Archives of Control Sciences
Volume 19(LV), 2009
No. 4, *pages 355–368*

Probabilistic design and tuning of LQ control

KVĚTOSLAV BELDA

Important point of each control design is a selection of suitable weight parameters, that balance various control aims assigned by user. This paper proposes the new way to set control parameters automatically by evaluation of system response in comparison with system model. Considering stochastic nature of real-life applications, fully probabilistic design is used. It employs complex information on controlled system behavior by means of probabilistic description.

Key words: adaptive control, closed-loop control, dynamic programming, state-space realization, tuneable filters

1. Introduction

The most general formulation of control problem is based on minimization of expected value of a suitably chosen loss function. The loss function is defined as a function of system inputs, outputs and desired behavior with respect to used control strategy, which is chosen in correspondence with a purpose of control.

Standard strategies select control actions that make the closed-loop behavior as close as possible to desired one by minimization of loss function including differences between actual and desired system outputs. One example of standard strategy is LQ control employing linear system model and quadratic criterion [2]. Its more general interpretation, fully probabilistic control [4] is presented here, with emphasis on control tuning. The proposed approach takes into account more complex information on controlled system behavior by probabilistic description of the closed-loop (Fig. 1) comprising controlled system and controller.

In fully probabilistic design, all aspects of the closed-loop, expected and desired inputs and outputs, are defined as probability density functions. Consequently, the probabilistic design may use more of available information contrary to standard control design. The standard control design may have an insufficient number of representative parameters or interpretations for the information available.

K. Belda is with the Institute of Information Theory and Automation Academy of Sciences of the Czech Republic, public non-university research institution, Pod Vodárenskou věží 4, 182 08 Prague 8, Czech Rep., e-mail: belda@utia.cas.cz

Received 30.06.2009.

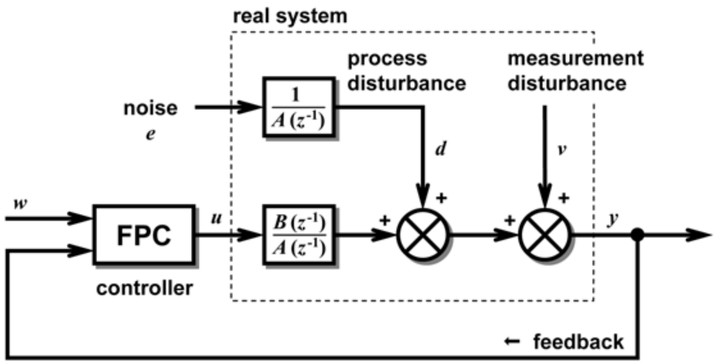

Figure 1. Block diagram of closed-loop.

Usually, the controller is based on user-provided parameters, which have the meaning of different balance weights in the evaluative loss function. They are called penalizations. Their physical meaning may be interpreted as the proportion between rigidity of input and output signals. This paper is focused on tuning of these on-line controller parameters. The tuning is based on actual system state and its behavior.

The on-line parameter tuning is motivated and geared to the mechatronic systems of particular type. The type of systems we focus on, are mechatronic systems representing a chain of different elements, causing inaccuracies. The important property of such systems is that combination of their elements has stochastic character. Therefore, probabilistic control design is particularly suitable for control design of such systems.

Two examples attached to this paper, demonstrate the algorithm. The practical application of the algorithm (e.g. manipulators-robots [8], [9]) would be in the same class as examples below.

The paper is organized as follows. In Section 2, the basic principles of fully probabilistic control design is outlined. Section 3 describes the idea of on-line fully probabilistic control tuning. In Section 4, the practical implementation of proposed design is demonstrated by examples.

2. Principles of fully probabilistic design

The aim of fully probabilistic design used in automatic control is to determine admissible strategy, which forces closed-loop behavior as close as possible to the desired user ideal. The closed-loop behavior involving behavior of controlled system and controller together (i.e. joint behaviour) is represented in fully probabilistic design by joint probability density functions (joint *pdf*s).

The following subsections outline in general individual steps of the design from starting assumptions, through optimization procedure to suitable probabilistic model representation in regard to fully probabilistic tuning.

2.1. Starting assumptions

From control point of view, the design is based on a specific optimization procedure, evaluation of which arises from several starting assumptions. The assumptions are connected with joint *pdf*s describing the closed-loop behavior. These joint *pdf*s are assumed to be operating on their domain i.e. on a set X^* of all values X. Considering the time for computation of control law and its digital realization, the *pdf*s are represented in discrete-time instants within appropriate finite discrete-time interval.

The design is provided for topical time instant or appropriate time interval respectively. For determination of admissible control strategy, the joint *pdf*s represent real and ideal closed-loop behavior:

- joint *pdf* that represents the real closed-loop behavior

$$\begin{aligned} f(X) &= f_N \equiv f(\mathbf{x}_{k+N}, u_{k+N-1}, \cdots, u_k, \mathbf{x}_k) \qquad (1) \\ &= \left\{ \prod_{j=k+1}^{k+N} f(\mathbf{x}_j|\mathbf{x}_{j-1}, u_{j-1})\, f(u_{j-1}, \mathbf{x}_{j-1}) \right\} f(\mathbf{x}_k) \end{aligned}$$

- joint *pdf* that represents the ideal closed-loop behavior

$$\begin{aligned} {}^I f(X) &= {}^I f_N \equiv {}^I f(\mathbf{x}_{k+N}, u_{k+N-1}, \cdots, u_k, \mathbf{x}_k) \qquad (2) \\ &= \left\{ \prod_{j=k+1}^{k+N} {}^I f(\mathbf{x}_j|\mathbf{x}_{j-1}, u_{j-1})\, {}^I f(u_{j-1}, \mathbf{x}_{j-1}) \right\} f(\mathbf{x}_k) \end{aligned}$$

These *pdf*s are considered to be defined for values in given time and their parameters to be valid within specific finite horizon N called control horizon. The label N represents the number of discrete time instants j from instant k within the horizon; i.e. $j = k+1, \cdots, k+N$; $u_{(\cdot)}$ are control actions.

Due to practical consequences, the *pdf*s arise from the assumption that succeeding system state $\mathbf{x}_j$ follows from previous system state $\mathbf{x}_{j-1}$ and system input u_{j-1} only. Thus, $\mathbf{x}_j$ is independent of past system states and system inputs. The assumptions are defined as follows:

$$f(\mathbf{x}_j|\mathbf{x}_{j-1}, u_{j-1}, \cdots, \mathbf{x}_0, u_0) = f(\mathbf{x}_j|\mathbf{x}_{j-1}, u_{j-1}) \qquad (3)$$

$$f(u_j|\mathbf{x}_j, u_{j-1}, \cdots, \mathbf{x}_0, u_0) = f(u_j|\mathbf{x}_j) \qquad (4)$$

$${}^I f(\mathbf{x}_j|\mathbf{x}_{j-1}, u_{j-1}, \cdots, \mathbf{x}_0, u_0) = {}^I f(\mathbf{x}_j|\mathbf{x}_{j-1}, u_{j-1}) \qquad (5)$$

$$^{I}f(u_j|\mathbf{x}_j,u_{j-1},\cdots,\mathbf{x}_0,u_0) \quad = \quad ^{I}f(u_j|\mathbf{x}_j) \tag{6}$$

where *pdf*s labeled by superscript ^{I} denote user requirements, i.e. user ideals. Thus, the *pdf*s (1) and (2), or *pdf*s (3) to (6) respectively, describe real and ideal behavior of individual parts of given closed-loop i.e. behavior of the system and controller; e.g. in instant $j = k+1$, the real and ideal system behavior is modeled by *pdf*s $f(\mathbf{x}_{k+1}|\mathbf{x}_k,u_k)$ and $^{I}f(\mathbf{x}_{k+1}|\mathbf{x}_k,u_k)$; and real and ideal controller behavior is modeled by *pdf*s $f(u_k|\mathbf{x}_k)$ and $^{I}f(u_k|\mathbf{x}_k)$, respectively.

2.2. Optimality criterion

To measure level of proximity of real and closed-loop behavior, the Kullback-Leibler divergence (*KL-divergence*) $\mathcal{D}(f\,||\,^{I}f)$ is used [3], [4]

$$\mathcal{D}(f\,||\,^{I}f) \equiv E\left\{\ln\frac{f(X)}{^{I}f(X)}\right\} = \int f(X)\ln\frac{f(X)}{^{I}f(X)}\,dX \tag{7}$$

In it, the pair of *pdf*s f and ^{I}f operate on their domains according to starting assumptions. From control point of view, the *KL-divergence* represents the loss function or optimality criterion. By its minimization, the optimal control law - optimal *pdf* $^{o}f(u_k|\mathbf{x}_k)$ of the *pdf* $f(u_k|\mathbf{x}_k)$ can be obtained as follows:

$$\{^{o}f(u_{j-1}|\mathbf{x}_{j-1})\}_{j=k+1}^{k+N} \in \underset{\{f(u_{j-1}|\mathbf{x}_{j-1})\}_{j=k+1}^{k+N}}{\arg\min} \mathcal{D}(f_N\,||\,^{I}f_N) \tag{8}$$

As indicated in (8), the task of design consists in minimization of *KL-divergence*. The following subsection outlines the minimization procedure, which leads to the optimal *pdf* of the controller and the optimal control law respectively.

2.3. Minimization procedure

This subsection presents minimization procedure briefly, the detail derivation is in [5]. Optimal *pdf* of the controller can be obtained using (8). From control theory point of view, considering the assumptions from subsection 2.1, the equation (8) can be interpreted as expression of specific dynamic programming procedure [1].

$$\min_{\{f(u_{j-1}|\mathbf{x}_{j-1})\}_{j=k+1}^{k+N}} \mathcal{D}(f_N\,||\,^{I}f_N) =$$

$$= \min_{\{f(u_{j-1}|\mathbf{x}_{j-1})\}_{j=k+1}^{k+N}} E\left\{\sum_{j=k+1}^{k+N} z_j\right\}$$

$$\cdots = \min_{\{f(u_k|\mathbf{x}_k)\}}\Big\{E(z_{k+1}) + \cdots$$

$$\min_{\{f(u_{k+N-2}|\mathbf{x}_{k+N-2})\}} \Bigg\{ E(z_{k+N-1})$$

$$+ \quad \min_{\{f(u_{k+N-1}|\mathbf{x}_{k+N-1})\}} \{E(z_{k+N})\}\Bigg\} \cdots \Bigg\} \tag{9}$$

where $z_j = \ln \frac{f_j(\mathbf{x}_j|\mathbf{x}_{j-1},u_{j-1})}{{}^If_j(\mathbf{x}_j|\mathbf{x}_{j-1},u_{j-1})}$ is j^{th} partial loss. This expression leads to the following *pdf* of optimal control:

$${}^of(u_k|\mathbf{x}_k) \quad = \quad \frac{1}{\gamma(\mathbf{x}_k)}\, {}^If(u_k|\mathbf{x}_k)\, e^{-\,\delta\,(u_k,\mathbf{x}_k)} \tag{10}$$

where $\delta(u_k,\mathbf{x}_k)$ and $\gamma(\mathbf{x}_k)$ are defined as follows

$$\delta(u_k,\mathbf{x}_k) \quad = \quad f(\mathbf{x}_{k+1}|u_k,\mathbf{x}_k) \ln \frac{f_j(\mathbf{x}_{k+1}|\mathbf{x}_k,u_k)}{{}^If_j(\mathbf{x}_{k+1}|\mathbf{x}_k,u_k)}\, d\mathbf{x}_{k+1} \tag{11}$$

$$\gamma(\mathbf{x}_k) \quad = \quad \int {}^If(u_k|\mathbf{x}_k)\, e^{-\,\delta(u_k,\mathbf{x}_k)} du_k \tag{12}$$

The definition of the parameters δ and γ arises from (9) leading to their recursive evaluation.

2.4. Real probabilistic model of controlled system

As formerly mentioned, the system behavior can be described by probability density function (*pdf*). Since the controlled system is influenced by different stochastic effects acting simultaneously in general conditions, let the system behavior has normally distributed character. This assumption follows from practical consequences, that sum of independent stochastic quantities arbitrarily distributed has approximately normal distribution [7].

Considering this practical assumption, the *pdf* denoted by $f(y)$ describing controlled system is defined as follows

$$\mathcal{N}(\mu_y, r_y):\; f(y) = \frac{1}{\sqrt{2\pi r_y}} e^{-\frac{(y-\mu_y)^2}{2r_y}} \tag{13}$$

where μ_y represents mean value, i.e. expected value of system output y ($\mu_y = E\{y\}$), $\sigma_y^2 = r_y$ denotes a dispersion (variance; $r_y = E\{(y-\mu_y)^2\}$).

In control design, these parameters are considered to be continuous in values and discrete in time. Their continuity follows from the character of the system. The discreteness in time is given by discrete realization of control, which naturally respects the time for its computation.

Internal structure of parameters mentioned above can be specified in more detail either as ARX model or as state-space model – the both models with normally distributed noise.

• ARX model [6] is defined as:

$$y_k = \underbrace{\sum_{i=1}^{n} b_i u_{k-i} - \sum_{i=1}^{n} a_i y_{k-i}}_{\mu_y} + e_{y_k},\ e_{y_k} \sim \mathcal{N}(0, r_y) \tag{14}$$

where n is an order and e_{y_k} is a model noise, which has a dispersion r_y.

• State-space model is defined as:

$$\mathbf{x}_{k+1} = \underbrace{\mathbf{A}\mathbf{x}_k + \mathbf{B}u_k}_{\mu_\mathbf{x}} + \mathbf{e}_{\mathbf{x}_k},\ \mathbf{e}_{\mathbf{x}_k} \sim \mathcal{N}(\mathbf{0}, \mathbf{R}) \tag{15}$$

$$y_k = \underbrace{\mathbf{C}\mathbf{x}_k}_{\mu_y} + \widetilde{e}_{y_k},\ \widetilde{e}_{y_k} \sim \mathcal{N}(0, \widetilde{r}_y) \tag{16}$$

Equations (15) and (16) represent general state-space notation, in which the state $\mathbf{x}_k$ may be available or not; e.g. it has not a physical interpretation and for control purposes it has to be estimated. To avoid this drawback, it is suitable to use so-called pseudo state-space model [2], which is a direct reinterpretation of ARX model (14).

Such reinterpretation means state-space model with nonminimal state, which contains only delayed values of inputs and outputs. The values of system inputs and outputs are known from measurement, therefore they need not be estimated. In spite of increase of elements of the state, the usage stays the same as in case of standard minimal state-space model.

An internal structure of the reinterpretation is defined as follows:

$$\mathbf{x}_k = \begin{bmatrix} u_{k-1} \\ \vdots \\ u_{k-n+1} \\ y_k \\ \vdots \\ y_{k-n+1} \end{bmatrix}, \mathbf{A} = \begin{bmatrix} 0 & \cdots & 0 & 0 & \cdots & 0 \\ 1 & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \ddots & 0 & 0 & \cdots & 0 \\ b_2 & \cdots & b_n & -a_1 & \cdots & -a_n \\ 0 & \cdots & 0 & 1 & \cdots & 0 \\ 0 & \ddots & 0 & 0 & \ddots & 0 \end{bmatrix}, \tag{17}$$

$$\mathbf{B} = [1,\ \cdots,\ 0,\ b_1,\ \cdots,\ 0]^T, \tag{18}$$

$$\mathbf{C} = [0,\ \cdots,\ 0,\ 1,\ 0,\ \cdots,\ 0],\ \widetilde{r}_y = 0 \tag{19}$$

Relation of the pseudo-state space model to ARX model is obvious from the following corollary:

$$y_k = \underbrace{\mathbf{C}\mathbf{A}\mathbf{x}_{k-1} + \mathbf{C}\mathbf{B}u_{k-1}}_{\mu_y} + \underbrace{\mathbf{C}\mathbf{e}_{\mathbf{x}_k}}_{e_{y_k}},\ e_{y_k} \sim \mathcal{N}(0, r_y) \tag{20}$$

Models (14); or (15) and (16); or (15) to (19) are suitable models for implementation of fully probabilistic control design.

3. Fully probabilistic control parameters tuning

This section introduces the problem of parameters in probabilistic controller (Section 3.1) and introduces the method of on-line tuning of these parameters based on data in closed loop (Section 3.2).

3.1. Control law and its parameters

General expression (10) is the *pdf* (Section 2), that represent the control law. To compute real parameters of this *pdf*, individual *pdf*s from assumptions (3), (5) and (6) have to be defined. These *pdf*s represent both real and ideal behavior of closed-loop (Fig. 1). Assuming model given by (15) to (19), i.e. finite memory and known parameters of appropriate distributions, then *pdf*s are defined as follows:

• *pdf* of the real controlled system output behavior

$$\mathcal{N}(\mu_y, r_y) : \ f(y_{k+1}|u_k, \mathbf{x}_k) = \\ = \frac{1}{\sqrt{2\pi r_y}} e^{-\frac{1}{2}(y_{k+1}-\mu_y)^T r_y^{-1}(y_{k+1}-\mu_y)} \tag{21}$$

• *pdf* of the ideal controlled system output behavior

$$\mathcal{N}({}^I\mu_y, {}^I r_y) : \ {}^I f(y_{k+1}|u_k, \mathbf{x}_k) = \\ = \frac{1}{\sqrt{2\pi {}^I r_y}} e^{-\frac{1}{2}(y_{k+1}-{}^I\mu_y)^T\, {}^I r_y^{-1}(y_{k+1}-{}^I\mu_y)} \tag{22}$$

where ideal ${}^I\mu_y$ is the desired output value w_{k+1};

• *pdf* of the ideal controlled system input behavior

$$\mathcal{N}({}^I\mu_u, {}^I r_u) : \ {}^I f(u_k|\mathbf{x}_k) = \\ = \frac{1}{\sqrt{2\pi {}^I r_u}} e^{-\frac{1}{2}(u_k-{}^I\mu_u)^T\, {}^I r_u^{-1}(u_k-{}^I\mu_u)} \tag{23}$$

where ${}^I\mu_u$ is assumed to be the previous action u_{k-1} and the dispersion ${}^I r_u$ can be viewed as a tuning parameter of the controller. For *pdf*s defined like that, the computation of *pdf* (10) leads to the following expressions:

$${}^o f(u_k|\mathbf{x}_k, u_{k-1}) = \\ = \frac{1}{\sqrt{2\pi {}^o r_u}} e^{-\frac{1}{2}\, {}^o r_u^{-1}\{u_k + {}^o r_u\, b\}^2} \tag{24}$$

$$= \frac{1}{\sqrt{2\pi {}^o r_u}} e^{-\frac{1}{2}\, {}^o r_u^{-1}\{u_k + \mathbf{k_x}\mathbf{x}_k - \sum_{j=k+1}^{k+N+1} k_{w_j} w_j - k_u u_{k-1}\}^2}$$

$$^{o}u_k \quad = \quad -\mathbf{k_x}\mathbf{x}_k + \sum_{j=k+1}^{k+N+1} k_{w_j} w_j + k_u u_{k-1} \tag{25}$$

where $^{o}u_k$ is the optimal control law with horizon N.

3.2. Principle of on-line tuning of control parameters

Dispersions $^{I}r_u$ and $^{I}r_y$ are very important for control, because they are determining factors for the gains $\mathbf{k_x}$, k_{w_j} and k_u in equation (24). In non-probabilistic control design (e.g. LQ control), their reciprocal values represent input penalization factor ($q_u = 1/^{I}r_u$) and output penalization factor ($q_y = 1/^{I}r_y$), which together adjust individual terms in loss-function. Their choice is based on experience or on experimental tuning. In fully probabilistic control design, interpretation of these quantities is more straightforward. The equations (22) and (23) imply that $^{I}r_u$ and $^{I}r_y$ represent noise dispersions for ideal distribution of the system and controller.

The algorithm proposed in this paper is intended for systems (e.g. mechatronic one), where the system model together with the noise can change substantially, possibly due to additional interference, that may occur randomly during the control. Inadequate choice of input and output penalizations or $^{I}r_u$ with $^{I}r_y$ respectively, can cause serious problems for controller. Unexpected system noise increase may force the controller to generate inputs, that are suddenly out of any reasonable physical range of the device or at least represent unreal magnitude change.

This may lead to serious device failures, e.g. system actuators (servo motors etc.) might not be able to achieve designed control interventions or may be damaged by them. It will be documented by figures in Section 4. In such cases, it would usually be acceptable to decrease control quality in order to achieve at least some reasonable value. Fully probabilistic control interpretation of penalization as dispersions can achieve it via on-line control tuning algorithm presented below.

3.3. Real implementation of on-line parameter tuning

On-line control tuning is based on the idea of changing $^{I}r_y$ so that its amplitude is proportional to the output dispersion r_y or practically to its estimate $\hat{r}_{y_i} = e_{y_i}\, e_{y_i}^T$, which is calculated from current data y_i. The effect is that during periods of increased output noise, output ideal is set to be less strict. It causes the output to be tracked less closely. This allows the input to stay in its reasonable constraints. However, current output dispersion can change very quickly causing big changes in $^{I}r_y$. In order to avoid this, $\hat{r}_{y_i}$ has to be filtrated. As a suitable filter, exponential forgetting is used. It can be defined as follows:

$$\widetilde{r}_{y_1} \quad = \quad (1-\lambda)\hat{r}_{y_1} \tag{26}$$

$$\widetilde{r}_{y_i} \quad = \quad \lambda\widetilde{r}_{y_{i-1}} + (1-\lambda)\hat{r}_{y_i}, \;\; i = 2, \cdots, k \tag{27}$$

where λ is a forgetting factor influencing quickness of weight decrease of individual contributions $\hat{r}_{y_i}$. The equations (26) and (27) can form one general expression:

$$\widetilde{r}_{y_k} = (1-\lambda)\sum_{i=1}^{k}\lambda^{k-i}\hat{r}_{y_i} \tag{28}$$

In order to find reasonable value for parameter λ, the suitable number of time instants ℓ has to be defined in correspondence to the character of control process. During these ℓ time instants, the contribution of $\hat{r}_{y_k}$ to $\widetilde{r}_{y_i}$ drops to the given level. Standard choice is to select the number of instants (denoted by $\ell_{1/2}$) that cause dropping the contribution of $\hat{r}_{y_k}$ to one half of the original value. It implies that $\ell_{1/2}$ satisfies the equation:

$$\lambda^{\ell_{1/2}}(1-\lambda)\hat{r}_{y_k} = \frac{1}{2}(1-\lambda)\hat{r}_{y_k} \tag{29}$$

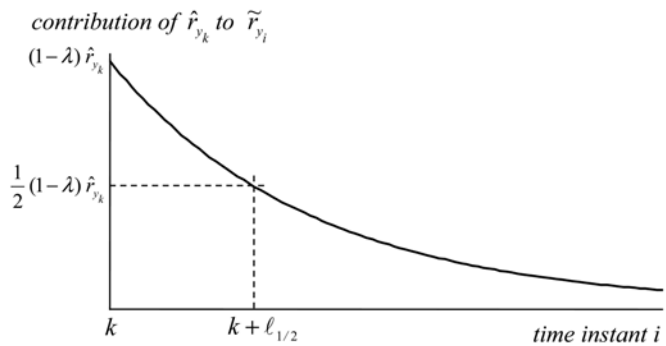

Figure 2. Trend of contribution of $\hat{r}_{y_k}$ to $\widetilde{r}_{y_i}$.

See Fig. 2 for illustration of this effect. Producing 'half-time' $\ell_{1/2}$ is user-friendly way to find a suitable value for constant λ, because user can easily imagine what is the time needed for a contribution of $\hat{r}_{y_k}$ to drop to one half. Consequently, suitable λ can be found like this:

$$\lambda = \left(\frac{1}{2}\right)^{\frac{1}{\ell_{1/2}}} \tag{30}$$

where $\ell_{1/2}$ is provided by the user.

4. Probabilistically tuned LQ control examples

This section demonstrates the presented fully probabilistic control design including the on-line tuning. Two representative examples are described here - one simulated and one real-time experiment. The aim is to illustrate improvements in control that follows from consequences of Section 3.2.

4.1. Simulated experiment - system given by ARX model

The simulated experiment considering structure in Fig. 1 was provided with simple system represented by ARX model $(n = 2, (14))$ forming the following transfer function:

$$G(z^{-1}) = \frac{B(z^{-1})}{A(z^{-1})} = \frac{0.0047z^{-1} + 0.0044z^{-2}}{1 - 1.8097z^{-1} + 0.8187z^{-2}}$$

$$\text{with } T_s = 0.1s \qquad (31)$$

The results are registered in Fig. 3 to Fig. 5.

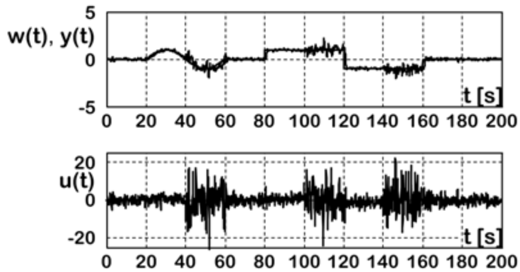

Figure 3. Simulated experiment: standard LQ control $q_y = 1000$ (desired and real system output $w(t)$ and $y(t)$; input $u(t)$; penalization $q_y(t)$).

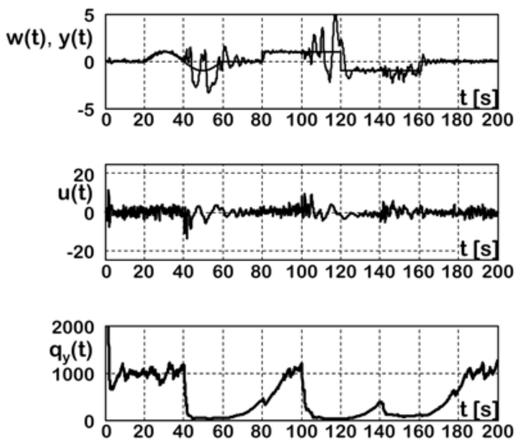

Figure 4. Simulated experiment: control designed by fully probabilstic approach including on-line tuning (desired and real system output $w(t)$ and $y(t)$; input $u(t)$; penalization $q_y(t)$).

The figures show a comparison of standard LQ controller and controller based on fully probabilistic design (Section 2) with on-line tuning (Section 3.2).

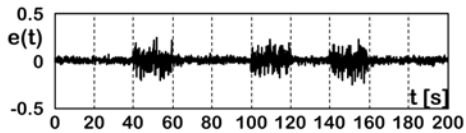

Figure 5. Simulated experiment: noise $e(t)$.

The noise $e \sim \mathcal{N}(\mu_e = 0,\ r_e = 0.02^2)$ simulating disturbance was artificially $4\times$ increased in intervals $\langle 40s; 60s\rangle$, $\langle 100s; 120s\rangle$, and $\langle 140s; 160s\rangle$. In LQ control, input $u(t)$ is alternating. In comparison, the probabilistic design with tuning maintains relatively stable $u(t)$ at the expense of control quality.

4.2. Real-life experiment - Application to gearbox system

The real system used for experiment consists of three wheels, which are mutually connected by two elastic belts (see Fig. 7(e)). Position of the wheel 1 is controlled by servo-motor, and the position of the wheel 3 is measured.

The experiment was arranged in analogical way as in previous simulated case. The system is controlled by adaptive controller with ARX model (14) with $n = 6$ (each wheel ≈ 2 orders). During control process, the discrepancy between model estimated and the real system occurs, and this causes input to change dramatically. This phenomenon can be suppressed by algorithm of adaptive controller tuning proposed in this paper (see Fig. 7(d)).

Fig. 6 to Fig. 7 demonstrates the controlled process behavior. Sub-figures (a), (b), and (c) show the control with different but constant output penalization (q_y). In all cases of constant q_y, the input magnitude changes rapidly. Furthermore, in case (c) the controller have not stabilized at all. With adaptive tuning proposed in this paper the changes in input are reasonably small, moreover, the output matches desired value much better. On top of that, the the system is much less susceptible to destabilizing as in Fig. 6(c).

5. Conclusion

The advanced on-line tuning was introduced in this paper. This is made possible by usage of the principles and practical aspects of fully probabilistic control design. This approach of design forms sound physical interpretation for tunable LQ controller parameters.

The design with tuning was applied and demonstrated both on simulated and real experiments. The representative results are documented and discussed in the section on application examples.

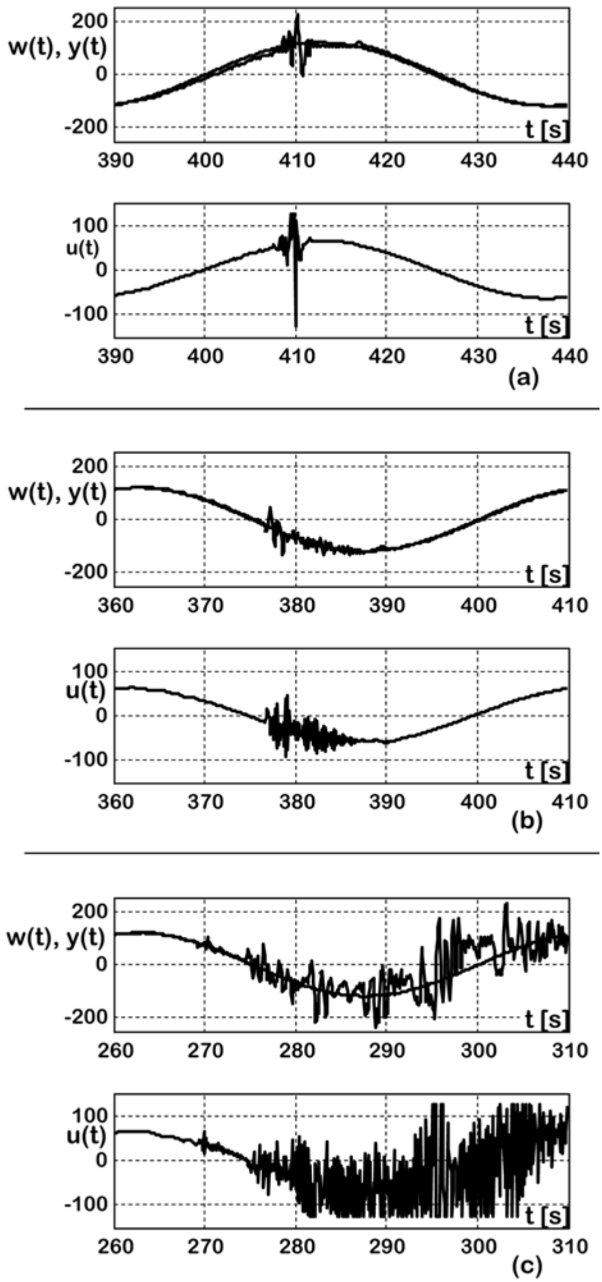

Figure 6. Real experiment: comparison of standard LQ control for different penalizations settings (a) $q_y = 1$, (b) $q_y = 100$ and (c) $q_y = 200$, respectively.

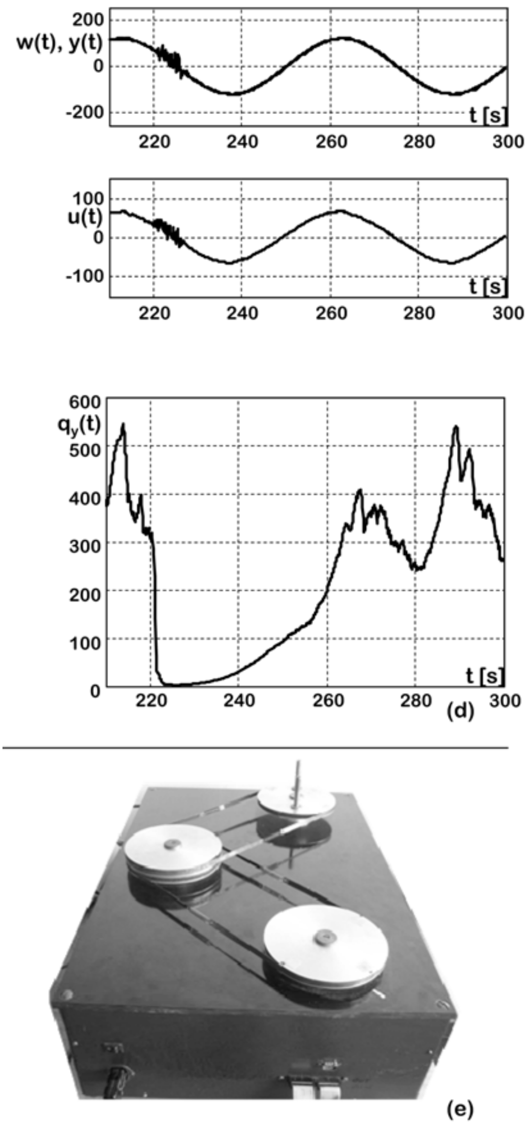

Figure 7. Real experiment: (d) control generated by probabilistic design with tuning (desired and real system output $w(t)$ and $y(t)$; input $u(t)$; penalization $q_y(t)$); (e) gear system.

References

[1] D. BERTSEKAS: Dynamic programming and optimal control, second ed. Athena Scientific, Nashua, US, New York, 2001.

[2] V. BOBÁL, J. BÖHM, J. FESSL and J. MACHÁČEK: Digital self-tuning controllers. Springer, 2005.

[3] M. KÁRNÝ: Towards fully probabilistic control design. *Automatica*, **32**(12), (1996), 1719-1722.

[4] M. KÁRNÝ, J. BÖHM, T. GUY, L. JIRSA, I. NAGY, P. NEDOMA and L. TESAŘ: Optimized bayesian dynamic advising, Theory and algorithms. Springer, 2006.

[5] M. KÁRNÝ and T. V. GUY: Fully probabilistic control design. *System & Control Letters*, **55** (2005), 259-265.

[6] V. PETERKA: *Trends and progress in system identification*, chapter: Bayesian approach to system identification. Pergamon Press, Oxford, 1981.

[7] H. TIJMS: Understanding probability: Chance rules in everyday life. Cambridge University Press, 2007.

[8] L.-W. TSAI: Robot analysis: The mechanics of serial and parallel manipulators. John Wiley & Sons, Inc., New York, 1999.

[9] E. WERNHOLT and S. GUNNARSSON: Nonlinear identification of a physically parameterized robot model. In *Proceedings of the 14th IFAC Symposium on System Identification*, Newcastle, Australia, (2006), 143-148.