THE ANTIBACKLASH TASK IN THE PATH CONTROL OF REDUNDANT PARALLEL ROBOTS

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Abstract: The paper deals with the possibility of solution of the antibacklash task in the new concept of the robots based on the parallel construction improved by redundant action. This type of the robot is generally described by Lagrange's equations of mixed type. On their base the used controls (Inverse Dynamics Control - IDC, Sliding Mode Control - SMC and Generalized Predictive Control - GPC) are designed. This paper discusses the two following ways. The first is based on solution of systems with the deficient rank matrix inversion (IDC, SMC) and the second is general utilization of the quadratic programming (GPC).

Keywords: Redundant parallel robot, backlash problem, matrix pseudoinversion, Quadratic programming.

1 INTRODUCTION

The area of the robots and manipulators is in the constant development caused by the fact that the robots are the basis of the most machine and production lines in the factories.

The uncompromising requirements on their new types are primarily high accuracy, high speed and price constraints. It means that the robot's structure, with considering the previous, must have high stiffness, good dynamic properties and acceptable price. The price includes requirements on design control and complexity of the construction.

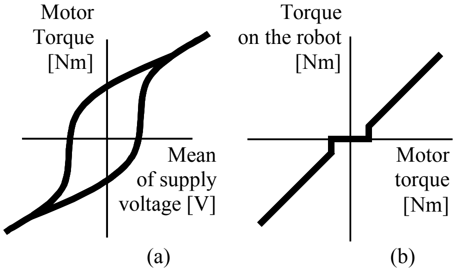
The main long - term conceptual problems, how to satisfy such requirements, are the following:

- the large moving masses during the robot movement,
- the backlashes and inaccuracies in the chain of the robot structure.

This paper deals with the possibility of solution of the antibacklash task in the new robot concept based on the parallel construction improved by redundant action. The results can be used both for once-redundantly actuated systems (section 2) and even for systems without any redundancy (section 3).

The antibacklash task is solved as additional requirement on control (the torques should have only one sign) within usually used control approaches (Inverse Dynamics Control IDC (Siciliano 1996), Sliding mode control SMC (Elmali 1992) and Generalized Predictive control GPC (Ordys 1993)), which are briefly described.

From general point of view, the mechanical systems powered from outside e.g. by direct current motor (DC motor) and consisting of sets of arms and joints (the most of the robots and manipulators) have drive backlashes (motor backlash) and gearing backlashes Fig.1.



(a) (b)

Fig. 1. The presumptive backlash characteristics occurred in a robot: (a) the DC motor hysteresis backlash; (b) the gearing backlash.

As a solution of the constrained control problem, pseudoinverse and quadratic programming has been chosen.

2 PSEUDOINVERSE SOLUTION

This section deals with two control approaches, which keep the redundancy of system. The first two subsections briefly introduce these approaches (they have been already introduced in detail in the previous papers (Siciliano 1996, Elmali 1992, Belda 2001)) and the last subsection explicates the solution of deficient rank system with necessity of pseudoinverse operation to which the approaches lead.

2.1 *Inverse dynamics control*

Consider mechanical system (robot manipulator) described by nonlinear differential equation

$$\ddot{\mathbf{y}} = -\mathbf{f}(\mathbf{y}, \dot{\mathbf{y}}) + \mathbf{B}(\mathbf{y})\,\mathbf{u} \tag{1}$$

The approach (IDC) is based on the idea to find a control vector **u** as a function of system state. The classical approach (Siciliano 1996) assumes that matrix B(y) is a full rank matrix which can be inverted. If it is valid, we can obtain the continuous control law as a function of the robot state in the form:

$$\mathbf{u} = (\mathbf{B}(\mathbf{y}))^{-1}(\ddot{\mathbf{y}} + \mathbf{f}(\mathbf{y}, \dot{\mathbf{y}})) \tag{2}$$

Such control leads to finding stabilizing control law for system with $\mathbf{q} = \ddot{\mathbf{y}}$

$$\mathbf{u} = (\mathbf{B}(\mathbf{y}))^{-1}(\mathbf{q} + \mathbf{f}(\mathbf{y}, \dot{\mathbf{y}})) \tag{3}$$

where **q** represents a new input vector to the whole robot control circuit. The nonlinear control law eq. (3) is termed as inverse dynamics control because it includes computation of the robot inverse dynamics itself. The system with this control is linear with respect to the new input **q**.

When the matrix **B** is singular as in our case, it can't be inverted. It is caused by redundant action. By using this property and algorithm for orthogonal-triangular decomposition, we have a possibility to compute control law and perform the antibacklash condition. The sequence is described in subsection 2.3.

2.2 Sliding mode control

Discrete type of the Sliding mode control (Elmali 1992) is derived analogically to the theory of stability in a continuous domain. Generally it is based on the 'switching' control action and the performance of Lyapunov stability theorem.

The state is driven towards a desired switching (sliding) hyperplane under Lyapunov control. The 'switching' maintains the state on this hyperplane, once it has been reached, in spite of perturbations. This method offers an advantage of accuracy at the cost of control dithering, which ensues from the 'switching' part.

Let us consider the nonlinear equation (1) which can be transformed and simply discretized by Taylor series with sampling period δ to the following state formulation:

$$\mathbf{X}(k+1) = \mathbf{A}(\mathbf{X}(k)) + \mathbf{B}(\mathbf{X}(k))\mathbf{u}(k) \tag{4}$$

With this state description, we can obtain control law in similar structure as in the previous section:

$$\mathbf{u}(k) = -(\mathbf{C}\mathbf{B}(k))^{-1}\{\mathbf{C}[\mathbf{A}(k) + \Psi(k) - \mathbf{X}_d(k+1)] - \mathbf{s}(k+1)\}$$

$$\mathbf{u}(k) = \qquad \tilde{\mathbf{B}}^{-1} \qquad\qquad (\mathbf{F}(\mathbf{X}, \mathbf{X}_d)) \tag{5}$$

where $\mathbf{s}(k+1) = e^{-p\delta}\mathbf{s}(k) - \mathbf{K}sign(\mathbf{s}(k))$ (6)

with considering $\mathbf{s}(k) = \boldsymbol{f}(\mathbf{X}(k) - \mathbf{X}_\mathrm{d}(k))$ (7)

is the choice of hyperplane. It satisfies Lyapunov stability theorem and $\boldsymbol{\Psi}(k)$ represents unknown perturbation, which can be estimated by

$$\boldsymbol{\Psi}(k-1) = \mathbf{X}_{\substack{actual\\topical}}(k) - \mathbf{A}(k-1) - \mathbf{B}(k-1)\mathbf{u}(k-1) \tag{8}$$

With the assumption that the dynamics of perturbation is considerably slower than discretization frequency and the order of the perturbation magnitude is much smaller, the estimation is valid.

Now we have defined control laws (IDC, SMC) and we can discuss the solution of their expressions.

2.3 *Solution of backlashes by pseudoinversion*

Consider now the eq. (3) and eq. (5) in the case that the inverse operation can't be provided i.e.

$$\mathbf{B}(\mathbf{y})\mathbf{u} = \mathbf{q} + \mathbf{f}(\mathbf{y}, \dot{\mathbf{y}}) \quad \text{and} \quad \tilde{\mathbf{B}}\mathbf{u}(k) = \mathbf{F}(\mathbf{X}, \mathbf{X}_d) \tag{9}$$

these equations have the same form as the ordinary system of the linear equations:

$$\mathbf{Ax} = \mathbf{B} \tag{10}$$

and it has an infinite number of solution. It is caused by deficient rank of matrix **A** (~ $\mathbf{B}(\mathbf{y})$, $\tilde{\mathbf{B}}$).

The approach for removing the backlashes is based on computation of the pseudoinverse operation and on the idea of the non-changing signs of the torques during the robot movement along the certain finite trajectory. The latter means that the switching of the torque signs disappears and the problem with backlashes should not exist.

The computation of the pseudoinverse operation gives the solution of the minimal length and some certain number of free parameters, which are used for change of undesirable signs of torques. This way, we obtain suitable solution, but it must be noted that this solution is not the same in the magnitude and it costs some additional energy and thus at least more powerful drives.

For showing the algorithm of pseudoinverse with eq. (10) the following theorems (Lawson 1974) are needed.

Theorem I.:

Suppose that **A** is an $m \times n$ matrix of rank k and that $\mathbf{A} = \mathbf{HRK}^{\mathrm{T}}$ where **H** is an $m \times m$ orthogonal matrix, **R** is an $m \times n$ matrix of the form $\mathbf{R} = \begin{bmatrix} \mathrm{R}_{11} & 0 \\ 0 & 0 \end{bmatrix}$ with $k \times k$ submatrix R_{11} of rank k and **K** is an $n \times n$ orthogonal matrix. Define the vector $\mathbf{H}^T\mathbf{b} = \mathbf{g} \equiv \begin{bmatrix} g_1 \\ g_2 \end{bmatrix} \begin{matrix} \}k \\ \}m-k \end{matrix}$ and introduce the new variable $\mathbf{K}^T\mathbf{x} = \mathbf{y} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \begin{matrix} \}k \\ \}n-k \end{matrix}$. And finally define $\widetilde{y}_1$ to be the unique solution of $\mathrm{R}_{11} y_1 = g_1$ then all solutions of system equations are of the form $\widehat{\mathbf{x}} = \mathbf{K} \begin{bmatrix} \widetilde{y}_1 \\ y_2 \end{bmatrix}$ where y_2 is arbitrary.

Note: This arbitrary vector is used for solution of backlashes.

Theorem II.:

Let **A** be an $m \times n$ matrix of rank k then there is an $m \times m$ orthogonal matrix **H** and an $n \times n$ orthogonal matrix **K** such that $\mathbf{H}^{\mathrm{T}}\mathbf{AK} = \mathbf{R}$, $\mathbf{A} = \mathbf{HRK}^{\mathrm{T}}$ where $\mathbf{R} = \begin{bmatrix} \mathrm{R}_{11} & 0 \\ 0 & 0 \end{bmatrix}$ and submatrix R_{11} is an $k \times k$ nonsingular triangular matrix.

These theorems give the mathematical relations on which the following algorithm is based.

$$\mathbf{QAP} = \mathbf{R} \equiv \begin{bmatrix} R_{11} & R_{12} \\ 0 & R_{22} \end{bmatrix} \begin{matrix} \}k \\ \}m-k \end{matrix} \qquad \begin{matrix} \underbrace{\quad}_{k} & \underbrace{\quad}_{n-k} \end{matrix} \tag{11}$$

$$\mathbf{Q}\,\mathbf{b} \;= \mathbf{c} \equiv \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} \begin{matrix} \}k \\ \}m-k \end{matrix} \tag{12}$$

$$[\mathrm{R}_{11}\ \ \mathrm{R}_{12}]\,\mathbf{K} = [\mathrm{W}\ 0]\ \ (\equiv \widetilde{\mathbf{A}}\,\mathbf{K} = [\mathrm{W}\ 0]) \tag{13}$$

$$\mathrm{W} y_1 = c_1 \qquad \Rightarrow\ y_1 \tag{14}$$

$$\mathbf{x} = \mathbf{PK} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \equiv \mathbf{PKy} \tag{15}$$

y_2 is arbitrary

$$\text{and} \quad \|\mathbf{b} - \mathbf{Ax}\| = \|c_2 - \mathrm{R}_{22} y_2\| \tag{16}$$

Note that expression (if $y_2 = 0$) $= \|c_2\|$.

The algorithm uses the orthogonal matrix **Q** and the permutation matrix **P** so that **R** is upper triangular and R_{11} is nonsingular. It was the first step to the solution of the backlashes. Now the sequence of suitable choice of an arbitrary vector y_2 follows.

The solution (15) can be divided for once redundantly determined system

$$\mathbf{x} = [\underbrace{\mathrm{PK}_{\mathrm{sub1}}}_{n\times n-1}\quad \underbrace{\mathrm{PK}_{\mathrm{sub2}}}_{n\times 1}] \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \begin{matrix} \}\,n-1\times 1 \\ \}\,1\times 1 \end{matrix} \tag{18}$$

$$\begin{aligned} \mathbf{x} &= \ \mathrm{PK}_{\mathrm{sub1}} \cdot y_1 \quad + \mathrm{PK}_{\mathrm{sub2}} \cdot y_2 \\ \mathbf{u} &= \underbrace{\mathbf{u}_1 (= \mathbf{x} \mid y_2 = 0)}_{\text{the minimal solution}} + \underbrace{\mathbf{u}_2 \quad \cdot y_2}_{\text{arbitrary part of the solution}} \end{aligned} \tag{19}$$

For defined safety bounds around the zero on torque axis, which can be represented by vector

$$\mathbf{rl} = [rl_1, rl_2, rl_3, rl_4]^{\mathrm{T}} \tag{20}$$

with meaning arising from Fig.2. ,

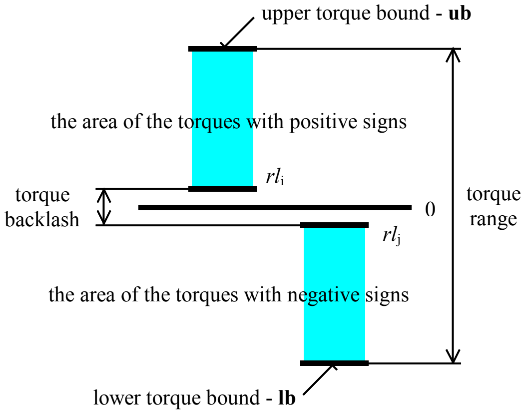


Fig. 2. The definition of backlashes and bounds.

the following sequence is accomplished.

Step1. find $u_1(i) \notin < rl_i, ub >$ for req. positive signs
$u_1(i) \notin < lb, rl_j >$ for req. negative signs

Step2. for such $\mathrm{u}_1(i)$ compute $y_2(i) = \dfrac{rl(i) - u_1(i)}{u_2(i)}$
(eq.(19) with substitution rl(i) for u).

Step3. for these $y_2(i)$ compute U(i) $\mathrm{U}(i) = \mathbf{u}_1 + \mathbf{u}_2 \cdot y_2(i)$

Step4. choice $\mathrm{U}^*(i)$ which satisfies $\|rl - \mathrm{U}(i)\| = \min$

Step5. check $\mathrm{U}^*(i)$ with considering to bounds (Fig.2) and provide hard restrictions.

Then final **u** equals $\mathrm{U}^*(i)$. Such result does not markedly change the properties of the robot control process, however, it changes the magnitude of all torques.

The minimal solution requires the minimum supply of the drive energy against result (Step5.) satisfying the antibacklash condition, where the required drive energy increases severalfoldly. The comparison will be shown in section 4.

Note that the sequence is valid for rank(**A**) = $k<m<n$. However, for rank($\widetilde{\mathbf{A}}$) = $m<n$ it is possible to use only shortened sequence from eq.(13) to eq. (15) without permutation **P**.

3 QUADRATIC PROGRAMMING SOLUTION

This section generally deals with adequately actuated systems, which must perform the condition of antibacklashes. It concerns only the simple example of the utilization of the quadratic programming (QP).

Firstly, the Generalized Predictive Control (GPC) is introduced, in which the antibacklash condition is implemented by QP algorithm.

3.1 Generalized predictive control

The Generalized predictive control (Ordys 1993) is a multi-step control based on local optimization of the quadratic criterion. This approach combines feedback-feedforward relation. For quadratic criterion, the linearized discrete state formula must be prepared (linearization, Valášek 1999) e.g. in this classical form:

$$\begin{aligned} \mathbf{X}(k+1) &= \mathbf{A}\,\mathbf{X}(k) + \mathbf{B}\,\mathbf{u}(k) \\ \mathbf{y}(k) &= \mathbf{C}\,\mathbf{X}(k) \end{aligned} \tag{21}$$

The base of predictive control is the expression of new unknown output values **y** from actual topical state **X**. Now we consider the N step prediction of **y** as follows

$$\hat{\mathbf{y}} = \mathbf{G}\,\mathbf{u} + \mathbf{f} \tag{22}$$

$$\text{where } \mathbf{G} = \begin{bmatrix} \mathbf{C} & \mathbf{B} \cdots & 0 \\ \vdots & \ddots & \vdots \\ \mathbf{CA}^{N-1}\mathbf{B} & \cdots & \mathbf{CB} \end{bmatrix} \text{ and } \mathbf{f} = \begin{bmatrix} \mathbf{CA} \\ \vdots \\ \mathbf{CA}^{N} \end{bmatrix} \mathbf{X}(k) \tag{23}$$

and further the quadratic criterion

$$\begin{aligned} J_k &= \mathcal{E}\left\{(\hat{\mathbf{y}} - \mathbf{w})^T (\hat{\mathbf{y}} - \mathbf{w}) + \mathbf{u}^T \boldsymbol{\lambda}\,\mathbf{u}\right\} = \\ &= \mathcal{E}\left\{(\mathbf{G}\,\mathbf{u} + \mathbf{f} - \mathbf{w})^T (\mathbf{G}\,\mathbf{u} + \mathbf{f} - \mathbf{w}) + \mathbf{u}^T \boldsymbol{\lambda}\,\mathbf{u}\right\} \end{aligned} \tag{24}$$

at certain instant k, with using N step prediction $\hat{\mathbf{y}}$. $\mathcal{E}$ is operator of mean value and $\boldsymbol{\lambda}$ is a penalization of actuator **u**.

On condition $J_k \overset{!}{=} \min$, we obtain the control law:

$$\mathbf{u} = \left(\mathbf{G}^T\mathbf{G} + \boldsymbol{\lambda}\right)^{-1} \mathbf{G}^T (\mathbf{w} - \mathbf{f}) \tag{25}$$

which can be already used.

It must be noted that only the first element $\mathbf{u}_k$ from vector **u** is used. If penalization $\boldsymbol{\lambda}$ is greater than zero, the matrix $\mathbf{G}^{\mathbf{T}} \cdot \mathbf{G}$ is regular and the problem with redundant action disappears.

The following subsection will take into account this case and it will show the utilization of the Quadratic programming for the specific category (as in the previous section) of the constraints – antibacklash condition.

3.2 Solution of backlashes by quadratic programming

There is only short introduction of the Quadratic programming here, because it has been already introduced in detail e.g. in the paper (Gill 1977). The main concern is how to form the constraining antibacklash inequalities.

Standard task of the Quadratic programming minimizes the quadratic purposive function with some linear constraints.

$$\underset{\mathbf{x}}{\text{minimize}} \left\{ \mathrm{F(x)} = \frac{1}{2}\mathbf{x}^{\mathrm{T}}\mathbf{H}\mathbf{x} + \mathbf{f}^{\mathrm{T}}\mathbf{x} \right\} \tag{26}$$

$$\text{subject to } \mathbf{A} \cdot \mathbf{x} \le \mathbf{b} \tag{27}$$

where **H** is an *n* x *n*, **f** an *n* vector, **A** is an *m* x *n* matrix and **b** is an *m* vector. The function F(**x**) is obtained from quadratic criterion eq. (24) as follows

$$\mathrm{F}(\mathbf{u}) = \frac{1}{2}\mathbf{u}^T \underbrace{\left(\mathbf{G}^T\mathbf{G} + \boldsymbol{\lambda}\right)}_{\mathbf{H}} \mathbf{u} + \underbrace{(\boldsymbol{f} - \mathbf{w})^T \mathbf{G}}_{\mathbf{f}^T} \mathbf{u} \tag{28}$$

Let us consider pair of torques:

$$\mathbf{u} = \begin{bmatrix} u(i) \in < rl(i), ub > \\ u(j) \in < lb, rl(j) > \end{bmatrix} \tag{29}$$

then the structure of the inequalities eq. (27) is such as this

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -1 & 0 \\ 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} u(i) \\ u(j) \end{bmatrix} \le \begin{bmatrix} ub \\ rl(j) \\ -rl(i) \\ -lb \end{bmatrix} \tag{30}$$

$$\mathbf{A} \quad \cdot \quad \mathbf{u} \quad \le \quad \mathbf{b}$$

After satisfying of all assumptions, the Quadratic programming gives always some solution, which is not optimal but the found solution of the full rank problem has the smallest aberration that can be attainable.

4 SIMULATION EXAMPLE AND CONCLUSION

For the simulation of the robot, some plan of the trajectory must be prepared and must be realizable for the robot. For example in our simulation, the trajectory composed of bisector segments and arc segments was chosen. The trajectory was time-parameterized with constant period. That is the matter of choice.

During the trajectory planning, the kinematic laws have been considered i.e. as a relationship between acceleration, velocity and position.

One example of the desired trajectory and its kinematic characterizations are shown in Fig. 3.

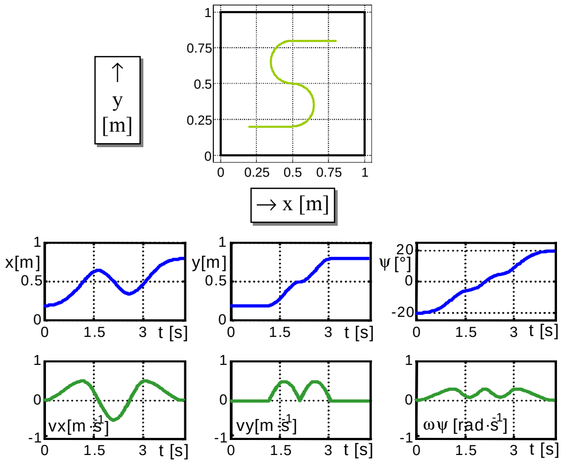


Fig.3. The desired trajectory with the kinematic characteristics.

As a test example, we consider one type of the redundantly actuated planar parallel robot (Fig.4).

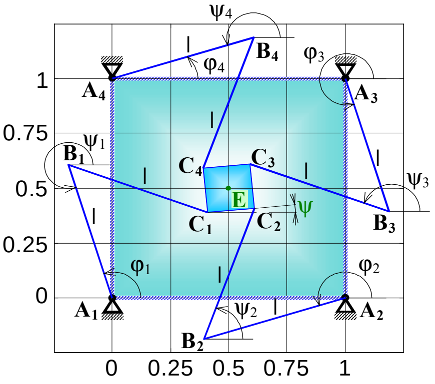


Fig. 4. Scheme of planar parallel robot with the most important geometrical description (The coordinates of center E of movable platform and its angle of winding ψ).

This configuration partly solves the question of moving masses, because all or almost all drives are located on the basic frame (i.e. the drives do not move with the robot). Moreover truss (parallel) construction of the robot leads to higher stiffness than in serial types. It is advantageous for accurate machining and positioning.

For the described trajectory above, the time histories of four torques are shown in Fig.5., firstly for unconstrained case and consecutively with satisfying of antibacklash condition for rl=[-1,1,-1,1].

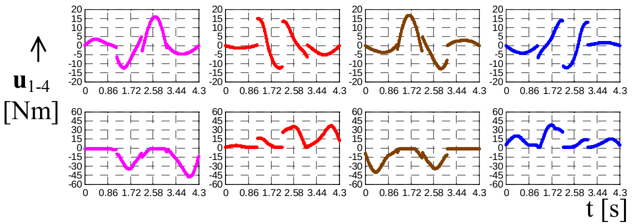


Fig.5. Unconstrained and constrained control (the time histories of four robot torques).

Therefore, the difference between the severable introduced approaches is not appreciable, the time histories of the aberrations are only compared.

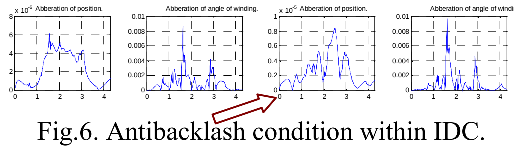


Fig.6. Antibacklash condition within IDC.

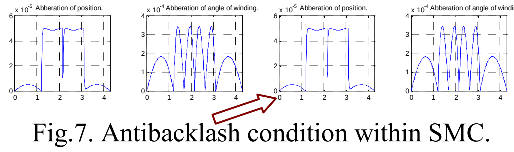


Fig.7. Antibacklash condition within SMC.

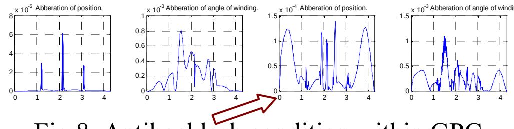


Fig.8. Antibacklash condition within GPC.

From figures it is appreciable that the former approach - solution of the deficient rank problem gives better result than the latter approach - quadratic programming. It is caused by fact that the first gives the optimal exact solution while the second gives only a suboptimal solution without utilization of the redundant property of the robot construction.

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5 REFERENCES

Ordys, A., Clarke, D. (1993). A state – space description for GPC controllers. INT. J. Systems SCI., vol.24, no.9, 1727 – 1744.

Valášek, M., Steinbauer, P. (1999). Nonlinear control of multibody systems. Euromech 404, 437 – 444.

Sciaviccio, L., Siciliano, B. (1996). Modeling and control of robot manipulators. New York: McGraw-Hill Co.

Elmali, H., Olgac, N. (1992). Sliding mode control with perturbation estimation (SMCPE): a new approach, INT. J. CONTROL, vol.56, no.4, 923-941.

Lawson, Ch., Hanson, R.J. (1974). Solving least squares problems Prentice-Hall, Inc.

Gill, P.E., Murray, W. (1977). Numerically stable methods for Quadratic programming. North-Holland Publish Company, Mathematical Programming 14, 349-372.

Böhm, J., Belda, K., Valášek, M. (2001). Study of control of planar redundant parallel robot. Proceedings of the IASTED Int. conference MIC 2001, 694-699.