SELF FINE TUNING OF LQ CONTROL WITH FIXED MODEL

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Abstract: Setup of suitable weight control parameters and their tuning with respect to topical controlled system behavior is a principal issue of every control design. This contribution proposes the efficient way for setup and tuning of the control parameters of LQ controllers automatically by evaluation of a controlled system response in comparison with a mathematical model following from a physical substance of that system. The controlled systems are considered as stochastic, which corresponds to a stochastic nature of real-life applications. In this paper, there will be explained general assumptions and conditions for the implementation of the proposed way. The theoretical results will be discussed and illustrated by several examples.

Keywords: LQ control, model-based control, control tuning, stochastic systems

1 INTRODUCTION

Setup and further parameters' tuning of used controller is a principal issue of every control design [Bobál et al., 2005]. Parameters of the controller (simply control parameters) balance various control aims assigned by user, determine the quality of a control process and character of a controller response. In engineering practice, there exist a lot of different setup control parameters' methods [Novák et al., 2003], the basis of which is dependent on presence of expert knowledge or some expert procedure. In almost all cases, they are only off-line one-off methods without a consideration of topical changes appearing usually in control process. Due to their time consumption, they are also not suitable methods for on-line continuous fine controller tuning and retuning.

If the focus is put on the model-based controllers – LQ controllers or different predictive controllers, thus they can respect unexpected changes by means of on-line identification methods. These methods provide an adaptation of the model parameters in the controller only, but they do not have any direct affect to the control parameters. Due to longer duration of the model parameter adaptation, the controller uses temporarily more or less unrealistic model, but still with the same weight as at presence of right realistic model. The same situation occurs in the case, if the fixed model, i.e. model with fixed model parameters based e.g. on mathematical-physical analysis, is considered for control design in the used controller. All of this leads to unreasonable and unsafe generation of control actions, which can damage the controller and the controlled system itself.

The chance events, unexpected changes may be caused e.g. by internal changes of the controlled system, increasing of the noise in the measurement or changing working conditions following from system surroundings. In most cases, they are sudden events requiring appropriate fast response of the controller consisting in its retuning or fine tuning.

This paper proposes systematic and efficient way for setup and on-line tuning of the control parameters of well known LQ controllers automatically. That way is based on evaluation of a controlled system response in comparison with a system model and the meaning relation to the control parameters. The controlled systems are considered here as stochastic systems corresponding to stochastic nature of real-life applications. The explanation will discuss also multidimensional cases, in which the cross relations of individual components or let as say signal channels occur and cannot be omitted. In those cases, the setup or tuning of the parameters with cross elements is not a simple task and it is not usually provided.

In the sections 2 and 3, there are a model definition and a discussion of quadratic criterion of LQ control including the function of its control parameters. The section 4 deals with control parameters' setup and on-line tuning and in the following sections 5 and 6, the results are demonstrated.

2 MATHEMATICAL MODEL DEFINITION

As formerly mentioned, the controlled system is considered to be stochastic in general. Since the system is influenced by different stochastic effects acting simultaneously in general conditions, let its behavior be considered to have normally distributed character and then the model describing the system has normally distributed output signals as well. This assumption follows from practical consequences, that the sum of independent stochastic quantities arbitrarily distributed has approximately normal distribution. Besides this practical assumption, considering digital LQ control, the discrete model is needed. Let us assume that the initial model in the form of differential equations, which follow from physical substance of controlled system, may be discretized to the form of difference equations. Let this way be indicated generally for multi-inputs multi-outputs systems (MIMO systems) via following expressions.

Firstly, the model of the MIMO system arising from mathematical-physical analysis is usually given by a set of differential equations

$$\mathbf{y}^{(n)} = \mathbf{f}(\mathbf{y}, \dot{\mathbf{y}}, \cdots, \mathbf{y}^{(n-1)}) + \mathbf{g}(\mathbf{y}, \dot{\mathbf{y}}, \cdots, \mathbf{y}^{(n-1)}, \mathbf{u}, \dot{\mathbf{u}}, \cdots, \mathbf{u}^{(n-1)}) \tag{1}$$

where $\mathbf{y}^{(\cdot)}$, $\mathbf{u}^{(\cdot)}$ are relevant derivatives of outputs and inputs and $\mathbf{f}(\cdot)$, $\mathbf{g}(\cdot)$ are system dynamics and input functions of appropriate dimensions given by a system order n and dimensions of outputs n_y and inputs n_u. For LQ control design purposes, multistep control strategy, the set of equations (1) is transformed to the set of linear differential equations

$$\mathbf{y}^{(n)} = \sum_{i=0}^{n-1} \mathbf{Bc}_i \, \mathbf{u}^{(i)} - \sum_{i=0}^{n-1} \mathbf{Ac}_i \, \mathbf{y}^{(i)} \tag{2}$$

and the parameters $\mathbf{Ac}_i$, $\mathbf{Bc}_i$ of which are consecutively discretized for a given sampling period Ts to the form of difference equations

$$\mathbf{y}(k) = \sum_{i=1}^{n} \mathbf{B}_i \, \mathbf{u}(k-i) - \sum_{i=1}^{n} \mathbf{A}_i \, \mathbf{y}(k-i) \tag{3}$$

The all equations above represent only deterministic relations of the system given by mathematical-physical system substance. However, the real systems contain number of stochastic components, which are usually involved into one noise term $\mathbf{e}_y$ as follows

$$\mathbf{y}(k) = \sum_{i=1}^{n} \mathbf{B}_i \, \mathbf{u}(k-i) - \sum_{i=1}^{n} \mathbf{A}_i \, \mathbf{y}(k-i) + \mathbf{e}_y(k) \tag{4}$$

The term $\mathbf{e}_y$ represents a stochastic uncertainty, which is not modeled, but it can be used in transferred interpretation for a quality evaluation of the model, i.e.

$$\mathbf{e}_y(k) = \mathbf{y}(k) - \sum_{i=1}^{n} \mathbf{B}_i \, \mathbf{u}(k-i) + \sum_{i=1}^{n} \mathbf{A}_i \, \mathbf{y}(k-i) = \mathbf{y}(k) - \boldsymbol{\mu}_y(k) \tag{5}$$

where term $\boldsymbol{\mu}_y(k) = \mathrm{E}\{\mathbf{y}(k)\}$ represents the deterministic relations, in stochastic point of view, expected values of the system outputs. Using this term, other descriptive statistics (e.g. error $\mathbf{e}_y = \{(\mathbf{y} - \boldsymbol{\mu}_y)\}$, dispersion $r_y = \mathrm{E}\{(y - \mu_y)^2\}$ or covariance matrix $\mathbf{C}_y = \mathrm{E}\{(\mathbf{y} - \boldsymbol{\mu}_y)^T (\mathbf{y} - \boldsymbol{\mu}_y)\}$) can be evaluated in the relation to further characteristics e.g. precision matrices, which will be applied to control tuning.

3 MODEL REORGANIZATION ADAPTED FOR CONTROL DESIGN

To cope with MIMO systems and to compose general simple algorithm of the controller, the model in the input-output autoregressive form (3) is not directly applicable, especially for high-order MIMO systems. Nowadays, more suitable models based on state-space are popular due to their transparency in the notation. However, they are generally limited by required knowledge of the state. If the state is directly measurable due to physical system substance, the state-space model is quite suitable. When the measurement is not possible and the state is unavailable, then it is helpful to consider so called state-space forms with non-minimal state [Peterka, 1981]. Even though, according to label 'non-minimal state', the dimensionality of the design matrices increased a little bit, so it is not a complication for control design based on LQ control strategy. Note, LQ design procedure works only with simple matrixes and multistep feature is realized through recursive algorithm against e.g. predictive control, where the type of equations of predictions increases not only by pure system dimension, but by prediction horizon multiplied by mentioned system dimension.

To outline the model reorganization based on state-space model with non-minimal state, let firstly the autoregressive model be renewed: assume, that the parameters are known from mathematical-physical analysis and expected values (mean) of stochastic components are zeros and that the controller reacts immediately against the controlled system, in model of which at least one time period delay is assumed as already indicated

$$\mathbf{y}(k) = \sum_{i=1}^{n} \mathbf{B}_i\, \mathbf{u}(k-i) - \sum_{i=1}^{n} \mathbf{A}_i\, \mathbf{y}(k-i) \tag{6}$$

let this model be transformed to the form of a predictor (for clearness partially itemized)

$$\mathbf{y}(k+1) = \mathbf{B}_2\, \mathbf{u}(k-1) \cdots + \mathbf{B}_n\, \mathbf{u}(k-n+1) - \mathbf{A}_1\, \mathbf{y}(k) - \mathbf{A}_2\, \mathbf{y}(k-1) \cdots - \mathbf{A}_n\, \mathbf{y}(k-n+1) + \mathbf{B}_1\, \mathbf{u}(k) \tag{7}$$

Then the suitable state-space form with non-minimal state corresponding to this model is given as

$$\begin{bmatrix} \mathbf{u}(k) \\ \vdots \\ \mathbf{u}(k-n+2) \\ \mathbf{y}(k+1) \\ \mathbf{y}(k) \\ \vdots \\ \mathbf{y}(k-n+2) \end{bmatrix} = \begin{bmatrix} \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & \cdots & \cdots & \mathbf{0} \\ \ddots & \ddots & \vdots & \vdots & \mathbf{0} & \mathbf{0} & \vdots \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} & \cdots & \cdots & \mathbf{0} \\ \mathbf{B}_2 & \cdots & \mathbf{B}_n & -\mathbf{A}_1 & -\mathbf{A}_2 & \cdots & -\mathbf{A}_n \\ \mathbf{0} & \cdots & \mathbf{0} & \mathbf{1} & \mathbf{0} & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots & \ddots & \ddots & \ddots & \vdots \\ \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{u}(k-1) \\ \vdots \\ \mathbf{u}(k-n+1) \\ \mathbf{y}(k) \\ \mathbf{y}(k-1) \\ \vdots \\ \mathbf{y}(k-n+1) \end{bmatrix} + \begin{bmatrix} \mathbf{1} \\ \vdots \\ \mathbf{0} \\ \mathbf{B}_1 \\ \mathbf{0} \\ \vdots \\ \mathbf{0} \end{bmatrix} \mathbf{u}(k) \tag{8}$$

i.e.

$$\mathbf{x}(k+1) = \mathbf{A}\, \mathbf{x}(k) + \mathbf{B}\, \mathbf{u}(k)$$

The state-space form is complemented by output equation

$$\mathbf{y}(k) = \begin{bmatrix} \mathbf{0} & \cdots & \mathbf{0} & \mathbf{1} & \mathbf{0} & \cdots & \mathbf{0} \end{bmatrix} \mathbf{x}(k)$$

i.e.

$$\mathbf{y}(k) = \mathbf{C}\, \mathbf{x}(k) \tag{9}$$

This state-space form is already well suited for control design based on LQ control strategy.

4 LQ CONTROL CRITERION AND PHYSICAL MEANING OF ITS PARAMETERS

LQ control represents multistep control strategy managing various MIMO systems. The basis of the design is a minimization of a quadratic criterion using dynamic programming. The general formulation of the criterion can be expressed as follows [Bobál et al., 2005]

$$J_k = E\Big[\sum_{j=k+1}^{k+N}\{(\mathbf{y}_{(j)}-\mathbf{w}_{(j)})^T\,\mathbf{Q}_\mathbf{y}^T\,\mathbf{Q}_\mathbf{y}\,(\mathbf{y}_{(j)}-\mathbf{w}_{(j)})+(\mathbf{u}_{(j-1)}-\mathbf{u}_\mathbf{r})^T\,\mathbf{Q}_\mathbf{u}^T\,\mathbf{Q}_\mathbf{u}\,(\mathbf{u}_{(j-1)}-\mathbf{u}_\mathbf{r})\}\Big] \quad (10)$$

minimization of which is performed by recursive dynamic programming procedure [Kárný, 96',2005]:

$$\min_{\mathbf{u}_k} J_k = \min_{\mathbf{u}_k}\{\,E\,[\,\|\,(\mathbf{y}_{(k+1)}-\mathbf{w}_{(k+1)})\,\mathbf{Q}_\mathbf{y}\,\|^2+\|\,(\mathbf{u}_{(k)}-\mathbf{u}_\mathbf{r})\,\mathbf{Q}_\mathbf{u}\,\|^2\,]+S_{k+1}^0\,\}$$

$$S_{k+1}^0=\min_{\mathbf{u}_{k+1}} J_{k+1} = \min_{\mathbf{u}_{k+1}}\{\,E\,[\,\|\,(\mathbf{y}_{(k+2)}-\mathbf{w}_{(k+2)})\,\mathbf{Q}_\mathbf{y}\,\|^2+\|\,(\mathbf{u}_{(k+1)}-\mathbf{u}_\mathbf{r})\,\mathbf{Q}_\mathbf{u}\,\|^2\,]+S_{k+2}^0\,\}$$

$$\vdots \qquad\qquad \vdots$$

$$S_{k+N-1}^0=\min_{\mathbf{u}_{k+N-1}} J_N = \min_{\mathbf{u}_{k+N-1}}\{\,E\,[\,\|(\mathbf{y}_{(k+N)}-\mathbf{w}_{(k+N)})\,\mathbf{Q}_\mathbf{y}\,\|^2+\|(\mathbf{u}_{(k+N-1)}-\mathbf{u}_\mathbf{r})\,\mathbf{Q}_\mathbf{u}\,\|^2\,]+S_{k+N}^0\}$$

(11)

where $\mathbf{w}(\cdot)$ is a vector of user set points, $\mathbf{u}_\mathbf{r}$ is a reference input signal or other control input, N is a prediction horizon, E is an operator of the expectation or mean and the terms $S_{k+i}^0,\ i=1,2,\cdots,N$ represent running losses of the criterion J_k.

Furthermore, in the criterion, there are two additional parameters $\mathbf{Q}_\mathbf{y}$ and $\mathbf{Q}_\mathbf{u}$ denoting weighting, control parameters or so called penalizations. As mentioned, they balance various control aims assigned by user, determine the quality of control process and character of controller response. Thus more precisely, penalization matrix $\mathbf{Q}_\mathbf{y}$ serves weighting of differences among expected system outputs and user set points, the higher values of which push the controller to generate control actions (usually energy- demanding) leading to more precise meeting the user set points (leading to higher precision) and vice versa. On the other hand, penalization matrix $\mathbf{Q}_\mathbf{u}$ weighs quantity, range or distribution of input energy, the lower values lead to higher, more independent control actions against the higher values of $\mathbf{Q}_\mathbf{u}$, which lead to zero control.

The elements of the control parameters or matrix parameters – penalizations themselves are usually specifically selected and being constant for whole run of control process; predominantly only elements on diagonals are non-trivial non-zero values. Furthermore, the control parameters only balance the criterion terms and they are mutually related. For their selection, their ratios are also important and significant. Due to the parameter physical interpretation described above, their internal values may be associated in logical way to the standard statistics introduced at the end of the section 2, specifically the elements of penalty matrix $\mathbf{Q}_\mathbf{y}$ correspond to the elements of precision matrix, inverse of which can be derived from covariance matrix of the prediction errors among the real system outputs versus their expected values; i.e. $\mathbf{e}_\mathbf{y} = \{(\mathbf{y} - \boldsymbol{\mu}_\mathbf{y})\}$.

The minimization procedure indicated in (11) forms following optimal control law or optimal control action vector implicitly including discussed control parameters:

$$^{o}\mathbf{u}_{(k)} = -\,\mathbf{k}_x\,\mathbf{x}_{(k)} + \sum_{j=k+1}^{k+N}\mathbf{k}_{w(j)}\,\mathbf{w}_{(j)}+\mathbf{k}_u\,\mathbf{u}_r \quad (12)$$

5 CONTROL PARAMETERS' SETUP AND ON-LINE TUNING

As was already mentioned and described in the previous section, LQ control has two main control parameters $\mathbf{Q}_\mathbf{y}$ and $\mathbf{Q}_\mathbf{u}$ called penalizations. Their inversions closely relate to dispersions r_u and r_y for single input single output systems (SISO systems) [Belda, 2009] or covariance matrices $\mathbf{C}_\mathbf{y}$ and $\mathbf{C}_\mathbf{u}$ for MIMO systems. They determine internal factors in the gains $\mathbf{k}_x$, $\mathbf{k}_{w(j)}$ and $\mathbf{k}_u$ in (12). Their selection is usually based on experience or on experimental tuning. However, considering the relation to the quality of the model, the selection is possible to be done more straightforwardly and can be provided on-line during control process. When the general MIMO system is taken into account, then covariance matrix $\mathbf{C}_\mathbf{y}$, in fact its inversion, represents precision matrix of the system model including cross relations among individual output signals and this matrix is proportional to output penalization $\mathbf{Q}_\mathbf{y}$. The matrix $\mathbf{C}_\mathbf{u}$ has similar interpretation, nevertheless for proper interpretation, note that in the ideal case, it, being diagonal, determines required (expected) square standard deviations of individual input (control) signals or in real case, the covariance matrix is confronted with additional measurement of really realized control actions, if it is possible.

This proposed idea is suitable for systems (e.g. mechatronic one), where the system model together with the noise can change substantially, possibly due to additional interference, that may occur randomly during the control. Inadequate choice of input and output penalizations can cause serious problems. Unexpected system noise increase may force the controller to generate inputs that are suddenly out of any reasonable physical range of the controlled system or at least represent unreal magnitude change. This may cause serious device failures, e.g. system actuators (servo motors etc.) might not be able to achieve designed control interventions or may be damaged. In such cases, where the model quality is low, it would usually be acceptable to decrease control quality in order to achieve at least some reasonable values. The reduction of control quality, i.e. smaller reflection of difference between real measured output and its appropriate set point, may be achieved just by adequate immediate on-line proposed control parameters' tuning.

The actual tuning is realized by on-line evaluation of the model quality, on the basis of which output penalization $\mathbf{Q}_\mathbf{y}$ is changed. Its elements are proportional to the output covariance matrix $\mathbf{C}_\mathbf{y}$ or practically to its estimate $\hat{\mathbf{C}}_{\mathbf{y}(i)} = \mathbf{e}_{\mathbf{y}(i)}\,\mathbf{e}^T_{\mathbf{y}(i)}$, which is calculated from current data $\mathbf{y}_{(i)}$. The effect is that during periods of increased output noise, output set point is set to be less strict. It causes the output to be tracked less closely. This allows the input to stay in its reasonable ranges. However, current output covariance matrix can change very quickly causing big changes of $\mathbf{Q}_\mathbf{y}$. In order to avoid this, $\hat{\mathbf{C}}_{\mathbf{y}(i)}$ has to be filtrated. As a suitable filter, exponential forgetting is proved as follows

$$\tilde{\mathbf{C}}_{\mathbf{y}(1)} = (1-\lambda)\,\hat{\mathbf{C}}_{\mathbf{y}(1)} \tag{13}$$

$$\tilde{\mathbf{C}}_{\mathbf{y}(i)} = \lambda\,\tilde{\mathbf{C}}_{\mathbf{y}(i-1)} + (1-\lambda)\,\hat{\mathbf{C}}_{\mathbf{y}(i)}, \quad i = 2,\cdots,k \;\;\Rightarrow\;\; \tilde{\mathbf{C}}_{\mathbf{y}(k)} = (1-\lambda)\sum_{i=1}^{k}\lambda^{k-i}\,\hat{\mathbf{C}}_{\mathbf{y}(i)} \tag{14}$$

where λ is a forgetting factor influencing quickness of weight decrease of individual contributions $\hat{\mathbf{C}}_{\mathbf{y}(i)}$. In order to find reasonable value for parameter λ, the suitable number of time instants ℓ has to be defined in correspondence to the character of control process. During these ℓ time instants, the contribution of $\hat{\mathbf{C}}_{\mathbf{y}(i)}$ to $\tilde{\mathbf{C}}_{\mathbf{y}(i)}$ drops to the given level. Standard choice is to select the number of instants (denoted by $\ell_{1/2}$) that cause dropping the contribution of $\hat{\mathbf{C}}_{\mathbf{y}(i)}$ to one half of its original. It implies that $\ell_{1/2}$ satisfies $\lambda^{\ell_{1/2}}(1-\lambda)\,\hat{\mathbf{C}}_{\mathbf{y}(k)} = \frac{1}{2}(1-\lambda)\,\hat{\mathbf{C}}_{\mathbf{y}(k)}$. So called 'half-time' $\ell_{1/2}$ is user friendly way for selection of the factor λ, because it can easily imagined what is the time needed for a contribution of $\hat{\mathbf{C}}_{\mathbf{y}(i)}$ to drop to one half.

6 APPLICATION EXAMPLES

The following figures demonstrate the behavior of untuned LQ control and set and tuned LQ control.

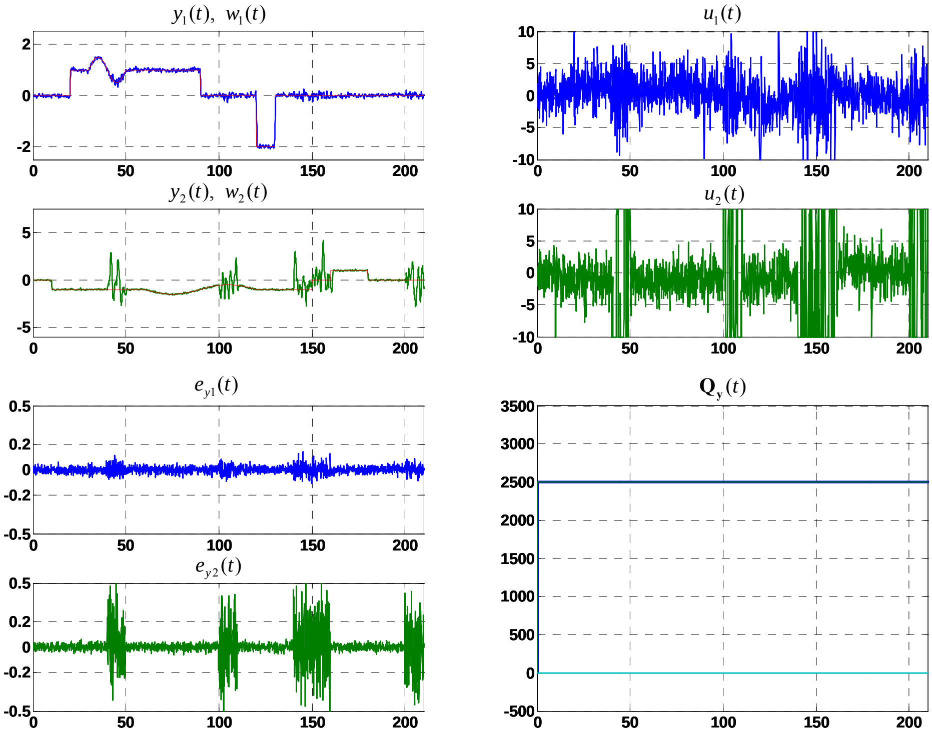

Figure 1 – Time histories of usual LQ control with fixed penalization matrix $\mathbf{Q_y}$

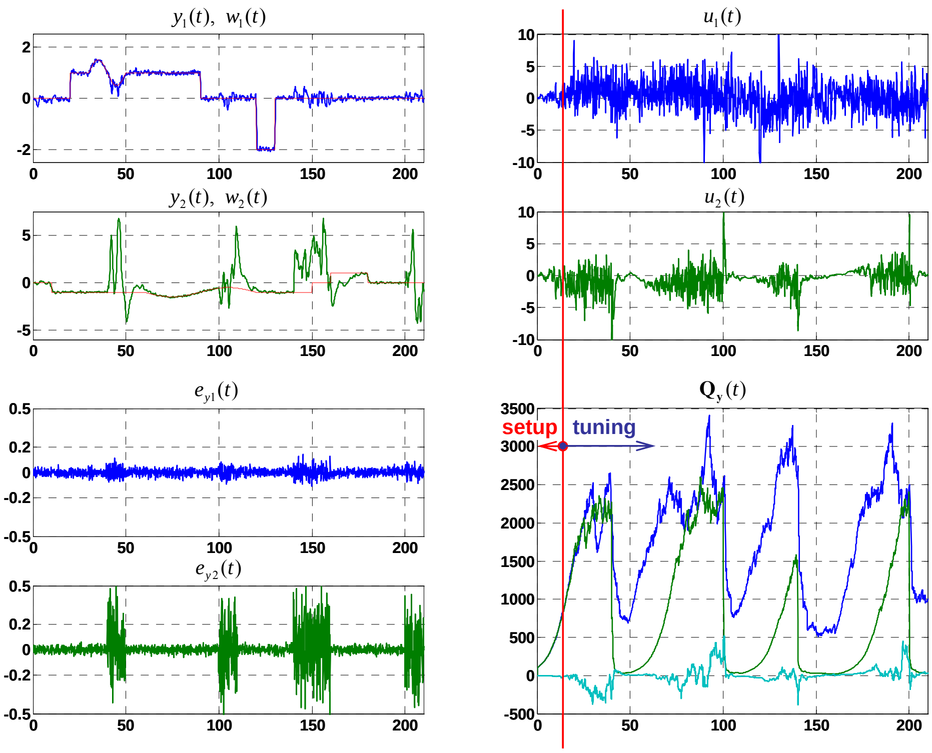

Figure 2 – Time histories of LQ control with setup and tuning of penalization matrix $\mathbf{Q_y}$

The general multidimensional digital LQ controller based on efficient square-root algorithm [Bobál et al., 2005] was applied to MIMO system of second order with two inputs and outputs. The figures 1 and 2 are organized according to the following matrix diagram:

$$\left[\begin{array}{c|c} y_1(t)-blue,\ w_1(t)-red & u_1(t)-blue \\ y_2(t)-green,\ w_2(t)-red & u_2(t)-green \\ \hline e_{y_1}(t)-blue & \mathbf{Q_y}\left(\begin{bmatrix}1,1-blue & 1,2-cyan \\ 2,1-cyan & 2,2-green\end{bmatrix}\right) \\ e_{y_1}(t)-green & \end{array}\right] \qquad (15)$$

where indicated variables have the meaning from previous sections and the color labels indicate color of the appropriate signal curves. Note, that the penalization $\mathbf{Q_u}$ was constant during the experiments.

It is obvious, when appropriate time histories are compared, that for different noise excitation in the first output channel increased two-times and in the second one four-times in appropriate time intervals, that the LQ controller response is quite different for both cases. During increased noise excitation, LQ controller with fix penalization generates exhausting control actions but due to physical limits cannot meet the set point signals. On the other hand, the set and tuned LQ controller takes into account increased noise excitation in impaired model adequateness indicated by increase of covariance matrix $\mathbf{C_y}$ or decreased of penalization $\mathbf{Q_y}$ respectively. In this case, the control actions remain between reasonable physical limits.

7 CONCLUSION

The paper deals with promising systematic and efficient way for self setup and fine tuning of LQ control. It is based on quality evaluation of the model of controlled systems. It uses descriptive statistics as expectation, dispersion or covariance matrices in the relation to precision matrices, which can be interpreted as proportional matrices to controller penalizations. In the paper, fixed model based on mathematical-physical analysis was considered.

References

KÁRNÝ, M. 1996. Towards fully probabilistic control design. *Automatica*, 32, 12, 1719-1722.

KÁRNÝ, M.; GUY, T. 2005. Fully probabilistic control design. *System & control letters*, 55, 259-265.

NOVÁK, M.; BÖHM, J.; NEDOMA, P. ; TESAŘ, L. 2003. Adaptive LQG controller tuning. *IEE Proc.-Control Theory Applications*, 150, 6, 655-665.

PETERKA, V. 1981. Bayesian approach to system identification. *Trends and Progress in System Identification*, (Eykhoff, P. ed.). Oxford: Pergamon Press, 239-304.

KÁRNÝ, M. *et al.* 2006. *Optimized Bayesian dynamic advising, Theory and algorithms.* Springer, ISBN 978-1-85233-928-9, 529 pp.

BELDA, K. 2009. Probabilistic design and tuning of LQ control. *Archives of Control Sciences*. Vol. 19, No. 4, 371-384.

BOBÁL, V.; BÖHM, J.; FESSL, J.; MACHÁČEK, J. 2005. *Digital Self-tuning Controllers*. London: Springer, ISBN 1-85233-980-2, 317 pp.

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