DECENTRALIZED AND CENTRALIZED CONTROL OF REDUNDANT PARALLEL ROBOT CONSTRUCTIONS

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Abstract

The fundamental task of the parallel robot constructions, especially redundantly actuated, is to provide effective and safe cooperation of all drives - actuators. This paper summarises the set of the available control approaches adjusted to redundant case: from simple decentralized control (PID (/PSD) controller with reduction of the unproductive part of Integral/Sum channels), to the simple centralized control (PID (/PSD) control with redistribution of adequate resultant fictitious force effects to really - used redundant actuators), and one example of the high level control approach (Generalized Predictive Control - GPC).

1 Introduction

The parallel robot constructions, in comparison with serial open-loop types, achieve higher stiffness, high load capacity, lower mass inertia etc. These properties, among others, predetermine the robots to the use within more powerful industrial applications performing accurate machining and positioning. Fundamental task of such parallel robot constructions, especially redundantly actuated, is how to provide effective and safe cooperation of all drives - actuators. This paper discusses and investigates the available approaches to the control adjusted to the redundant case:

- simple decentralized control (the independent PID (/PSD) control with reduction of the unproductive part of I/S channels of the controller),
- simple centralized control (the independent PID (/PSD) control with redistribution of adequate resultant fictitious actuators to the really used redundant drive configuration),
- and one example of the high level control approach (Generalized Predictive Control - GPC; ensuring the optimal cooperation of all actuators both adequate and redundant).

The robots - manipulators are multibody systems, which can be described by Lagrange's equations, in redundant case, of mixed type. These equations lead to the differential - algebraic equations (DAE) in the following form:

$$\begin{aligned} \mathbf{M}\ddot{\mathbf{s}} - \mathbf{\Phi}_s^T \boldsymbol{\lambda} &= \mathbf{g} + \mathbf{T}\mathbf{u} \\ \mathbf{f}(\mathbf{s}(t)) &= \mathbf{0} \end{aligned} \tag{1}$$

where $\mathbf{M}$ is a mass matrix, $\mathbf{s}$ is a vector of physical coordinates (their number is higher than the number of degrees of freedom), $\mathbf{\Phi}_s$ is an overall Jacobian of the system, $\boldsymbol{\lambda}$ are Lagrange's multipliers, $\mathbf{g}$ is a vector of right sides, matrix $\mathbf{T}$ transforms the inputs $\mathbf{u}$ (n torques) into n drives and $\mathbf{f}(\mathbf{s}(t)) = \mathbf{0}$ represents geometrical constrains.

The physical coordinates $\mathbf{s}$ consist of the independent coordinates $\mathbf{x}$ (Cartesian coordinates of the fix point of the cutting tool or gripper), drives' (actuators') coordinates $\mathbf{q}_1$ and other auxiliary geometrical coordinates $\mathbf{q}_2$.

Let us consider the possibility to transform the model (1) into independent coordinates $\mathbf{x}$ [3]. As follows, the DAE robot model is transformed to the ordinary differential model (ODE). It means that the Lagrange's multipliers disappear and design of the robot control becomes considerably simpler. The resulting model of the robot system is the following:

$$\mathbf{R}^T\mathbf{MR}\ddot{\mathbf{x}} + \mathbf{R}^T\mathbf{M}\dot{\mathbf{R}}\dot{\mathbf{x}} = \mathbf{R}^T\mathbf{g} + \mathbf{R}^T\mathbf{Tu} \tag{2}$$

It is very important to note, firstly, that the Jacobian matrix $\mathbf{R}$ is the basis of the null space of the overall Jacobian $\mathbf{\Phi}_s$ and thus it satisfies the expression

$$\mathbf{\Phi}_s\mathbf{R} = \mathbf{R}^T\mathbf{\Phi}_s^T = \mathbf{0} \text{ and } \dot{\mathbf{s}} = \mathbf{R}\dot{\mathbf{x}} \rightarrow \ddot{\mathbf{s}} = \mathbf{R}\ddot{\mathbf{x}} + \dot{\mathbf{R}}\dot{\mathbf{x}} \tag{3}$$

and, secondly, the Jacobian $\mathbf{R}$ can be decomposed into submatrices $\mathbf{R}_{\mathbf{q}^1}$, $\mathbf{R}_{\mathbf{q}^2}$ and $\mathbf{R}_\mathbf{x}=\mathbf{I}_\mathbf{x}$. Relation between $\dot{\mathbf{q}}_1$ and $\dot{\mathbf{x}}$ expressed as

$$\dot{\mathbf{q}}_1 = \mathbf{R}_{\mathbf{q}1}\dot{\mathbf{x}} \quad \left(\equiv \frac{d\mathbf{q}_1}{dt} = \mathbf{R}_1 \cdot \frac{d\mathbf{x}}{dt}\right) \tag{4}$$

will be useful in the following sections, describing simple decentralized and centralized control. $\mathbf{R}_1$ can be obtained either directly as null space of $\mathbf{\Phi}_s$ [2] or from geometrical relation like

$$\mathbf{R}_1 = \left[\frac{\partial \mathbf{q}_1(\mathbf{x})}{\partial x_1}, \cdots, \frac{\partial \mathbf{q}_1(\mathbf{x})}{\partial x_n}\right]\Bigg|_{\substack{n = \text{number of independent} \\ \text{coordinates } = \\ = \text{ degrees of freedom}}} \tag{5}$$

2 Simple decentralized control

The simplest control strategy, which can be taken into account, is view on the robots – manipulators, powered by group of the independent systems (drives - actuators), controlled separately, as a set of single-input / single-output systems. The mutual interactions among all actuators due to varying configurations during the robot's motion are involved as disturbance in each system. The graphical representation corresponds with the classical PID/PSD feedback control. However, in case of the parallel robots, mainly redundantly actuated, some problem of the unproductive part of integral/sum control channels must be solved. It does not occur at serial open-loop structures. One example of the planar classical and parallel structure is in Figure 1.

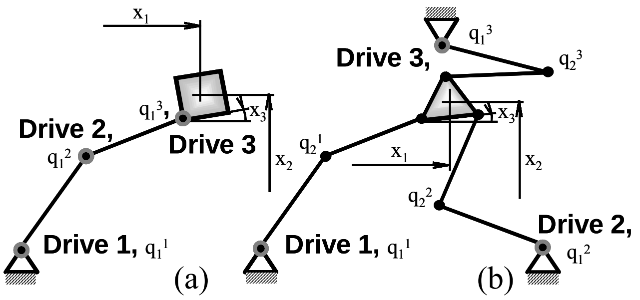

Figure 1: Example of classical (a) and parallel (b) planar robot structure.

Undesirable unproductive part of I/S channels is caused by inaccuracies in mechanism. It means that the drive coordinates $\mathbf{q}_1$ designed from independent (Cartesian) coordinates $\mathbf{x}$ in certain cases cannot be attainable. This causes unpredictable increase of I/S channels, which does not contribute to motion and moreover leads to instability of the whole robot system. Therefore, some additional block must be added to the control circuit to reduce this undesired property.

From mathematical point of view the following problem is solved:

$$\underset{\min|\mathbf{u}|}{\mathbf{A}\ \mathbf{u}\ =\ \mathbf{b}} \tag{6}$$

where $\mathbf{A}$ is $\mathbf{R}^T\mathbf{T}$, generally horizontal rectangular matrix and $\mathbf{u}$ are actuators – inputs to the robot system (desiderative moments) and $\mathbf{b}$ is a vector of generalized force effects. The subsequent lines show how to obtain reducing projection in view of $\mathbf{u}$.

The quadratic criterion is used and optimised:

$$J = \frac{1}{2}\mathbf{u}^T\mathbf{u} + \boldsymbol{\lambda}^T(\mathbf{Au} - \mathbf{b}) \overset{!}{=} \min \tag{7}$$

$$\frac{\partial J}{\partial \mathbf{u}} = \mathbf{0} \rightarrow \mathbf{u} + \mathbf{A}^T\boldsymbol{\lambda} = \mathbf{0} \Rightarrow \mathbf{u} = -\mathbf{A}^T\boldsymbol{\lambda} \tag{8}$$

$$\frac{\partial J}{\partial \boldsymbol{\lambda}} = \mathbf{0} \rightarrow \mathbf{Au} - \mathbf{b} = \mathbf{0} \tag{9}$$

by substituting (8) to (9) the parameter $\boldsymbol{\lambda}$ is obtained

$$\boldsymbol{\lambda} = -\left(\mathbf{AA}^T\right)^{-1}\mathbf{b} \tag{10}$$

and by back substitution to (8) when considering the start Equation (6), the final result is:

$$\mathbf{u}_{red} = \mathbf{A}^T\left(\mathbf{AA}^T\right)^{-1}\mathbf{Au}, \quad \text{with assumption that } |\mathbf{A}^T\left(\mathbf{AA}^T\right)^{-1}\mathbf{A}| \le |\mathbf{I}| \tag{11}$$

Now it is possible to see the described theory from graphical point of view. The projection is applied only on I/S channels of the controller, which can be construed as an independent parallel configuration of single PID controllers – Figure 2.

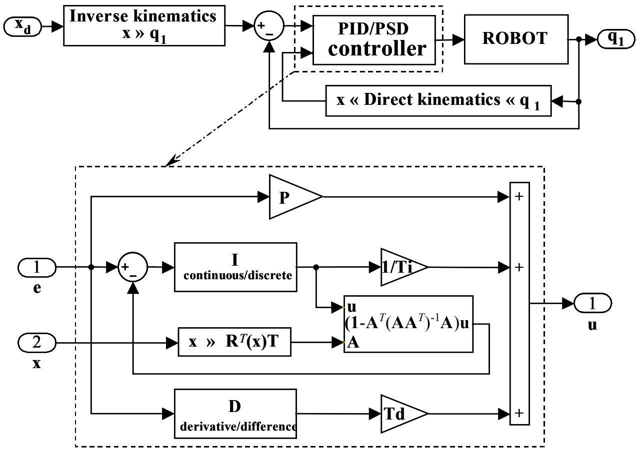

Figure 2: Simple decentralized control circuit.

Figure 3 compares the time histories in both ideal, geometrically accurate, case (c, d) and in real case (a, b) with geometrical inaccuracies.

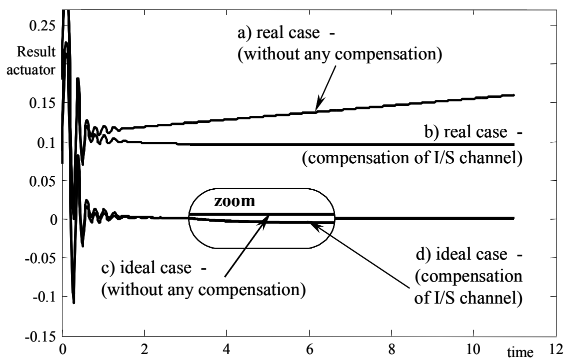

Figure 3: Presumptive trend of one actuator in stop sequence (example of the compensation).

In the ideal situation (c) the integral/sum part levelled off at certain magnitude, which was integrated during the whole control process. In case (d) this unproductive part is reduced/compensated to zero value. The cases (a, b) are caused by integration/sum of the lasting fictitious control error, which appears from geometrical inaccuracies in parallel construction (interaction of the actuators in close-loop systems). (Note: in serial open-loop robot constructions this undesirable property does not occur).

3 Centralized control

The described decentralized control in the previous section takes into account interactions and connection effects among all parts of the robot construction as disturbances influencing in each single drive system.

However, as shown by the dynamic model Equation (2), the robot-manipulator is not a set of n independent systems, but it is one multibody system with m inputs (drives - actuators) and n outputs (the independent Cartesian coordinates; n is equal to the number of degrees of freedom) interacting among them by means of the nonlinear kinematic and dynamic relations.

The first subsection will show the simple centralized control, which is formed from classical PID cascade control. This approach is a connecting link between decentralized and centralized control.

And the second subsection will introduce one of the possible high-level control approaches – Generalized Predictive Control (GPC) derived for redundant parallel robot construction.

3.1 Simple centralized control

Although this control approach uses the same parallel (independent) PID/PSD configuration, it controls different signals.

The approach is based on the control of the independent Cartesian coordinates $\mathbf{x}$. The controller designs fictional actuators acting directly in the fix point of the tool (or gripper). These fictional actuators are consecutively recomputed to the appropriate values. They are expecting from the drives, in order to perform the desired movement. Figure 4 shows this situation.

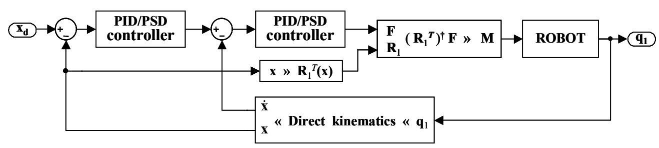

Figure 4: Simple centralized control.

Utilization of the centralized control has one important advantage. The all-independent Cartesian coordinates within workspace of the robot are always achievable and they do not depend on any recomputation. Thus, there does not occur any unpredictable increase of I/S channels, which can damage the drives.

In the independent Cartesian space, the generalized forces/moments **F** are formed. They all together give an overall fictive effect needed for required movement of the robot. This effect is redistributed on real values **M** pursued by the drives. This redistribution is based on the principle of the virtual works. The following lines imply this.

The derivation is firstly focused on inverse kinematics. Let us consider the relation between Cartesian and drives' coordinates:

$$\mathbf{q}_1 = f_1(\mathbf{x}) \tag{12}$$

and its time derivation:

$$\dot{\mathbf{q}}_1 = \mathbf{R}_{\mathbf{q}1}\dot{\mathbf{x}} \quad (\equiv \frac{d\mathbf{q}_1}{dt} = \mathbf{R}_1 \cdot \frac{d\mathbf{x}}{dt}) \tag{13}$$

$$\mathbf{R}_1 = \left[\frac{\partial \mathbf{q}_1(\mathbf{x})}{\partial x_1}, \cdots, \frac{\partial \mathbf{q}_1(\mathbf{x})}{\partial x_n}\right]\Bigg|_{\substack{n = \text{number of independent} \\ \text{coordinates=} \\ = \text{ degrees of freedom}}} \tag{14}$$

Now the principle of the virtual works is applied (in matrix form):

$$\mathbf{M}^T\mathbf{q}_1 = \mathbf{F}^T\mathbf{x} \tag{15}$$

$$\mathbf{M}^T\Delta\mathbf{q}_1 = \mathbf{F}^T\Delta\mathbf{x} \rightarrow \mathbf{M}^T d\mathbf{q}_1 = \mathbf{F}^T d\mathbf{x}\Big|_{\substack{\Delta\mathbf{q}_1 \to 0 \Rightarrow \Delta\mathbf{q}_1 = d\mathbf{q}_1 \\ \Delta\mathbf{x} \to 0 \Rightarrow \Delta\mathbf{x} = d\mathbf{x}}} \tag{16}$$

$$\mathbf{M}^T \frac{d\mathbf{q}_1}{d\mathbf{x}} = \mathbf{F}^T \Leftrightarrow \left(\frac{d\mathbf{q}_1}{d\mathbf{x}}\right)^T \mathbf{M} = \mathbf{F} \Leftrightarrow \mathbf{R}_1^T\mathbf{M} = \mathbf{F} \tag{17}$$

Since the matrix $\mathbf{R}_1{}^T$ is not generally square, the Equation (17) can be solved by the right pseudoinversion

$$\mathbf{M} = \mathbf{R}_1\left(\mathbf{R}_1^T\ \mathbf{R}_1\right)^{-1}\mathbf{F} \tag{18}$$

or by pseudoinversion uses the orthogonal-triangular decomposition [5].

3.2 High level control approach - GPC

High level controls use knowledge of the dynamic model of the system (2) and they globally optimise whole control design. One of them is Generalized Predictive Control (GPC).

The Predictive control [4, 2] is a multi-step control based on local optimisation of the quadratic criterion, where the linarized equation or state formula is used (i.e. only the nearest future control signal is evaluated). This approach admits combination of feedback~feedforward parts.

As mentioned above, for the quadratic criterion, the nonlinear model (2) must be linearized [6] and converted from continuous to discrete time. This model transformation enables us to consider the discrete state formula in the following form:

$$\begin{aligned} \mathbf{X}(k+1) &= \mathbf{A}\,\mathbf{X}(k) + \mathbf{B}\,\mathbf{u}(k) \\ \mathbf{x}(k) &= \mathbf{C}\,\mathbf{X}(k) \end{aligned} \tag{19}$$

where **X** is composed as $\mathbf{X} = [\mathbf{x}, \dot{\mathbf{x}}]^T$ and **x** agrees with Equation (2). The base of predictive control is the expression of new unknown output values **x** from actual topical state **X**. The following lines imply it.

$$\begin{aligned} \mathbf{x}(k) &= \mathbf{C}\ \ \mathbf{X}(k) \\ \mathbf{X}(k+1) &= \mathbf{A}\ \ \mathbf{X}(k) + \qquad \mathbf{B}\,\mathbf{u}(k) \\ \hat{\mathbf{x}}(k+1) &= \mathbf{CA}\ \ \mathbf{X}(k) + \mathbf{C}\quad \mathbf{B}\,\mathbf{u}(k) \\ &\ \ \vdots \qquad\qquad\qquad \vdots \\ \hat{\mathbf{X}}(k+N) &= \mathbf{A}^N\mathbf{X}(k) + \mathbf{A}^{N-1}\mathbf{B}\,\mathbf{u}(k) + \cdot + \mathbf{B}\,\mathbf{u}(k+N-1) \\ \hat{\mathbf{x}}(k+N) &= \mathbf{CA}^N\mathbf{X}(k) + \mathbf{CA}^{N-1}\mathbf{B}\,\mathbf{u}(k) + \cdot + \mathbf{CB}\,\mathbf{u}(k+N-1) \end{aligned} \tag{20}$$

then the prediction of **x** is the following

$$\hat{\mathbf{x}} = \mathbf{G}\,\mathbf{u} + \mathbf{f} \tag{21}$$

$$\mathbf{G} = \begin{bmatrix} \mathbf{C} & \mathbf{B} & \cdots & \mathbf{0} \\ \vdots & & \ddots & \vdots \\ \mathbf{CA}^{N-1}\mathbf{B} & & \cdots & \mathbf{CB} \end{bmatrix} \text{ and } \mathbf{f} = \begin{bmatrix} \mathbf{CA} \\ \vdots \\ \mathbf{CA}^N \end{bmatrix}\mathbf{X}(k) \tag{22}$$

Now we have enough information for performance of the optimisation of the quadratic criterion.

The quadratic criterion is optimised at certain instant k, using predictions of $\mathbf{x}$ ($\hat{\mathbf{x}} = [\hat{\mathbf{x}}_{k+1} \cdots \hat{\mathbf{x}}_{k+N}]^T$)

$$J_k = \mathcal{E}\left\{(\hat{\mathbf{x}} - \mathbf{w})^T (\hat{\mathbf{x}} - \mathbf{w}) + \mathbf{u}^T \boldsymbol{\lambda}\, \mathbf{u}\right\} = \\ = \mathcal{E}\left\{(\mathbf{G}\,\mathbf{u} + \mathbf{f} - \mathbf{w})^T (\mathbf{G}\,\mathbf{u} + \mathbf{f} - \mathbf{w}) + \mathbf{u}^T \boldsymbol{\lambda}\, \mathbf{u}\right\} \tag{23}$$

where $\mathcal{E}$ is operator of mean value, N is horizon of prediction, $\mathbf{x}$ is vector of outputs, $\mathbf{w}$ are desired values, $\boldsymbol{\lambda}$ is penalization of input and $\mathbf{u}$ is vector of robot inputs. Condition is

$$J_k \overset{!}{=} \min \tag{24}$$

$$\Rightarrow \mathbf{u} = \left(\mathbf{G}^T\mathbf{G} + \boldsymbol{\lambda}\right)^{-1} \mathbf{G}^T (\mathbf{w} - \mathbf{f}) \tag{25}$$

This control law (25) can be already used. It must be noted that only the first element $\mathbf{u}_k$ from vector $\mathbf{u}$ is used. If penalization $\boldsymbol{\lambda}$ is greater than zero, the matrix $\mathbf{G}^T \cdot \mathbf{G}$ is regular and the problem with redundant action disappears. Theoretical case of zero penalization $\boldsymbol{\lambda}$ can be again solved by pseudoinvesion.

When the constraint of the actuators is required, the quadratic programming can be used [7].

Graphical representation of system with Predictive control is in Figure 5.

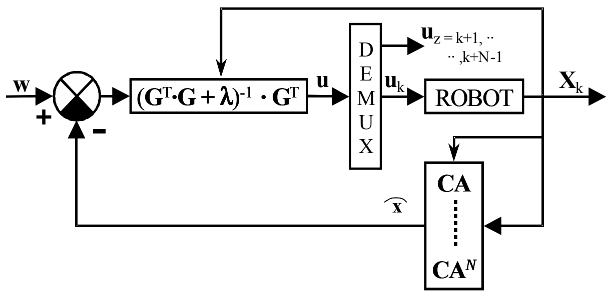

Figure 5: Centralized control (GPC control).

4 Illustrative tests on one prototypic parallel robot construction

For the real tests and simulations of the described control approaches, the realizable trajectory composed of the bisector segments and the arc segments was chosen.

The trajectory was time–parameterised with constant period. That is the matter of the choice.

During the trajectory planning, the kinematic laws have been considered i.e. as a relationship among acceleration, velocity and position.

The chosen desired trajectory and its kinematic characterizations are shown in Figure 6.

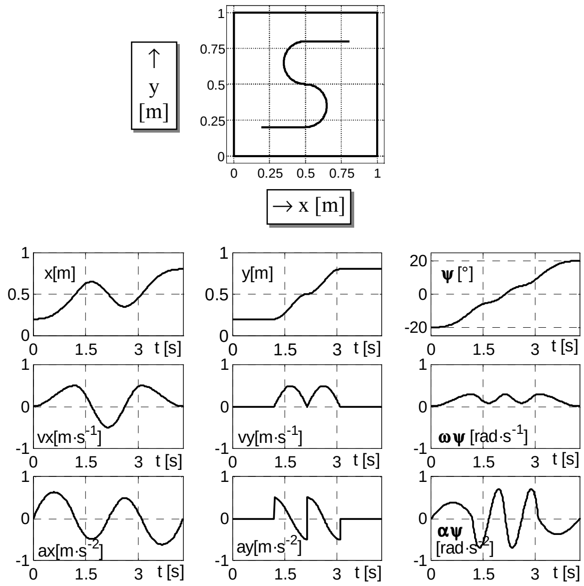

Figure 6: The desired trajectory with the kinematic characterisations: positions, velocities, accelerations.

As a test example of the parallel robot construction, let us consider one type of the redundantly actuated planar parallel robot construction - Figure 7.

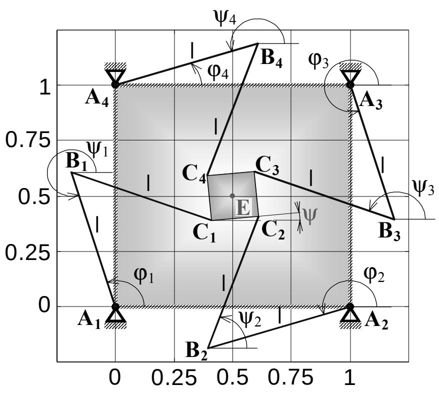

Figure 7: Scheme of one planar redundant parallel robot construction.

This configuration partly solves the question of moving masses, because all or almost all drives are located on the basic frame (i.e. the drives do not move with the robot). Moreover, truss (parallel) construction of the robot leads to higher stiffness than in serial types. It is advantageous especially for accurate machining and positioning.

For the described trajectory above, this section shows the time histories of four torques (actuators – drives) designed according to the described control approaches - Figure 8.

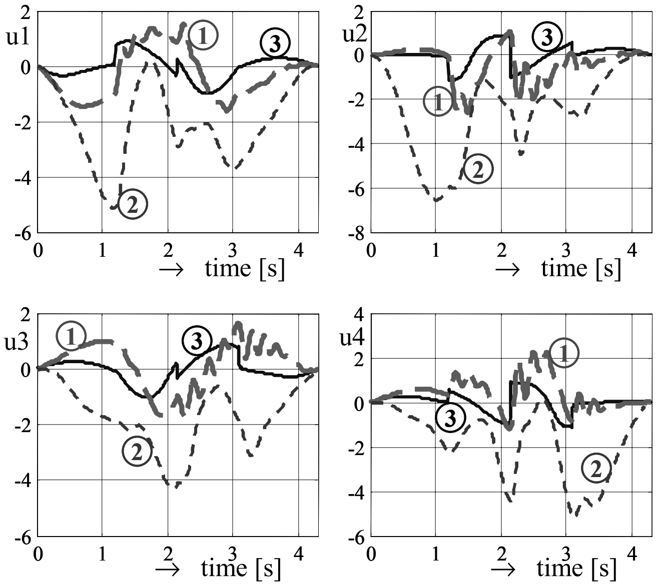

Figure 8: Time histories of simulation of four torques (values generated by drives-actuators); (1) decentralized PSD control (error $\approx 10^{-3}$m), (2) centralized PSD control (error $\approx 10^{-3}$m), (3) Predictive control (GPC) (error $\approx 10^{-5}$m).

In Figure 8, the time histories of torques are compared on four drives, located in the corners of the basic frame. Setting of the parameters of the controllers of each control approach is not the same task. Number of the setting parameters is also different. For tested parallel robot construction, Figure 4, the decentralized control has three parameters (although generally 4x3 P, I/S, D), but the character of control signals is the same, the 3 is sufficient.

The second approach, simple centralized control is not so simple for setting, because it generally represents 18 parameters, but four signals have similar character (position x, y and velocity $\dot{x}, \dot{y}$), therefore the number of parameters can be reduced on 12.

The last approach has the simplest setting, because it has only two parameters (horizon N_1 and penalization λ) and their choice is not dramatic problem as in the previous approaches. Predictive control achieves the best compliance of the planed trajectory ($\approx 10^{-5}$m), but it requires linear or linearized dynamic model of the robot.

5 Conclusion

This paper summarizes the set of the available control approaches adjusted for parallel robot constructions considering the supposable existence of redundant actuators. The paper briefly introduces application of simple decentralized and centralized approach.

The simple decentralized control are being successfully tested on the real robot application and the other controls, (after promising simulations) are under preparation also for real tests.

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