Influence of short sales and margin requirements on portfolio efficiency - a DEA-risk approach

Martin Branda [1]

Abstract

We focus on efficiency of assets and portfolios available to investors on financial markets. We employ diversification consistent DEA-risk models with CVaR deviations as the inputs and expected rate of return as the output. Moreover, we allow short selling and take into account margin requirements. Our model is then employed in an empirical study where selected assets from US stock market are investigated. The sample approximation technique is used to deal with the multivariate skew-normal distribution of random returns.

Key words

Financial efficiency, data envelopment analysis, short sales, margin requirements, CVaR deviation

JEL Classification: C61, D81, G11

1. Introduction

Data Envelopment Analysis (DEA) was introduced by Charnes et al. (1978) as a tool for accessing efficiency of homogeneous decision-making (production) units, which consume multiple inputs and produce several outputs, cf. Cooper et al. (2011). In general, characteristics with lower values preferred to higher are considered as the inputs, whereas characteristics with preferred higher values are used as the outputs. Murthi et al. (1997) found an analogy in finance and considered mutual funds as the decision making units. Risk measures (standard deviation), expense ratio and loadings were considered as the inputs, whereas gross return was used as the output.

The traditional DEA models were extended to take into account dependencies between considered investment opportunities leading to the class of diversification-consistent (DC) DEA models. The first DC DEA model was proposed by Briec et al. (2004) under Markowitz's (1952) mean-variance framework. The model was extended by considering the skewness as an additional input, see Briec et al. (2007), Joro and Na (2006). Recently, Lamb and Tee (2012) introduced a general class of diversification-consistent models where arbitrary number of coherent risk measures can be used as the inputs and several return measures serve as the outputs. The models were further extended by Branda (2013a, 2013b) to take into account various concept from multiobjective optimization and various choices of the set of investment opportunities including limited diversification. Branda and Kopa (2012, 2014) investigated relations between DEA models and stochastic dominance efficiency tests.

Most of the studies considered the case where short sales are not allowed or are allowed, however without taking into account the margin requirements. Recently, the margin requirements were taken into consideration by Ding et al. (2014), who worked with the mean-variance efficiency. We continue in this direction and formulate a DC DEA-risk model with

[1] RNDr. Martin Branda, Ph.D., Charles University in Prague, Faculty of Mathematics and Physics, Ke Karlovu 3, 121 16 Prague 2; AND Academy of Sciences of the Czech Republic, Institute of Information Theory and Automation, Pod Vodárenskou věží 4, 182 08, Prague.
E-mail: branda@karlin.mff.cuni.cz

CVaR deviations, short sales and margin requirements. The model leads to a large scale linear programming problem under the assumption that the random asset returns have a finite discrete distribution. However, in the numerical study we employ the multivariate skew-normal distribution, cf. Azzalini and Dalla Valle (1996). To solve the DEA models, we use the sample approximation technique where simulated samples are used to approximate the true distribution by a discrete one. The final scores are then based on several samples. Moreover, we discuss the stability of ranking.

This paper is organized as follows. In Section 2, we propose the basic notation and discuss the choice of the set of available investment opportunities. Moreover, we define the deviation measure based on the conditional value at risk. In Section 3, we propose diversification-consistent DEA-risk model and its linear programming distribution for discretely distributed returns. The DEA model is then employed in Section 4 to access efficiency of selected assets from US stock market. Section 5 concludes the paper.

2. Preliminaries and notation

We consider n assets with random rates of return R_i, $i=1, \dots, n,$ with finite mean value. The most traditional choice of set of investment opportunities, where we enable to combine the assets into a portfolio which is composed exclusively from the considered assets and do not enable short sales, is as follows:

$$\mathcal{R} = \left\{ \sum_{i=1}^{n} R_i x_i \; : \; \sum_{i=1}^{n} x_i = 1, \; x_i \; \geq 0 \right\}.$$

If short sales are allowed (SSA), then the set simplifies to

$$\mathcal{R}^{SSA} = \left\{ \sum_{i=1}^{n} R_i x_i \; : \; \sum_{i=1}^{n} x_i = 1 \right\}.$$

However, we will consider the following set of investment opportunities where short sales are allowed with margin requirements (MR):

$$\mathcal{R}^{MR} = \left\{ \begin{array}{c} \sum_{i=1}^{n} R_i x_i + \left(1 - \sum_{i=1}^{n} x_i\right) r : 1 - \sum_{i=1}^{n} x_i + \gamma \sum_{i=1}^{n} x_i^+ \geq \beta \sum_{i=1}^{n} x_i^- , \\ x_i = x_i^+ - x_i^-, \; x_i^+, x_i^- \geq 0 \end{array} \right\},$$

where $\beta \geq 1$ denotes the margin requirement level on short position for risky assets and $\gamma \geq 0$ the nonnegative discount rate for the long position of the owned risky assets. If α is set to zero, then only cash can be used for margins. Moreover, the cash invested over budget has to be borrowed for interest rate r or the remaining not-invested cash can be deposit for the same rate. Similar set of investment opportunities was considered by Ding et al. (2014) under Markowitz's (1952) mean-variance framework.

A general deviation measure derived from the conditional value at risk can be defined using the following minimization formula, cf. Rockafellar et al. (2006):

$$CVaRdev_\alpha(R) = \min_{\eta} \frac{1}{1-\alpha} \mathbb{E}\left[\max\{(1-\alpha)(R-\eta), \alpha(\eta - R)\} \right],$$

for an investment opportunity represented by a random rate of return R with finite mean value and for level $\alpha \in (0,1)$. The measure is called CVaR deviation or a deviation from α-quantile, see also Ogryczak and Ruszczynski (2002).

3. DEA-risk model with margin requirements

In this section, we propose a new diversification consistent DEA-risk model where K CVaR deviations on different levels α_k are used as the inputs and the expected return as the only output. In general, the model can be written as:

$$\begin{aligned} &\min_{\theta,R} \theta \\ &\mathbb{E}[R] \geq \mathbb{E}[R_0], \\ &CVaRdev_{\alpha_k}(R) \leq \theta \cdot CVaRdev_{\alpha_k}(R_0), k = 1, \dots, K, \\ &R \in \mathcal{R}^{MR}. \end{aligned}$$

We say that investment opportunity R_0 is efficient if and only if the optimal value is equal to one, otherwise it is inefficient. The model enables full diversification and short sales due to the choice of the set of available investment opportunities $\mathcal{R}^{MR}$.
Under a discrete distribution of the returns with S equiprobable realizations $r_{i,s}$, the model can be reformulated as a (large) linear programming problem:

$$\begin{aligned} &\min_{\theta,x_i,u_{s,k},\eta_k,y_{s,k},z_{s,k}} \theta \\ &\frac{1}{S}\sum_{s=1}^{S}\sum_{i=1}^{n} x_i \cdot (r_{i,s} - r) \geq \frac{1}{S}\sum_{s=1}^{S} r_{0,s} - r, \\ &\frac{1}{S}\sum_{s=1}^{S} u_{s,k} \leq \sum_{s=1}^{S} r_{0,s} y_{s,k} - \frac{\alpha_k}{1-\alpha_k}\sum_{s=1}^{S} r_{0,s} z_{s,k}, k = 1, \dots, K, \\ &\sum_{i=1}^{n} x_i\, r_{i,s} - \eta_k \leq u_{s,k}, k = 1, \dots, K, s = 1, \dots, S, \\ &\frac{\alpha_k}{1-\alpha_k}\left(\eta_k - \sum_{i=1}^{n} x_i\, r_{i,s}\right) \leq u_{s,k}, k = 1, \dots, K, s = 1, \dots, S, \\ &\sum_{s=1}^{S} y_{s,k} - \frac{\alpha_k}{1-\alpha_k}\sum_{s=1}^{S} z_{s,k} = 0, k = 1, \dots, K, \\ &y_{s,k} + z_{s,k} = \frac{\theta}{S}, k = 1, \dots, K, s = 1, \dots, S, \\ &y_{s,k}, z_{s,k} \geq 0, \\ &1 - \sum_{i=1}^{n} x_i + \gamma \sum_{i=1}^{n} x_i^+ \geq \beta \sum_{i=1}^{n} x_i^-, \\ &x_i = x_i^+ - x_i^-, x_i^+, x_i^- \geq 0, i = 1, \dots, n. \end{aligned}$$

The first constraint corresponds to the expected returns, whereas the next six groups of constraints express the relation between CVaR deviations. Note that the CVaR deviations for benchmark R_0 are substituted by their dual expressions using nonnegative variables $y_{s,k}, z_{s,k}$, see Branda (2014) for details. Auxiliary nonnegative variables $u_{j,s}$ are used to model the maximum in the definition of CVaR deviations. Below, we consider $K = 4$ inputs with the levels $\alpha_k \in \{0.75, 0.9, 0.95, 0.99\}$. The last two constraints define the set of investment opportunities.

4. Empirical study

In this section, we employ the proposed diversification-consistent DEA model together with the sample approximation technique, see Shapiro et al. (2009) for a general introduction, to access efficiency of selected US assets observed monthly from January 2009 to December 2013. The historical prices were available through the function *FinancialData* in Wolfram Mathematica 9. We consider Boeing (BA), Coca-Cola (KO), JPMorgan Chase (JPM), Oracle (ORCL), Microsoft (MSFT), Nike (NKE), Intel (INTC), Apple (AAPL). We employ the multivariate skew-normal distribution to model the asset returns, see Azzalini and Dalla Valle (1996) for details. The parameters of the returns distribution were estimated using the R packages *sn* and *fAssets*. The same packages were also used to simulate the Monte-Carlo samples. To test performance of the sample approximation technique, 100 independent samples of 1000 return realizations are used.

The modelling system GAMS and the solver CPlex were used to solve the optimization problems. The results of the DEA models can be found in Tables 1-4. Tables 1 and 3 show basic descriptive statistics and 95% confidence intervals of the resulting efficiency scores based on the samples for different values of the discount rate γ. In the first case ($\gamma = 0$), no discount of risky investments is possible, thus short sales have to be fully guaranteed by cash. In the second case ($\gamma = 0{,}5$), fifty percent of the value of the risky assets can be used to cover the margin requirements. Note that the resulting ranking is based on the mean scores and does not change with different γ. The stability of the ranking can be verified from Tables 2 a 4, which show counts of differences in ranking based on the considered samples compared with the ranking based on the mean score. The only efficient asset - Apple - is classified as efficient based on all samples. Also the ranking of the first inefficient asset - Nike - is quite stable and the asset was not ranked as second for 9 samples only. The misclassification for the other assets does not exceed one rank in most cases.

Table 1: Descriptive statistics of efficiency scores (discount rate $\gamma = 0$)

	BA	KO	JPM	ORCL	MSFT	NKE	INTC	AAPL
Mean	0,563	0,507	0,398	0,428	0,489	0,681	0,533	1,000
Ranking	**3**	**5**	**8**	**7**	**6**	**2**	**4**	**1**
St.dev.	0,051	0,063	0,069	0,064	0,067	0,061	0,061	0,000
Minimum	0,413	0,347	0,245	0,270	0,318	0,508	0,381	1,000
Maximum	0,679	0,674	0,570	0,622	0,692	0,819	0,766	1,000
Conf. int. Lb	0,573	0,520	0,412	0,441	0,502	0,693	0,545	1,000
Conf. int. Ub	0,553	0,495	0,385	0,416	0,476	0,669	0,521	1,000

Table 2: Differences in ranking (discount rate $\gamma = 0$)

Difference	BA	KO	JPM	ORCL	MSFT	NKE	INTC	AAPL
4	0	0	2	0	2	0	0	0
3	0	3	0	0	5	0	0	0
2	0	14	7	10	13	0	2	0
1	2	21	28	20	29	0	23	0
0	**52**	**21**	**63**	**41**	**39**	**91**	**31**	**100**
-1	31	18	0	29	9	6	27	0
-2	12	18	0	0	3	2	13	0
-3	3	5	0	0	0	1	4	0

Table 3: Descriptive statistics of efficiency scores (discount rate $\gamma = 0{,}5$)

	BA	KO	JPM	ORCL	MSFT	NKE	INTC	AAPL
Mean	0,559	0,507	0,398	0,428	0,489	0,672	0,533	0,873
Ranking	**3**	**5**	**8**	**7**	**6**	**2**	**4**	**1**
St.dev.	0,049	0,063	0,069	0,064	0,067	0,057	0,061	0,038
Minimum	0,413	0,347	0,245	0,270	0,318	0,507	0,381	0,789
Maximum	0,676	0,674	0,569	0,615	0,690	0,811	0,764	0,975
Conf. int. Lb	0,569	0,520	0,411	0,441	0,502	0,683	0,545	0,881
Conf. int. Ub	0,550	0,495	0,384	0,416	0,476	0,661	0,521	0,866

Table 4: Differences in ranking (discount rate $\gamma = 0{,}5$)

Difference	BA	KO	JPM	ORCL	MSFT	NKE	INTC	AAPL
-4	0	0	2	0	2	0	0	0
-3	0	3	0	0	5	0	0	0
-2	0	16	7	10	13	0	2	0
-1	2	19	28	20	29	0	24	0
0	**49**	**21**	**63**	**41**	**39**	**91**	**31**	**100**
1	33	18	0	29	9	6	26	0
2	13	18	0	0	3	2	13	0
3	3	5	0	0	0	1	4	0

5. Conclusion

In this paper, we have formulated a new diversification-consistent DEA-risk model based on CVaR deviations. The set of available investment opportunities allows short selling and takes into account the margin requirements. We have employed the proposed DEA model to access efficiency of selected assets from US stock market. Using the sample approximation technique, we have verified the stability of the efficiency classification and of the assets ranking.

Acknowledgement The research was supported by the Czech Science Foundation (GAČR) under the project GP13-03749P.

References

[1] Azzalini, A., Dalla Valle, A. (1996). The multivariate skew-normal distribution. *Biometrika* 83 (4), 715–726.

[2] Basso, A. and Funari, S. (2001). A data envelopment analysis approach to measure the mutual fund performance. *European Journal of Operational Research* 135 (3), 477–492.

[3] Branda, M. (2013a). Diversification-consistent data envelopment analysis with general deviation measures. *European Journal of Operational Research* 226 (3), 626–635.

[4] Branda, M. (2013b). Reformulations of input-output oriented DEA tests with diversification. *Operations Research Letters* 41 (5), 516–520.

[5] Branda, M. (2014). Sample approximation techniques for DEA-risk efficiency tests. Accepted to *MME conference proceedings* 2014.

[6] Branda, M. and Kopa, M. (2012). DEA-risk efficiency and stochastic dominance efficiency of stock indices. *Finance a úvěr – Czech Journal of Economics and Finance* 62 (2), 106–124.

[7] Branda, M. and Kopa, M. (2014). On relations between DEA-risk models and stochastic dominance efficiency tests. *Central European Journal of Operations Research* 22 (1), 13–35.

[8] Briec, W., Kerstens, K. and Lesourd, J.-B. (2004). Single period Markowitz portfolio selection, performance gauging and duality: a variation on the Luenberger shortage function. *Journal of Optimization Theory and Applications* 120 (1), 1–27.

[9] Briec, W., Kerstens, K., Jokung, O. (2007). Mean-variance-skewness portfolio performance gauging: a general shortage function and dual approach. *Management Science* 53, 135–149.

[10] Charnes, A., Cooper, W. and Rhodes, E. (1978). Measuring the efficiency of decision-making units. *European Journal of Operational Research* 2 (6), 429–444.

[11] Cooper, W.W., Seiford, L.M. and Zhu, J. (2011). *Handbook on data envelopment analysis*. Springer, New York.

[12] Ding, H., Zhou, Z., Xiao, H., Ma, C., Liu, W. (2014). Performance Evaluation of Portfolios with Margin Requirements. *Mathematical Problems in Engineering*, vol. 2014, Article ID 618706, 8 pages, 2014. doi:10.1155/2014/618706

[13] Joro, T., Na, P. (2006). Portfolio performance evaluation in a mean-variance-skewness framework. *European Journal of Operational Research* 175 (1), 446–461.

[14] Lamb, J.D. and Tee, K-H. (2012). Data envelopment analysis models of investment funds. *European Journal of Operational Research* 216 (3), 687–696.

[15] Markowitz, H.M. (1952). Portfolio selection. *The Journal of Finance* 7 (1), 77–91.

[16] Murthi, B.P.S., Choi, Y.K. and Desai, P. (1997). Efficiency of mutual funds and portfolio performance measurement: a non-parametric approach. *European Journal of Operational Research* 98 (2), 408–418.

[17] Ogryczak, W., Ruszczynski, A. (2002). Dual stochastic dominance and related mean-risk models. *SIAM Journal on Optimization* 13, 60–78.

[18] Rockafellar, R.T., Uryasev, S., Zabarankin, M. (2006). Generalized deviations in risk analysis. *Finance and Stochastics* 10, 51–74.

[19] Shapiro, A., Dentcheva, D., Ruszczyński, A. (2009). *Lectures on stochastic programming: Modeling and theory*. SIAM, Philadelphia.