The Antibacklash Task in the Path Control of Redundant Parallel Robots

(subtopic: preparation of real-time control)

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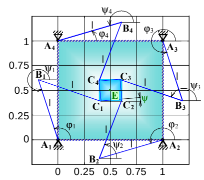
Abstract

The draft paper deals with the possibility to solve the antibacklash task of the redundant parallel robots. This type of the robot is generally described by Lagrange's equations of mixed type, on which base the used controls (Inverse Dynamics Control - IDC, Sliding Mode Control - SMC and Generalized Predictive Control - GPC) are designed. This paper discusses the two following ways. The first is based on solution of systems with the deficient rank matrix inversion (IDC, SMC) and the second is general utilization of the quadratic programming (GPC).



1 Introduction

This paper deals with the possibility of solution of the antibacklash task in the new robot concept based on the parallel construction improved by redundant action. The results can be used both for once-redundantly actuated systems (section 2) and even for systems without any redundancy (section 3).

The antibacklash task is solved as additional requirement on control (the torques should have only one sign) within usually used control approaches (Inverse Dynamics Control IDC (Siciliano 1996), Sliding mode control SMC (Elmali 1992) and Generalized Predictive control GPC (Ordys 1993)), which are briefly described.

From general point of view, the mechanical systems powered from outside e.g. by direct current motor (DC motor) and consisting of sets of arms and joints (the most of the robots and manipulators) have drive backlashes (motor backlash) and gearing backlashes Fig.1.

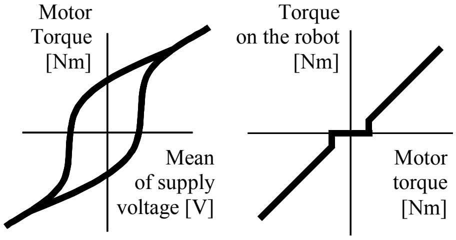


Fig. 1. The presumptive backlash characteristics occurred in a robot:
(a) the DC motor hysteresis backlash;
(b) the gearing backlash.

As a solution of the constrained control the pseudoinverse and quadratic programming has been chosen.

2 Pseudoinverse solution

The first two subsections briefly introduce IDC and SMC approaches (they have been already introduced in detail in the papers (Siciliano 1996, Elmali 1992, Belda 2001)) and the last subsection explicates the solution of deficient rank system on which these approaches lead.

2.1 Inverse dynamics control

Consider mechanical system (robot manipulator) described by nonlinear differential equation

$$\ddot{\mathbf{y}} = -\mathbf{f}(\mathbf{y}, \dot{\mathbf{y}}) + \mathbf{B}(\mathbf{y})\,\mathbf{u} \tag{1}$$

The approach (IDC) is based on the idea to find a control vector **u** as a function of system state. The classical approach (Siciliano 1996) assumes that matrix B(y) is a full rank matrix which can be inverted. If it is valid, we can obtain the continuous control law as a function of the robot state in the form:

$$\mathbf{u} = (\mathbf{B}(\mathbf{y}))^{-1}(\ddot{\mathbf{y}} + \mathbf{f}(\mathbf{y}, \dot{\mathbf{y}})) \tag{2}$$

The nonlinear control law eq. (2) is termed as inverse dynamics control because it includes computation of the robot inverse dynamics itself. The system with this control is linear with respect the new input – second derivation of **y**.

When the matrix **B** is singular as in our case, it can't be inverted. It is caused by redundant action. By using this property and algorithm for orthogonal-triangular decomposition, we have a possibility to compute control law and perform the antibacklash condition together. The sequence is described in subsection 2.3.

2.2 Sliding mode control

Discrete type of the Sliding mode control (Elmali 1992) is derived analogically to the theory of stability in a continuous domain. Generally it is based on the 'switching' control action and the performance of Lyapunov stability theorem.

The state is driven towards a desired switching (sliding) hyperplane under Lyapunov control. The 'switching' maintains the state on this hyperplane, once it has been reached, in spite of perturbations.

Let us consider the nonlinear equation (1), which can be transformed and simply discretized by Taylor series with sampling period δ to the following state formulation:

$$\mathbf{X}(k+1) = \mathbf{A}(\mathbf{X}(k)) + \mathbf{B}(\mathbf{X}(k))\mathbf{u}(k) \tag{3}$$

With this state description, we can obtain control law in similar structure as in the previous section:

$$\mathbf{u}(k) = -(\mathbf{C}\mathbf{B}(k))^{-1}\{\mathbf{C}[\mathbf{A}(k) + \Psi(k) - \mathbf{X}_d(k+1)] - \mathbf{s}(k+1)\} \qquad \mathbf{u}(k) = \tilde{\mathbf{B}}^{-1}\ (\mathbf{F}(\mathbf{X}, \mathbf{X}_d)) \tag{4}$$

Now we have defined control laws (IDC, SMC) and we can discuss the solution of their expressions.

2.3 Solution of backlashes by pseudoinversion

Consider now the eq. (2) and eq. (4) in the case that the inverse operation can't be provided. These equations have the same form as the ordinary system of the linear equations:

$$\mathbf{A}\mathbf{x} = \mathbf{B} \tag{5}$$

and it has an infinite number of solution. It is caused by deficient rank of matrix **A**.

The approach for removing the backlashes is based on computation of the pseudoinverse operation and on the idea of the non-changing signs of the torques during the robot movement along the certain finite trajectory.

The computation of the pseudoinverse operation gives the solution of the minimal length and some certain number of free parameters, which are used for change of undesirable signs of torques. This way, we obtain suitable solution, but it must be noted that this solution is not the same in the magnitude and it costs some additional energy and thus at least more powerful drives.

For showing the algorithm of pseudoinverse with eq. (5) the following theorems (Lawson 1974) are needed.

Theorem I.:

Suppose that **A** is an m x n matrix of rank k and that $\mathbf{A} = \mathbf{HRK}^T$ where **H** is an m x m orthogonal matrix, **R** is an m x n matrix of the form $\mathbf{R} = \begin{bmatrix} \mathrm{R}_{11} & 0 \\ 0 & 0 \end{bmatrix}$ with k x k submatrix R_{11} of rank k and **K** is an n x n orthogonal matrix. Define the vector $\mathbf{H}^T\mathbf{b} = \mathbf{g} \equiv \begin{bmatrix} g_1 \\ g_2 \end{bmatrix} \begin{matrix} \}k \\ \}m-k \end{matrix}$ and introduce the new variable $\mathbf{K}^T\mathbf{x} = \mathbf{y} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \begin{matrix} \}k \\ \}n-k \end{matrix}$. And finally define $\tilde{y}_1$ to be the unique solution of $\mathrm{R}_{11} y_1 = g_1$ then all solutions of system equations are of the form $\hat{\mathbf{x}} = \mathbf{K}\begin{bmatrix} \tilde{y}_1 \\ y_2 \end{bmatrix}$ where y_2 is arbitrary.

Note: This arbitrary vector is used for solution of backlashes.

Theorem II.:

Let **A** be an m x n matrix of rank k then there is an m x m orthogonal matrix **H** and an n x n orthogonal matrix **K** such that $\mathbf{H}^T\mathbf{AK} = \mathbf{R}$, $\mathbf{A} = \mathbf{HRK}^T$ where $\mathbf{R} = \begin{bmatrix} \mathrm{R}_{11} & 0 \\ 0 & 0 \end{bmatrix}$ and submatrix R_{11} is an k x k nonsingular triangular matrix.

These theorems give the mathematical relations on which the following algorithm is based.

$$(\mathbf{QAPx} =)\ \mathbf{Rx} = \mathbf{c}\ (= \mathbf{Qb}) \tag{6}$$

$$[\mathrm{R}_{11}\ \mathrm{R}_{12}]\,\mathbf{K} = [\mathrm{W}\ 0]\ (\equiv \tilde{\mathbf{A}}\,\mathbf{K} = [\mathrm{W}\ 0]) \ \Rightarrow\ \ \mathrm{W}y_1 = c_1 \ \ \Rightarrow\ y_1 \tag{7}$$

$$\mathbf{x} = \mathbf{PK}\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \equiv \mathbf{PKy} \qquad y_2 \text{ is arbitrary} \tag{8}$$

The algorithm uses the orthogonal matrixes **Q**, **K** and the permutation matrix **P** so that **R** and **W** are upper triangular and R_{11} is nonsingular. It was the first step to the solution of the backlashes. Now the sequence of suitable choice of an arbitrary vector y_2 follows.

The solution (8) can be divided for once redundantly determined system

$$\mathbf{x} = [\mathrm{PK}_{\mathrm{sub1}} \quad \mathrm{PK}_{\mathrm{sub2}}]\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \tag{9}$$

$$\begin{aligned} \mathbf{x} &= \ \mathrm{PK}_{\mathrm{sub1}} \cdot y_1 \qquad + \mathrm{PK}_{\mathrm{sub2}} \cdot y_2 \\ \mathbf{u} &= \mathbf{u}_1 (= \mathbf{x} \,|\, y_2 = 0) + \quad \mathbf{u}_2 \quad \cdot y_2 \end{aligned} \tag{10}$$

the minimal solution   arbitrary part of the solution

For defined safety bounds around the zero on torque axis, which can be represented by vector

$$\mathbf{rl} = [rl_1, rl_2, rl_3, rl_4]^T \tag{11}$$

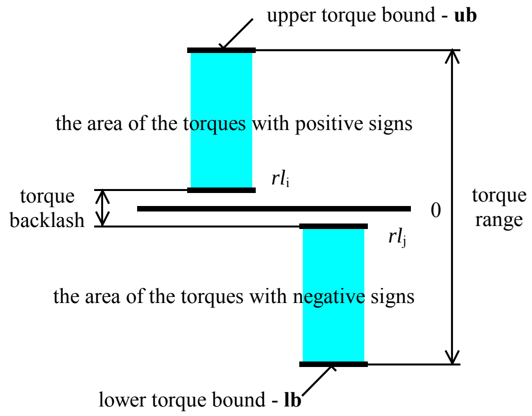


Fig. 2. The definition of backlashes and bounds.

the following sequence is being applied.

Step1. find $u_1(i) \notin <rl_i, ub>$ for req. positive signs
$u_1(i) \notin <lb, rl_j>$ for req. negative signs

Step2. for such $u_1(i)$ compute $y_2(i) = \dfrac{rl(i) - u_1(i)}{u_2(i)}$
(eq.(10) with substitution rl(i) for u).

Step3. for these $y_2(i)$ compute U(i)
$U(i)=\mathbf{u}_1+\mathbf{u}_2 \cdot y_2(i)$

Step4. choice $U^*(i)$ which satisfies $\| rl - U(i) \| = \min$

Step5. check $U^*(i)$ with considering to bounds (Fig.2) and provide hard restrictions.

Then final **u** equals $U^*(i)$. Such result does not markedly change the properties of the robot control process, however, it changes the magnitude of all torques. The minimal solution requires the minimum supply of the drive energy against result (Step5.) satisfying the antibacklash condition, where the required drive energy increases severalfoldly.

3. Quadratic programming Solution

This section generally introduces the Generalized Predictive Control (GPC) which leads on adequately actuated problem and concerns with only the simple example of the utilization of the quadratic programming (QP) for performing of the antibacklash condition.

3.1 Generalized predictive control (GPC)

The Generalized predictive control (Ordys 1993) is a multi-step control based on local optimization of the quadratic criterion. For quadratic criterion, the linearized discrete state formula must be prepared in this classical form:

$$\begin{aligned} \mathbf{X}(k+1) &= \mathbf{A}\,\mathbf{X}(k) + \mathbf{B}\,\mathbf{u}(k) \\ \mathbf{y}(k) &= \mathbf{C}\,\mathbf{X}(k) \end{aligned} \tag{12}$$

The base of predictive control is the expression of new unknown output values **y** from actual topical state **X**. Now we consider the N step prediction of **y** as follows

$$\hat{\mathbf{y}} = \mathbf{G}\,\mathbf{u} + \mathbf{f} \text{ where } \mathbf{G} = \begin{bmatrix} \mathbf{C} & \mathbf{B} \cdots & 0 \\ \vdots & \ddots & \vdots \\ \mathbf{CA}^{N-1}\mathbf{B} & \cdots & \mathbf{CB} \end{bmatrix} \text{ and } \mathbf{f} = \begin{bmatrix} \mathbf{CA} \\ \vdots \\ \mathbf{CA}^N \end{bmatrix} \mathbf{X}(k) \tag{13}$$

and further the quadratic criterion

$$\begin{aligned} J_k &= \mathcal{E}\left\{ (\hat{\mathbf{y}} - \mathbf{w})^T (\hat{\mathbf{y}} - \mathbf{w}) + \mathbf{u}^T \lambda\, \mathbf{u} \right\} = \\ &= \mathcal{E}\left\{ (\mathbf{G}\,\mathbf{u} + \mathbf{f} - \mathbf{w})^T (\mathbf{G}\,\mathbf{u} + \mathbf{f} - \mathbf{w}) + \mathbf{u}^T \lambda\, \mathbf{u} \right\} \end{aligned} \tag{14}$$

where N is a horizon of prediction $\hat{\mathbf{y}}$. $\mathcal{E}$ is operator of mean value and $\boldsymbol{\lambda}$ is a penalization of **u**.

On condition $J_k \overset{!}{=} \min$, we obtain the control law: $\mathbf{u} = (\mathbf{G}^T\mathbf{G} + \lambda)^{-1} \mathbf{G}^T (\mathbf{w} - \mathbf{f})$ (15)

It must be noted that only the first element $\mathbf{u}_k$ from vector **u** is used. If penalization $\boldsymbol{\lambda}$ is greater than zero, the matrix $\mathbf{G}^T \cdot \mathbf{G}$ is regular and the problem with redundant action disappears.

3.2 Solution of backlashes by quadratic programming

This subsection takes into account fact of regularity of matrix $\mathbf{G}^{T}\cdot\mathbf{G}$ and briefly introduces utilization of the Quadratic programming (the references on it were noted in previous papers). The main concern is how to form the constraining antibacklash inequalities.

Standard task of the Quadratic programming minimizes the quadratic purposive function with some linear constraints.

$$\underset{\mathbf{x}}{\text{minimize}}\left\{ \mathrm{F}(\mathrm{x}) = \frac{1}{2}\mathbf{x}^{T}\mathbf{H}\mathbf{x} + \mathbf{f}^{T}\mathbf{x} \right\} \qquad \text{subject to} \quad \mathbf{A}\cdot\mathbf{x} \leq \mathbf{b} \tag{16}$$

where $\mathbf{H}$ is an n x n, $\mathbf{f}$ an n vector, $\mathbf{A}$ is an m x n matrix and $\mathbf{b}$ is an m vector. The function F($\mathbf{x}$) is obtained from quadratic criterion eq. (24) as follows

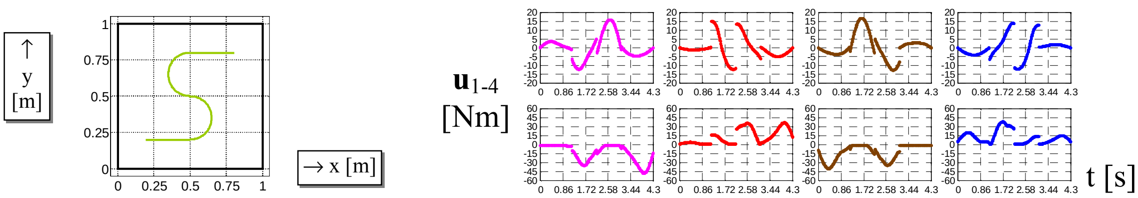
$$\mathrm{F}(\mathbf{u}) = \frac{1}{2}\mathbf{u}^{T}\underbrace{\left(\mathbf{G}^{T}\mathbf{G}+\lambda\right)}_{\mathbf{H}}\mathbf{u} + \underbrace{\left(f-\mathbf{w}\right)^{T}\mathbf{G}\,\mathbf{u}}_{\mathbf{f}^{T}} \tag{17}$$

$$\text{e.g.: for } \mathbf{u} = \begin{bmatrix} u(i) \in < rl(i), ub > \\ u(j) \in < lb, rl(j) > \end{bmatrix} \text{ the structure of } \mathbf{A}\cdot\mathbf{x} \leq \mathbf{b} \text{ is } \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -1 & 0 \\ 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} u(i) \\ u(j) \end{bmatrix} \leq \begin{bmatrix} ub \\ rl(j) \\ -rl(i) \\ -lb \end{bmatrix} \tag{18}$$

After satisfying of all assumptions, the Quadratic programming gives always some solution, which is not optimal but the found solution of the full rank problem has the smallest aberration that can be attainable.

4. Simulation example and illustration of preparation of real-time control

For the simulation of the robot, some plan of the trajectory must be prepared. One example of the desired trajectory is shown in overall Fig. 3.



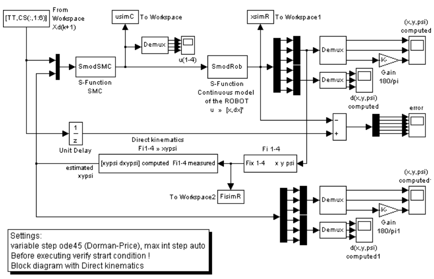


Fig.3.: Trajectory; the time histories of four torques, firstly without constrains and consecutively with satisfying of antibacklash condition for rl = [-1,1,-1,1].

Fig.4.: Example of real-time circuit with using MATLAB SIMULINK environment.

Designed solutions are suitable for classical parallel robots (QP prog.) and for parallel robots with one redundancy (Nonmin. s.). At present the algorithms are being prepared for real application Fig.4.

References

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