SINGULAR CASES OF THE PLANAR PARALLEL ROBOT

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Summary: *Singular cases – singularities of the robot are defined as positions in which the robot loses kinematic definiteness. The loss appears in both direct and inverse kinematic transformation. Existence of the singularities in the robot workspace causes sharp changes in profiles of the velocities and accelerations and changes in force effects. The objective of this paper is to investigate singularities (singular positions) of one parallel robot structure and to simply classify such singular cases. The analysis of the singularities is important for real application, i.e. for control design, which should ensure effective and safe cooperation of all drives – actuators in the robot structure.*

1. Introduction

Singular cases – singularities (Stejskal, V., 1997; Tsai, L.-W., 1999) of the robot are defined as positions in which the robot loses kinematic definiteness. The loss appears in both direct and inverse kinematic transformation. Existence of the singularities in the robot workspace causes sharp changes in profiles of the velocities and accelerations and changes in force effects. The objective of this paper is to investigate singularities (singular positions) of one parallel robot structure (Neugebauer, R. ed., 2002; Valášek, M. et al., 2002) and to simply classify such singular cases. The analysis of the singularities is important for real application, i.e. for control design (Kock, S., & Schumacher, W., 2000) which should ensure effective and safe cooperation of all drives – actuators in the robot structure. Thus, from control point of view, in singular positions the robot (mechanical structure) loses controllability and user can not influence this unsafe situation.

The structure of the paper is the following. In section 2, the geometrical relations in parallel structure are described. Then, the paper continues by section 3, which deals with singular analysis using the relations from section 2. The next section 4, section of examples, presents singular analysis for real parallel structure with given parameters. The paper is concluded by simple classification of singular cases in section 5 and conclusions in section 6.

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2. Description of the geometrical relations in parallel robot structure

Let us consider the robot with *n* degrees of freedom (DOF) e.g. the robot with 3 DOF shown in Fig. 1. The robot consists of movable platform defined by dimensions d_1, d_3 and angle β, workspace or fixed platform defined by the fixed pivots A_1, A_2, A_3, and arms, which are characterized by their lengths l_1, l_3, l_5 (external arms) l_2, l_4, l_6 (internal arms) and their appropriate orientation angles $\varphi_1, \varphi_2, \varphi_3$ and ψ_1, ψ_2, ψ_3 respectively.

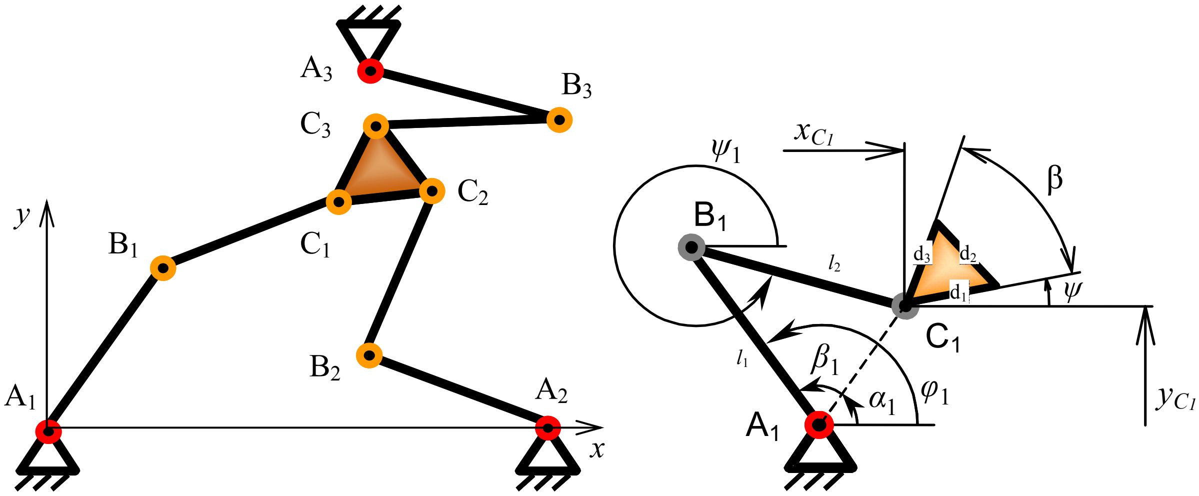


Fig. 1. Scheme of planar parallel robot structure.

The coordinates appearing in the robot structure may be divided into drive **z** (φ_i), operational **q** (x_{C1}, y_{C1}, ψ) and other auxiliary $\mathbf{z}_a$ (ψ_i) coordinates. All these coordinates are either independent (their number equals to DOF) or dependent (remaining coordinates) together called physical coordinates (Stejskal, V. & Valášek, M., 1996). Let us consider possibility to reduce number of structural equations (1 a) to number of degrees of freedom and that the equations represent relations only between operational **q** and drive **z** coordinates. Then, the relations can be generally expressed as follows:

for positions

$$\mathbf{f}(\mathbf{q}, \mathbf{z}, \mathbf{z}_a) = \mathbf{0} \quad \text{reduced to} \quad \mathbf{f}(\mathbf{q}, \mathbf{z}) = \mathbf{0} \qquad (1\text{ a, b})$$

for velocities

$$\mathbf{\Phi}_\mathbf{q}(\mathbf{q}, \mathbf{z})\,\dot{\mathbf{q}} + \mathbf{\Phi}_\mathbf{z}(\mathbf{q}, \mathbf{z})\,\dot{\mathbf{z}} = \mathbf{0} \qquad (2)$$

for accelerations

$$\dot{\Phi}_\mathbf{q}(\mathbf{q}, \mathbf{z})\,\dot{\mathbf{q}} + \Phi_\mathbf{q}(\mathbf{q}, \mathbf{z})\,\ddot{\mathbf{q}} + \dot{\Phi}_\mathbf{z}(\mathbf{q}, \mathbf{z})\,\dot{\mathbf{z}} + \Phi_\mathbf{z}(\mathbf{q}, \mathbf{z})\,\ddot{\mathbf{z}} = \mathbf{0} \qquad (3)$$

The equations (1 b), (2) and (3) represent the main kinematic relations in the structure. Now we can start investigating the robot singularities.

3. Singular analysis

To analyze singularities of the robot in Fig. 1, let us consider the equations (1 b), (2) and (3) adjusted for its coordinates. As mentioned, existence of singularities in the robot workspace causes sharp changes in profiles of velocities and accelerations and changes in force effects. Therefore when analyzing the singularities, we search situations, in which the system of equations (2) or (3) has infinity number of the solutions or has no solution. The situations appear when the systems of equations lead to undefined relations between drive and operational coordinates. It can be investigated by evaluating the determinants of the Jacobian matrixes ($\boldsymbol{\Phi}_\mathbf{q}, \boldsymbol{\Phi}_\mathbf{z}$) from equation system (2) (Stejskal, V., 1997). Thus, if the equation system (2) has full rank Jacobian matrixes, then the topical robot configuration is properly determined.

For the robot in Fig. 1, structural relations (geometrical constrains) can be most simply written as follows:

$$\mathbf{f}(\mathbf{q}, \mathbf{z}, \mathbf{z}_a) = \mathbf{0} \tag{4}$$

$$\begin{aligned}
&x_{C_1} - l_1 \cos\varphi_1 - l_2 \cos\psi_1 = 0\\
&y_{C_1} - l_1 \sin\varphi_1 - l_2 \sin\psi_1 = 0\\
&x_{C_1} + d_1 \cos\psi - l_3 \cos\varphi_2 - l_4 \cos\psi_2 - x_{A_2} = 0\\
&y_{C_1} + d_1 \sin\psi - l_3 \sin\varphi_2 - l_4 \sin\psi_2 = 0\\
&x_{C_1} + d_3 \cos(\psi+\beta) - l_5 \cos\varphi_3 - l_6 \cos\psi_3 - x_{A_3} = 0\\
&y_{C_1} + d_3 \cos(\psi+\beta) - l_5 \sin\varphi_3 - l_6 \sin\psi_3 - y_{A_3} = 0
\end{aligned} \tag{5}$$

The structural system (5) consists of the six equations, which contain the nine physical coordinates $[\varphi_1, \varphi_2, \varphi_3, \psi_1, \psi_2, \psi_3, x_{C_1}, y_{C_1}, \psi]$. However, the robot has only three degrees of freedom thus the three equations are dependent. Therefore, we rewrite the system to adequate form having just only three equations:

$$\mathbf{f}(\mathbf{q}, \mathbf{z}) = \mathbf{0} \tag{6}$$

$$\begin{aligned}
f_1 &= (x_{C_1} - l_1 \cos\varphi_1)^2 + (y_{C_1} - l_1 \sin\varphi_1)^2 - l_2^2 = 0\\
f_2 &= (x_{C_1} + d_1 \cos\psi - l_3 \cos\varphi_2 - x_{A_2})^2 + (y_{C_1} + d_1 \sin\psi - l_3 \sin\varphi_2)^2 - l_4^2 = 0\\
f_3 &= (x_{C_1} + d_3 \cos(\psi+\beta) - l_5 \cos\varphi_3 - x_{A_3})^2 + (y_{C_1} + d_3 \cos(\psi+\beta) - l_5 \sin\varphi_3 - y_{A_3})^2 - l_6^2 = 0
\end{aligned} \tag{7}$$

Then we can determine the Jacobian matrices of the robot system:

$$\frac{\partial \mathbf{f}(\mathbf{q}, \mathbf{z})}{\partial \mathbf{q}}\,\dot{\mathbf{q}} + \frac{\partial \mathbf{f}(\mathbf{q}, \mathbf{z})}{\partial \mathbf{z}}\,\dot{\mathbf{z}} = \mathbf{0}$$

$$\Phi_\mathbf{q}(\mathbf{q}, \mathbf{z})\,\dot{\mathbf{q}} + \Phi_\mathbf{z}(\mathbf{q}, \mathbf{z})\,\dot{\mathbf{z}} = \mathbf{0} \tag{8}$$

$$\text{Jacobian matrix } \mathbf{\Phi_z} \text{ (drive coordinates)} = \begin{bmatrix} \frac{\partial f_1}{\partial \varphi_1} & \frac{\partial f_1}{\partial \varphi_2} & \frac{\partial f_1}{\partial \varphi_3} \\ \frac{\partial f_2}{\partial \varphi_1} & \frac{\partial f_2}{\partial \varphi_2} & \frac{\partial f_2}{\partial \varphi_3} \\ \frac{\partial f_3}{\partial \varphi_1} & \frac{\partial f_3}{\partial \varphi_2} & \frac{\partial f_3}{\partial \varphi_3} \end{bmatrix} =$$

$$= \begin{bmatrix} 2x_{C_1} l_1 \sin\varphi_1 - 2y_{C_1} l_1 \cos\varphi_1 & 0 & 0 \\ 0 & 2l_3(x_{C_1} + d_1\cos\psi - \sin\varphi_2)\sin\varphi_2 - 2l_3(y_{C_1} + d_1\sin\psi)\cos\varphi_2 & 0 \\ 0 & 0 & 2l_3(x_{C_1} + d_1\cos\psi - \sin\varphi_2)\sin\varphi_2 - 2l_3(y_{C_1} + d_1\sin\psi)\cos\varphi_2 \end{bmatrix} \quad (9)$$

$$\text{Jacobian matrix } \mathbf{\Phi_q} \text{ (operational coordinates)} = \begin{bmatrix} \frac{\partial f_1}{\partial x_{C_1}} & \frac{\partial f_1}{\partial y_{C_1}} & \frac{\partial f_1}{\partial \psi} \\ \frac{\partial f_2}{\partial x_{C_1}} & \frac{\partial f_2}{\partial y_{C_1}} & \frac{\partial f_2}{\partial \psi} \\ \frac{\partial f_3}{\partial x_{C_1}} & \frac{\partial f_3}{\partial y_{C_1}} & \frac{\partial f_3}{\partial \psi} \end{bmatrix} =$$

$$= \begin{bmatrix} 2x_{C_1} - 2l_1\cos\varphi_1 & 2y_{C_1} - 2l_1\sin\varphi_1 & 0 \\ 2x_{C_1} - 2d_1\cos\psi - 2l_3\cos\varphi_2 - 2x_{A_2} & 2y_{C_1} - 2d_1\sin\psi - 2l_3\sin\varphi_2 & 2d_1(-x_{C_1} + l_3\cos\varphi_2 + x_{A_2})\sin\psi - 2d_1(-y_{C_1} + l_3\sin\varphi_2)\cos\psi \\ 2x_{C_1} + 2d_3\cos(\psi+\beta) - 2l_5\cos\varphi_3 - 2x_{A_3} & 2y_{C_1} + 2d_3\sin(\psi+\beta) - 2l_5\sin\varphi_3 - 2y_{A_3} & 2d_3(-x_{C_1} + l_5\cos\varphi_3 + x_{A_3})\sin(\psi+\beta) - 2d_3(-y_{C_1} + l_5\sin\varphi_3 + y_{A_3})\cos(\psi+\beta) \end{bmatrix} \quad (10)$$

To evaluate the determinants of the Jacobian matrixes effectively, some additional equations are needed. It means that for the evaluating we need direct (explicit) relation between drive and operational coordinates, at least in one direction (direction is arbitrary). Equations (5) do not provide it. Therefore, we must return to the scheme of the robot in Fig. 1.

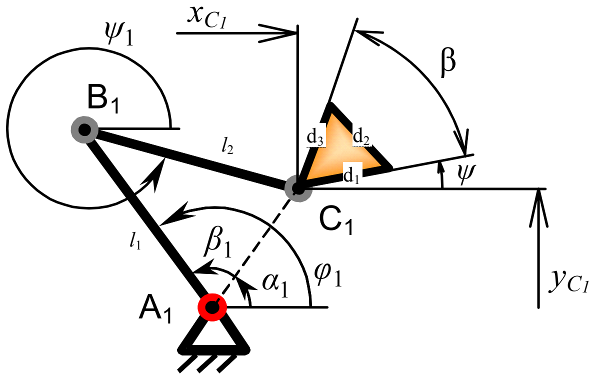


Fig. 2. Detail of the first arm group of the parallel robot; description for determining the explicit relation between the coordinates.

For this parallel structure, the explicit relation can be determined in the following way. When considering the Fig. 2, then for first arm group we can write:

$$\varphi_1(x_{C_1}, y_{C_1}, \psi) = \arctan\left(\frac{y_{C_1} - y_{A_1}}{x_{C_1} - x_{A_1}}\right) + \arccos\left(\frac{l_1^2 + (y_{C_1} - y_{A_1})^2 + (x_{C_1} - x_{A_1})^2 - l_2^2}{2l_1\sqrt{(y_{C_1} - y_{A_1})^2 + (x_{C_1} - x_{A_1})^2}}\right) = \alpha_1 + \beta_1 \quad (11\text{ a})$$

And similarly, it is possible to write relations for remaining arm groups:

$$\varphi_2(x_{C_1}, y_{C_1}, \psi) = \arctan\left(\frac{y_{C_1} + d_1 \sin\psi - y_{A_2}}{x_{C_1} + d_1 \cos\psi - x_{A_2}}\right) + \arccos\left(\frac{l_3^2 + (y_{C_1} + d_1 \sin\psi - y_{A_2})^2 + (x_{C_1} + d_1 \cos\psi - x_{A_2})^2 - l_4^2}{2l_3\sqrt{(y_{C_1} + d_1 \sin\psi - y_{A_2})^2 + (x_{C_1} + d_1 \cos\psi - x_{A_2})^2}}\right) = \alpha_2 + \beta_2 \quad (11\text{ b})$$

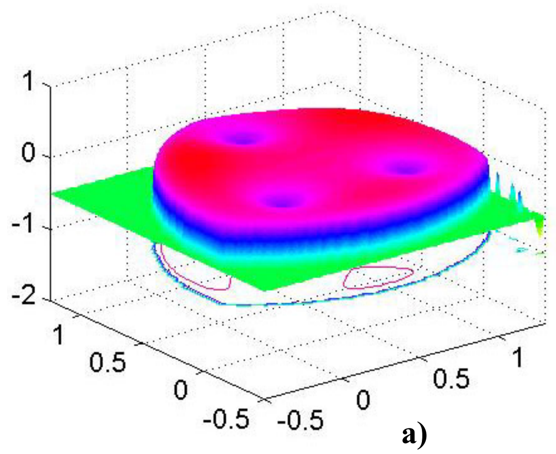
$$\varphi_3(x_{C_1}, y_{C_1}, \psi) = \arctan\left(\frac{y_{C_1} + d_3 \sin(\psi+\beta) - y_{A_3}}{x_{C_1} + d_3 \cos(\psi+\beta) - x_{A_3}}\right) + \arccos\left(\frac{l_5^2 + (y_{C_1} + d_3 \sin(\psi+\beta) - y_{A_3})^2 + (x_{C_1} + d_3 \cos(\psi+\beta) - x_{A_3})^2 - l_6^2}{2l_5\sqrt{(y_{C_1} + d_3 \sin(\psi+\beta) - y_{A_3})^2 + (x_{C_1} + d_3 \cos(\psi+\beta) - x_{A_3})^2}}\right) = \alpha_3 + \beta_3 \quad (11\text{ c})$$

The functions $\varphi_1(x_{C_1}, y_{C_1}, \psi)$, $\varphi_2(x_{C_1}, y_{C_1}, \psi)$ and $\varphi_3(x_{C_1}, y_{C_1}, \psi)$ express inverse kinematic transformation of the parallel robot structure, i.e. transformation of operational coordinates to drive coordinates. Now, the operational (Cartesian) coordinates may be selected as independent and definitional coordinates for computation of the determinants.

4. Examples

Let us consider the following structural parameters for planar parallel robot as illustrated in Fig. 1: lengths of arms $l_1 = l_3 = l_5 = l_2 = l_4 = l_6 = 0.636$ m; dimensions of the movable platform $d_1 = d_3 = 0.2$ m, and angle $\beta = \pi/3$; Cartesian coordinates of vertexes of triangular workspace (fix platform) $x_{A_1} = 0$ m; $y_{A_1} = 0$ m; $x_{A_2} = 1$ m; $y_{A_2} = 0$ m; $x_{A_3} = 0.5$ m; $y_{A_3} = 1$ m.

The following figures show values of individual determinants in the robot workspace. The figures are drawn for constant angle of movable platform $\psi = 0$.



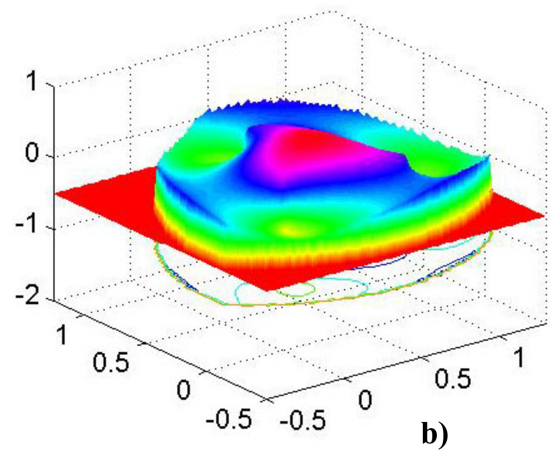
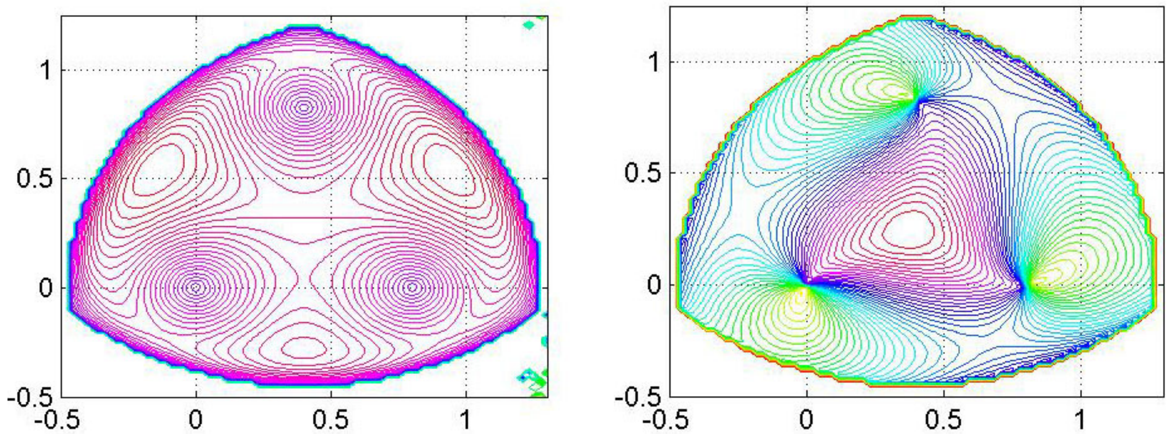


Fig. 3. Spatial graphs: **a)** values of determinant Φ_z (det(Φ_z) vertical axis) and **b)** values of determinant Φ_q (det(Φ_q) vertical axis).



a) **b)**

Fig. 4. Planar contour graphs **a)** values of determinant $\Phi_\mathbf{z}$ and **b)** values of determinant $\Phi_\mathbf{q}$.

In neighborhoods of the fixed pivots A_1 [0, 0], A_2 [1, 0], A_3 [0.5, 1] (Fig. 1), the singular points appear (Fig. 4 and Fig. 5). Singular points around A_2 and A_3 are shifted to the left side by proper projection of side $d_i = 0.2$ m of the movable platform.

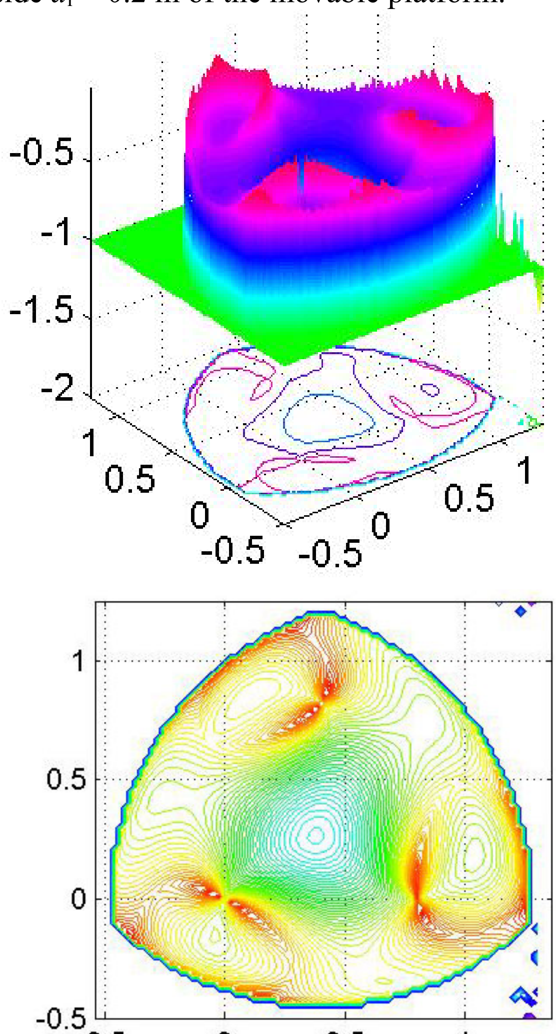


Fig. 5. Spatial graph and its planar contours for combined singularities; ($\psi = 0$).

5. Classification of singular cases

According to determinants of individual Jacobian matrixes and computational tests we can define three types of singularities:

- Singularities of the first case (***Inverse Kinematic Singularities***).

Inverse kinematic singularities occur when one of the diagonal elements of $\mathbf{\Phi_z}$ disappeared. Consequently, an inverse kinematic singularity arises whenever any group of arm i.e. $((\overline{A_iB_i};\overline{B_iC_i}),\ i = 1, 2, 3)$ is in a fully stretched-out or folded-back configuration. The manipulator loses 1, 2, or 3 degrees of freedom according to configuration if one, two, or three arm groups are fully-stretched-out or folded-back. In the configurations, the changes of working elements (input – external arms) do not cause any change of movable platform.

- Singularities of the second case (***Direct Kinematic Singularities***).

Direct kinematic singularities occur when the determinant of $\mathbf{\Phi_q}$ goes to zero. The robot loses controllability in certain configuration when the directions of arms $\overrightarrow{B_iC_i},\ i = 1, 2, 3$ intersect in one point of movable platform (e.g. Fig. 6: ψ = -54.2°). In the point, the platform can slightly rotate in spite of locked drives, i.e. it has one degree of freedom. Or the different configuration appears, when directions of arms $\overrightarrow{B_iC_i},\ i = 1, 2, 3$ are parallel to one another. In this configuration, the movable platform can perform infinitesimal translations perpendicular to the parallel arms.

- Singularities of the third case (***Combined Singularities***).

A combined singularity occurs when the both determinants $\mathbf{\Phi_z}$ and $\mathbf{\Phi_q}$ equal to zero. The case is caused by dependence of structural relations (geometrical constrains).

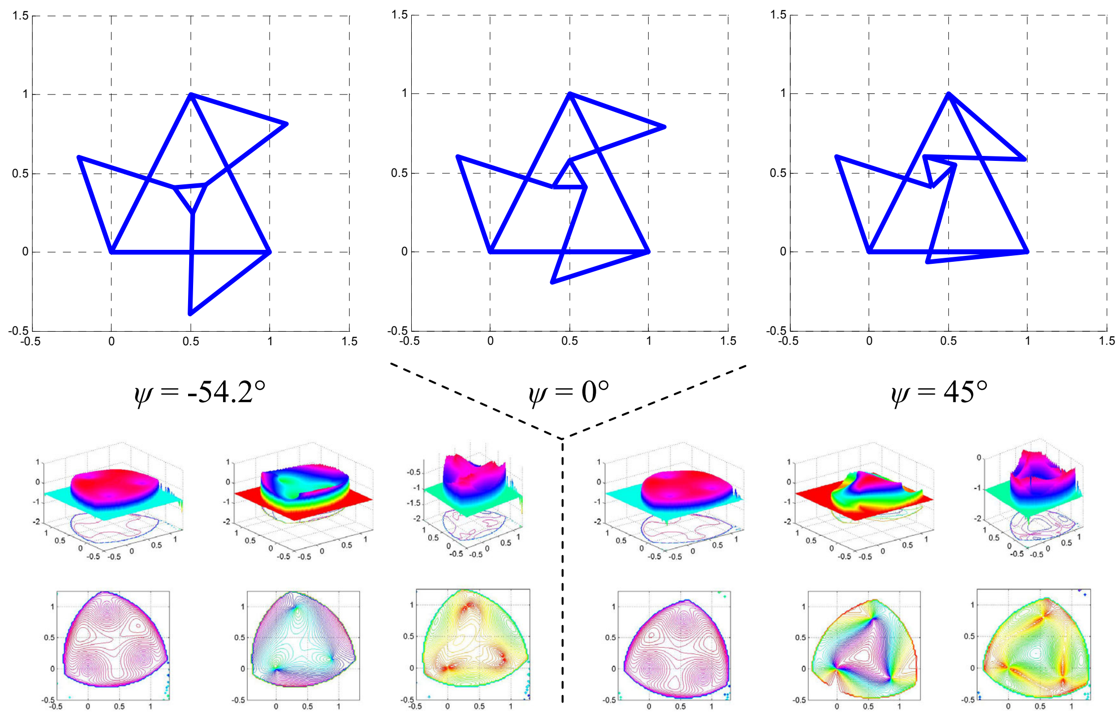


Fig. 6. Comparison of determinant values for different angles of movable platform, determinants $\mathbf{\Phi_z}$, $\mathbf{\Phi_q}$ and their combinations (position x_{C1} = 0.4 m, y_{C1} = 0.4 m).

6. Conclusions

The paper deals with the analysis and simple classification of the singularities in workspace of one prototype of the planar parallel robot. It describes obtaining the kinematic relations (geometrical constrains) and shows their use just in investigating the problem of singularities.

7. References

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