

Non-stationary Autoregressive Model for On-line Detection of Inter-area Oscillations in Power Systems

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Abstract—This paper addresses early on-line detection of inter-area electro-mechanical oscillations in power systems using dynamic data such as currents, voltages and angle differences measured across transmission lines in real time. The main objective is to give the transmission operator qualitative information regarding stability margins. In our approach, the observed signal is modeled with the non-stationary second order autoregressive model. Bayesian estimation of the system is based on the forgetting approach. The stability margins are obtained as posterior probabilities that the poles of the estimated system are unstable. The approach is demonstrated on real retrospective data recorded in a 500 kV power grid and voltage data obtained by numerical simulations.

Index Terms—Autoregressive processes; Kalman filtering; Power system stability; Probability.

I. INTRODUCTION

POWER systems are characterized by many modes of electro-mechanical oscillations caused by interactions among its components. During such oscillations, mechanical kinetic energy is exchanged between synchronous generators as electric power flows through the network. For example, one generator rotor could swing relative to another. The inter-area modes are usually associated with groups of machines swinging relative to other groups across a relatively weak transmission line. These impacts cause oscillations in state variables of the electric system such as voltage, current, power and frequency, which are conventionally measured by PMU devices (Phasor Measurement Units).

The amplitudes of the swinging state variables are mainly determined by the following factors:

- The position of the subsystem in the whole power system.
- The distribution of the natural damping elements such as series resistance of the lines, and shunt resistance of the loads.
- The number and position of special damping controllers, e.g. Power System Stabilizer (PSS) and controllers of different FACTS devices (SVC, TCSC, etc).

This work is supported by the grants of the Federal Agency for Science and Innovation of the Russian Federation: #1857.2008.8 of the Leading Scientific School of the Russian Federation and by the governmental contract #02.527.11.0004, by Russian Foundation for Basic Research (project 09-08-91-330) and GA CR project 102/08/P250. This work is fulfilled as part of Russian-Czech Inter-Academia project No. 67.

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PSS controllers are among the most effective and robust solutions [9], SVCs are also widely used for stabilization by modulating voltage at strategic locations of the power system (see e.g. the example of Mexico System [9], [7]).

There are two distinct types of problematic oscillations in power systems: local mode oscillations and inter-area oscillations. Local mode oscillations occur when a generator (or group of generators) under voltage regulator control at a station is oscillating against the rest of the system. Inter-area oscillations involve combinations of many machines on one part of a system swinging against machines on another part of the system. It is to be noted that the local mode of oscillations are well damped by the traditional PSS controllers, but normally fails the inter-area ones [10].

In this paper, we address the problem of robust detection of inter-area oscillations. This problem needs more sophisticated approaches in order to ensure accurate monitoring of system dynamics and reliable detection of dangerous oscillations with noise-polluted PMU measurements. Note that oscillations themselves are not necessarily dangerous as long as they do not become unstable [1]. The key objective of this paper is to design an algorithm evaluating stability margins.

The paper is organized as follows. In Section 1, we provide the motivation of our studies, next we briefly discuss the related techniques employed for on-line oscillations prediction. In Section 2, we present oscillations detection algorithm based on regularized exponential forgetting suitable for non-stationary data analysis of power systems. In Section 3, we apply the designed technique for real retrospective data corresponding to inter-area oscillations event recorded in 500kV power grid and for the data obtained by numerical simulations. The concluding remarks and possible ways for improvements of the proposed techniques are briefly listed in Section 4.

II. PROBLEM STATEMENT

A. State-of-the-art Techniques

The deregularization of the power market has caused substantial demand for the development of new tools for electro-mechanical oscillations prediction. Many classical non-adaptive algorithms such as Yule-Walker, Burgs, lattice and Prony's methods (see e.g. [5], [6] and [3]) have been applied in the field. Also recursive least squares (RLS) and least mean squares (LMS) methods are typical solutions. Recently, Kalman filtering techniques has been employed by Korba et al [2] for on-line oscillations prediction.

B. Techniques Comparison

These methods are typically based on treatment of the underlying process as a linear system. Detection of the oscillations is based on two basic results of linear systems theory:

- 1) poles of oscillating linear systems have non-zero imaginary components,
- 2) poles of unstable linear systems have absolute values greater than 1.

We use these facts to detect oscillations as follows: (i) the observed process is locally approximated by a linear system, (ii) parameters of the linear system are estimated, (iii) poles of the system are computed from the estimates, and (iv) stability and oscillatory behavior is analyzed.

The methods mentioned above differ typically in (i) and (ii). For example, detection methods based on fixed windows assume that all data in the window were generated by the estimated linear system with identical weight. An alternative is represented by RLS with discounting which assumes exponential decrease of importance of older data records. The difference in (ii) is typically in the assumption whether the variance of the measurement noise is known (Kalman filter) or unknown (RLS-type methods). However, the methods rarely differ in (iii) and (iv) where a point estimate of the poles is being analyzed. We aim to address this issue.

The traditional point estimate approach provides a single option for all poles for given time without any uncertainty bounds on the result. Thus, it is hard to assess the reliability of this value. In this paper, we continue our studies [13] concerned with the Bayesian approach, i.e. we develop full posterior density of the parameters of the linear system and transform this density to density on poles.

III. OSCILLATIONS DETECTION ALGORITHM

We represent the signal as a second-order linear system with unknown time-variant parameters:

$$y_t = a_t y_{t-1} + b_t y_{t-2} + c_t + \sigma_t e_t \quad (1)$$

where y_t is the observed signal, a_t, b_t, c_t, σ_t are its unknown parameters, and e_t is Gaussian noise with zero mean and unit variance, $e_t = \mathcal{N}(0, 1)$.

In the probabilistic formulation, (1) defines probability density function (pdf) of the observed random variable y_t :

$$p(y_t | y_{t-1}, y_{t-2}, a_t, b_t, c_t, \sigma_t) = \mathcal{N}(a_t y_{t-1} + b_t y_{t-2} + c_t, \sigma_t^2). \quad (2)$$

Estimation of system (2) with stationary parameters is a well known task in statistics, with posterior density of Normal-inverse-Gamma type. Extension of this approach to a non-stationary system can be achieved by specification of the parameter evolution model, $p(a_t, b_t, c_t, \sigma_t | a_{t-1}, b_{t-1}, c_{t-1}, \sigma_{t-1})$. The specific choice of such a model yields a Bayesian filtering task, which can be solved in some cases by the Kalman filter.

However, we consider a simpler alternative, known as forgetting (or discounting). In this approach, the time-variant

system is treated similarly to the time-invariant system, but the resulting sufficient statistics is multiplied by a constant ϕ , $0 < \phi < 1$. In effect, the delayed data records, y_{t-k} , are weighted by ϕ^k , which is equivalent to the application of an exponential weighting. However, this simple approach has some shortcomings, such as numerical instability when the data are not informative.

We will apply an improved version of forgetting, [11] where regularized exponential forgetting is formalized as follows:

$$\begin{aligned} p(a_t, b_t, c_t, \sigma_t | y_1, \dots, y_t) &\propto p(y_t | y_{t-1}, y_{t-2}, a_t, b_t, c_t, \sigma_t) \\ &\times p(a_{t-1}, b_{t-1}, c_{t-1}, \sigma_{t-1} | y_1, \dots, y_{t-1})^\phi \\ &\times \bar{p}(a_{t-1}, b_{t-1}, c_{t-1}, \sigma_{t-1} | y_1, \dots, y_{t-1})^{1-\phi}. \end{aligned} \quad (3)$$

Here, $\bar{p}(\cdot)$ denotes an *alternative* probability of the parameters. This probability expresses an alternative (prior) knowledge about location of the parameters.

A. Posterior density

One advantage of (3) is that for system (1) it preserves posterior density of the Normal-inverse-Gamma type,

$$p(a_t, b_t, c_t, \sigma_t) = \mathcal{NiG}(V_t, \nu_t), \quad (4)$$

the statistics of which are recursively computed as follows:

$$\begin{aligned} V_t &= \phi V_{t-1} + [y_t, y_{t-1}, y_{t-2}, 1]' [y_t, y_{t-1}, y_{t-2}, 1] + \\ &+ (1 - \phi) \bar{V}, \end{aligned} \quad (5)$$

and $\nu_t = \phi \nu_{t-1} + 1 + (1 - \phi) \bar{\nu}$. Here, $\bar{V}, \bar{\nu}$ denote statistics of the alternative pdf.

Important moments of this posterior density are mean value,

$$[\hat{a}_t, \hat{b}_t, \hat{c}_t]' = [V_{2,1}, V_{3,1}, V_{4,1}] C, \quad (6)$$

$$C = \begin{bmatrix} V_{2,2} & V_{2,3} & V_{2,4} \\ V_{3,2} & V_{3,3} & V_{3,4} \\ V_{4,2} & V_{4,3} & V_{4,4} \end{bmatrix}^{-1}$$

which is equivalent to the result of RLS with discounting (under the choice of $\bar{V} = 0$). Covariance of the autoregressive parameters is:

$$\text{cov}([a, b, c]) = \frac{V_{1,1}}{\nu_t - 7} C. \quad (7)$$

Since the main parameters of interest are parameters $[a_t, b_t]$ we marginalize (4) to obtain marginal density of Student-t type. An important property of this density is that it is not as sharply concentrated as a Gaussian, since it assigns higher probability to values distant from the mean. The difference is greatest for $\nu_t < 20$, which arise for $\phi < 0.95$. Since we usually choose $\phi > 0.95$ we consider (6) and (7) to be an adequate approximation. For more details, see [12].

B. Prior distribution

The prior distribution should reflect information that is not available in the processed data. Typically, information about possible ranges of parameters, their expected ratios, etc. The same is true about the alternative density $\bar{p}(\cdot)$ in (3). Note that non-informative data $y_t = y_{t-1}, \forall t$ can be explained by infinitely many combinations of parameters a_t, b_t, c_t . In standard RLS, this degeneracy would manifest itself by the loss of rank of matrix (5) yielding numerically unstable estimates (6). The stabilized forgetting avoids this danger via positive definite matrix \bar{V}_t . In case of non-informative data, the estimates are fully determined by this matrix.

In all our simulations we made the following choice:

$$\bar{V}_t = \text{diag}([1e-2, 1e-3, 1e-3, 1e-5]).$$

Under this choice, the expected values of parameters are $[\hat{a}_t, \hat{b}_t, \hat{c}_t, \hat{\sigma}_t] = [0, 0, 0, 0.0033] \pm [3, 3, 36, 0.0067]$. This encodes the following information:

- the linear coefficients, a_t and b_t , are assumed to be closer to zero than the estimated constant c_t
- the variance of observations σ_t is small.

The influence of this alternative distribution on the posterior (5) is negligible when the data are informative.

C. Detection of oscillations

The probability of unstable oscillation is computed as the probability of unstable oscillating poles of the system. The poles of the system (1) are:

$$p_{1,2} = \frac{a_t \pm \sqrt{a_t^2 + 4b_t}}{2}.$$

The system is oscillating when the poles are imaginary (i.e. $a_t^2 < -4b_t$) and the system is unstable when $|p_{1,2}| > 1$, i.e.

$$|p_{1,2}| = \left| \left(\frac{a_t}{2} \right)^2 - \frac{a_t^2 + 4b_t}{4} \right| = |b_t| > 1.$$

These two inequalities define a space over which we need to integrate the posterior density (4). This demanding task was addressed via Monte Carlo sampling in [13].

In this paper, we propose to use an approximation motivated by the fact that we are most often interested in detecting oscillations around the stability boundary, $b_t = -1$. Around this boundary, the first inequality is approximated by $|a_t| < 2$. Moreover, we expect the oscillation to be slower than the sampling frequency, i.e. $a_t > 0$ we can only test probability of $a_t < 2$. Under this approximation and approximation of the posterior by a Gaussian density, the probability of unstable oscillation is computed as

$$\begin{aligned} Pr(\text{unstab. oscil.}) &= Pr(a_t < 2)Pr(b_t < -1) \\ &= \frac{1}{2} \left(1 - \text{erf} \frac{\hat{a}_t - 2}{\sqrt{2\text{var}(a_t)}} \right) \frac{1}{2} \left(1 - \text{erf} \frac{\hat{b}_t + 1}{\sqrt{2\text{var}(b_t)}} \right) \end{aligned} \quad (8)$$

where erf is the error function. This approximation tends to underestimate the risk of oscillations since the approximate area of integration is smaller than the correct one, see Fig. 1.

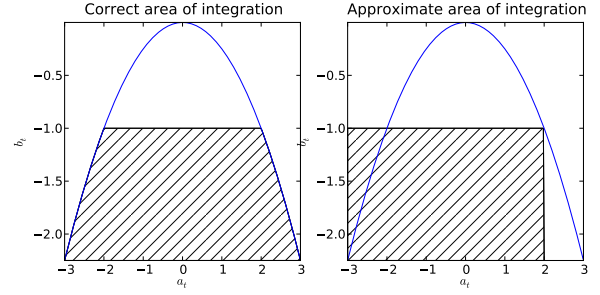


Fig. 1. Illustration of integration area for evaluation of oscillation risk.

Thus, it can serve as a lower bound of the true risk. On the other hand, probability of instability

$$\begin{aligned} Pr(\text{instability}) &= Pr(b_t < -1) \\ &= \frac{1}{2} \left(1 - \text{erf} \frac{\hat{b}_t + 1}{\sqrt{2\text{var}(b_t)}} \right) \end{aligned} \quad (9)$$

may serve as an upper bound.

Final oscillation detection algorithm is then as follows:

Off-line: choose initial alternative statistics, $\bar{V}, \bar{\nu}$ and forgetting factor ϕ .

On-line: at each time t do:

- 1) update statistics V, ν using (5),
- 2) compute posterior mean and variance via (6) and (7)
- 3) compute probability of unstable oscillations using (8) or (9)

IV. RESULTS ANALYSIS

In order to verify the designed techniques, two different sets of data are considered. The first data set is from real PMU measurements. The second data set is obtained by means of numerical simulations.

A. Real data analysis

The first case study represents analysis of a real retrospective time series. In particular, time series of power flow (see Fig. 2) has been registered using PMU devices from 500 kV power grid [4] with sampling period of 20 ms. Note that the condition of instability (9) allows to detect instability of the system, however, the condition of unstable oscillation (8) provides smoother estimates without the outlier artifact at the end of the instability region.

B. Model data analysis

The second case study represents analysis of data obtained by means of numerical simulations. A classical model of the two-area system shown in Fig. 3 was used for inter-area oscillations modeling.

Test system parameters are represented in [15]. Area 1 and Area 2 are sending and receiving subsystems accordingly. White noise having signal-to-noise ratio 40db and spectral width 0.5 Hz (accidental load variation) was added to the load which makes the simulations more complicated and realistic.

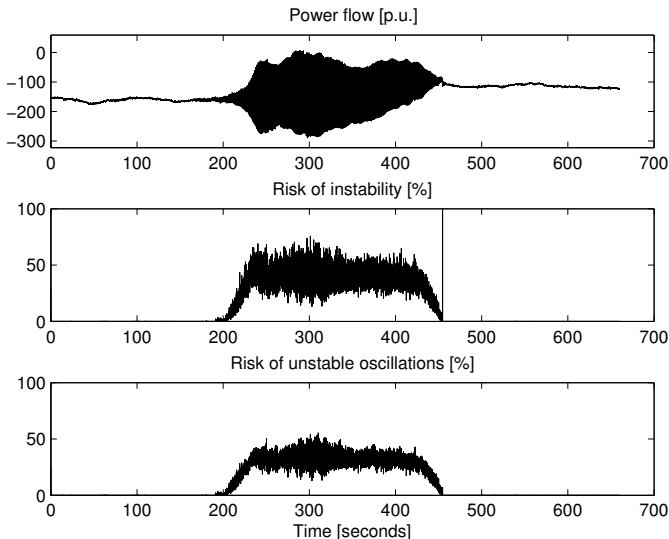


Fig. 2. Analysis of real data with $\phi = 0.97$. **Top**: observed power flow through 500kV transmission line. **Centre**: Risk of instability (9) **Bottom**: Risk of unstable oscillations (8).

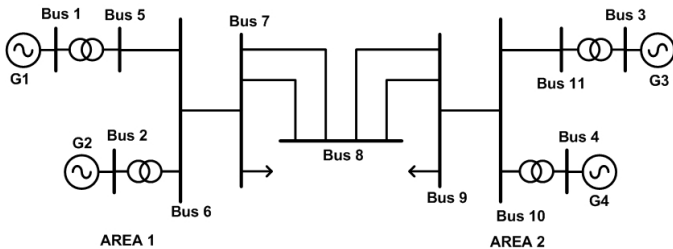


Fig. 3. An 11-bus test system

The sampling period is 0.02 second. During the simulation, the following scenario of changing an active power flow from Area 1 to Area 2 has been studied:

- From 50 to 115 second, increase of active power output of G2 from 500 MW to 660 MW. Active power increase rate 2.5 MW per second;
- From 165 to 230 second, decrease of active power output of G2 from 660 MW to 500 MW. Active power decrease rate 2.5 MW per second.

Variation of voltage on Bus 8 during the simulation process and corresponding risk of instability are displayed in Fig. 4.

Nowadays, PMU measurement technologies provide a good observability of real-time processes in power systems. It is to be noted that when having started, the inter-area oscillations can be easily detected by a system operator. However, correctly assessing where they actually start is a non-trivial task. The computed probability of both real and modeled oscillations marks the start of the oscillations quite clearly. Thus, after detailed studies it is possible that the considered method can be adopted to give the transmission operator qualitative information regarding stability margins.

V. CONCLUSION

We addressed the problem of robust detection of inter-area electro-mechanical unstable oscillations in power systems

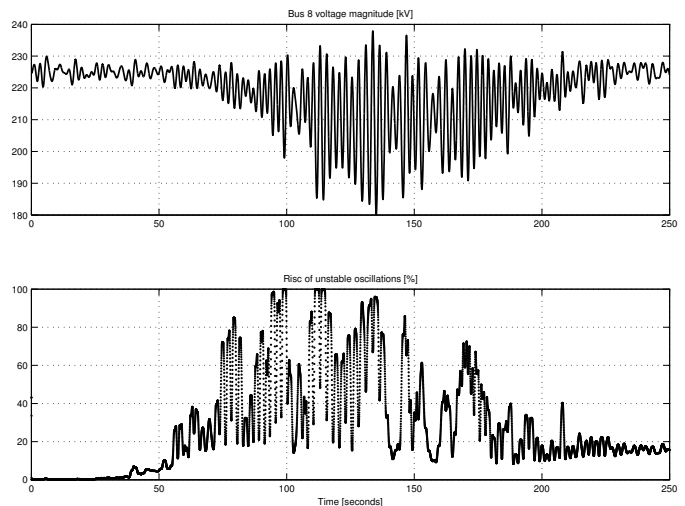


Fig. 4. Numerical simulation. Results for voltage amplitude data. **Top**: observed data. **Bottom**: computed probability of unstable oscillations.

using dynamic data registered by PMU. Oscillating systems have been modeled by a second order autoregressive system. The stability is assessed by probability of unstable poles of the estimated approximate linear system.

The algorithm was tested on recorded retrospective data, in particular power flow through 500kV transmission line [4] was employed. We found that the computed probability of oscillations clearly marks the start of the unstable oscillations. We conjecture that the designed algorithm has an advantage over the conventional approach based on Kalman filtering, because it is able to estimate covariance of the observation σ_t . For deeper understanding of its properties, the algorithm has been tested on voltage data obtained by numerical simulations of well studied and understood scenarios.

Our current work involves search for a physically based model of parameter evolution which would allow more accurate prediction of future behavior of the system. A framework for comprehensive comparative analysis of proposed approach is currently under construction.

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