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Fully probabilistic control design

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Abstract

Stochastic control design chooses the controller that makes the closed-loop behavior as close as possible to the desired one. The fully probabilistic design describes both the closed loop and its desired behavior in probabilistic terms and uses Kullback–Leibler divergence as their proximity measure. This approach: (i) unifies stochastic control design methodology; (ii) provides explicit minimizer.

The paper completes the previous solutions of various particular cases by formulating and solving the fully probabilistic control design in the general, discrete-time, state-space setting.

Keywords: Stochastic control design; Fully probabilistic design; State-space models

1. Introduction

The standard stochastic control problem is formulated as minimization of an expected loss function with respect to feedback control strategies, e.g. [1,2]. This minimization can be interpreted as an attempt to influence selected characteristics of the joint distribution of variables occurring in the optimized closed loop. This interpretation provides an alternative formulation: *the joint distribution of closed-loop variables should be forced to be as close as possible to their desired distribution*. The above formulation of the control design problem, called *fully probabilistic design* (FPD), was proposed in [7]. It has been extended to systems modeled by finite probabilistic mixtures [8].

The approach has the following special features.

- Explicit minimizer is available in the pair of operations (minimization, expectation) that are applied sequentially when optimizing via stochastic dynamic programming [7].
- Employing the Kullback–Leibler divergence together with case-specific desired distribution makes the resulting loss function to take into account both the deterministic and stochastic properties of the controlled system.
- The use of the multi-modal desired distribution provides a well-justified and feasible multiple-objective control design [10,4].

The listed features indicate a significant application potential of the FPD and justify attempts to formulate it as generally as possible. Up to now, the published versions of the FPD considered only controlled problems with the observable variables. Relaxation of this restriction is the main contribution of the current paper, which provides the fully probabilistic control design coping with stochastic state-space models.

Section 2 prepares the necessary notions and notations. Section 3 contains the main result, i.e., the FPD in the state-space setting. Concluding remarks close the paper.

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2. Preliminaries

The following notations are used throughout the paper: $\equiv$ is an equality by definition; X^* denotes a set of X-values; $\mathring{X}$ means cardinality of a finite set X^*; $f(\cdot|\cdot)$ stands for a probability density function (pdf) that is assumed to exist; t labels discrete-time moments, $t \in t^* \equiv \{1, \ldots, \mathring{t}\}$; $\mathring{t} < \infty$ denotes a given control horizon; $d_t = (y_t, u_t)$ is the data record at time t consisting of an observed system output y_t and of an optional system input u_t; x_t denotes an unobserved system state; d, x are assumed to be finite-dimensional; $X(t)$ stands for the sequence $(X_1, \ldots, X_t)$, $X(t) \in \{d(t), y(t), u(t), x(t)\}$.

The following adopted simplifications are also used.

- Names of arguments distinguish respective pdfs. No formal distinction is made between a random variable, its realization and an argument of a pdf.
- Integrals used are always definite and multivariate ones. The integration domain coincides with the support of the pdf in its argument.

The FPD exploits the *Kullback–Leibler* (*KL*) *divergence* $\mathscr{D}(f\|\tilde{f})$ [12] that measures the proximity of a pair of pdfs f, $\tilde{f}$ acting on a set X^*. It is defined as follows:

$$\mathscr{D}(f\|\tilde{f}) \equiv \int f(X) \ln\left(\frac{f(X)}{\tilde{f}(X)}\right) \mathrm{d}X. \tag{1}$$

The KL divergence has the following key property

$$\mathscr{D}(f\|\tilde{f}) \geqslant 0, \quad \mathscr{D}(f\|\tilde{f}) = 0 \text{ iff } f = \tilde{f} \text{ almost everywhere on } X^*. \tag{2}$$

The joint pdf $f(d(\mathring{t}), x(\mathring{t})|x_0, d(0))f(x_0|d(0)) = f(d(\mathring{t}), x(\mathring{t})|x_0)f(x_0)$ of all random variables involved is known to be the most complete probabilistic description of the controlled closed loop. In it, x_0 is an initial uncertain state and $d(0)$ denotes the prior information available before the choice of the first input. Habitually, $d(0)$ is considered only implicitly.

The chain rule for pdfs [14] implies the following decomposition of the joint pdf representing the complete probabilistic description of the closed-loop behavior

$$\begin{aligned} &f(d(\mathring{t}), x(\mathring{t})|x_0) \\ &\quad = \prod_{t \in t^*} f(y_t|u_t, x(t), d(t-1))f(x_t|u_t, x(t-1), d(t-1))f(u_t|x(t-1), d(t-1)). \end{aligned} \tag{3}$$

The chosen order of conditioning distinguishes the *observation model* $f(y_t|u_t, x(t), d(t-1))$, the *state evolution model* $f(x_t|u_t, x(t-1), d(t-1))$ and the general *randomized control law* $f(u_t|x(t-1), d(t-1))$. The collection of control laws over $t \in t^*$ forms *control strategy*.

The following assumptions are adopted on particular factors of the closed-loop decomposition (3).

Assumptions

Distribution of the system state x_t is determined by the current system input u_t and the previous system state x_{t-1} only, i.e.,

$$f(x_t|u_t, x(t-1), d(t-1)) = f(x_t|u_t, x_{t-1}).$$

Distribution of the observed system output y_t is determined by the current system input u_t and the current system state x_t only, i.e.,

$$f(y_t|u_t, x(t), d(t-1)) = f(y_t|u_t, x_t).$$

Admissible control strategies, generating the system input u_t from the observed data history $d(t-1)$ and ignoring the unobserved states $x(t-1)$, are considered, i.e., $f(u_t|x(t-1), d(t-1)) = f(u_t|d(t-1))$.

With these Assumptions, the closed-loop description (3) reduces to

$$f(d(\mathring{t}), x(\mathring{t})|x_0) = \prod_{t \in t^*} f(y_t|u_t, x_t)f(x_t|u_t, x_{t-1})f(u_t|d(t-1)). \tag{4}$$

Note that the omission of $d(t-1)$ in the first two items of the Assumptions is unnecessary but it makes the explanations more transparent and closer to a usual understanding of the state-space models.

3. Problem formulation and solution

The control aim and constraints are quantified by the so-called *ideal pdf* that represents the desired joint distribution of the considered closed-loop variables. It is constructed in the way analogous to (4) with user-specified factors distinguished by the superscript ^{I}

$$ {}^I f(d(\mathring{t}), x(\mathring{t})|x_0)\,{}^I f(x_0) = \prod_{t\in t^*} {}^I f(y_t|u_t, x_t)\,{}^I f(x_t|u_t, x_{t-1})\,{}^I f(u_t|d(t-1)) f(x_0), $$

where ideal pdfs ${}^I f(y_t|u_t, x_t)$, ${}^I f(x_t|u_t, x_{t-1})$ and ${}^I f(u_t|d(t-1))$ describe the desired observation model, state evolution model and control law, respectively.

The prior pdf on initial states x_0^* cannot be influenced by the optimized control strategy so that it is left to its fate, i.e., ${}^I f(x_0) = f(x_0)$.

To formulate the FPD concisely and to simplify some formal manipulations, the following shorthand notation is used

$$ f_t \equiv f(d(t), x(t)|x_0) f(x_0), \quad {}^I f_t \equiv {}^I f(d(t), x(t)|x_0) f(x_0). \tag{5} $$

Under the adopted Assumptions, Section 2, the FPD is formulated as follows.

Find admissible control strategy minimizing the KL divergence $\mathscr{D}\left(f_{\mathring{t}} \,\|\, {}^I f_{\mathring{t}}\right)$

Solution of the FPD requires the solution of stochastic filtering problem in the closed control loop.

Proposition 1 (*Stochastic filtering in closed control loop*). *Let the prior pdf $f(x_0)$ be given and the Assumptions hold. Then, the pdf $f(x_t|d(t))$, determining the* state estimate, *and the pdf $f(x_t|u_t, d(t-1))$, determining the* state prediction, *evolve according to the coupled equations*

$$ \textit{Time updating} \quad f(x_t|u_t, d(t-1)) = \int f(x_t|u_t, x_{t-1}) f(x_{t-1}|d(t-1))\,\mathrm{d}x_{t-1}, $$

$$ \textit{Data updating} \quad f(x_t|d(t)) = \frac{f(y_t|u_t, x_t) f(x_t|u_t, d(t-1))}{\underbrace{\int f(y_t|u_t, x_t) f(x_t|u_t, d(t-1))\,\mathrm{d}x_t}_{f(y_t|u_t, d(t-1))}}. $$

The stochastic filtering does not depend on the used admissible control strategy $\{f(u_t|d(t-1))\}_{t\in t^*}$ but on the generated inputs only.

Proof. The pdf $f(x_0|d(0)) = f(x_0)$ is given. Let us assume that we have already got $f(x_{t-1}|d(t-1))$ for a generic $t \in t^*$. Then the time updating is implied by the following identities

$$ \begin{aligned} f(x_t|u_t, d(t-1)) &\underbrace{=}_{marginalization} \int f(x_t, x(t-1)|u_t, d(t-1))\,\mathrm{d}x(t-1) \\ &\underbrace{=}_{chain\ rule} \int f(x_t|u_t, x(t-1), d(t-1)) f(x(t-1)|u_t, d(t-1))\,\mathrm{d}x(t-1) \\ &\underbrace{=}_{Assumptions} \int f(x_t|u_t, x_{t-1}) f(x(t-1)|d(t-1))\,\mathrm{d}x(t-1) \\ &\underbrace{=}_{marginalization} \int f(x_t|u_t, x_{t-1}) f(x_{t-1}|d(t-1))\,\mathrm{d}x_{t-1}. \end{aligned} $$

The data updating completes the recursion and it is implied by the identities

$$f(x_t|d(t)) \underbrace{=}_{conditioning} \frac{f(x_t, y_t, u_t|d(t-1))}{f(y_t, u_t|d(t-1))} \underbrace{=}_{chain\ rule} \frac{f(y_t|u_t, x_t, d(t-1))f(x_t|u_t, d(t-1))f(u_t|d(t-1))}{f(y_t|u_t, d(t-1))f(u_t|d(t-1))} \underbrace{=}_{Assumptions} \frac{f(y_t|u_t, x_t)f(x_t|u_t, d(t-1))}{\int f(y_t|u_t, x_t)f(x_t|u_t, d(t-1))\,\mathrm{d}x_t}. \quad \square$$

Proposition 2 (*Solution of the FPD*). *Let both the joint pdf $f(x(\mathring{t}), d(\mathring{t})|x_0)$ and its ideal counterpart ${}^I f(x(\mathring{t}), d(\mathring{t})|x_0)$ meet the Assumptions.*

Then, the optimal admissible control strategy minimizing $\mathscr{D}\left(f_{\mathring{t}}||{}^I f_{\mathring{t}}\right)$ is randomized one given by the pdfs

$$ {}^o f(u_t|d(t-1)) = {}^I f(u_t|d(t-1)) \frac{\exp[-\omega(u_t, d(t-1))]}{\gamma(d(t-1))}, \quad t \in t^*, \tag{6}$$

$$\gamma(d(t-1)) \equiv \int {}^I f(u_t|d(t-1)) \exp[-\omega(u_t, d(t-1))]\,\mathrm{d}u_t.$$

Starting with $\gamma(d(\mathring{t})) \equiv 1$, the functions $\omega(u_t, d(t-1))$ are generated recursively for $t = \mathring{t}, \mathring{t}-1, \ldots, 1$ in the backward manner, as follows:

$$\omega(u_t, d(t-1)) \equiv \int \Omega(u_t, d(t-1), x_{t-1}) f(x_{t-1}|d(t-1))\,\mathrm{d}x_{t-1}$$

with $f(x_t|d(t))$ updated according to Proposition 1 and

$$\Omega(u_t, d(t-1), x_{t-1}) \equiv \int f(y_t|u_t, x_t) f(x_t|u_t, x_{t-1}) \ln\left(\frac{f(y_t|u_t, x_t) f(x_t|u_t, x_{t-1})}{\gamma(d(t))^I f(y_t|u_t, x_t)^I f(x_t|u_t, x_{t-1})}\right) \mathrm{d}y_t\,\mathrm{d}x_t.$$

Proof. The result is implied by the following sequence of equalities in which we use the definition of the KL divergence (1), its basic property (2), Fubini theorem on multiple integration [15], marginalization and normalization of pdfs and the chain rule for them [14] together with conditional independencies expressed by the adopted Assumptions. The definition $\gamma(d(\mathring{t})) \equiv 1$ and the shorthand notation (5) are used. Moreover, the notation of differentials is simplified by taking $\mathrm{d}x(t) \equiv \mathrm{d}x(t)\mathrm{d}x_0$.

The KL divergence can be viewed as an expectation of the additive loss function with the tth partial loss equal to

$$\ln\left(\frac{f(y_t|u_t, x_t) f(x_t|u_t, x_{t-1}) f(u_t|d(t-1))}{{}^I f(y_t|u_t, x_t)^I f(x_t|u_t, x_{t-1})^I f(u_t|d(t-1))}\right).$$

This observation, together with the definition $\gamma(d(\mathring{t})) = 1$, implies the following identity

$$\mathscr{D}_{\mathring{t}} \equiv \min_{\{f(u_t|d(t-1))\}_{t=1}^{\mathring{t}}} \mathscr{D}(f_{\mathring{t}}||{}^I f_{\mathring{t}}) = \min_{\{f(u_t|d(t-1))\}_{t=1}^{\mathring{t}-1}} \left\{ \mathscr{D}(f_{\mathring{t}-1}||{}^I f_{\mathring{t}-1}) + \min_{f(u_{\mathring{t}}|d(\mathring{t}-1))} \int f_{\mathring{t}-1} \left[\int f(y_{\mathring{t}}|u_{\mathring{t}}, x_{\mathring{t}}) f(x_{\mathring{t}}|u_{\mathring{t}}, x_{\mathring{t}-1}) f(u_{\mathring{t}}|d(\mathring{t}-1)) \times \ln\left(\frac{f(y_{\mathring{t}}|u_{\mathring{t}}, x_{\mathring{t}}) f(x_{\mathring{t}}|u_{\mathring{t}}, x_{\mathring{t}-1}) f(u_{\mathring{t}}|d(\mathring{t}-1))}{\gamma(d(\mathring{t}))^I f(y_{\mathring{t}}|u_{\mathring{t}}, x_{\mathring{t}})^I f(x_{\mathring{t}}|u_{\mathring{t}}, x_{\mathring{t}-1})^I f(u_{\mathring{t}}|d(\mathring{t}-1))}\right) \mathrm{d}y_{\mathring{t}}\,\mathrm{d}x_{\mathring{t}}\,\mathrm{d}u_{\mathring{t}} \right] \mathrm{d}x(\mathring{t}-1)\,\mathrm{d}d(\mathring{t}-1) \right\}. \tag{7}$$

Let us deal with the second term in the expression (7), which depends on the last member $f(u_{\mathring{t}}|d(\mathring{t}-1))$ of the optimized admissible strategy. In this term, the factor $A_{\mathring{t}}$, defined as the expression in the square brackets, can be rearranged as follows

$$
\begin{aligned}
A_{\mathring{t}} \equiv & \int f(y_{\mathring{t}}|u_{\mathring{t}}, x_{\mathring{t}})(x_{\mathring{t}}|u_{\mathring{t}}, x_{\mathring{t}-1}) f(u_{\mathring{t}}|d(\mathring{t}-1)) \\
& \times \ln\left(\frac{f(y_{\mathring{t}}|u_{\mathring{t}}, x_{\mathring{t}}) f(x_{\mathring{t}}|u_{\mathring{t}}, x_{\mathring{t}-1}) f(u_{\mathring{t}}|d(\mathring{t}-1))}{\gamma(d(\mathring{t}))^I f(y_{\mathring{t}}|u_{\mathring{t}}, x_{\mathring{t}})^I f(x_{\mathring{t}}|u_{\mathring{t}}, x_{\mathring{t}-1})^I f(u_{\mathring{t}}|d(\mathring{t}-1))}\right) \mathrm{d}y_{\mathring{t}}\, \mathrm{d}x_{\mathring{t}}\, \mathrm{d}u_{\mathring{t}} \\
= & \int f(u_{\mathring{t}}|d(\mathring{t}-1)) \left[\ln\left(\frac{f(u_{\mathring{t}}|d(\mathring{t}-1))}{{}^I f(u_{\mathring{t}}|d(\mathring{t}-1))}\right) \right. \\
& \left. + \underbrace{\int f(y_{\mathring{t}}|u_{\mathring{t}}, x_{\mathring{t}}) f(x_{\mathring{t}}|u_{\mathring{t}}, x_{\mathring{t}-1}) \ln\left(\frac{f(y_{\mathring{t}}|u_{\mathring{t}}, x_{\mathring{t}}) f(x_{\mathring{t}}|u_{\mathring{t}}, x_{\mathring{t}-1})}{\gamma(d(\mathring{t}))^I f(y_{\mathring{t}}|u_{\mathring{t}}, x_{\mathring{t}})^I f(x_{\mathring{t}}|u_{\mathring{t}}, x_{\mathring{t}-1})}\right) \mathrm{d}y_{\mathring{t}}\, \mathrm{d}x_{\mathring{t}} \right]}_{\Omega(u_{\mathring{t}}, d(\mathring{t}-1), x_{\mathring{t}-1})} \mathrm{d}u_{\mathring{t}}.
\end{aligned}
$$

This rearrangement uses Fubini theorem, non-presence of $y_{\mathring{t}}$, $x_{\mathring{t}}$ in the optimized pdf $f(u_{\mathring{t}}|d(\mathring{t}-1))$ and normalization $\int f(y_{\mathring{t}}|u_{\mathring{t}}, x_{\mathring{t}})(x_{\mathring{t}}|u_{\mathring{t}}, x_{\mathring{t}-1})\, \mathrm{d}y_{\mathring{t}}\, \mathrm{d}x_{\mathring{t}} = 1$.

With the adopted notation (5) and the introduced function $\Omega(u_{\mathring{t}}, d(\mathring{t}-1), x_{\mathring{t}-1})$, the term $B_{\mathring{t}} \equiv \int f_{\mathring{t}-1}[A_{\mathring{t}}]\, \mathrm{d}x(\mathring{t}-1)\, \mathrm{d}d(\mathring{t}-1)$ in (7) influenced by $f(u_{\mathring{t}}|d(\mathring{t}-1))$ becomes

$$
\begin{aligned}
B_{\mathring{t}} \equiv \int f(d(\mathring{t}-1)) \Bigg\{ & \int f(u_{\mathring{t}}|d(\mathring{t}-1)) \left[\ln\left(\frac{f(u_{\mathring{t}}|d(\mathring{t}-1))}{{}^I f(u_{\mathring{t}}|d(\mathring{t}-1))}\right) \right. \\
& \left. + \underbrace{\int f(x(\mathring{t}-1)|d(\mathring{t}-1)) \Omega(u_{\mathring{t}}, d(\mathring{t}-1), x_{\mathring{t}-1})\, \mathrm{d}x(\mathring{t}-1) \right]}_{\omega(u_{\mathring{t}}, d(\mathring{t}-1))} \mathrm{d}u_{\mathring{t}} \Bigg\}\, \mathrm{d}d(\mathring{t}-1).
\end{aligned} \tag{8}
$$

To get (8), the chain rule $f(d(\mathring{t}-1), x(\mathring{t}-1)|x_0) f(x_0) = f(x(\mathring{t}-1), x_0|d(\mathring{t}-1)) f(d(\mathring{t}-1))$, Fubini theorem, non-occurrence of $x(\mathring{t}-1)$ and x_0 in the control law $f(u_{\mathring{t}}|d(\mathring{t}-1))$, and the normalization $\int f(x(\mathring{t}-1), x_0|d(\mathring{t}-1))\, \mathrm{d}x(\mathring{t}-1) = 1$ are sequentially exploited.

As the function $\Omega(u_{\mathring{t}}, d(\mathring{t}-1), x_{\mathring{t}-1})$ does not depend on $x(\mathring{t}-2)$ and x_0, these superfluous variables can be integrated out from the joint pdf. Hence, the function $\omega(\cdot)$ defined in (8) can be evaluated using the pdf determining the state estimate only

$$
\omega(u_{\mathring{t}}, d(\mathring{t}-1)) = \int f(x_{\mathring{t}-1}|d(\mathring{t}-1)) \Omega(u_{\mathring{t}}, d(\mathring{t}-1), x_{\mathring{t}-1})\, \mathrm{d}x_{\mathring{t}-1}.
$$

Proposition 1 has shown that the filtering result $f(x_{\mathring{t}-1}|d(\mathring{t}-1))$ does not depend on the optimized admissible strategy. The optimized $f(u_{\mathring{t}}|d(\mathring{t}-1))$ enters only the functional in the compound brackets in (8) as follows

$$\int f(u_{\mathring{t}}|d(\mathring{t}-1)) \ln \left(\frac{f(u_{\mathring{t}}|d(\mathring{t}-1))}{\frac{{}^{I}f(u_{\mathring{t}}|d(\mathring{t}-1))\exp[-\omega(u_{\mathring{t}},d(\mathring{t}-1))]}{\gamma(d(\mathring{t}-1))}} \right) \mathrm{d}u_{\mathring{t}} \underbrace{- \ln \left(\int {}^{I}f(u_{\mathring{t}}|d(\mathring{t}-1)) \exp[-\omega(u_{\mathring{t}}, d(\mathring{t}-1))] \, \mathrm{d}u_{\mathring{t}} \right)}_{-\ln(\gamma(d(\mathring{t}-1)))}. \quad (9)$$

By adding and subtracting $\ln(\gamma(d(\mathring{t}-1)))$ in (9), the first term in (9) has become a conditional version of the KL divergence. The basic property (2) of the KL divergence and independence of $\ln(\gamma(d(\mathring{t}-1)))$ on the optimized $f(u_{\mathring{t}}|d(\mathring{t}-1))$ imply that this expression is minimized by the claimed pdf (6)

$${}^{o}f(u_{\mathring{t}}|d(\mathring{t}-1)) = \frac{{}^{I}f(u_{\mathring{t}}|d(\mathring{t}-1)) \exp[-\omega(u_{\mathring{t}}, d(\mathring{t}-1))]}{\gamma(d(\mathring{t}-1))}.$$

The expression (8) evaluated for this pdf provides the reached minimum

$$-\int f(d(\mathring{t}-1)) \ln(\gamma(d(\mathring{t}-1))) \, \mathrm{d}d(\mathring{t}-1) = -\int f_{\mathring{t}-1} \ln(\gamma(d(\mathring{t}-1))) \, \mathrm{d}x(\mathring{t}-1) \, \mathrm{d}d(\mathring{t}-1).$$

Inserting this minimum into (7) shows that $-\ln(\gamma(d(\mathring{t}-1)))$ enters $\mathscr{D}(f_{\mathring{t}-1}\|{}^{I}f_{\mathring{t}-1})$ at the same place as $-\ln(\gamma(d(\mathring{t})))=0$ enters $\mathscr{D}(f_{\mathring{t}}\|{}^{I}f_{\mathring{t}})$. Thus, the whole procedure can be repeated for the decreasing horizons $\mathring{t}-1, \mathring{t}-2, \ldots, 1$. $\square$

4. Concluding remarks

This paper completes the development of the fully probabilistic control design by covering the control design for the stochastic state-space model. Advantageous properties of the FPD, see [7,8,10,4] and the brief outline in Introduction, apply to the treated case, too. For instance, the formula (6) implies that the support of the ideal control law ${}^{I}f(u_t|d(t-1))$ includes the support of the designed control law ${}^{o}f(u_t|d(t-1))$, i.e. the ideal pdf ${}^{I}f(u_t|d(t-1))$ may set hard constraints on the range of inputs or their changes.

Besides, the unified probabilistic treatment opens a novel way of distributed control by adopting technology developed in connection with graphical models [5,11] for creating the compound probabilistic models.

In a sense, the initial phase of development of the FPD theory is concluded here by extending the FPD to the state-space models that allow for a richer and more realistic modeling of the controlled system than the input–output models. A further development will include the following directions.

- *Applications of the FPD to specific models*.
  For instance, the complete linear-quadratic-Gaussian design with the Gaussian state-space model is obtained if all pdfs considered within the problem are Gaussian. This allows the rich analytical results available, for instance, general notions like controllability and other “abilities”, [13] to be widely exploited by the FPD.
  On the other hand, the general theory of the FPD contributes to a deeper understanding of the “traditional” notions. For example, the interpretation of penalization matrices in quadratic loss function as precision matrices of the ideal pdf helps to choose their values, cf. [7].
  Algorithmic novelty is expected from the applications to the linear dynamic models with the uniform noise. A sort of dynamic linear programming will arise.
- *Computational and algorithmic aspects*.
  In our opinion, a further development should predominantly concentrate on the algorithmic aspects. The computational complexity barrier that is ever present in any stochastic design is the primary problem. The fact that the minimizer is found explicitly reduces this problem to simpler, but still very hard, multivariate integration and an approximate interpolation of the involved multivariate functions. The recently proposed stationary version of FPD [9] is expected to help further on. The practical solvability of the underlying evaluation problems is indicated by an existence of the solutions for the case of probabilistic mixtures, see the cited references and the vast amount work done in this respect world-wide, e.g. [3,6].

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