ADVANCED CONTROL FOR REDUNDANT PARALLEL ROBOTS

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Abstract: One of the trends in robotics is a study of parallel structures and their control, lending, in some situation, to control of systems with more inputs than outputs (over-actuated, drive-redundant systems). The simples control approach considered means taking the robots as a set of single input-output systems (setSISO); decentralized control design. As an auspicious alternative is model-based approach i.e. centralized control for instance Generalized Predictive Control (GPC). It pursues global design of control actions corresponding with actual requirement to robot movement. In the paper, the square-root form of GPC in both absolute and incremental algorithm is presented and compared with decentralized approach.

Keywords: Redundancy, PID/PSD control, Predictive Control, Control Insensitivity.

1 INTRODUCTION

Development of new robot constructions closely relates with the design of new approaches to their control. One of the topical trends in robotics is a study of certain promising parallel structures of the robots – manipulators and consecutively the design of their control, leading, in certain situation, to control of systems with more inputs than outputs (over-actuated or drive-redundant systems). Their concept is shown in Fig. 1.

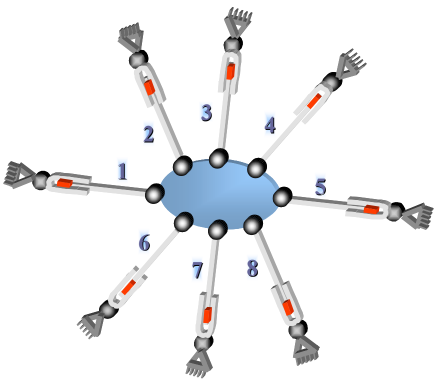


Fig. 1. Concept of the redundant parallel robots.

Parallel robots (structures) can be simply understood as movable truss constructions or as a movable platform, generally representing the place of gripper for fixing or gripping, supported by more beams. They are characterized by close loops, interconnected through the platform. This configuration affords significant improvement in stiffness, dynamics and accuracy of the robots. These properties, among others, predetermine the robots to the use within more powerful industrial applications performing accurate machining and positioning. Fundamental task of such parallel robot constructions, especially redundantly actuated, is how to provide effective and safe control of all drives - actuators.

The simples control approach (Sciavicco et al. 1996) considered means taking the robots (serial and parallel) as a set of single input-output systems (setSISO) – – decentralized design. Mutual interaction among “independent” inputs is considered as disturbances entering each system of setSISO of the structure.

As an auspicious alternative, model-based approach so-called centralized control comes forward. It pursues global design of control actions corresponding with actual requirement to robot movement - optimizes energy consumption. The approach provides cooperation of all drives in parallel structure even in redundant case. In the paper, as an illustration, the Generalized Predictive Control (GPC) is used (Ordys et al. 1993). It is presented in square-root form of both absolute and incremental algorithm.

The paper compares the mentioned approaches (decentralized and centralized) and shows results achieved on one laboratory robot prototype.

2 MODEL COMPOSITION

The important issue is a choice and arrangement of suitable model of the robot for given control method, used either for simulative control tests or for real control design. Since the robot is a mechanical body, the classical equations of motion and their suitable modification can be used.

The mathematical model can be composed on the base of Lagrange's equations, in general, of mixed type. These equations lead to the differential - algebraic equations (DAE) in the following form:

$$\begin{aligned} \mathbf{M}\ddot{\mathbf{s}} - \mathbf{\Phi}_s^T \boldsymbol{\lambda} &= \mathbf{g} + \mathbf{T}\mathbf{u} \\ \mathbf{f}(\mathbf{s}(t)) &= \mathbf{0} \end{aligned} \tag{1}$$

where $\mathbf{M}$ $(m \times m)$ is a mass matrix, $\mathbf{s}$ $(m \times 1)$ is a vector of physical coordinates (their number m is higher than the number of degrees of freedom n), $\mathbf{\Phi}_s$ $((m\text{-}n) \times m)$ is an overall Jacobian of the system, $\boldsymbol{\lambda}$ $((m\text{-}n) \times 1)$ are Lagrange's multipliers, $\mathbf{g}$ $(m \times 1)$ is a vector of right sides, $\mathbf{T}$ $(m \times r)$ is an unitary matrix adjusting the dimension of inputs $\mathbf{u}$ $(r \times 1)$, and $\mathbf{f}(\mathbf{s}(t)) = \mathbf{0}$ $((m\text{-}n) \times 1)$ represents geometrical constrains. For redundant case the number of inputs r is higher than the number of degrees of freedom n $(r > n)$.

The physical coordinates $\mathbf{s}$ consist of the independent coordinates $\mathbf{x}$ (Cartesian coordinates of the cutting tool or gripper), coordinates of drives (inputs) $\mathbf{q}_1$ and other auxiliary geometrical coordinates $\mathbf{q}_2$.

For control design, we search for the most compact notation in our case in independent coordinates. Let us consider the possibility of such transformation (Stejskal et al. 1996). As follows, the DAE robot model is transformed to the ordinary differential model (ODE). It means that the Lagrange's multipliers disappear and design of the robot control becomes considerably simpler. The resulting model of the robot system is the following:

$$\mathbf{R}^T\mathbf{M}\mathbf{R}\ddot{\mathbf{x}} + \mathbf{R}^T\mathbf{M}\dot{\mathbf{R}}\dot{\mathbf{x}} = \mathbf{R}^T\mathbf{g} + \mathbf{R}^T\mathbf{T}\mathbf{u} \tag{2}$$

That transformation is based on the Jacobian $\mathbf{\Phi}_s$, described by matrix $\mathbf{R}$ fulfiling

$$\mathbf{\Phi}_s\mathbf{R} = \mathbf{R}^T\mathbf{\Phi}_s^T = \mathbf{0} \tag{3}$$

and

$$\dot{\mathbf{s}} = \mathbf{R}\dot{\mathbf{x}} \;\rightarrow\; \ddot{\mathbf{s}} = \mathbf{R}\ddot{\mathbf{x}} + \dot{\mathbf{R}}\dot{\mathbf{x}} \tag{4}$$

wherefrom matrix $\mathbf{R}$ is obtained:

$$\mathbf{R} = \left[\frac{\partial \mathbf{s}(\mathbf{x})}{\partial x_1}, \cdots, \frac{\partial \mathbf{s}(\mathbf{x})}{\partial x_n}\right] \tag{5}$$

As there are more control actions $\mathbf{u}$ $(r \times 1)$ in redundant case, it is possible to introduce the independent force equivalent $\mathbf{h}$ $(n \times 1)$ in such way that

$$\mathbf{R}^T\mathbf{T}\mathbf{u} = \mathbf{h} \tag{6}$$

that is not uniquely solvable for $\mathbf{u}$ $(r \times 1)$, $\mathbf{h}$ $(n \times 1)$ and $r > n$. The model form (2) with modification (6) represents the suitable base not only for simulative tests but mainly for the following control design.

3 DECENTRALIZED CONTROL

The simplest control approach considered means taking the robots and manipulators, powered by group of independent drives /actuators/, separately controlled, as a set of single input - single output systems (setSISO) (Sciavicco et al. 1996). Mutual interactions among all drives, caused by different positions during the robot movement, are included as disturbances entering each "single" system constituting the robot.

In that view, the classical PID/PSD feedback control scheme can be used. If the scheme is applied, serious problem of mutual conflict of drives may occur (Valášek et al. 2002). It is indicated by unpredictable increase of integral/sum (I/S) channels in controller. Undesirable unproductive part of I/S channels is caused by the fact, that kinematic description is never perfect, i.e. it does not represent exactly the real kinematics of redundant parallel structure, given by production and partly by topical technological conditions.

Due to drive redundancy there exist no unique transformation between coordinates of drives $\mathbf{q}_1$ and independent coordinates $\mathbf{x}$ here. There exists only inverse relation $\mathbf{q}_1 = \boldsymbol{f}(\mathbf{x})$. It means that $\mathbf{q}_1$ coordinates are dependent. This relation is never fully matched in view of inaccurate dimensions of the robot. The PID/PSD controllers try to achieve zero errors for all dependent drive coordinates $\mathbf{q}_1$, but it is not possible. This fact causes the increase of I/S channels in controllers to saturation, i.e. undesirable increase of expected values in drives. Adding a new block into control circuit can solve this issue and reduce this undesirable property.

Idea of the solution is the following: local decentralized controllers compute magnitudes of actuators $\mathbf{u}$ for drives and then some operation as a certain projection is applied to these magnitudes. The projection transforms the actuators to independent space (i.e. it computes so-called general force effects), where the undesirable effects are eliminated, and consecutively the inverse projection recomputes them (free of unproductive components) back to required magnitudes of actuators in drives.

From mathematical viewpoint we solve the following task:

$$\underset{\min|\mathbf{u}|}{\mathbf{A}\;\mathbf{u}} = \mathbf{b} \tag{7}$$

where $\mathbf{A}$ $(n \times r)$ is $\mathbf{R}^T\mathbf{T}$, generally horizontal rectangular matrix (input matrix in robot model), $\mathbf{u}$ $(r \times 1)$ is a vector of actuators – inputs in robot system (e.g. magnitudes of requisite torques on shafts of drives /motors/), and $\mathbf{b}$ $(n \times 1)$ is a vector of general forces $\mathbf{h}$.

The following lines show derivation of the reductive projection (solution of the task (6) or (7) respectively) in view of $\mathbf{u}$.

The quadratic criterion can be used and by its optimization, the projection is obtained.

The form of the criterion is the following:

$$J=\frac{1}{2}\mathbf{u}^T\mathbf{u}+\boldsymbol{\lambda}^T(\mathbf{Au}-\mathbf{b})\overset{!}{=}\min \tag{8}$$

and its optimization:

$$\frac{\partial J}{\partial \mathbf{u}}=\mathbf{0}\rightarrow \mathbf{u}+\mathbf{A}^T\boldsymbol{\lambda}=\mathbf{0}\Rightarrow \mathbf{u}=-\mathbf{A}^T\boldsymbol{\lambda} \tag{9}$$

$$\frac{\partial J}{\partial \boldsymbol{\lambda}}=\mathbf{0}\rightarrow \mathbf{Au}-\mathbf{b}=\mathbf{0} \tag{10}$$

by insertion (9) to (10) we obtain the parameter $\boldsymbol{\lambda}$:

$$\boldsymbol{\lambda}=-(\mathbf{AA}^T)^{-1}\mathbf{b} \tag{11}$$

Then the back substitution of (11) into (9) with consideration of initial equation (7) gives the final result in the following form:

$$\mathbf{u}_{red}=\mathbf{A}^T(\mathbf{AA}^T)^{-1}\mathbf{Au} \quad (\text{for } \min_{|\mathbf{u}|}\mathbf{A\,u}=\mathbf{b}) \tag{12}$$

The proof of the result (12) for (7) is as follows. Let us use SVD decomposition of matrix $\mathbf{A}$: $\mathbf{A}=\mathbf{USV}^T$ (where $\mathbf{S}$ is diagonal and $\mathbf{U}$ and $\mathbf{V}^T$ are orthogonal matrixes). SVD decomposition helps us to simplify and to evaluate the product in (12).

$$\begin{aligned}\mathbf{A}^T(\mathbf{AA}^T)^{-1}\mathbf{A}&=\mathbf{VS}^T\mathbf{U}^T(\mathbf{USV}^T\mathbf{VS}^T\mathbf{U}^T)^{-1}\mathbf{USV}^T=\\&=\mathbf{VS}^T\mathbf{U}^T(\mathbf{USS}^T\mathbf{U}^T)^{-1}\mathbf{USV}^T=\mathbf{VS}^T(\mathbf{SS}^T)^{-1}\mathbf{SV}^T=\\&=\mathbf{VS}^{\dagger}\mathbf{V}^T\end{aligned} \tag{13}$$

If we analyze the product $\mathbf{VS}^{\dagger}\mathbf{V}^T$, thus $\mathbf{V}$ is orthogonal matrix and product $\mathbf{S}^{\dagger}$ $(r\times r)$ is diagonal and moreover unitary deficient rank matrix /note: $\mathbf{S}$ is $(n\times r)$/ i.e.:

$$\mathbf{A}^T(\mathbf{AA}^T)^{-1}\mathbf{A}=\mathbf{V}\ diag([1110])\ \mathbf{V}^T \tag{14}$$

On the base of (14) we can compare the norm of $\mathbf{u}$ and $\mathbf{u}_{red}$ from (12):

$$\left|\mathbf{A}^T(\mathbf{AA}^T)^{-1}\mathbf{A}\right|=\left|\mathbf{V}\ diag([1110])\ \mathbf{V}^T\right|=1 \tag{15}$$

$$\begin{aligned}|\mathbf{A}^T(\mathbf{AA}^T)^{-1}\mathbf{Au}|&\le|\mathbf{A}^T(\mathbf{AA}^T)^{-1}\mathbf{A}\,||\,\mathbf{u}\,|=1\,|\,\mathbf{u}\,|\\ |\mathbf{u}_{red}|&\le|\mathbf{u}|\end{aligned} \tag{16}$$

The following figure (Fig. 2) shows the situation of one I/S channel without and with projection.

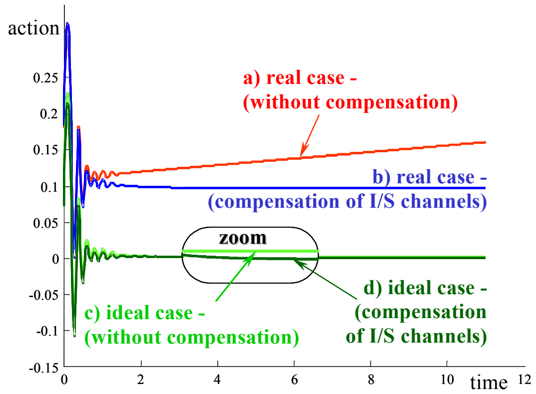


Fig. 2. Comparison of trends of one control action channel in appropriate drive during stop sequence (example of compensation by reductive projection).

In ideal case c) I/S channel is leveled off on certain magnitude, which was being integrated during whole control process. In case d) the unproductive part is reduced to zero level. The real cases a), b) are caused by integration (sum) of steady control error, arising from geometrical inaccuracies in redundant parallel structure.

Since the undesirable unproductive part is generated only in I/S channels of the controllers, then the separate compensation of these channels is sufficient. It means that the compensative block is added directly to controller (Fig. 4). If the block was situated behind whole controller, in such a case the saturation of I/S channels ($\rightarrow\infty$) would appear and real channel would be out of its range without any impact of reduction projection. Resultant controller can be composed as an independent parallel configuration of separate PID/PSD controllers with certain internal modification. Corresponding scheme is in Fig. 3.

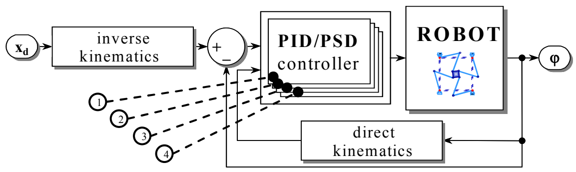


Fig. 3. Scheme of simple decentralized control.

Block of the projection determines unproductive part, which is subtracted from the input of I/S channel. It provides smooth compensation. Application of compensation I/S channel as well as the whole internal configuration of one individual controller (drive (1)) is shown in Fig. 13.

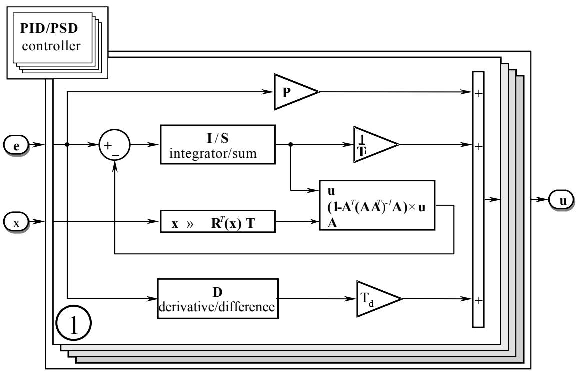


Fig. 4. Internal structure of PID/PSD controller with compensative (projective) block.

4 CENTRALIZED CONTROL

The described decentralized control in the previous section takes into account interactions and connection effects among all parts of the robot construction as disturbances influencing in each single drive system.

However, as shown by the dynamic model equation (2), the robot-manipulator is not a set of independent systems, but it is one multibody system with r inputs (drives - actuators) and n outputs (the independent Cartesian coordinates; n is equal to the number of degrees of freedom) interacting among them by means of the nonlinear kinematic and dynamic relations. Since these relations are known, there is a possibility to use them directly for the design of control.

As an example of such approach are high level techniques using knowledge of the dynamic model of the system (2) and which globally optimize whole control process. One of them is Generalized Predictive Control (GPC).

The Generalized Predictive Control (Ordys et al. 1993) is a multi-step control based on local optimization of the quadratic criterion, where the linearized equation or state formula is used (i.e. only the nearest future control signal is evaluated). This approach admits combination of feedback~feedforward parts and, by its multi-step character, offers influence of generated control actions.

The first subsection 4.1 will show preparation of prediction model for both absolute and incremental algorithm.

And the second subsection 4.2 will explain square-root predictive algorithm (Generalized Predictive Control (GPC)) for the mentioned prediction models derived for redundant parallel robot constructions.

4.1 Model for prediction

As mentioned previously, for derivation of GPC the nonlinear model (2) with modification (6)

$$\mathbf{R}^T\mathbf{M}\mathbf{R}\ddot{\mathbf{x}} + \mathbf{R}^T\mathbf{M}\dot{\mathbf{R}}\dot{\mathbf{x}} = \mathbf{R}^T\mathbf{g} + \mathbf{h} \tag{17}$$

and following simplification

$$\ddot{\mathbf{x}} = \mathbf{f}(\mathbf{x},\dot{\mathbf{x}}) + \mathbf{g}(\mathbf{x})\mathbf{h} \tag{18}$$

suitably transformed to state-space formulation using state vector $\dot{\mathbf{X}} = [\mathbf{x},\dot{\mathbf{x}}]^T$ description

$$\begin{aligned} \dot{\mathbf{X}} &= \mathbf{f}(\mathbf{X}) + \mathbf{g}(\mathbf{X})\mathbf{h} \\ \mathbf{x} &= \mathbf{C}\ \mathbf{X} \end{aligned} \tag{19}$$

must be linearized (Valášek et al. 1999) and converted from continuous to discrete domain (note: to preserve the traditional control notation in next part of this section, the symbol $\mathbf{u}$ is used instead of $\mathbf{h}$)

$$\begin{aligned} \mathbf{X}(k+1) &= \mathbf{A}\,\mathbf{X}(k) + \mathbf{B}\,\mathbf{u}(k) \\ \mathbf{x}(k) &= \mathbf{C}\,\mathbf{X}(k) \end{aligned} \tag{20}$$

The base of the predictive approach is an expression of new unknown output values $\mathbf{x}$ from actual topical state $\mathbf{X}$ for a considered horizon of prediction N. The following lines imply the expression firstly for absolute and consecutively for incremental GPC algorithms.

$$\begin{aligned} \mathbf{x}(k) &= \mathbf{C}\quad \mathbf{X}(k) \\ \hat{\mathbf{X}}(k+1) &= \mathbf{A}\ \ \mathbf{X}(k) + \qquad \mathbf{B}\mathbf{u}(k) \\ \hat{\mathbf{x}}(k+1) &= \mathbf{C}\mathbf{A}\ \ \mathbf{X}(k) + \mathbf{C}\qquad \mathbf{B}\mathbf{u}(k) \\ &\vdots \qquad\quad \vdots \qquad\qquad \vdots \\ \hat{\mathbf{X}}(k+N) &= \mathbf{A}^N\mathbf{X}(k) + \ \mathbf{A}^N\ \mathbf{B}\mathbf{u}(k) + \cdot + \ \mathbf{B}\mathbf{u}(k+N-1) \\ \hat{\mathbf{x}}(k+1) &= \mathbf{C}\mathbf{A}^N\mathbf{X}(k) + \mathbf{C}\mathbf{A}^{N-1}\mathbf{B}\mathbf{u}(k) + \cdot + \mathbf{C}\mathbf{B}\mathbf{u}(k+N-1) \end{aligned}$$

then the prediction of $\mathbf{x}$ is the following

$$\hat{\mathbf{x}} = \mathbf{f} + \mathbf{G}\mathbf{u} \tag{21}$$

with meaning

$$\hat{\mathbf{x}} = [\hat{\mathbf{x}}(k+1),\, \hat{\mathbf{x}}(k+2),\, \cdots,\, \hat{\mathbf{x}}(k+N)]^T \tag{22}$$

$$\mathbf{u} = [\mathbf{u}(k),\, \mathbf{u}(k+1),\, \cdots,\, \mathbf{u}(k+N-1)]^T \tag{23}$$

$$\mathbf{f} = \begin{bmatrix} \mathbf{C}\mathbf{A} \\ \vdots \\ \mathbf{C}\mathbf{A}^N \end{bmatrix} \mathbf{X}(k),\ \mathbf{G} = \begin{bmatrix} \mathbf{C} & \mathbf{B} & \cdots & \mathbf{0} \\ \vdots & & \ddots & \vdots \\ \mathbf{C}\mathbf{A}^{N-1}\mathbf{B} & & \cdots & \mathbf{C}\mathbf{B} \end{bmatrix} \tag{24}$$

For incremental algorithm we must firstly modify equation (20) with $\mathbf{u}(k)$=$\mathbf{u}(k-1) + \Delta\mathbf{u}(k)$

$$\begin{aligned} \mathbf{X}(k+1) &= \mathbf{A}\mathbf{X}(k) + \mathbf{B}\mathbf{u}(k-1) + \mathbf{B}\Delta\mathbf{u}(k) \\ \mathbf{x}(k) &= \mathbf{C}\mathbf{X}(k) \end{aligned} \tag{25}$$

to which corresponds

$$\begin{aligned} \hat{\mathbf{X}}(k+1) &= \mathbf{A}\ \ \mathbf{X}(k) + \qquad\qquad \mathbf{B}\mathbf{u}(k-1) + \\ &+ \qquad\qquad \mathbf{B}\Delta\mathbf{u}(k) \\ \hat{\mathbf{x}}(k+1) &= \mathbf{C}\mathbf{A}\ \ \mathbf{X}(k) + \mathbf{C} \qquad\qquad \mathbf{B}\mathbf{u}(k-1) + \\ &+ \mathbf{C} \qquad\qquad \mathbf{B}\Delta\mathbf{u}(k) \\ &\vdots \qquad \vdots \qquad \vdots \qquad\qquad \vdots \\ \hat{\mathbf{X}}(k+N) &= \mathbf{A}^N\mathbf{X}(k) + \ (\mathbf{A}^{N-1} + \cdot + \mathbf{A} + \mathbf{1})\mathbf{B}\mathbf{u}(k-1) + \\ &+ \ (\mathbf{A}^{N-1} + \cdot + \mathbf{A} + \mathbf{1})\mathbf{B}\Delta\mathbf{u}(k) + \cdot + \ \mathbf{B}\Delta\mathbf{u}(k+N-1) \end{aligned}$$

$$\begin{aligned} \hat{\mathbf{x}}(k+N) &= \mathbf{C}\mathbf{A}^N\mathbf{X}(k) + \mathbf{C}(\mathbf{A}^{N-1} + \cdot + \mathbf{A} + \mathbf{1})\mathbf{B}\mathbf{u}(k-1) + \\ &+ \mathbf{C}(\mathbf{A}^{N-1} + \cdot + \mathbf{A} + \mathbf{1})\mathbf{B}\Delta\mathbf{u}(k) + \cdot + \mathbf{C}\mathbf{B}\Delta\mathbf{u}(k+N-1) \end{aligned}$$

then the prediction of $\mathbf{x}$ is the following

$$\hat{\mathbf{x}} = \mathbf{f} + \mathbf{f}_r\,\mathbf{u}(k-1) + \mathbf{G}_r\,\Delta\mathbf{u} \tag{26}$$

with meaning

$$\hat{\mathbf{x}} = [\hat{\mathbf{x}}(k+1),\, \hat{\mathbf{x}}(k+2),\, \cdots,\, \hat{\mathbf{x}}(k+N)]^T \tag{27}$$

$$\Delta\mathbf{u} = [\Delta\mathbf{u}(k),\, \Delta\mathbf{u}(k+1),\, \cdots,\, \Delta\mathbf{u}(k+N-1)]^T \tag{28}$$

$$\mathbf{f} = \begin{bmatrix} \mathbf{C}\mathbf{A} \\ \vdots \\ \mathbf{C}\mathbf{A}^N \end{bmatrix} \mathbf{X}(k),\ \mathbf{f}_r = \begin{bmatrix} \mathbf{C}\,\mathbf{B} \\ \vdots \\ \mathbf{C}(\mathbf{A}^{N-1} + \cdot + \mathbf{A} + \mathbf{1})\mathbf{B} \end{bmatrix} \tag{29}$$

$$\mathbf{G}_r = \begin{bmatrix} \mathbf{C} & \mathbf{B} & \cdots & \mathbf{0} \\ & \vdots & \ddots & \vdots \\ \mathbf{C}(\mathbf{A}^{N-1} + \cdot + \mathbf{A} + \mathbf{1})\mathbf{B} & & \cdots & \mathbf{C}\mathbf{B} \end{bmatrix} \tag{30}$$

Now we can proceed to next section, dealing with the real derivation of predictive algorithms.

4.2 *Predictive control algorithm (derivation of GPC)*

The derivation of control law with the model configuration (19) or (20) respectively, which, in real computation, needs matrixes with smaller dimensions and moreover, if the penalization $\boldsymbol{\lambda}$ is nonzero value, it keeps redundant properties (if it exists). It also can be used for accomplishment of additional control requirements.

Furthermore, the advantages of root form of the quadratic criterion are used in this part. They are marked out by compact notation and good preparation for operations with huge matrixes.

To start derivate Predictive Control in the root form, let us unify predictive models (21) and (26) arisen from previous section as

$$\hat{\mathbf{x}} = \bar{\mathbf{f}} + \bar{\mathbf{G}}\bar{\mathbf{u}} \tag{31}$$

The equation (31) expressed generally the both models saving their meaning. Then the derivation can be provided for the both simultaneously.

For predictive control we use quadratic criterion

$$J_k = \mathcal{E}\left\{(\hat{\mathbf{x}}-\mathbf{w})^T(\hat{\mathbf{x}}-\mathbf{w}) + \bar{\mathbf{u}}^T\boldsymbol{\lambda}^T\boldsymbol{\lambda}\,\bar{\mathbf{u}}\right\} \tag{32}$$

which we rewrite in square root form

$$J_k = \left[[\hat{\mathbf{x}}-\mathbf{w}]^T, \bar{\mathbf{u}}^T\right]\begin{bmatrix}\mathbf{1} & \mathbf{0}\\ \mathbf{0} & \boldsymbol{\lambda}\end{bmatrix}^T\begin{bmatrix}\mathbf{1} & \mathbf{0}\\ \mathbf{0} & \boldsymbol{\lambda}\end{bmatrix}\begin{bmatrix}\hat{\mathbf{x}}-\mathbf{w}\\ \bar{\mathbf{u}}\end{bmatrix} = \mathbf{J}^T\mathbf{J} \tag{33}$$

Now we can work only with the square root

$$\mathbf{J} = \begin{bmatrix}\mathbf{1} & \mathbf{0}\\ \mathbf{0} & \boldsymbol{\lambda}\end{bmatrix}\begin{bmatrix}\hat{\mathbf{x}}-\mathbf{w}\\ \bar{\mathbf{u}}\end{bmatrix} = \begin{bmatrix}\mathbf{1} & \mathbf{0}\\ \mathbf{0} & \boldsymbol{\lambda}\end{bmatrix}\left(\begin{bmatrix}\hat{\mathbf{x}}\\ \bar{\mathbf{u}}\end{bmatrix} - \begin{bmatrix}\mathbf{w}\\ \mathbf{0}\end{bmatrix}\right) \tag{34}$$

$\mathbf{J}$ is a column vector with its Euclidean norm being the cost value of the root of the scalar criterion (32).

Firstly, the expression for prediction is substituted in the root (34) and it is adjusted to the form, where control actions **u** are in a separate term

$$\mathbf{J} = \begin{bmatrix}\hat{\mathbf{x}}\\ \boldsymbol{\lambda}\bar{\mathbf{u}}\end{bmatrix} - \begin{bmatrix}\mathbf{w}\\ \mathbf{0}\end{bmatrix} = \begin{bmatrix}\bar{\mathbf{f}}+\bar{\mathbf{G}}\bar{\mathbf{u}}\\ \boldsymbol{\lambda}\bar{\mathbf{u}}\end{bmatrix} - \begin{bmatrix}\mathbf{w}\\ \mathbf{0}\end{bmatrix}$$

$$\mathbf{J} = \begin{bmatrix}\bar{\mathbf{G}}\\ \boldsymbol{\lambda}\end{bmatrix}\bar{\mathbf{u}} + \begin{bmatrix}\bar{\mathbf{f}}\\ \mathbf{0}\end{bmatrix} - \begin{bmatrix}\mathbf{w}\\ \mathbf{0}\end{bmatrix} \tag{35}$$

Consecutively, we look for such **u**, which minimizes the Euclidean norm. This is fulfilled, if the **J** is annulled

$$\begin{bmatrix}\bar{\mathbf{G}}\\ \boldsymbol{\lambda}\end{bmatrix}\bar{\mathbf{u}} + \begin{bmatrix}\bar{\mathbf{f}}\\ \mathbf{0}\end{bmatrix} - \begin{bmatrix}\mathbf{w}\\ \mathbf{0}\end{bmatrix} = \mathbf{0} \tag{36}$$

which is an overdetermined system of equations (more rows tan columns) for the optimal control **u**:

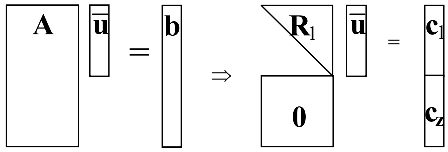
$$\mathbf{A}\quad\bar{\mathbf{u}} - \quad\mathbf{b}\quad = \mathbf{0} \tag{37}$$

For solution, the orthogonal triangular decomposition (Lawson et al. 1974) is used. It reduces excess rows of matrix **A** $[(2\cdot N\cdot n)\times(N\cdot n)]$ and elements of vector **b** $[2\cdot N\cdot n]$ (N is an horizon, n is a number of DOF) to upper triangular matrix **R** and vector **c** as follows:

$$\mathbf{A}\,\bar{\mathbf{u}} = \mathbf{b} \qquad /\mathbf{Q}^T$$

$$\mathbf{Q}^T\mathbf{A}\,\bar{\mathbf{u}} = \mathbf{Q}^T\mathbf{b}$$

$$\mathbf{R}\quad\bar{\mathbf{u}} = \mathbf{c} \tag{38}$$



(39)

Vector $\mathbf{c}_Z$ is a residuum vector, whose Euclidean norm $|\mathbf{c}_Z|$ is equal to the square root value of the criterion (33).

For the solution, we need only upper part of the system (39), which can be simply solved for unknown **u**

$$\begin{aligned}\mathbf{R}_1\,\bar{\mathbf{u}} &= \mathbf{c}_1\\ \bar{\mathbf{u}} &= (\mathbf{R}_1)^{-1}\mathbf{c}_1\end{aligned} \tag{40}$$

Since the matrix $\mathbf{R}_1$ has a form of upper triangle, we can use back-run procedure. By such computation we obtain only fictitious general force effects $\bar{\mathbf{u}} = \mathbf{h}$ or its increments $\bar{\mathbf{u}} = \Delta\mathbf{h}$, from which only the first subvector (k^{th} step) is used respectively. Thus, for incremental algorithm the force effects from previous time step must be added

$$\mathbf{h}(k) = \mathbf{h}(k-1) + \bar{\mathbf{u}}(k)\Big|_{\bar{\mathbf{u}}(k) = \Delta\mathbf{h}(k)} \tag{41}$$

Then, it order to obtain the real actuators **u**, the equation (6) must be solved based on **h** for **u**. As mentioned, its solution is not unique. It generally represents deficient rank system, where we can use pseudo-inverse operation (Lawson et al. 1974).

A graphical representation of Predictive Control is shown in Fig. 5.

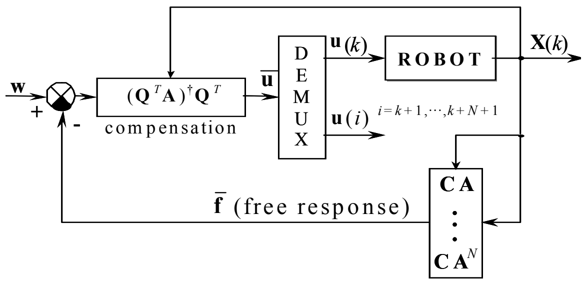


Fig. 5. Scheme of control circuit with Generalized Predictive Control (GPC) and robot.

Scheme generally represents the both square root algorithms. The recomputation (6) is hidden in the block "DEMUX".

5 SIMULATIONS AND CONTROL IN REALITY

For simulations and also for real control was considered the parallel structure shown in Fig. 6.

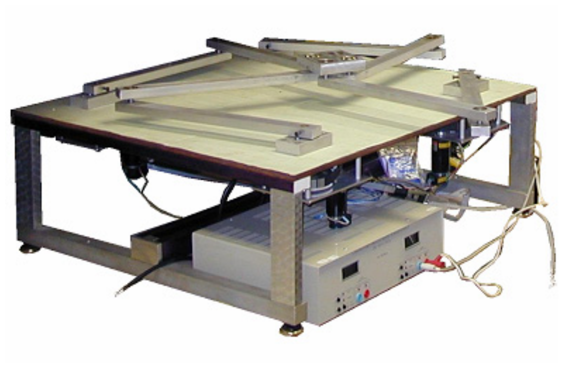


Fig. 6. Laboratory model of planar redundant parallel robot.

For laboratory tests the standard hardware and software tools (Real Time Workshop, MATLAB, SIMULINK, dSPACE, DSP, DC motors).

The following figures show results for trajectory in Fig. 7. It was planned according to classical kinematic laws and it provides smooth and continuous trends of velocities and continuous and segmentally smooth trends of accelerations.

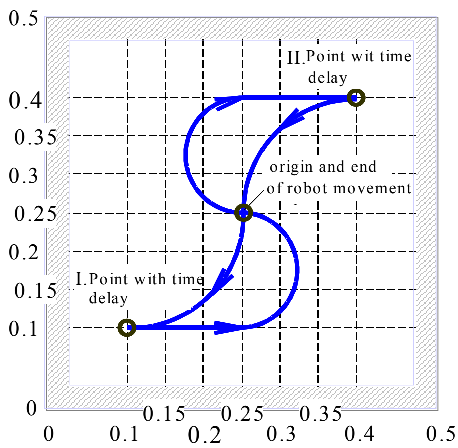


Fig. 7. Trajectory used for the tests.

Fig. 8 and Fig. 9 show trends of actuators (vertical axes) in following order corresponding with geometrical locations of the drives:

$$\begin{matrix} [u_4[Nm]] & [u_3[Nm]] \\ [u_1[Nm]] & [u_2[Nm]] \end{matrix} \qquad (42)$$

The time horizontal axes are considered in seconds.

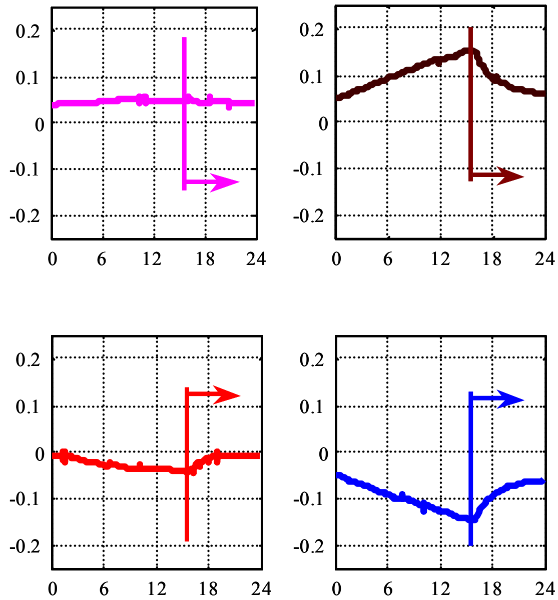


Fig. 8. Increase of undesirable part in I/S channels $(0 \div 15.8\ \text{s})$ and compensation $(15.8\,\text{s} \rightarrow)$ during control (or staying) on final position ($[x = 0.25$ m; $y = 0.25$ m; ψ - 90 °])

Fig. 8 corresponds with theory and its explanatory figure (Fig. 2) in section 3. The conflicts of drives occur in the most cases after certain delay, when, by influence of kinematic description inaccuracy (discrepancies of laboratory model with kinematic description), the effect of inaccuracy is accumulated in sum channel of PSD controller. Due to compensation according to eq. (12) the decrease and consecutively stabilization of actuator values appear.

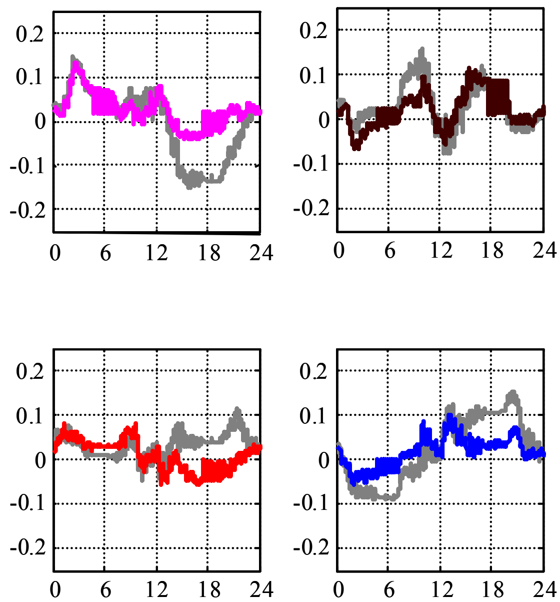


Fig. 9. Comparison of time histories of torques with Predictive Control and PSD control. (Time histories in the same color mark decentralized PSD control).

Predictive control algorithm achieves comparable results with PSD control and furthermore it reaches the lowest load of the motors (property of multi-step strategy, Fig. 9). The same level of qualitative results is caused by hardware constraints (accuracy of sensors tracking the motion of the robot. However, during the real tests the question of steady control error appears. It can be caused by absence of integral element. Thus the incremental approach is derived. At present, it is successfully simulated and it gives identical results as are discussed above.

6 CONCLUSION

The redundant parallel structures represent promising solutions of industrial robots or machine tool applications. Their control is not a simple and fully investigated problem. The paper describes utilization of classical PID/PSD modified by compensative block decreasing the influences in setSISO concept and advanced high-level model-based design presented by General Predictive Control, which solves directly drive cooperation (removes possible drive fighting) in redundant parallel structures and which is able to match additional control requirements.

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7 REFERENCES

Lawson, Ch., and Hanson, R.J. (1974). *Solving least square problems*. Prentice Hall: New York.

Ordys, A., and Clarke, D. (1993). A state space description for GPC controllers. In: *Int. J. Systems*. Vol. 24. pp. 1727-1744. Taylor & Francis Ltd.

Sciavicco, L. and Siciliano, B. (1996). *Modeling and control of robot manipulators*. The McGraw-Hill Companies, Inc. New York.

Stejskal, V. and Valášek, M. (1996). *Kinematics and Dymanics of Machinery*. Marcel Dekker Inc. New York.

Valášek, M., Belda, K. and Florian, M. (2002). Control and calibration of redundantly actuated parallel robots, Conf. on parallel Kinematics, Chemnitz, Verlag Wissenschaftliche Scripten.

Valášek, M. and Steinbauer, P. (1999). Nonlinear control of multibody systems. *Euromech 404*. pp. 437-444. Portugal.