_j \mid \mathbf{x}_j) \end{gathered} \tag{2}$$

Assuming that $f(\mathbf{x}_k) = {}^I f(\mathbf{x}_k)$ is a prior *pdf* on initial state $\mathbf{x}_k$:

- joint *pdf* that represents the real closed-loop behavior

$$\begin{aligned} f(X) = f_N &\equiv f(\mathbf{x}_{k+N}, u_{k+N-1}, \mathbf{x}_{k+N-1}, u_{k+N-2}, \cdots, \mathbf{x}_{k+1}, u_k \mid \mathbf{x}_k)\, f(\mathbf{x}_k) \\ &= \left\{ \prod_{j=k+1}^{k+N} f(\mathbf{x}_j \mid \mathbf{x}_{j-1}, u_{j-1})\, f(u_{j-1} \mid \mathbf{x}_{j-1}) \right\} f(\mathbf{x}_k) \end{aligned} \tag{3}$$

- joint *pdf* that represents the ideal closed-loop behavior

$$\begin{aligned} {}^I f(X) = {}^I f_N &\equiv {}^I f(\mathbf{x}_{k+N}, u_{k+N-1}, \mathbf{x}_{k+N-1}, u_{k+N-2}, \cdots, \mathbf{x}_{k+1}, u_k \mid \mathbf{x}_k)\, f(\mathbf{x}_k) \\ &= \left\{ \prod_{j=k+1}^{k+N} {}^I f(\mathbf{x}_j \mid \mathbf{x}_{j-1}, u_{j-1})\, {}^I f(u_{j-1} \mid \mathbf{x}_{j-1}) \right\} f(\mathbf{x}_k) \end{aligned} \tag{4}$$

These *pdf*s are considered to be defined for values in given time and their parameters to be valid within specific finite horizon N called control horizon. The label N represents the number of discrete time instants j from instant k within the horizon; i.e. $j = k+1, \cdots, k+N$, $u_{(\cdot)}$ are control actions. Moreover, joint *pdf* labeled by superscript ^{I} denotes user requirements, i.e. desired values. They will be introduced and defined in implementation section.

Individual *pdf*s in (3) and (4) describe real and ideal behavior of individual parts of given closed-loop i.e. behavior of the system and controller; e.g. in instant $j = k+1$, the real and ideal system behavior is modeled by *pdf*s $f(\mathbf{x}_{k+1} | \mathbf{x}_k, u_k)$ and ${}^I f(\mathbf{x}_{k+1} | \mathbf{x}_k, u_k)$; and real and ideal controller behavior is modeled by *pdf*s $f(u_k | \mathbf{x}_k)$ and ${}^I f(u_k | \mathbf{x}_k)$, respectively.

Note, the determination of optimal control law – optimal *pdf* ${}^O f(u_k | \mathbf{x}_k)$ of the *pdf* $f(u_k | \mathbf{x}_k)$ is the task of fully probabilistic design:

$$\{{}^O f(u_{j-1} | \mathbf{x}_{j-1})\}_{j=k+1}^{k+N} \in \operatorname{Arg} \min_{\{f(u_{j-1} | \mathbf{x}_{j-1})\}_{j=k+1}^{k+N}} \mathcal{D}(f_N \| {}^I f_N) \tag{5}$$

In detail, the equation (5) together with equation (1) are initial forms for algorithmic representation of fully probabilistic control design (minimization of *KL-divergence*), which is needed for practical utilization. The *KL-divergence* in (1) can be viewed as an expectation of the additive loss function including j^{th} partial loss, which is defined as

$$\ln\left(\frac{f(\mathbf{x}_j | \mathbf{x}_{j-1}, u_{j-1})\, f(u_{j-1} | \mathbf{x}_{j-1})}{{}^I f(\mathbf{x}_j | \mathbf{x}_{j-1}, u_{j-1})\, {}^I f(u_{j-1} | \mathbf{x}_{j-1})} \right) \tag{6}$$

The algorithm minimizing the *KL-divergence* arises from the following expression [Kárný et al., 2005]

$$\begin{aligned}
\mathcal{D}_k \equiv & \min_{\{f(u_{j-1} | \mathbf{x}_{j-1})\}_{j=k+1}^{k+N}} \mathcal{D}(f_N \| {}^I f_N) = \min_{\{f(u_{j-1} | \mathbf{x}_{j-1})\}_{j=k+1}^{k+N}} \Bigg\{ \mathcal{D}(f_{N-1} \| {}^I f_{N-1}) \\
& + \min_{f(u_{k+N-1} | \mathbf{x}_{k+N-1})\}} \int \cdots \int f_{N-1} \Bigg[\iint f(\mathbf{x}_{k+N} | \mathbf{x}_{k+N-1}, u_{k+N-1})\, f(u_{k+N-1} | \mathbf{x}_{k+N-1}) \\
& \times \ln\left(\frac{f(\mathbf{x}_{k+N} | \mathbf{x}_{k+N-1}, u_{k+N-1})\, f(u_{k+N-1} | \mathbf{x}_{k+N-1})}{\gamma(\mathbf{x}_{k+N})\, {}^I f(\mathbf{x}_{k+N} | \mathbf{x}_{k+N-1}, u_{k+N-1})\, {}^I f(u_{k+N-1} | \mathbf{x}_{k+N-1})} \right) d\mathbf{x}_{k+N}\, du_{k+N-1} \Bigg] \\
& d\mathbf{x}_{k+N-1}\, du_{k+N-2} \cdots d\mathbf{x}_{k+1}\, du_k \Bigg\}
\end{aligned} \tag{7}$$

which can successively be minimized by searching for its minimizing factors in following way.

$$\mathcal{D}_k \equiv \min_{\{f(u_{j-1}\,|\,\mathbf{x}_{j-1})\}_{j=k+1}^{k+N}} \Big\{ \mathcal{D}(\, f_{N-1} \,||\, {}^{I}f_{N-1}\,) + \tag{8}$$

$$\min_{f(u_{k+N-1}\,|\,\mathbf{x}_{k+N-1})\}} \int \cdots \int f_{N-1}\,[A_N]\, d\,\mathbf{x}_{k+N-1}\, d\,u_{k+N-2}\, \cdots d\,\mathbf{x}_{k+1}\, d\,u_k \Big\}$$

The equality (8) represents initial equality (7), only the elements inside of square brackets are labeled as term A_N. Focusing on evaluation of this term, a specific auxiliary function $\delta(\cdot)$ is suitable to be defined as follows

$$\delta(u_{j-1},\mathbf{x}_{j-1}) \equiv \int f(\mathbf{x}_j \,|\, u_{j-1}, \mathbf{x}_{j-1}) \ln\left(\frac{f(\mathbf{x}_j \,|\, u_{j-1}, \mathbf{x}_{j-1})}{\gamma(\mathbf{x}_j)\; {}^{I}f(\mathbf{x}_j \,|\, u_{j-1}, \mathbf{x}_{j-1})} \right) d\,\mathbf{x}_j \tag{9}$$

This definition helps us in further description of minimizing process. Continuing in minimization of (7), where outside multiple integration, also present in the form (8), is suitable again to be marked as additional term B_N as

$$B_N \equiv \int \cdots \int f_{N-1}\,[A_N]\, d\,\mathbf{x}_{k+N-1}\, d\,u_{k+N-2}\, \cdots d\,\mathbf{x}_{k+1}\, d\,u_k \tag{10}$$

Minimization of term B_N, in which the auxiliary function $\delta(\cdot)$ is considered, leads to the expression of the last auxiliary function labeled as $\gamma(\cdot)$

$$\gamma(\mathbf{x}_{j-1}) \equiv \int {}^{I}f\,(u_{j-1} \,|\, \mathbf{x}_{j-1})\; e^{-\,\delta\,(u_{j-1},\,\mathbf{x}_{j-1})}\, du_j \tag{11}$$

initiating as $\gamma(\mathbf{x}_{k+N}) \equiv 1$

and optimal control strategy

$${}^{O}f(u_{j-1} \,|\, \mathbf{x}_{j-1}) = \frac{1}{\gamma(\mathbf{x}_{j-1})}\; {}^{I}f(u_{j-1} \,|\, \mathbf{x}_{j-1})\; e^{-\,\delta\,(u_{j-1},\mathbf{x}_{j-1})} \tag{12}$$

Finally, the algorithm of fully probabilistic control design is defined by expressions (9), (11) and (12). The algorithm runs recursively for $j = k+N, k+N-1, \cdots, k+1$ in this backward manner. In essence, the algorithm of fully probabilistic design generates auxiliary functions $\delta(\cdot)$ and $\gamma(\cdot)$, which sequentially determine optimal admissible control strategies $\{{}^{O}f(u_{j-1} \,|\, \mathbf{x}_{j-1})\}_{j=k+1}^{k+N}$ expressed by equation (12). These strategies minimize *KL-divergence* $\mathcal{D}(f \,||\, {}^{I}f)$.

3 PRACTICAL MODEL REPRESENTATION

As was formerly mentioned, the stochastic system behavior can be generally described by probability density function (*pdf*). If the system behavior has a normally distributed character, then *pdf* function has the following form

$$\mathcal{N}(\mu_y, r_y): \quad f(y) = \frac{1}{\sqrt{2\pi r_y}} e^{-\frac{(y-\mu_y)^2}{2 r_y}} \tag{13}$$

where μ_y represents mean value, i.e. expected value of system output y ($\mu_y = E\{y\}$), r_y denotes a dispersion (variance; $r_y = E\{(y-\mu_y)^2\}$). These parameters are considered in a control design as quantities continuous in values and discrete in time. The former follows from character of the system and the latter is given by discrete realization of control, which naturally respects time for its computation. Mentioned parameters have direct relation both to ARX model [Peterka, 1981] with normally distributed noise:

$$y_k = \underbrace{\sum_{i=1}^{n} b_i u_{k-i} - \sum_{i=1}^{n} a_i y_{k-i}}_{\mu_y} + e_{y_k}, \qquad e_{y_k} \sim \mathcal{N}(0, r_y) \tag{14}$$

where n is a system order and e_{y_k} is an output noise, which has a dispersion r_y;
but also to its appropriate state-space model, respectively:

$$\mathbf{x}_{k+1} = \underbrace{\mathbf{A}\mathbf{x}_k + \mathbf{B}u_k}_{\mu_{\mathbf{x}}} + \mathbf{e}_{\mathbf{x}_k}, \quad \mathbf{e}_{\mathbf{x}_k} \sim \mathcal{N}(\mathbf{0}, \mathbf{R}), \quad y_k = \underbrace{\mathbf{C}\mathbf{x}_k}_{\mu_y} + e_{y_k}, \quad e_{y_k} \sim \mathcal{N}(0, r_y) \tag{15}$$

Equation (15) represents general state-space notation, in which the state $\mathbf{x}_k$ may be available or not; e.g. it has not a physical interpretation and had to be estimated. To avoid this drawback, it is suitable to use so-called pseudo state-space model [Bobál et al., 2005], i.e. state-space model with non-minimal state, which contains only delayed values of inputs and outputs, then the state and state-space elements are defined as follows

$$\mathbf{x}_k = \begin{bmatrix} u_{k-1} \\ \vdots \\ u_{k-n+1} \\ y_k \\ \vdots \\ y_{k-n+1} \end{bmatrix}, \; \mathbf{A} = \begin{bmatrix} 0 & \cdots & 0 & 0 & \cdots & 0 \\ 1 & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \ddots & 0 & 0 & \cdots & 0 \\ b_2 & \cdots & b_n & -a_1 & \cdots & -a_n \\ 0 & \cdots & 0 & 1 & \cdots & 0 \\ 0 & \ddots & 0 & 0 & \ddots & 0 \end{bmatrix}, \; \mathbf{B} = \begin{bmatrix} 1 \\ \vdots \\ 0 \\ b_1 \\ \vdots \\ 0 \end{bmatrix} \text{ and } \mathbf{C} = [\,0 \; \cdots \; 0 \; 1 \; \cdots \; 0\,] \tag{16}$$

In this case, since the system output y_k is directly present in the system state $\mathbf{x}_k$, it is possible to omit output equation $y_k = \mathbf{C}\mathbf{x}_k + e_{y_k}$ and to project output noise e_{y_k} in state noise $\mathbf{e}_{\mathbf{x}_k}$ as follows

$$\mathbf{e}_{\mathbf{x}_k} = [\,0,\ \cdots,\ 0,\ 1,\ 0,\ \cdots\,]^T e_{y_k} \tag{17}$$

The described models (14) and (15) covered in probability density function (13) are initial models for real implementation of fully probabilistic control design.

4 IMPLEMENTATION OF FULLY PROBABILISTIC CONTROL DESIGN

Let us start from theoretical implications listed in section 2 defined for simplicity for one-step ahead strategy, i.e. $N=1$ and let them be written in time instant $j=k+1$

$$\delta(u_k,\mathbf{x}_k)=\int f(\mathbf{x}_{k+1}\,|\,u_k,\mathbf{x}_k)\ln\left(\frac{f(\mathbf{x}_{k+1}\,|\,u_k,\mathbf{x}_k)}{\gamma(\mathbf{x}_{k+1})\ {}^{I}\!f(\mathbf{x}_{k+1}\,|\,u_k,\mathbf{x}_k)}\right)d\,\mathbf{x}_{k+1}\,,\ \gamma(\mathbf{x}_{k+1})\equiv 1 \tag{18}$$

$$\gamma(\mathbf{x}_k)=\int {}^{I}\!f\,(u_k\,|\,\mathbf{x}_k)\ e^{-\,\delta\,(u_k,\,\mathbf{x}_k)}\ du_k \tag{19}$$

$${}^{O}\!f(u_k\,|\,\mathbf{x}_k)=\frac{1}{\gamma(\mathbf{x}_k)}\ {}^{I}\!f(u_k\,|\,\mathbf{x}_k)\,e^{-\,\delta\,(u_k,\mathbf{x}_k)} \tag{20}$$

As was mentioned, the equations above, main steps of fully probabilistic control design are formulated generally for arbitrary probability distribution. However, from computational reasons, the normal distribution is considered. To use the algorithm, it is necessary, at first, to define individual probability density functions occurred in it.

The probability density functions (*pdf*s) define both real and ideal closed-loop (Figure 1) behavior. On the assumptions of finite memory of the behavior, known distribution and its parameters, and state-space model (15), the *pdf*s functions have following form:

- *pdf* representing the real controlled system behavior

$$f(\mathbf{x}_{k+1}\,|\,u_k,\mathbf{x}_k)=\frac{1}{\sqrt{2\pi}}\,\mathbf{R}^{-\frac{1}{2}}\,e^{-\frac{1}{2}(\mathbf{x}_{k+1}-\mu_{\mathbf{x}})^T\,\mathbf{R}^{-1}\,(\mathbf{x}_{k+1}-\mu_{\mathbf{x}})} \tag{21}$$

- *pdf* representing the ideal controlled system behavior

$${}^{I}\!f(\mathbf{x}_{k+1}\,|\,u_k,\mathbf{x}_k)=\frac{1}{\sqrt{2\pi}}\,\mathbf{R}^{-\frac{1}{2}}\,e^{-\frac{1}{2}(\mathbf{x}_{k+1}-{}^{I}\!\mu_{\mathbf{x}})^T\,\mathbf{R}^{-1}\,(\mathbf{x}_{k+1}-{}^{I}\!\mu_{\mathbf{x}})} \tag{22}$$

- *pdf* representing the ideal controller behavior

$${}^{I}\!f(\,u_k\,|\,\mathbf{x}_k)=\mathcal{N}\,({}^{I}\!\mu_u,{}^{I}r_u)=\frac{1}{\sqrt{2\pi\,{}^{I}r_u}}\ \ e^{-\frac{1}{2}(u_{k+1}-{}^{I}\!\mu_u)^T\,r_u^{-1}\,(u_{k+1}-{}^{I}\!\mu_u)} \tag{23}$$

where for simplicity ${}^{I}\!\mu_u=0$ is assumed and the dispersion ${}^{I}r_u$ can be viewed as a tuning parameter of the controller. For that defined *pdf*s, the steps (18) - (20) lead to the following expressions

$$\delta(u_k,\mathbf{x}_k,)=\frac{1}{2}(\mu_{\mathbf{x}}-{}^{I}\!\mu_{\mathbf{x}})^T\,\mathbf{R}^{-1}\,(\mu_{\mathbf{x}}-{}^{I}\!\mu_{\mathbf{x}}),\quad \mu_{\mathbf{x}}=\mathbf{A}\,\mathbf{x}_k+\mathbf{B}\,u_k\,,\quad {}^{I}\!\mu_{\mathbf{x}}=\mathbf{w}_{k+1} \tag{24}$$

$$\gamma(\mathbf{x}_k,\mathbf{w}_{k+1})=e^{-\frac{1}{2}\ln(1+({}^{I}r_u^{-\frac{1}{2}})^T\,\mathbf{B}^T\mathbf{R}^{-1}\,\mathbf{B}\,{}^{I}r_u^{-\frac{1}{2}})}\times e^{-\frac{1}{2}\mathbf{x}_k^T\mathbf{A}^T\mathbf{S}\,\mathbf{A}\,\mathbf{x}_k\,-\,\mathbf{x}_k^T\mathbf{A}^T\mathbf{S}\,\mathbf{w}_{k+1}\,-\,\frac{1}{2}\mathbf{w}_{k+1}^T\mathbf{S}\,\mathbf{w}_{k+1}} \tag{25}$$
$$\mathbf{S}\equiv(\mathbf{R}^{-1}-\mathbf{R}^{-1}\mathbf{B}({}^{I}r_u^{-1}+\mathbf{B}^T\mathbf{R}^{-1}\mathbf{B})^{-1}\mathbf{B}^T\mathbf{R}^{-1})$$

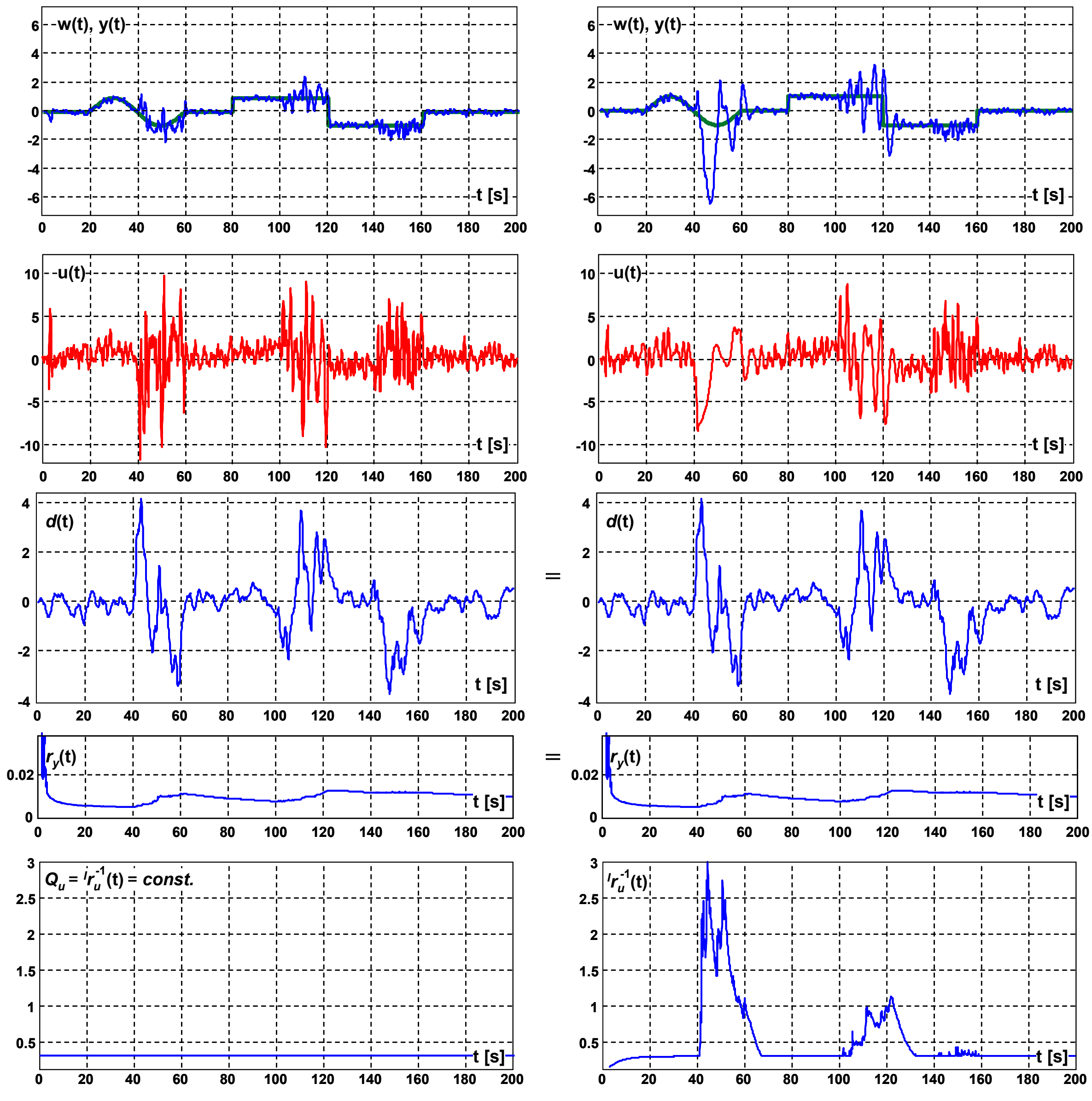
$${}^{O}\!f(u_k\,|\,\mathbf{x}_k,\mathbf{w}_{k+1})=\frac{1}{\sqrt{2\pi}}\ {}^{O}r_u^{-\frac{1}{2}}\ e^{-\frac{1}{2}\ {}^{O}r_u\,\{u_k+{}^{O}r_u^{-1}\,\mathbf{B}^T\mathbf{R}^{-1}(\mathbf{A}\,\mathbf{x}_k-\mathbf{w}_{k+1})\}^2}\ ,\quad {}^{O}r_u=({}^{I}r_u^{-1}+\mathbf{B}^T\mathbf{R}^{-1}\,\mathbf{B})^{-1} \tag{26}$$

$${}^{O}u_k=-\,{}^{O}r_u^{-1}\,\mathbf{B}^T\,\mathbf{R}^{-1}\,(\mathbf{A}\mathbf{x}_k-\mathbf{w}_{k+1})=-\,\mathbf{K}_{\mathbf{x}}\mathbf{x}_k+\mathbf{K}_{\mathbf{w}}\mathbf{w}_{k+1} \tag{27}$$

where ${}^{O}u_k$ is searched optimal control, here as a result of one-step ahead control strategy.

5 APPLICATION EXAMPLES

In this section, the possibilities of fully probabilistic design will be described and demonstrated on benchmark experiment specified as follows: let Gaussian stochastic system be modeled by models described in section 3 and let its stochastic part be generated by Gaussian noise, amplitude of which is randomly increased in spurts. The aim of such experiment is to eliminate sharp chattering of the controller attempting to compensate the noise rise. In practice, random fleeting noise rise can cause the damage of the actuator, which realizes control actions. The following figures will compare the behavior of the controller, which does not involve the full probabilistic description with the controller considering control strategy with fully probabilistic design. Both these controllers are adaptive, but only the latter considers probabilistic information. The information arises from adaptive procedure – identification of the model – in the form of values of system dispersions, by which controller stiffness is modified and increased control action chattering is damped down.



(a) (b)

Figure 2 - Comparison: (a) standard LQG control; (b) control designed by fully probabilistic design (real and desired system output; system input; process disturbance; system dispersion, ideal ${}^I r_u$)

Other application is a self-tuning process of the controllers via pre-simulations, during which the values of the controller parameters are initialized and subsequently optimized [Novák et al., 2003] using principles of fully probabilistic approach.

6 CONCLUSION

The paper briefly outlines the principles of fully probabilistic control design, from which the steps of the algorithmic representation were derived. Furthermore, from practical point of view, the implementation of this algorithmic representation was applied to Gaussian stochastic systems. However, generally such representation is usable for arbitrary distribution. Under the assumption of Gaussian stochastic systems, the resulting expressions are comparable with LQG controller. Nevertheless, the idea of probabilistic design casts different physical interpretation of the controller tuning parameters, representing them in probabilistic concept, e.g. dispersions of ideal close-loop behavior. This result was documented in section on application examples.

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