

14 Fusion of Blurred Images

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CONTENTS

I. Introduction.....	423
II. Multichannel Image Acquisition Models.....	425
III. Piecewise Ideal Imaging.....	426
A. Application in Confocal Microscopy	427
IV. Uniformly Blurred Channels	429
A. Alternating Minimization Algorithm.....	431
1. Regularization of the Image $Q(u)$	432
2. Regularization of the Blurs $R(h)$	432
3. Iterative Minimization Algorithm	433
4. Experiment with Artificial Data	434
5. Experiment with Real Data	435
V. Slightly Misregistered Blurred Channels	437
A. Maximum <i>a Posteriori</i> Probability Algorithm	439
1. Experiment with Misregistered Images.....	440
VI. Heavily Misregistered Blurred Channels	441
VII. Channels with Space-Variant Blurring	444
VIII. Conclusion	446
Acknowledgments	447
References.....	447

I. INTRODUCTION

In general, the term *fusion* means an approach to extraction of information spontaneously adopted in several domains. The goal of image fusion is to integrate combinations of complementary multisensor, multitemporal, and multiview information into one new image containing information of which the quality could not be achieved otherwise. The term *quality* depends on the application requirements.

Image fusion has been used in many application areas. In remote sensing and in astronomy,^{1,2} multisensor fusion is used to achieve high spatial and spectral resolutions by combining images from two sensors, one of which has high spatial

46 resolution and the other, high spectral resolution. Numerous fusion applications
47 have appeared in medical imaging (see Ref. 3 or 4 for instance) such as
48 simultaneous evaluation of a combination of computer tomography (CT), nuclear
49 magnetic resonance (NMR), and positron emission tomography (PET) images. In
50 the case of multiview fusion, a set of images of the same scene taken by the same
51 sensor but from different viewpoints is fused to obtain an image with higher
52 resolution than the sensor normally provides, or to recover the three-dimensional
53 representation of the scene (shape from stereo). The multitemporal approach
54 recognizes two different aims. Images of the same scene are acquired at different
55 time instances either to find and evaluate changes in the scene or to obtain a less
56 degraded image of the scene. The former aim is common in medical imaging,
57 especially in change detection of organs and tumors, and in remote sensing for
58 monitoring land or forest exploitation. The acquisition period is usually months
59 or years. The latter aim requires the different measurements to be much closer to
60 each other, typically in the scale of seconds, and possibly under different
61 conditions. Our motivation for this work came from this area.

62 We assume that several images of the same scene called *channels* are
63 available. We further assume all channels were acquired by the same sensor (or
64 by different sensors of the same type) but under different conditions and
65 acquisition parameters. Thus, all channels are of the same modality and represent
66 similar physical properties of the scene.

67 Since imaging sensors and other devices have their physical limits and
68 imperfections, the acquired image represents only a degraded version of the
69 original scene. Two main categories of degradations are recognized: color (or
70 brightness) degradations and geometric ones. The former degradations are caused
71 by such factors as incorrect focus, motion of the scene, media turbulence, noise,
72 and limited spatial and spectral resolution of the sensor; they usually result in a
73 blurring of the image. The latter degradations originate from the fact that each
74 image is a two-dimensional projection of a three-dimensional world. They cause
75 deformations of object shapes and other spatial distortions of the image.

76 Individual channels are supposed to be degraded in different ways because of
77 differences in acquisition parameters and imaging conditions (see [Figure 14.1](#)
78 for multichannel acquisition scheme). There are many sources of corruption and
79 distortion that we have to cope with. Light rays (or other types of electromagnetic
80 waves) reflected by objects on the scene travel to measuring sensors through a
81 transport medium, for example, the atmosphere. Inevitably, each transport
82 medium modifies the signal in some way. The imaging system is thus subject to
83 blurring due to the medium's rapidly changing index of refraction, the finite
84 broadcast bandwidth and the object motion. The source of corruption and its
85 characteristics are often difficult to predict. In addition, the signal is corrupted
86 inside a focusing set after reaching the sensor. This degradation is inherent to
87 the system and cannot be bypassed, but it can often be measured and accounted
88 for; typical examples are a range of lens imperfections. Finally, the signal must
89 be stored on photographic material or first digitized with Charge-Coupled
90 Devices (CCDs) and then stored. In both cases the recording exhibits a number

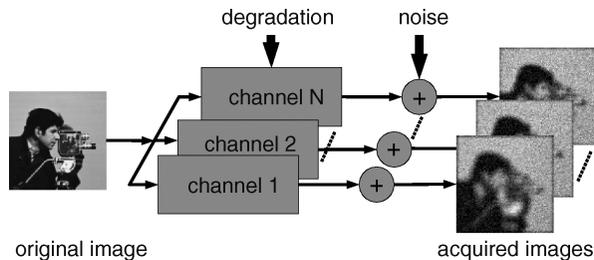


FIGURE 14.1 Multichannel acquisition model: the original scene is captured by N different channels which are subject to various degradations.

of degradations. Digital imaging systems suffer from low resolution and low sensitivity to the input signal, which are imposed by a finite number of intensity levels and a finite storage capacity. In analog systems, resolution artifacts are caused by the limited size of photographic material grain. Random noise is another crucial factor that severely affects the quality of image acquisition. In all real applications, measurements are degraded by noise. By utilizing suitable measuring techniques and appropriate devices, it can be considerably diminished, but unfortunately never cancelled.

Analysis and interpretation of degraded images is the key problem in real applications, because the degradations are, in principle, inevitable. A very promising approach to image quality enhancement is to fuse several channels with different degradations together in order to extract as much useful information as possible.

II. MULTICHANNEL IMAGE ACQUISITION MODELS

Regardless of its particular type, image degradations can be mathematically described by an operator based on an ideal representation of the scene. More formally, let $u(x, y)$ be an ideal image of the scene and let $z_1(x, y), \dots, z_N(x, y)$ be acquired channels. The relation between each z_i and u is expressed as

$$z_i(x, y) = D_i(u(x, y)) + n_i(x, y) \quad (14.1)$$

where D_i is an unknown operator describing the image degradations of the i th channel and n_i denotes additive random noise. In the ideal situation, D_i would equal identity and n_i would be zero for each i . The major goal of the fusion is to obtain an image \hat{u} as a “good estimate” of u ; that means \hat{u} , in some sense, should be a better representation of the original scene than each individual channel z_i .

The fusion methodology depends significantly on the type of degradation operators D_i . In this work we focus on the cases where each operator D_i is a composition of image blurring and of geometric deformations caused by imaging geometry.

Under these assumptions, Equation 14.1 becomes

$$z_i(\tau_i(x, y)) = \int h_i(x, y, s, t)u(s, t)ds dt + n_i(x, y) \quad (14.2)$$

where $h_i(x, y, s, t)$ is called the *point spread function* (PSF) of the i th imaging system at location (x, y) and τ_i stands for the co-ordinate transform, describing geometric differences between the original scene and the i th channel (in simple cases, τ_i is limited to rotation and translation, but in general complex nonlinear deformations may be present too). Having N channels, Equation 14.2 can be viewed as a system of N integral equations of the first kind. Even if all h_i s were known and neither geometric deformations nor noise were present, this system would not be generally solvable. In the sequel we simplify the model (2) by additional constraints and we show how to fuse the channels (that is, how to estimate the original scene) in these particular cases. The constraints are expressed as some restrictive assumptions on the PSFs and on the geometric deformations. We review five basic cases covering most situations occurring in practice.

III. PIECEWISE IDEAL IMAGING

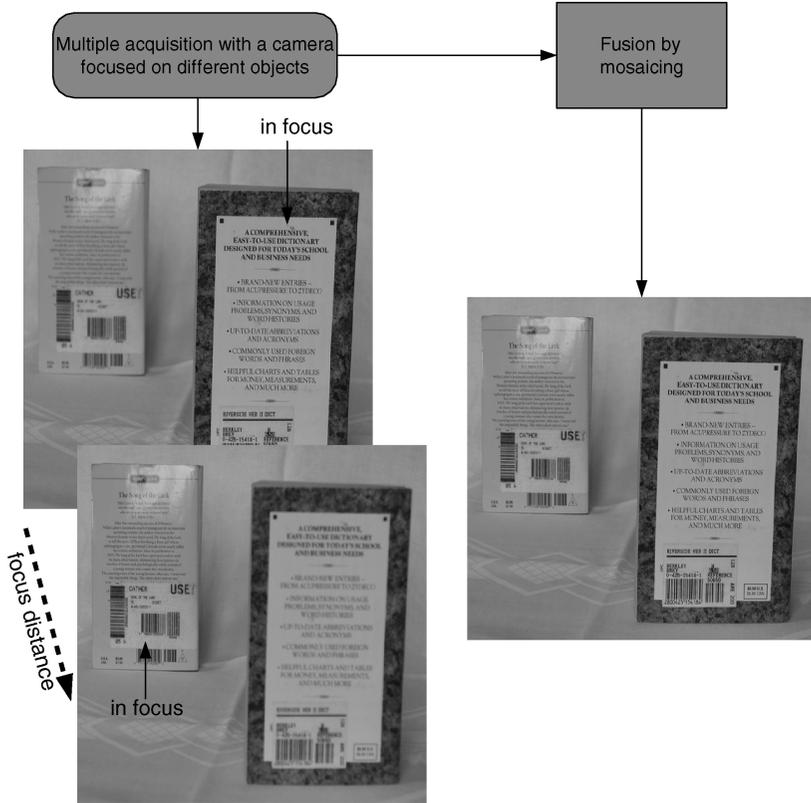
In this simplest model, the PSF of each channel is supposed to be piecewise space-invariant and every point (x, y) of the scene is assumed to be acquired undistorted in (at least) one channel. No geometric deformations are assumed.

More precisely, let $\Omega = \bigcup_{k=1}^K \Omega_k$ be a support of image function $u(x, y)$, where Ω_k are its disjoint subsets. Let h_i^k be a local PSF acting on the region Ω_k in the i th channel. Since every h_i^k is supposed to be space-invariant (that is, $h_i^k(x, y, s, t) = h_i^k(x - s, y - t)$), the imaging model is defined as

$$z_i(x, y) = (u * h_i^k)(x, y) \Leftrightarrow (x, y) \in \Omega_k \quad (14.3)$$

where $*$ stands for convolution and for each region Ω_k there exists channel j such that $h_j^k(x - s, y - t) = \delta(x - s, y - t)$.

This model is applicable in so-called *multifocus imaging*, when we photograph a static scene with a known piecewise-constant depth and focus channel-by-channel on each depth level. Image fusion then consists of comparing the channels in the image domain^{5,6} or in the wavelet domain,^{7,8} identifying the channel in which the pixel (or the region) is depicted undistorted and, finally, mosaicing the undistorted parts (no deconvolution is performed in this case, see Figure 14.2). To find the undistorted channel for the given pixel, a local focus measure is calculated over the pixel neighborhood and the channel which maximizes the focus measure is chosen. In most cases, the focus measures used are based on the idea of measuring the quantity of high frequencies of the image. It corresponds with an intuitive expectation that the blurring suppresses high frequencies regardless of the particular PSF. Image variance,⁹ energy of a Fourier spectrum,¹⁰ norm of image gradient,⁹ norm of image Laplacian,⁹ image



Q3 FIGURE 14.2 Two-channel piecewise ideal imaging: in each channel, one book is in focus while the other one is out of focus. Image fusion is performed by mosaicing the channel regions which are in focus.

moments,¹¹ and energy of high-pass bands of a wavelet transform^{7,8,12} belong to the most popular focus measures. Fusion in the image domain is seriously affected by the size of the neighborhood on which the focus measure is calculated. On the other hand, fusion in the wavelet domain is very sensitive to translation changes in the source images.

A. APPLICATION IN CONFOCAL MICROSCOPY

A typical application area where piecewise ideal imaging appears is confocal microscopy of three-dimensional samples. Since the microscope has a very narrow depth of field, several images of the sample differing from each other by the focus distance are taken. Each of them shows in focus only the parts of the sample that are a certain distance from the lens, while other parts are blurred by an out-of-focus blur of various extents. These image layers form the so-called

226 *stack image* of the sample. To obtain a focused image of the whole sample is
 227 beyond the scope of the microscope; the only possibility to get it employs a fusion
 228 of the stack image.

229 If the focal step used in the acquisition process is less than or equal to the
 230 depth of field of the microscope, then the assumptions of piecewise ideal imaging
 231 are fulfilled and fusion by mosaicing the undistorted parts of the individual layers
 232 can be applied.

233 A crucial question is how to find, for each pixel, the layer in which the given
 234 pixel (together with its neighborhood) depicted is least distorted or undistorted.
 235 Among the focus measures mentioned above, wavelet-based methods gave the
 236 best results. Their common idea is to maximize the energy of high-pass bands.
 237 Most wavelet-based focus measures ignore low-pass band(s), but Kautsky¹²
 238 pointed out that the energy of low-pass bands also reflects the degree of image
 239 blurring and, that considering both low-pass and high-pass bands increases the
 240 discrimination power of the focus measurement. We adopted and modified the
 241 idea from Ref. 12, and proposed to use a product of energies contained in low-
 242 pass and high-pass bands as a local focus measure

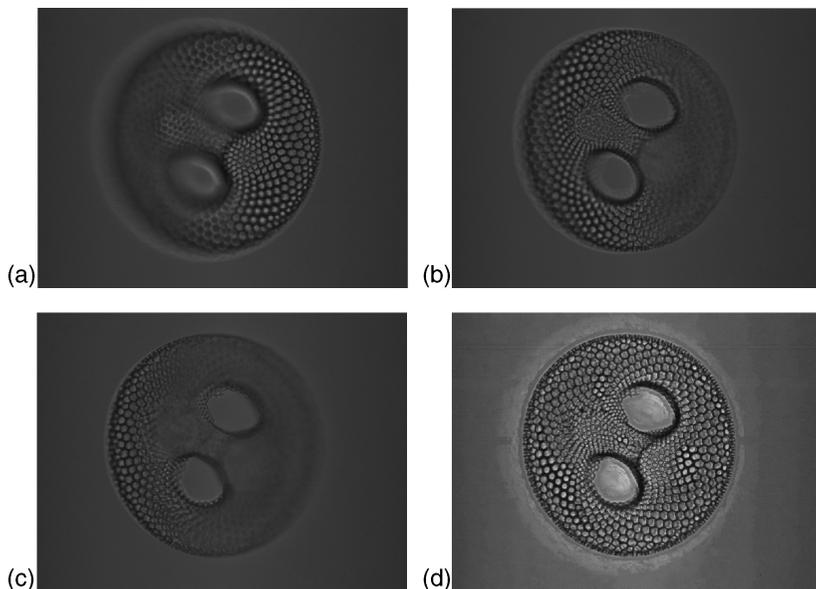
$$243 \quad 244 \quad 245 \quad 246 \quad \varrho_i(x, y) = \|w_L(z_i)\|^2 \|w_H(z_i)\|^2 \quad (14.4)$$

247 Both energies are calculated from a certain neighborhood of the point (x, y) ; high-
 248 pass band energy $\|w_H(z_i)\|^2$ is calculated as the mean from three high-pass bands
 249 (in this version, we used decomposition to depth one only).

250 The fusion of the multifocus stack $\{z_1, \dots, z_N\}$ is conducted in the wavelet
 251 domain as follows. First, we calculate the wavelet decomposition of each image
 252 z_i . Then a decision map $M(x, y)$ is created in accordance with a max-rule
 253 $M(x, y) = \arg \max_i \varrho_i(x, y)$. The decision map is the size of the subband, that is, a
 254 quarter of the original image, and it tells us from which image the wavelet
 255 coefficients should be used. The decision map is applied to all four bands and,
 256 finally, the fused image is obtained by inverse wavelet transform.

257 The performance of this method is shown here in fusion of microscopic
 258 images of a unicellular water organism (see [Figure 14.3](#)). The total number of the
 259 stack layers was 20; three of them are depicted. The fused image is shown on the
 260 bottom right.

261 In several experiments similar to this one we tested various modifications of
 262 the method. The definition 14.4 can be extended for deeper decompositions but it
 263 does not lead to any improvement. We compared the performance of various
 264 wavelets and studied the influence of the wavelet length. Short wavelets are too
 265 sensitive to noise, while long wavelets do not provide enough discrimination
 266 power. Best results on this kind of data were obtained by biorthogonal wavelets.
 267 Another possible modification is to calculate the decision map separately for each
 268 band but this also did not result in noticeable refinement. We also tested the
 269 performance of other fusion techniques. The proposed method always produced a
 270 visually sharp image, assessed by the observers as the best or one of the best.



290 **FIGURE 14.3.** Fusion of a multifocus microscope image: three out of 20 layers in the
291 multifocus stack and the result of the fusion in the wavelet domain (bottom right).
292

293 We can thus conclude that this method is very suitable for fusion of multifocus
294 microscope images.
295

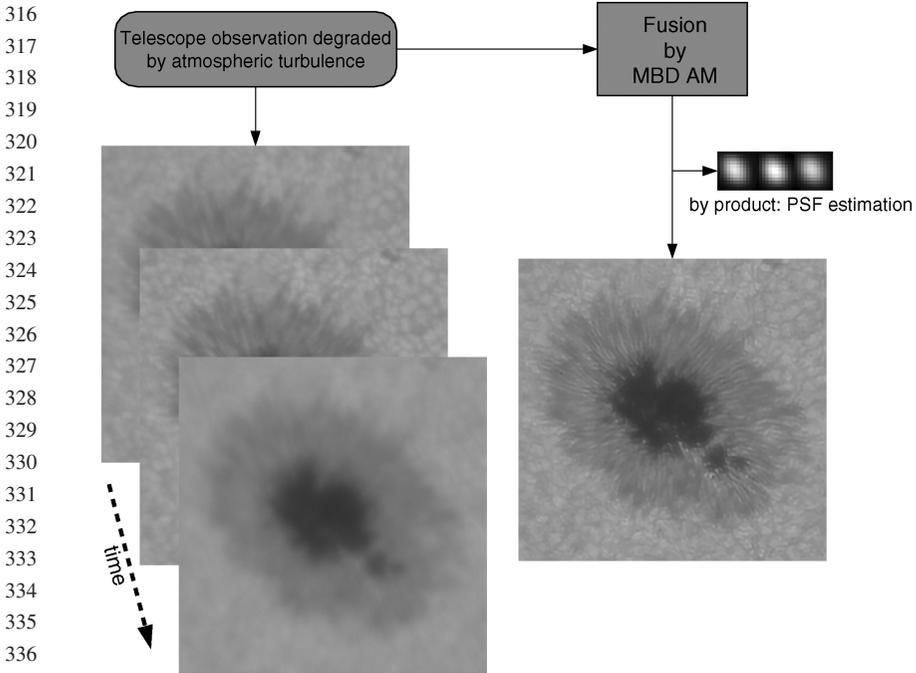
296 297 IV. UNIFORMLY BLURRED CHANNELS

298 An acquisition model with uniformly blurred channels assumes that every PSF h_i
299 is space-invariant within the channel, that is, $h_i(x, y, s, t) = h_i(x - s, y - t)$.
300 Equation 14.2 then turns into the form of “traditional” convolution in each
301 channel with no geometric deformations:
302

$$303 \quad z_i(x, y) = (u * h_i)(x, y) + n_i(x, y) \quad (14.5)$$

304
305
306 This model describes, for instance, photographing a flat static scene with different
307 (but always wrong) focuses, or repetitively photographing a scene through a
308 turbulent medium whose optical properties change between the frames (see
309 [Figure 14.4](#)). The image fusion is performed via multichannel blind deconvolution
310 (MBD). It should be noted that if the PSFs were known, then this task would
311 turn into the classical problem of *image restoration* which has been considered in
312 numerous publications, see Ref. [13](#) for a survey.

313 Blind deconvolution in its most general form is an unsolvable problem. All
314 methods proposed in the literature inevitably make some assumptions about the
315 PSFs h_i and/or the original image $u(x, y)$. Different assumptions give rise to



338 **FIGURE 14.4** Images of a sunspot taken by a ground-based telescope and blurred due to
 339 perturbations of wavefronts in the Earth atmosphere. The perturbations vary in time which
 340 leads to different blurring of the individual frames. The resulting image was fused by the
 341 MBD-AM algorithm described in Section IV.A.

342
343 various deconvolution methods. There are two basic approaches to solving the
 344 MBD problem. The first one is to separately treat each channel by any single-
 345 channel deconvolution method and then to combine the results; the other is to
 346 employ deconvolution methods that are multichannel in their nature.

347 Numerous single-channel blind deconvolution methods have been
 348 published extensively in the literature in the last two decades (see Ref. 14 or
 349 15 for a basic survey). However, their adaptation to the MBD problem cannot
 350 reach the power of intrinsic multichannel methods and this approach seems to be
 351 a “dead-end”.

352 The development of intrinsic multichannel methods has begun just recently.
 353 One of the earliest methods¹⁶ was designed particularly for images blurred by
 354 atmospheric turbulence. Harikumar and Bresler^{17,18} proposed indirect algorithms
 355 (EVAM), which first estimate the PSFs and then recover the original image by
 356 standard nonblind methods. Giannakis and Heath^{19,20} (and at the same time
 357 Harikumar and Bresler²¹) developed another indirect algorithm based on
 358 Bezout’s identity of coprime polynomials which finds inverse filters and, by
 359 convolving the filters with the observed images, recovers the original image.
 360 Pillai and Liang²² have proposed another intrinsically multichannel method

361 based on the greatest common divisor which is, unfortunately, even less
 362 numerically stable than the previous methods. Pai and Bovik^{23,24} came with two
 363 direct multichannel restoration algorithms that directly estimate the original
 364 image from the null space or from the range of a special matrix. To reach higher
 365 robustness, Šroubek²⁵ proposed an iterative deconvolution method which
 366 employs anisotropic regularization of the image and between-channel regular-
 367 ization of the PSFs.

369 A. ALTERNATING MINIMIZATION ALGORITHM

370 In this section, we present an alternating minimization algorithm for
 371 multichannel blind deconvolution (MBD-AM) and we demonstrate that it is a
 372 powerful tool for image fusion in the case of uniformly blurred channels.
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374 Since the blind deconvolution problem is ill posed with respect to both u and
 375 h_i , a constrained minimization technique is required to find the solution of
 376 Equation 14.5. Constraints are built on prior knowledge that we have about the
 377 system. Typical assumptions valid for the majority of real acquisition processes
 378 are the following: n_i is supposed to have zero mean and the same variance σ^2
 379 in each channel, and the PSFs are supposed to preserve the overall brightness (mean
 380 intensity) of the image. The imposed constraints then take the forms

$$381 \frac{1}{|\Omega|} \int_{\Omega} (h_i * u - z_i)^2 dx = \sigma^2 \quad (14.6)$$

$$382 \int_{\Omega} h(x) dx = 1 \quad (14.7)$$

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 388 (To simplify the notation, we drop the two-dimensional co-ordinates (x, y) or, if
 389 required, we only write x .)

390 Let $Q(u)$ and $R(h)$ denote some regularization functionals of the estimated
 391 original image u and blurs $h \equiv \{h_1, \dots, h_N\}$, respectively. The constrained
 392 minimization problem is formulated as $\min_{u,h} Q(u) + \gamma R(h)$ subject to Equation
 393 14.6 and Equation 14.7. The unconstrained optimization problem, obtained by
 394 means of Lagrange multipliers, is to find u and h which minimize the functional

$$395 E(u, h) = \frac{1}{2} \sum_{i=1}^N \|h_i * u - z_i\|^2 + \lambda Q(u) + \gamma R(h) \quad (14.8)$$

396 where λ and γ are positive parameters which penalize the regularity of the
 397 solutions u and h . The crucial questions are how to construct functionals Q and R
 398 and whether the global minimum can be reached. We propose an alternating
 399 minimization algorithm that iteratively searches for a minimum of Equation 14.8.

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 402
 403 **Q1** Constraint Equation 14.7 was dropped since it will be automatically satisfied in
 404 the algorithm if the initial blurs satisfy the constraint. We now proceed the
 405 discussion with possible choices of $Q(u)$ and $R(h)$.

1. Regularization of the Image $Q(u)$

Regularization of Equation 14.5 with respect to the image function can adopt various forms. The classical approach of Tichonov chooses $Q(u) = \int_{\Omega} |\nabla u(x)|^2 dx$, where ∇u denotes the gradient of u . Apart from easy implementation, this regularization is not suitable, since the L_2 norm of the image gradient penalizes too much the gradients corresponding to edges and an oversmoothing effect is observed. In real images, object edges create sharp steps that appear as discontinuities in the intensity function. It is the space of bounded variation (BV) functions that is widely accepted as a proper setting for real images. Rudin²⁶ first demonstrated very good anisotropic denoising properties of the total variation (TV) $Q_{TV}(u) = \int |\nabla u(x)| dx$. Existence and uniqueness of the minimum of TV is possible only in the BV-space, in which case ∇u denotes the gradient of u in the distributional sense. The same holds true for a more general case of convex functions of measures

$$Q_{\phi}(u) = \int \phi(|\nabla u(x)|) dx$$

where ϕ is a strictly convex, nondecreasing function that grows at most linearly. Examples of $\phi(s)$ are $s(TV)$, $\sqrt{1+s^2} - 1$ (hyper-surface minimal function) or $\log(\cosh(s))$. For nonconvex functions nothing can be said about the existence of the minimum. Nevertheless, nonconvex functions, such as $\log(1+s^2)$, $s^2/(1+s^2)$ or $\arctan(s^2)$ (Mumford–Shah functional²⁷), are often used since they provide better results for segmentation problems.

2. Regularization of the Blurs $R(h)$

Regularization of the blurs h_i 's directly follows from our model, Equation 14.5, and can be derived from the mutual relations of the channels. The blurs are assumed to have finite support S of the size (s_1, s_2) and certain channel disparity is necessary. The disparity is defined as weak coprimeness of the channel blurs, which states that the blurs have no common factor except a scalar constant. In other words, if the channel blurs can be expressed as a convolution of two subkernels then there is no subkernel that is common to all blurs. An exact definition of weakly coprime blurs can be found in Ref. 20. The channel coprimeness is satisfied for many practical cases, since the necessary channel disparity is mostly guaranteed by the nature of the acquisition scheme and random processes therein. We refer the reader to Ref. 18 for a relevant discussion.

Under the assumption of channel coprimeness, we can see that any two correct blurs h_i and h_j satisfy $\|z_i * h_j - z_j * h_i\|^2 = 0$ if the noise term in Equation 14.5 is omitted. We therefore propose to regularize the blurs by

$$R(h) = \frac{1}{2} \sum_{1 \leq i < j \leq N} \|z_i * h_j - z_j * h_i\|^2 \quad (14.9)$$

This regularization term does not penalize spurious factors, that is, $f * h_i$ for any factor f are all equivalent. We see that the functional $R(h)$ is convex but far from

451 strictly convex. The dimensionality of the null space of $R(\hat{h})$ is proportional to
 452 the degree of size overestimation of \hat{h}_i with respect to the size of the original blurs
 453 h_i 's. Therefore to use the above regularization, we have to first estimate S of the
 454 original blurs and impose this support constraint in R . The size constraint is
 455 imposed automatically in the discretization of R , which is perfectly plausible since
 456 the calculations are done in the discrete domain anyway. An exact derivation of
 457 the size of the null space is given in Ref. 18.

458
 459 **3. Iterative Minimization Algorithm**

460 We consider the following minimization problem

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$$E(u, h) = \frac{1}{2} \sum_{i=1}^N \|h_i * u - z_i\|^2 + \frac{\gamma}{2} \sum_{1 \leq i < j \leq N} \|z_i * h_j - z_j * h_i\|^2 + \lambda \int_{\Omega} \phi(|\nabla u(x)|) dx \tag{14.10}$$

469 $E(u, h)$, as a function of variables u and h , is not convex due to the convolution in
 470 the first term. On the other hand, the energy function is convex with respect to u if
 471 h is fixed and it is convex with respect to h if u is fixed. The minimization
 472 sequence (u^n, h^n) can be thus built by alternating between two minimization
 473 subproblems

474 **Q2**
$$u^n = \arg \min_u E(u, h^{n-1}) \quad \text{and} \quad h^n = \arg \min_h E(u^n, h) \tag{14.11}$$

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 477 for some initial h^0 with the rectangular support S . The advantage of this scheme
 478 lies in its simplicity, since for each subproblem a unique minimum exists that can
 479 be easily calculated. However, we cannot guarantee that the global minimum is
 480 reached this way, but thorough testing indicates good convergence properties of
 481 the algorithm for many real problems.

482 The solution of the subproblem, Equation 14.10, formally satisfies the Euler–
 483 Lagrange equation

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$$\frac{\partial E}{\partial u} = \sum_{i=1}^N h'_i * (h_i * u - z_i) - \lambda \operatorname{div} \left(\frac{d\phi(|\nabla u|)}{d|\nabla u|} \frac{\nabla u}{|\nabla u|} \right) = 0 \tag{14.12}$$

489 where the prime means mirror reflection of the function, that is, $h'_i(x, y) =$
 490 $h_i(-x, -y)$. One can prove (see for example Ref. 28) that a unique solution exists
 491 in the BV-space, where the image gradient is a measure. To circumvent the
 492 difficulty connected with implementing the measure and with the nonlinearity of
 493 the divergence term in Equation 14.12, the solution can be found by relaxing ϕ
 494 and following a half-quadratic algorithm originally proposed in Ref. 29 and
 495 generalized for convex functions of measures in Ref. 28.

496 The solution of the subproblem, Equation 14.11, formally satisfies the Euler–
497 Lagrange equations

$$498 \frac{\partial E}{\partial h_k} = u' * (u * h_k - z_k) - \gamma \sum_{\substack{i=1 \\ i \neq k}}^N z_i' * (z_i * h_k - z_k * h_i) = 0, \quad k = 1, \dots, N \quad (14.3)$$

502 This is a set of linear equations and thus finding h is a straightforward task.

503 It is important to note that the algorithm runs in the discrete domain and that a
504 correct estimation of the weighting constants, λ , and γ , and mainly of the blur
505 support S is crucial. In addition, the algorithm is iterative and the energy
506 (Equation 14.9) as a function of the image and blurs does not have one minimum,
507 so the initial guess g^0 plays an important role as well. The positive weighting
508 constants λ and γ are proportional to the noise levels σ and can be calculated in
509 theory from the set of Equation 14.6, Equation 14.12 and Equation 14.13 if the
510 noise variance is known. This is, however, impossible to carry out directly and
511 techniques such as generalized cross validation must be used instead. Such
512 techniques are computationally very expensive and we suggested an alternative
513 approach in Ref. 25 which uses bottom limits of λ and γ . Estimation of the size of
514 the blur support S is even more vexatious. Methods proposed in Refs. 18,20
515 provide a reliable estimate of the blur size only under ideal noise-free conditions.
516 In the noisy case they suggest a full search, that is, for each discrete rectangular
517 support S estimate the blurs and compare the results.

519 4. Experiment with Artificial Data

520 First, we demonstrate the performance of the MBD–AM algorithm on images
521 degraded by computer-generated blurring and noise and we compare the results
522 with two recent methods — Harrikumar’s EVAM and Pai’s method.

523 For the evaluation, we use the percentage mean-square error of the fused
524 image \hat{u} , defined as

$$525 \text{PMSE}(u) \equiv 100 \frac{\|\hat{u} - u\|}{\|u\|} \quad (14.14)$$

526 Although the mean-square error does not always correspond to visual evaluation
527 of the image quality, it has been commonly used for quantitative evaluation and
528 comparison.

529 A test image of size 250×250 in Figure 14.5(a) was first convolved with four
530 7×7 PSFs in Figure 14.5(b) and then white Gaussian noise at five different levels
531 (SNR = 50, 40, 30, 20, and 10 dB, respectively) was added. This way, we
532 simulated four acquisition channels ($N = 4$) with a variable noise level. The size
533 of the blurs and the noise level were assumed to be known. All three algorithms
534 were therefore started with the correct blur size $S = (7, 7)$. In the case of
535 MBD–AM, λ and γ were estimated as described in Ref. 25 and the starting
536 position h^0 was set to the delta functions. The reconstructed images and blurs are

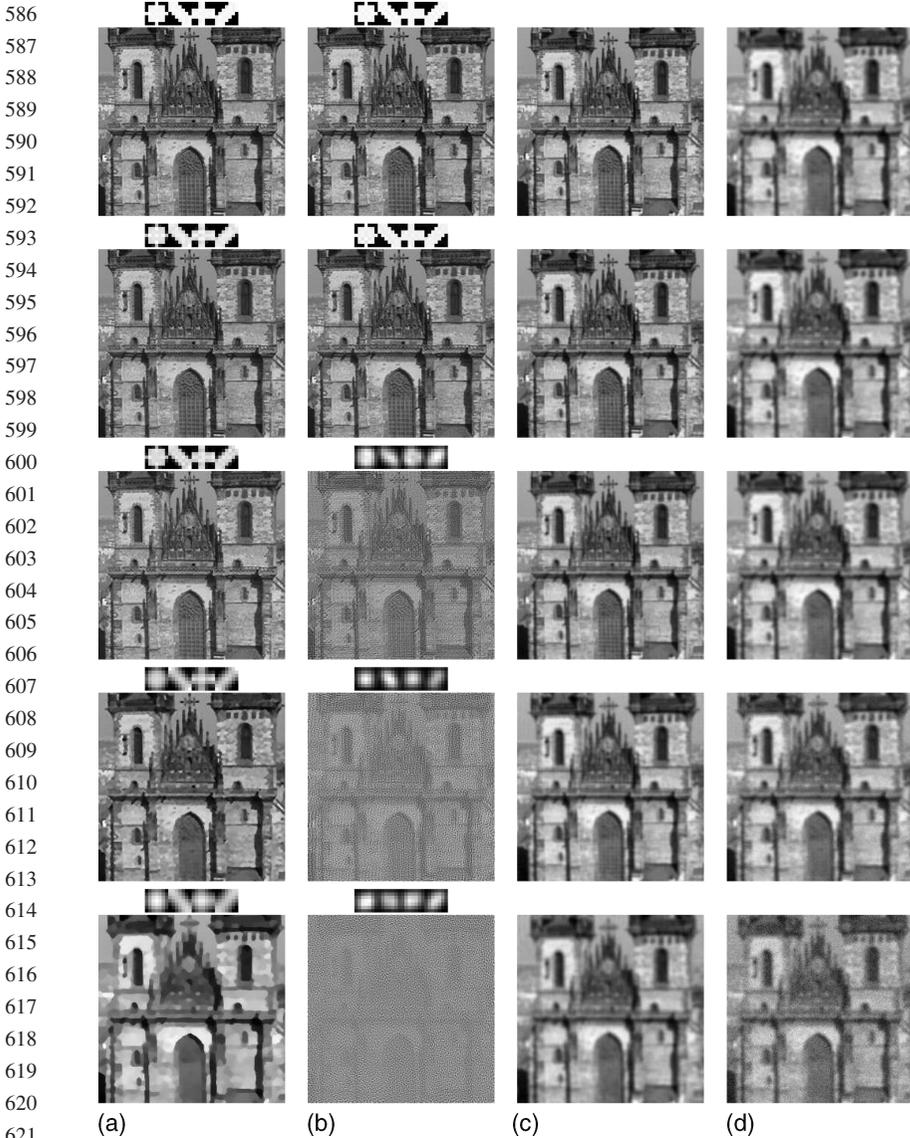


561 **FIGURE 14.5** Synthetic data: (a) original 250×250 image of the Tyn church, (b) four **Q3**
562 7×7 PSFs, and (c) four blurred channels.

563
564 shown in [Figure 14.6](#) with the percentage mean-square errors summarized in
565 [Table 14.1](#). Note that Pai's method reconstructed only the original image and not
566 the blurs. The performance of the EVAM method quickly decreases as SNR
567 decreases, since noise is not utilized in the derivation of this method. Pai's
568 method shows superior stability but for lower SNR the reconstructed images are
569 still considerably blurred. Contrary to the previous two methods, the MBD-AM
570 algorithm is stable and performs well even for lower SNRs (20 dB, 10 dB). One
571 slight drawback is that the output increasingly resembles a piecewise constant
572 function which is due to the variational regularization $Q(u)$.
573

574 575 5. Experiment with Real Data

576
577 The following experiment was conducted to test the applicability of MBD-AM
578 on real data. Four images of a bookcase were acquired with a standard digital
579 camera focused to 80 cm (bookcase in focus), 40, 39, and 38 cm distance,
580 respectively. The acquired data were stored as low resolution 640×480 24 bit
581 color images and only the central rectangular part of the green band of size
582 250×200 was used for the fusion. The central part of the first image, which
583 captures the scene in focus, is shown in [Figure 14.7\(a\)](#). Three remaining images,
584 [Figure 14.7\(c\)](#), were used as the input for the MBD-AM algorithm. The
585 parameter $\lambda = 1.6 \times 10^{-4}$ was estimated experimentally by running the



622 **FIGURE 14.6** Reconstruction of the test image and blurs from four degraded images
623 **Q3** using (a) MBD-AM, (b) EVAM, and (c) Pai's method. The first of the four degraded
624 channels is in column (d) for comparison. From top to bottom SNR = 50, 40, 30, 20, and
625 10 dB, respectively.

626
627 algorithm with different λ 's and selecting the most visually acceptable results.
628 The parameter γ was calculated as described in Ref. 25. A defocused camera
629 causes image degradation approximately modeled by cylindrical blurs. A
630 cepstrum analysis in Ref. 30 was used to estimate diameters of these blurs, which

TABLE 14.1

Performance of the MBD-AM, the EVAM and the Pai's Algorithms on the Data in Figure 14.5

SNR (dB)	MBD-AM	EVAM	Pai
50	0.93	0.99	5.09
40	2.61	2.99	7.87
30	5.17	24.1	10.9
20	10.2	35.7	13.9
10	15.3	38.3	16.4

The table shows percentage mean-square error of the fused image.

were determined to be around eight pixels. Obtained results after ten iterations are shown in Figure 14.7(b). Further iterations did not produce any visual enhancement. Simple visual comparison reveals that the letters printed on shelf backs are more legible in the restored image but still lack the clarity of the focused image, and that the reconstructed blurs resemble the cylindrical blurs as was expected. It is remarkable how successful the restoration was, since one would expect that the similarity of blurs would violate the coprimeness assumption. It is believed that the algorithm would perform even better if a wider disparity between blurs was assured.

V. SLIGHTLY MISREGISTERED BLURRED CHANNELS

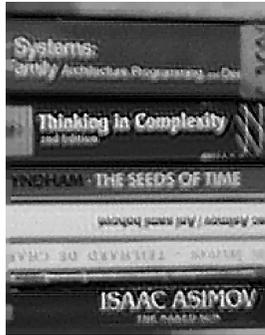
This is a generalization of the previous model which allows between-channel shifts (misregistrations) of extent up to a few pixels.

$$z_i(x + a_i, y + b_i) = (u * h_i)(x, y) + n_i(x, y) \quad (14.15)$$

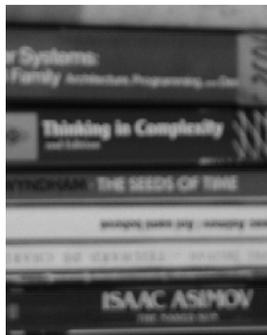
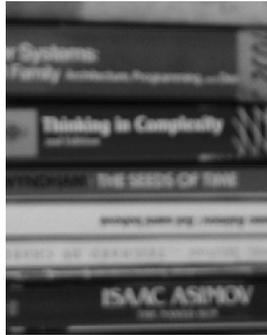
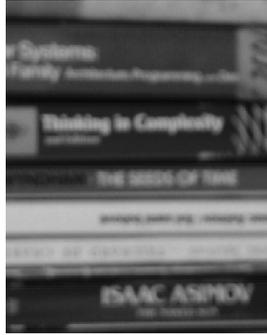
where a_i, b_i are unknown translation parameters of the i th channel. This model is applicable in numerous practical tasks when the scene or the camera moves slightly between consecutive channel acquisitions (see Figure 14.8). Such a situation typically occurs when the camera is subject to vibrations or in multitemporal imaging if the scene is not perfectly still. Sometimes a subpixel between-channel shift is even introduced intentionally in order to enhance spatial resolution of the fused image (this technique is called *superresolution imaging*, see Ref. 31 for a survey and other references).

Images degraded according to this model cannot be fused by the methods mentioned in Section IV.A. If they were applied, the channel misregistrations would lead to strong artifacts in the fused image. On the other hand, the misregistrations considered in this model are too small to be fully removed by image registration techniques (in case of blurred images, registration methods

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(b)



(c)



(a)

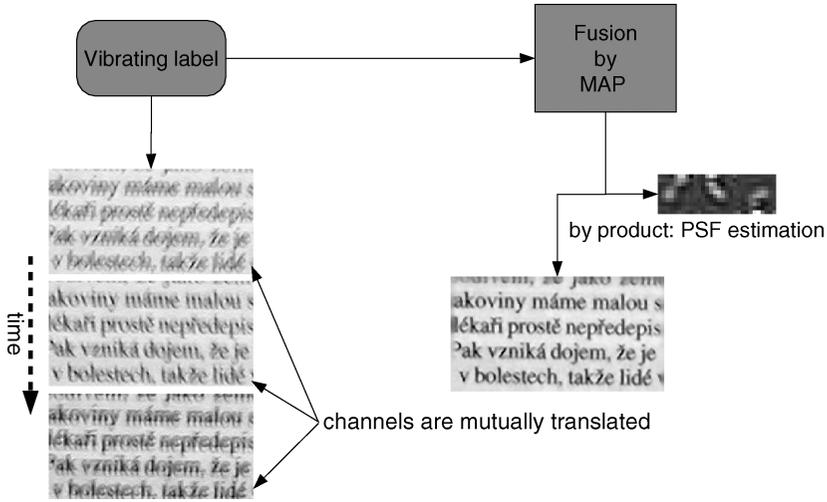


FIGURE 14.8 Multiple acquisition of a vibrating text label. Motion blur of various kind and small spatial misalignments of the individual frames can be observed. The fused image was achieved by the MAP algorithm described in Section I.

usually can suppress large spatial misalignments but seldom reach subpixel accuracy).

Fusion of images degraded according to this model requires special blind deconvolution methods, which can — in addition to the deconvolution itself — identify and compensate the between-channel misregistration. A successful method based on a stochastic approach is described below.

A. MAXIMUM A POSTERIORI PROBABILITY ALGORITHM

Equation 14.15 can be expressed into equivalent form

$$z_i(x, y) = u(x, y) * h_i(x - a_i, y - b_i) + n_i(x, y) \tag{14.16}$$

which can be further rewritten as

$$z_i(x, y) = (u * g_i)(x, y) + n_i(x, y) \tag{14.17}$$

where g_i is a shifted version of the original PSF $h_i : g_i(x, y) = h_i(x - a_i, y - b_i)$. We can therefore work only with g_i and use the MBD-AM algorithm (Equation 14.10 and Equation 14.11). In this case, the estimate of the blur size has to include

Q4 FIGURE 14.7 Real bookcase images: (a) 250×250 image acquired with the digital camera set to the correct focus distance of 80 cm, (b) MBD-AM fused image along with 10×10 estimated blurs obtained from three images, and (c) image of false focus distances 40 cm, 39 cm and 38 cm, after 10 iterations and $\lambda = 1.6 \times 10^{-4}$.

766 also the maximum shift between the channels. Since this is difficult to determine,
 767 standard MBD techniques including MBD-AM in its present form cannot be
 768 applied.

769 To overcome the difficulties connected with the parameter estimation, we
 770 adopt in Ref. 32 a stochastic approach to the minimization problem. The
 771 restoration can be formulated then as a maximum *a posteriori* (MAP) estimation.
 772 We assume that the matrices u , $g = \{g_1, \dots, g_N\}$ and $z = \{z_1, \dots, z_N\}$ are random
 773 vector fields with given probability density functions (PDFs) $p(u)$, $p(g)$ and $p(z)$,
 774 respectively, and we look for such realizations of u and g which maximize the *a*
 775 *posteriori* probability $p(u, g|z)$. The MAP estimation is equivalent to minimizing
 776 $-\log(p(u, g|z))$. The only two assumptions that we must make in addition to those
 777 in the energy minimization problem are: u and g are supposed to be statistically
 778 independent and n_i is white (that is, uncorrelated) Gaussian noise. Using the
 779 Bayes rule, the relation between *a priori* densities $p(u)$, $p(g)$ and the *a posteriori*
 780 density is $p(u, g|z) \propto p(z|u, g)p(u)p(g)$. The conditional PDF $p(z|u, g)$ follows
 781 from our model, Equation 14.5, and from our assumption of white noise. The blur
 782 PDF $p(g)$ can be derived from the regularization $R(g)$, which is also Gaussian
 783 noise with a covariance matrix that can be easily calculated. If the image PDF
 784 $p(u)$ is chosen in such a way that $-\log(p(u)) \propto Q(u)$ then the MAP estimation is
 785 almost identical to the minimization problem, Equation 14.9, for $\lambda = \sigma^2$ and
 786 $\gamma = |S|/2$ and we can use the alternating iterative algorithm. To improve stability
 787 of the algorithm against the overestimation of S and thus handle inaccurate
 788 registration, it suffices to add the constraint of positivity $h(x) > 0$ to Equation
 789 14.7 and perform in Equation 14.11 the minimization subject to the new
 790 constraints.

791 Setting appropriately initial blurs can help our iterative algorithm to
 792 converge to the global minimum. This issue is especially critical for the case
 793 of overestimated blur size. One can readily see that translated versions of the
 794 correct blurs are all equivalent as long as they fit into our estimated blur size.
 795 We have seen that the regularization of the blurs R is unable to distinguish
 796 between the correct blurs and the correct blurs convolved with an arbitrary
 797 spurious factor. This has a negative impact on the convergence mainly if
 798 channel misalignment occurs, since new local minima appear for blurs that
 799 cope with the misalignment by convolving the correct blurs with an
 800 interpolating kernel. To get closer to the correct solution, we thus propose
 801 to set the initial blurs g^0 to delta functions positioned at the centers of gravity
 802 of blurs $\hat{g} = \arg \min R(g)$. This technique enables us to compensate for the
 803 channel shifts right from the start of the algorithm and get away from the
 804 incorrect interpolated solutions.

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1. Experiment with Misregistered Images

Although the MAP fusion method can also be applied to registered channels, its
 main advantageous property, that discriminates it from other methods, is the

811 ability to fuse channels which are not accurately registered. This property is
 812 illustrated by the following experiment.

813 The 230×260 test image in [Figure 14.9\(a\)](#) was degraded with two different
 814 5×5 blurs and noise of $\text{SNR} = 50$ dB. One blurred image was shifted by 5×5
 815 pixels and then both images were cropped to the same size; see [Figure 14.9\(c\)](#).
 816 The MAP algorithm was initialized with the overestimated blur size 12×12 . The
 817 fused image and the estimated blur masks are shown in [Figure 14.10](#). Recovered
 818 blurs contain negligible spurious factors and are properly shifted to compensate
 819 for the misregistration. The fused image is, by visual comparison, much sharper
 820 than the input channels and very similar to the original, which demonstrates
 821 excellent performance. This conclusion is supported also by the real experiment
 822 shown in [Figure 14.8](#), where both blurring and shift were introduced by object
 823 vibrations. Unlike the input channels, the text on the fused image is clearly
 824 legible.

826 VI. HEAVILY MISREGISTERED BLURRED CHANNELS

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 828 This model is a further generalization of the previous model. Blurring of each
 829 channel is still uniform and is modeled by a convolution, but significant
 830 misregistrations between the channels are allowed.

$$831 \quad z_i(\tau_i(x, y)) = (u * h_i)(x, y) + n_i(x, y) \quad (14.18)$$

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 833
 834 In this model there are almost no restrictions on the extent and the type of τ_i ; it
 835 may have a complex nonlinear form (the only constraint is that the individual
 836 frames must have sufficient overlap in the region of interest). This is a very
 837 realistic model of photographing a flat scene, where the camera moves in three-
 838 dimensional space in an arbitrary manner (see [Figure 14.11](#)).

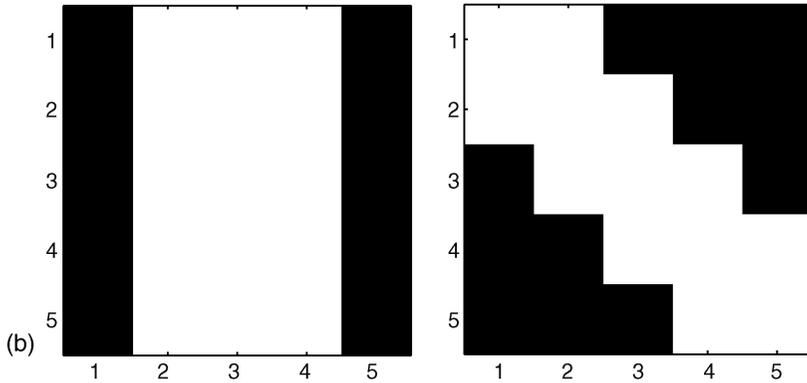
839 Because of the complex nature of τ_i , it cannot be compensated for during the
 840 deconvolution step. Thus, fusion of images degraded according to this model is a
 841 two-stage process — it consists of image registration (spatial alignment)
 842 followed by MBD discussed in the previous section. Since all deconvolution
 843 methods require either perfectly aligned channels (which is not realistic) or allow,
 844 at most, small shift differences, the registration is a crucial step of the fusion.

845 Image registration in general is a process of transforming two or more images
 846 into a geometrically equivalent form. It eliminates the degradation effects caused
 847 by geometric distortion. From a mathematical point of view, it consists of
 848 approximating τ_i^{-1} and of resampling the image. For images which are not
 849 blurred, the registration has been extensively studied in the recent literature (see
 850 [Ref. 33](#) for a survey). However, blurred images require special registration
 851 techniques. They can, as well as the general-purpose registration methods, be
 852 divided in two groups — global and landmark-based ones. Regardless of the
 853 particular technique, all feature extraction methods, similarity measures, and
 854 matching algorithms used in the registration process must be insensitive to image
 855 blurring.

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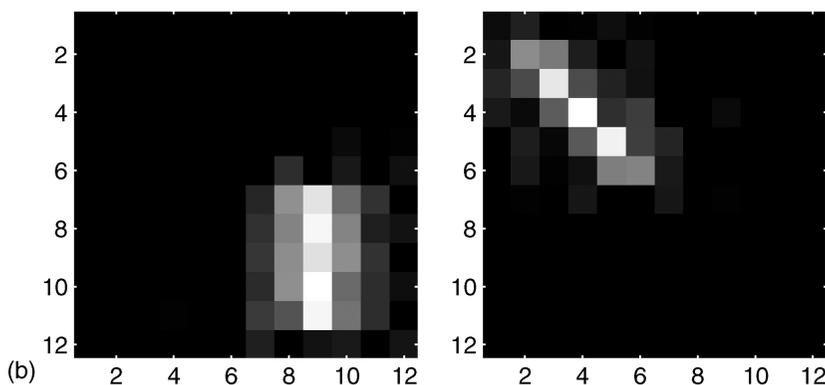


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FIGURE 14.9 (a) original test image 230×260 pixels, (b) two 5×5 PSFs, and (c) blurred and shifted images.

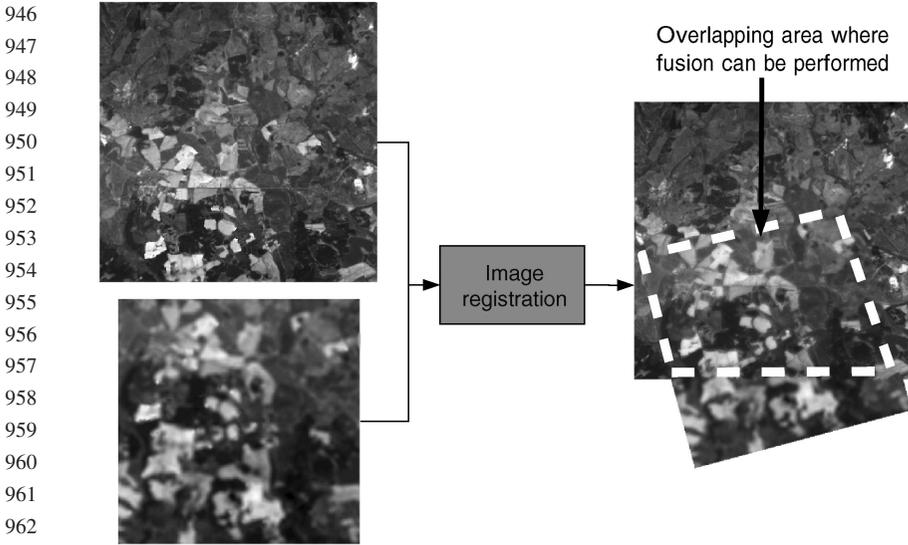


926 **Q3** FIGURE 14.10 MAP image fusion: (a) fused image and (b) estimated blur masks with
927 between-channel shift.
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930 Global methods do not search for particular landmarks in the images. They
931 try to estimate directly the between-channel translation and rotation. Myles and
932 Lobo³⁴ proposed an iterative method working well if a good initial estimate of the
933 transformation parameters is available. Zhang et al.,^{35,36} proposed to estimate the
934 registration parameters by bringing the channels into canonical form. Since blur-
935 invariant moments were used to define the normalization constraints, neither the
936 type nor the level of the blur influences the parameter estimation. Kubota et al.³⁷
937 proposed a two-stage registration method based on hierarchical matching, where
938 the amount of blur is considered as another parameter of the search space. Zhang
939 and Blum³⁸ proposed an iterative multiscale registration based on optical flow
940 estimation in each scale, claiming that optical flow estimation is robust to image
941 blurring. All global methods require considerable (or even complete) spatial
942 overlap of the channels to yield reliable results, which is their major drawback.

943 Landmark-based blur-invariant registration methods have appeared very
944 recently, just after the first paper on the moment-based blur-invariant features.³⁹
945 Originally, these features could only be used for registration of mutually



964 **FIGURE 14.11.** Two satellite images differing from one another by amount of blur due to
965 different spatial resolution and by shift, rotation, and scaling (left). After the registration
966 (right), the MAP fusion algorithm from the previous section can be applied on the
967 overlapping area. The registration of these images was performed in IMARE Toolbox⁵⁰ by
968 means of invariant-based method,⁴² courtesy of Barbara Zitová.

969 shifted images.^{40,41} The proposal of their rotational-invariant version⁴² in
970 combination with a robust detector of salient points⁴³ led to registration methods
971 that are able to handle blurred, shifted and rotated images.^{44,45}

972 Although the above-cited registration methods are very sophisticated and can
973 be applied to almost all types of images, the result rarely tends to be perfect. The
974 registration error usually varies from subpixel values to a few pixels, so only
975 fusion methods sufficiently robust to between-channel misregistration can be
976 applied to channel fusion.
977

978 VII. CHANNELS WITH SPACE-VARIANT BLURRING

979 This model comprises space-variant blurring of the channels as well as nonrigid
980 geometric differences between the channels.
981

$$982 \quad z_i(\tau_i(x, y)) = \int h_i(x, y, s, t)u(s, t)ds dt + n_i(x, y) \quad (14.19)$$

983
984 The substantial difference from the previous models is that image blurring is no
985 longer uniform in each frame and thus it cannot be modeled as a convolution. Here,
986 the PSF is a function of spatial co-ordinates (x, y) which makes the channel
987 degradation variable depending on the location. This situation typically arises
988 when photographing a three-dimensional scene by a camera with a narrow depth of
989 field. Differently blurred channels are obtained by changing the focus distance of
990

the camera (see [Figure 14.12](#)). Unlike piecewise ideal imaging, the depth of the scene can vary in a continuous manner and the existence of at least one “ideal” picture for each location is not guaranteed. Another example is photographing a dynamic scene where different parts move by different velocity and/or in different directions (see [Figure 14.13](#)).

Space-variant blurring is not a simple extension of the previous models. It requires qualitatively new approaches and methods. As for the previous model, the image fusion consists of image registration and multichannel blind deblurring but there is a significant difference. While the registration methods can be in principle the same, the techniques used here in the second step must be able to handle space-variant blurring.

Up until now, no papers have been published on multichannel space-variant deblurring. There are, however, a few papers on single-channel space-variant image deblurring, usually originating from space-invariant deconvolution methods. Guo et al.⁴⁶ proposed to divide the image into uniformly blurred regions (if possible) and then to apply a modified expectation-maximization algorithm in each region. You and Kaveh⁴⁷ considered parameterized PSF and used anisotropic regularization for image deblurring. Cristobal and Navarro⁴⁸

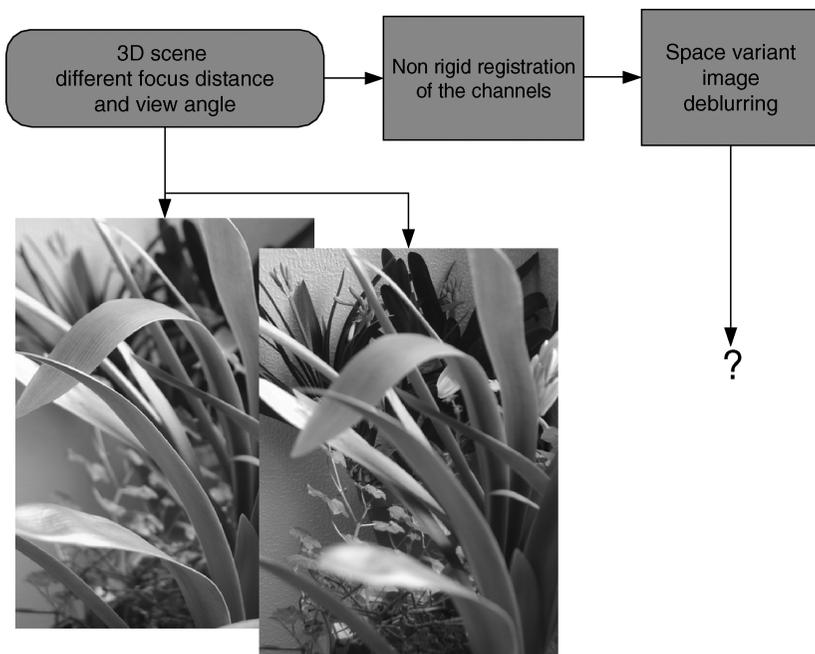


FIGURE 14.12 Space-variant blurring of the channels. Two pictures of a complex three-dimensional scene taken with a variably focused camera. The camera also changed its position and viewing angle between the acquisitions which lead to projective geometric deformation between the channels.

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FIGURE 14.13 Space-variant degradation of the channels due to motion blur.

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applied multiscale Gabor filters to the restoration. However, the extension of the above methods to the multichannel framework is questionable.

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No doubt the solution to this problem is a big challenge for image fusion for the near future. Prospective methods should employ all available *a priori* information, such as a depth map or relief model. They may comprise depth-based, defocus-based or depth- and defocus-based segmentation of the input channels in order to find regions of the same type of blur. Nevertheless, a general solution probably does not exist.

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VIII. CONCLUSION

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In this chapter, we presented an overview of image fusion methods for the case where the input channels are blurred, noisy, and geometrically different. One has to face this problem in various application areas where the picture of the scene is taken under nonideal conditions. Mathematically, this task is ill posed and cannot be resolved by inverting all degradation factors. The only solution is multiple acquisition of the scene and consequent fusion of all acquired channels. It is believed that if the channel degradations are different, the channels can be fused together in such a way that the information missing in one channel can be supplemented by the others. The fusion approaches and methods differ from each other according to the type of assumed degradations of the channels. Here we classified the possible degradations into five major groups: piecewise ideal imaging, uniform blurring, slight and heavy channel misregistration, and space-variant blurring of the channels. For each category, except the last one, we presented reliable fusion methods whose performance was experimentally verified.

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ACKNOWLEDGMENTS

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1216 **Author Queries**

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1218 *JOB NUMBER: 9295*

1219 *CHAPTER: Fusion of Blurred Images*

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1222 **Q1** “Constraint (7) was dropped...” I have assumed (7) refers to Equation
1223 (7) and changed to Equation 14.7 to conform to CRC style. Please
1224 confirm this is correct. If so, should the wording of this phrase be
1225 changed to “The constraint of Equation 14.7 was dropped...”?

1226

1227 **Q2** Since there are two equations with the same number so the equations are
reordered. Kindly check.

1228

1229 **Q3** Please check the figure quality.

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1231 **Q4** Please check whether the permission line is needed or not for this
1232 figure. If so, please provide.

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