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Academy of Sciences of the Czech Republic
Institute of Information Theory and Automation

# RESEARCH REPORT

Václav Slimáček:

# Mathematical Model of a Production Line Checkpoint — Simulation Program

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ÚTIA AV ČR, P. O. Box 18, 182 08 Prague, Czech Republic
Telex: 122018 atom c, Fax: (+42) (2) 688 4903
E-mail: utia@utia.cas.cz

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# 1 Program description

As a part of the analysis of the processes at the production line checkpoint, the simulation program, which can simulate the processing of valves at the production line checkpoint and the related process of obtaining rewards, was created. This simulation program is able to simulate not only the specific processes from the introduced model of a production line checkpoint, but also general semi-Markov processes and related processes of obtaining rewards. The program also performs the basic analysis of the embedded Markov chain.

The simulation program runs in the Matlab environment and the algorithms used in the analysis were adapted from [2] and is a continutation of [4].

The input parameters of this simulation program are (in the following order):

- Matrix of transition probabilities.
  This matrix represents the transition probabilities between the possible states of the simulated process. It must be a stochastic matrix, i.e., a square matrix with all elements in the interval $[0, 1]$ and the sums of elements in each row must equal to 1.

- Vector of initial distribution.
  This vector represents the probability of the choice of the initial state of the simulated process. It must be a stochastic row vector (i.e., all entries must be in the interval $[0, 1]$ and the sum of all entries must equal to 1) with the same number of columns as the matrix of transition probabilities.

- Cell array containing specifications of distributions of transition times.
  This cell array specifies the probability distribution and its parameters used for the simulation of the transition times between the states of the simulated process. The dimensions of this cell array must be the same as the dimensions of the matrix of transition probabilities. Each cell of this array corresponding to the nonzero value of the transition probability must contain a row vector with four columns. The first value in this row vector specifies the probability distribution (0 denotes the log-normal distribution, 1 denotes the Weibull distribution), the other three values specify the scale, the shape and the location parameter (in that order) of the distribution. Since the simulated random variables represent transition times, only nonnegative values of the location parameters are allowed. The ranges of values of other parameters are specified in [5]. The cells corresponding to the zero transition probabilities can be arbitrary.

- Matrix of rewards.
  This matrix represents the rewards obtained for transitions from one state to another. It is a real valued matrix with the same dimensions as the matrix of transition probabilities.

- End time.
  This value represents the duration of the simulated process. It is a nonnegative real value. The end time should be specified in the same time units as used for specification of the distributions of the transition times.

The created simulation program performs the analysis of the matrix of transition probabi-lities, i. e, it determines the classes of the embedded Markov chain and whether these classes are transient or recurrent. The program computes the period of each recurrent class and the limiting distribution for each of aperiodic recurrent classes (each recurrent class forms a Markov subchain). The program also computes the matrices using the transient classes $[I-U]^{-1}$ and $[I-U]^{-1} \cdot Y$ which determine the mean time spent in transient classes and probabilities of absorption in recurrent classes ([3], [1]). The output from this analysis is printed out on the screen.

After the analysis, the simulation of the process is performed. First of all, the initial state is chosen and then the process evolves according to the transition probabilities specified in the matrix of transition probabilities. The time spend in each state is generated with use of the cell array containing the specification of the distributions of the transition times and the obtained reward is determined from the corresponding element of the matrix of rewards. The simulation runs until the total time of running the simulating process exceed the specified end time.

The output is the matrix with three rows and unspecified number of columns. The first row contains the information about the state of the process, the second row represents the time, which the simulated process spent in the state indicated in the first row and the third row represents the reward obtained for the transition from the state in the first row to the next state of the process (which is specified in the first row of the next column).

# 2 Simulations

The processing of valves at the production line checkpoint based on the model introduced in [5], which was adapted to the results from the analysis of the real data, was also simulated by the simulation program introduced in the previous section in order of verifying the analytical results and demonstration of the functionality of the simulation program.

The maximal number of possible successive repairs was $K = 3$, which defines the number of states of the simulated process as 6. The initial state of the simulated process was the state "Preprocessing". The duration of the simulated process was chosen as 9600 minutes, which corresponds to 20 work shifts lasting 8 hours. According to [5], the probability of successful repair was set up to $p = 0.597$ and the cost of repair and value of valve was set up to $c = 30$ and $C = 80$ respectively.

The log-normal distribution with the shape parameter $\sigma = 0.25$ was chosen for simulation of all transition times, and the values of the scale and location parameters were tuned up so they represented the real process with the means defined by the equations, see [5]. That is, the scale parameter $\mu = -1.42$ and the location parameter $\kappa = 0.5$ were used for the simulation of the time spent in the state "Preprocessing", the scale parameter $\mu = -0.72$ and the location parameter $\kappa = 1$ were used for the simulation of the time spent in the one of the repairing states and the scale parameter $\mu = -2.33$ and the location parameter $\kappa = 0.4$ were used for the simulation of the time spent in the state "Reject" and in the state "Functional valve".

The matrices used as the input to the program were:

```
%matrix of transition probabilities
P = [0, 0.403, 0,     0,     0,     0.597;...
     0, 0,     0.403, 0,     0,     0.597; ...
     0, 0,     0,     0.403, 0,     0.597; ...
```

```
     0, 0,     0,     0,     0.403, 0.597; ...
     1, 0,     0,     0,     0,     0; ...
     1, 0,     0,     0,     0,     0];

%vector of initial distribution
alpha = [1 0 0 0 0 0];

%cell array containing specifications of distributions of transition times
tau = {0 [0 -1.42 0.25 0.5] 0 0 0 [0 -1.42 0.25 0.5];...
       0 0 [0 -0.72 0.25 1] 0 0 [0 -0.72 0.25 1];...
       0 0 0 [0 -0.72 0.25 1] 0 [0 -0.72 0.25 1];...
       0 0 0 0 [0 -0.72 0.25 1] [0 -0.72 0.25 1];...
       [0 -2.33 0.25 0.4] 0 0 0 0 0;...
       [0 -2.33 0.25 0.4] 0 0 0 0 0};

%matrix of rewards
rewards= [0 -30 0 0 0 80;...
          0 0 -30 0 0 80;...
          0 0 0 -30 0 80;...
          0 0 0 0 -80 80;...
          0 0 0 0 0 0;...
          0 0 0 0 0 0];

%end time
Time = 9600;
```

The simulation program is then executed by

```
output=MCsimul(P,alpha,tau,rewards,Time);
```

and the output is saved in the three rows matrix "output".

During the simulation, which lasted 1.275 seconds, 4345 valves were processed. The number of repairs which one valve absolved till the full functionality and number of discarded valves are summarized in the following table:

| Number of repairs | 0 | 1 | 2 | 3 | Reject |
|---|---|---|---|---|---|
| Number of valves | 2597 | 1013 | 443 | 178 | 114 |

The average number of repairs, which one valve had to absolve, was 0.639 and the sample variance of the number of repairs was 0.838. The probability of discarding a valve (estimated from the relative frequencies) was 0.026.

The average time of processing one valve was 2.208 minutes and the sample variance of the average time of processing one valve was 1.903. The average time between two discarded valves was 83.574 minutes.

The proportions of time which process spent in particular states were

$$P_0 = 0.339, \quad P_1 = 0.275, \quad P_2 = 0.114, \quad P_3 = 0.046, \quad P_4 = 0.006, \quad P_5 = 0.220.$$

The total reward was 246110.

The average reward from one valve was 56.642 and the sample variance of the reward from one valve was 2003.386.

The average reward per time unit was 25.506.

As can be seen, all of these results agree very well with the analytical results, for detail, see [5].

# References


[1] J. Jansen and R. Manca: *Applied Semi-Markov Processes.* Springer, 2006.

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[3] C. D. Mayer: *Matrix Analysis and Applied Linear Algebra.* Society for Industrial and Applied Mathematics, 2000.

[4] J. Michálek and M. Šiman: Vyhodnocování markovských řetězců pomocí funkce $MR \cdot m$ v Matlabu. Technical Report No. 2288, ÚTIA AV ČR, November 2010.

[5] V. Slimáček: Mathematical model of a production line checkpoint. Technical Report No. 2300, ÚTIA AV ČR, July 2011.