ESTIMATION OF VAR AND CVAR FROM FINANCIAL DATA USING SIMULATED ALPHA-STABLE RANDOM VARIABLES

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ABSTRACT

It is of great importance for those in charge of measuring and managing financial risk to analyse financial data by determining a certain probabilistic model. These data usually possess distribution with tails heavier than those of normal distribution. The class of α-stable distributions can be chosen for modelling financial data since this probabilistic model is able to capture asymmetry and heavy tails. In this paper, mixed α-stable model is applied for the analysis of return data of Lithuanian pension funds that usually contain a significant number of zero values. The distribution fitting and simulation algorithm are also described. Risk measures VaR (Value-at-Risk) and CVaR (Conditional Value-at-Risk) are chosen to evaluate the characteristics of return data, especially the degree of heavy tails. VaR and CVaR are estimated from return data, then computed from simulated data when using mixed α-stable law and finally compared to the measures obtained using α-stable model and Gaussian model. The empirical results of the simulation model performance are discussed.

INTRODUCTION

Stable distributions are the class of probability laws that have become a versatile tool in financial modelling. Their application to financial engineering is reasoned that stable distributions generalize the normal distribution and capture asymmetry and heavy tails, which are frequently observed in financial data (Kabasinskas et al. 2009; Kim et al. 2011; Xu et al. 2011). Adequate distributional fitting of empirical financial series is very important in forecasting and supporting investment decisions. Stable distributions are also proposed as a suitable model for many types of physical and economic systems.

The fact that financial series have a heavy-tailed distribution may be essential to a financial risk manager. Financial theory has long-recognized the interaction of risk and reward for decision making. Incorrect risk evaluation leads to non-optimal financial solutions (Sorwar and Dowd 2010; Serbinenko and Emmenegger 2007; Glantz and Kissell 2014). Quantile based measures of risk, such as Value-at-Risk (VaR) and Conditional Value-at-Risk (CVaR), may be considerably different if estimated for a heavy-tailed distribution (Ortobelli et al. 2005; Rockafellar and Uryasev 2000). This is particularly true for the highest quantiles of the distribution induced by adverse market movements (Bradley and Taqqu 2003; Asimit et al. 2013). This phenomenon is usually observed during global financial crisis. Nonstationarity, external and internal risk shocks are increasingly felt in young and emerging finance markets since national economies become increasingly interdependent. This leads to the modelling of heavy tailed data (Ibragimov et al. 2013).

This paper presents the model for the special case of financial markets known as “daily zero return" inherent in young markets (Kabasinskas et al. 2009; Belovas et al. 2007). The Baltic States, other Central and Eastern Europe countries have small emerging markets where the number of daily zero returns can reach ninety percent. From the modelling view point, it is presumed that mixed α-stable law with included additional parameter for zero returns can outperform α-stable distribution while simulating financial data returns and estimating risk measures VaR and CVaR.

In this paper, the comparative analysis is done while judging empirical and simulated values of risk measures for normal, α-stable and mixed α-stable distributions. The simulation model is verified by comparing simulated values of VaR and CVaR with theoretical ones. The case study is performed on modelling the returns of pension funds that are rather young markets in Lithuania. That’s why the probability of zero returns is rather high. The influence of tail probability on simulation results is also presented.

THEORETICAL BACKGROUD

This section shortly presents the notations for normal, α-stable and mixed α-stable distributions. The estimation of distribution parameters and simulation procedures are also given for each case.

Value-at-risk (VaR) measure is probably one of the most widely used for calculating risk charges. In statistical terms, VaR is a quantile of distribution. For financial asset returns (Stoyanov et al. 2013), VaR is defined as the minimal value of return at a given confidence level $(1-\varepsilon)$, or tail probability $\varepsilon \in (0,1)$, is defines as:

$$\mathrm{VaR}_{\varepsilon}(X) = -\inf\{x : \Pr(X \le x) \ge \varepsilon\} = -F_X^{-1}(\varepsilon). \quad (1)$$

The drawback of VaR is that it makes use of the cut-off point corresponding to the tail probability ε and does not measure any information beyond this point. The Conditional Value-at-Risk (CVaR) corrects for this. It is an average of VaRs and is more sensitive to the tail behaviour of asset returns (Stoyanov et al. 2013):

$$\mathrm{CVaR}_{\varepsilon}(X) = \tfrac{1}{\varepsilon}\int_0^{\varepsilon} \mathrm{VaR}_t(X)\mathrm{d}t. \quad (2)$$

If particular probability distribution is assumed, the theoretical expressions for Equations (1), (2) can be derived and will be referenced in the following sections.

Normal Distribution and Risk Measures

Normal distribution approximates many natural phenomena. The notation $N(\mu,\sigma)$ means normally distributed values with mean μ and standard deviation σ. The values of parameters are usually estimated using maximum likelihood method. Normal distribution is often used as a reference for many probability problems.

The normally distributed values can be simulated by one of methods described in (Wallace 1996).

The exact formulas for risk measures VaR and CVaR in the case of normal distribution are given in e.g. reference (Yamai and Yoshiba 2002).

α-Stable Distribution and Risk Measures

α-stable distribution belongs to the models for heavy tailed data. It is characterized by four parameters: α – index of stability, σ – scale parameter, β – skewness parameter, μ – location parameter. The parameters are restricted to the range $\alpha \in (0, 2]$, $\beta \in [-1, 1]$, $\sigma \in (0, \infty)$, $\mu \in \Re$. In financial applications, parameter α is usually more than 1; this is essential requirement to guarantee that the theoretical mean or expectation will exist. Shortly, the notation $S_\alpha(\sigma, \beta, \mu)$ is used to denote the class of stable laws. Generally, the characteristic function $\phi_X(t)$ of random variable X, which is distributed by α-stable law, is

$$\phi_X(t) = \begin{cases} \exp\left\{-\sigma^\alpha |t|^\alpha \left(1 - i\beta\left(\tan\frac{\pi\alpha}{2}\right)(\operatorname{sign} t)\right) + i\mu t\right\}, & \alpha \ne 1; \\ \exp\left\{-\sigma^\alpha |t|^\alpha \left(1 - i\beta\frac{\pi}{2}(\operatorname{sign} \ln|t|)\right) + i\mu t\right\}, & \alpha = 1. \end{cases}$$

The index of stability α determines the rate at which the tails decay. If $\alpha = 2$ the characteristic function in given equation reduces to the characteristic function of the normal distribution.

The estimation of α-stable law parameters is complicated because of the lack of closed-form density function in general. From the class of numerical methods, one of quantile methods (McCulloch 1986) or characteristic function methods (Kogon and Williams 1998) can be applied.

There are many approaches that have been proposed in the literature for simulating sequences of α-stable random variables. The paper (Chambers et al. 1976) presents the simulation algorithm which is rather quick and accurate.

Theoretical expressions of VaR and CVaR for α-stable distribution are given in (Stoyanov et al. 2006).

Mixed α-Stable Distribution

Mixed α-stable distribution was applied for modelling the financial data in the paper (Kabasinskas et al. 2009). The additional parameter $p \in [0, 1]$ is included to model the zeros with a certain probability, i.e.

$$X^M = \begin{cases} 0, & p < u; \\ S_\alpha(\sigma, \beta, \mu), & p \ge u; \end{cases}$$

where u is uniform random variable $u \sim U(0,1)$.

The probability density function of mixed α-stable distribution is given as

$$f^M(x) = p \cdot \delta(x) + (1-p) \cdot f_\alpha(x)$$

where $f_\alpha(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \phi_X(t) \cdot e^{-ixt}\mathrm{d}t$ is probability density function of α-stable distribution expressed through its characteristic function, $\delta(x)$ is Dirac delta function.

While estimating the parameters of mixed α-stable law, the maximum likelihood method is applied (Kabasinskas et al. 2012). It is time consuming, but the implementation of parallel algorithms can allow us to get results in an adequate time even for long data series.

We propose such scheme to simulate random data sequence X following mixed α-stable law:

- Generate random value $u \sim U(0,1)$;
- Compare u and p:
  - If $u > p$, then $X = 0$;
  - If $u \le p$, then generate $X \sim S_\alpha(\sigma, \beta, \mu)$ by employing the algorithm for simulating α-stable random value (Chambers et al. 1976).

This procedure will be applied for simulating random variables in the experimental study.

In the case of mixed α-stable distribution, theoretical expressions for VaR and CVaR currently are not presented in the scientific literature.

CASE STUDY: MODELLING THE RETURNS OF PENSION FUNDS

This paper focus on the data analysis of performance of Lithuanian private pension funds employing modelling technique. 18 pension funds are currently operating in Lithuanian market.

Data analysis of pension funds is carried out using historical fund unit values during period 02/01/2007 – 31/12/2013, recalculating them into the rate of return. Pension funds can be classified into several categories according to the investment allocation part into shares (Liutvinavičius and Sakalauskas 2011):

- Funds of conservative investments – no risky funds:

DNB pensija 1	DNBP1
ERGO konservatyvusis	ERGOK
Finasta konservatyvaus investavimo	FKI
Finasta Nuosaikus	FN
SEB pensija 1	SEBP1
Swedbank pensija 1	SWEDP1

- Funds with a small amount into shares (up to 30%) – low risk funds:

DNB pensija 2	DNBP2
Finasta augančio pajamingumo	FAP
Swedbank Pensija 2	SWEDP2

- Funds with a medium amount into shares (up to 70%) – intermediate risk funds:

DNB pensija 3	DNBP3
ERGO balans	ERGOB
Finasta aktyvaus investavimo	FAI
Finasta Subalansuotas	FS
SEB Pensija 2	SEBP2
Swedbank Pensija 3	SWEDP3
Swedbank Pensija 4	SWEDP4

- Funds with a large amount into shares (up to 100%) – high risk funds:

Finasta Racionalios rizikos	FRR
SEB pensija 3	SEBP3

The pension fund was randomly chosen to reveal the distribution characteristics of return data by displaying them in QQ plot (Figure 1). One can see that the distribution of these data has two heavy tails, especially the left one, showing the asymmetry also. In practice this means much bigger possible losses than profits.

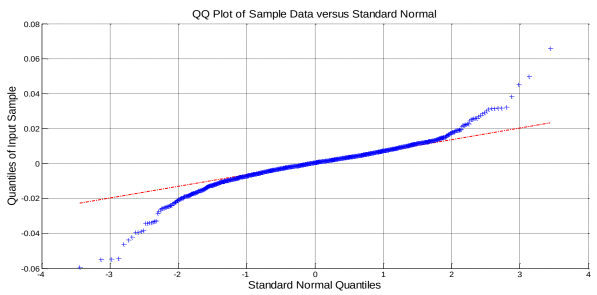

Figure 1: Pension Fund's Return Data Versus Standard Normal Distribution

Simulation and Analysis

The experiment includes the following steps:

- Estimation the empirical values of VaR and CVaR using Equation (1) and (2) from the sample data of pension fund returns;
- Fitting the distribution for return data of funds by employing a particular algorithm for parameter estimation;
- Generation of trials using the fitted distribution;
- Estimation of simulated VaR, CVaR measures and comparative analysis using Mean Absolute Percentage Error (MAPE):

$$\text{MAPE} = \frac{100\%}{n} \sum_{k=1}^{n} \left| \frac{A_k - M_k}{A_k} \right|; \qquad (3)$$

where A_k – actual value, M_k – simulated value, n – sample size, $k = \overline{1, n}$;

- Model performance and sensitivity analysis.

Fitting the Distribution for Pension Fund Returns

Normal distribution, α-stable and mixed α-stable distributions are fitted to the data of pension fund returns. The values of estimated parameters are given in Table 1.

Table 1: Estimates of Distribution Parameters

	Mixed α-stable distribution / α-stable distribution						*Normal distribution*		
	p	α	β	μ	σ		μ	σ	
Pension funds of conservative investments									
DNBP1	0,93843	1,59964	-0,06678	0,00019	0,00043		0,00017	0,00086	*
		1,53277	0,07800	0,00019	0,00040				
ERGOK	0,95724	1,36879	-0,12256	0,00008	0,00068		0,00013	0,00150	*
		1,32228	-0,07962	0,00009	0,00063				
FKI	0,93045	1,18195	0,15238	0,00034	0,00026	*	0,00022	0,00070	*
		1,19581	0,25403	0,00035	0,00027	*			
FN	0,88027	1,31185	-0,12014	0,00013	0,00017	*	0,00013	0,00043	*
		1,41334	0,12039	0,00015	0,00016	*			
SEBP1	0,96351	1,61310	-0,07432	0,00010	0,00084		0,00012	0,00160	*
		1,56067	-0,03556	0,00011	0,00079				
SWEDP1	0,79190	1,13500	0,04470	0,00015	0,00024	*	0,00009	0,00064	*
		1,05467	0,17696	0,00037	0,00035	*			
Pension funds with a small amount into shares									
DNBP2	0,98003	1,76366	-0,51559	0,00011	0,00120		0,00016	0,00192	*
		1,74716	-0,44057	0,00011	0,00117				
FAP	0,99030	1,55146	-0,22419	0,00018	0,00142		0,00019	0,00298	*
		1,54124	-0,19888	0,00019	0,00140				
SWEDP2	0,97319	1,67636	-0,23580	0,00009	0,00117		0,00010	0,00210	*
		1,64052	-0,17287	0,00009	0,00112				
Pension funds with a medium amount into shares									
DNBP3	0,98860	1,76518	-0,52343	0,00006	0,00222		0,00016	0,00356	*
		1,75466	-0,47027	0,00007	0,00219				
ERGOB	0,99031	1,54972	-0,32891	-0,00001	0,00206		0,00012	0,00395	*
		1,53898	-0,30292	0,00000	0,00203				
FAI	0,99430	1,55509	-0,34310	-0,00005	0,00238		0,00007	0,00511	*
		1,55059	-0,32353	-0,00003	0,00236				
FS	0,97377	1,44267	-0,14348	0,00002	0,00181		-0,00002	0,00446	*
		1,40450	-0,10465	0,00004	0,00173				
SEBP2	0,99430	1,58576	-0,29153	-0,00001	0,00254		0,00009	0,00497	*
		1,57989	-0,27175	0,00001	0,00251				
SWEDP3	0,98974	1,66397	-0,36530	-0,00004	0,00205		0,00007	0,00364	*
		1,64873	-0,32942	-0,00003	0,00202				
SWEDP4	0,98746	1,60376	-0,30191	-0,00021	0,00344		0,00003	0,00635	*
		1,58704	-0,26817	-0,00018	0,00338				
Pension funds with a large amount into shares									
FRR	0,99716	1,56845	-0,34228	-0,00045	0,00450		-0,00021	0,01009	*
		1,57208	-0,32081	-0,00036	0,00450				
SEBP3	0,99488	1,59734	-0,28270	-0,00017	0,00482		0,00006	0,00938	*
		1,59242	-0,25552	-0,00012	0,00479				

* - Goodness-of-fit hypothesis is rejected

Goodness-of-fit hypothesis which tests whether a given distribution is not significantly different from one hypothesized is also performed. Table 1 shows that normal distribution was rejected in all cases, α -stable distribution, as well as mixed α -stable distribution, are rejected for marked three pension funds.

The obtained estimates of distribution parameters are used to generate trials.

Analysis of Estimated VaR and CVaR Measures

The tail probability was set equal to $\varepsilon = 0.05$. It means that the left tail of distribution is explored. In simulation, 1000 trials of size 1755 were chosen as enough number, since the simulated values of risk measures were close to theoretical values.

The results of estimated VaR and CVaR as measure of loss from simulation model are given in Tables 2-3. To compare empirical values of risk measures with simulated ones, MAPE is computed in the categories of pension funds, as well as also in total, using Equation 3. Table 2 shows that mixed α-stable law has outperformed other distributions for VaR estimation because of smaller MAPE value. But for CVaR risk measure (Table 3), one can see that normal distribution is the most adequate. Mixed-stable and stable distributions exhibit fat tails, especially for small alphas like in cases SWEDP1, FKI, FN etc., that's why the expectation in the tail may be much bigger than in empirical case.

Table 2: Estimates of VaR Measure

VaR ε**=0,05**	*Normal distribution*	*α–stable distribution*	*Mixed α–stable distribution*	*Empirical data*
Pension funds of conservative investments				
DNBP1	0,00125	0,00096	0,00099	0,00100
ERGOK	0,00234	0,00229	0,00230	0,00223
FKI	0,00093	0,00078	0,00073	0,00064
FN	0,00058	0,00038	0,00047	0,00035
SEBP1	0,00252	0,00221	0,00222	0,00218
SWEDP1	0,00096	0,00191	0,00079	0,00084
MAPE	**9,38757**	**9,23606**	**3,25276**	
Pension funds with a small amount into shares				
DNBP2	0,00300	0,00305	0,00308	0,00297
FAP	0,00471	0,00413	0,00417	0,00437
SWEDP2	0,00336	0,00307	0,00311	0,00300
MAPE	**1,17846**	**0,56495**	**0,65694**	
Pension funds with a medium amount into shares				
DNBP3	0,00571	0,00582	0,00587	0,00573
ERGOB	0,00640	0,00642	0,00646	0,00643
FAI	0,00833	0,00744	0,00752	0,00731
FS	0,00737	0,00590	0,00595	0,00595
SEBP2	0,00810	0,00759	0,00763	0,00760
SWEDP3	0,00593	0,00582	0,00585	0,00597
SWEDP4	0,01044	0,01029	0,01037	0,01043
MAPE	**2,54394**	**0,45569**	**0,48060**	
Pension funds with a large amount into shares				
FRR	0,01681	0,01421	0,01437	0,01376
SEBP3	0,01538	0,01437	0,01452	0,01501
MAPE	**1,37247**	**0,41640**	**0,42817**	
Total MAPE	**14,48244**	**10,67310**	**4,81847**	

Table 3: Estimates of CVaR Measure

CVaR ε**=0,05**	*Normal distribution*	*α-stable distribution*	*Mixed α-stable distribution*	*Empirical data*
Pension funds of conservative investments				
DNBP1	0,00159	0,00270	0,00244	0,00198
ERGOK	0,00294	0,00913	0,00841	0,00364
FKI	0,00121	0,00492	0,00488	0,00161
FN	0,00075	0,00204	0,00232	0,00092
SEBP1	0,00316	0,00537	0,00497	0,00362
SWEDP1	0,00121	0,01876	0,00585	0,00149
MAPE	**6,32753**	**95,86953**	**46,73124**	
Pension funds with a small amount into shares				
DNBP2	0,00377	0,00596	0,00596	0,00446
FAP	0,00589	0,01071	0,01153	0,00773
SWEDP2	0,00419	0,00697	0,00661	0,00523
MAPE	**3,28828**	**5,85772**	**6,07064**	
Pension funds with a medium amount into shares				
DNBP3	0,00712	0,01126	0,01118	0,00844
ERGOB	0,00795	0,01699	0,01658	0,01020
FAI	0,01036	0,01909	0,01932	0,01345
FS	0,00913	0,01881	0,01726	0,01217
SEBP2	0,01006	0,01853	0,02006	0,01289
SWEDP3	0,00737	0,01313	0,01255	0,00877
SWEDP4	0,01297	0,02462	0,02389	0,01529
MAPE	**7,71297**	**19,49623**	**18,63030**	
Pension funds with a large amount into shares				
FRR	0,02080	0,03429	0,03545	0,02684
SEBP3	0,01911	0,03373	0,03402	0,02399
MAPE	**2,37988**	**3,79815**	**4,10483**	
Total MAPE	**19,70866**	**125,02164**	**75,53701**	

The second reason why normal random values are more adequate choice rather than mixed-stable and stable ones, may be a small sample size of historical return data of pension funds. If the sample size is too small even one simulated extreme value may influence the expectation. In this case median tail loss could be better risk measure (Chernobai et al. 2007, page 237). Moreover, returns of SWEDP1, FKI and FN do not fit any tested distribution (normal, mixed-stable and stable) and they are not correctly modelled by these probability laws.

If cases, which have been rejected after distribution fitting are not included to analysis, MAPE for VaR is 2,27286 (stable) and 2,25518 (mixed stable); MAPE for CVaR is 50,65747 (stable) and 47,31476 (mixed stable).

Sensitivity Analysis for Tail Probability

The analysis is continued to explore the influence of tail probability ε on accuracy of VaR measure (Figure 2) and CVaR measure (Figure 3).

From both figures we can conclude that MAPE decreases if tail probability ε increases when mixed α -stable law is applied. The same distribution is recommended to model the underlying series if VaR is measured. Concerning CVaR, normal distribution is recommended because of smallest MAPE when it is computed while ignoring the inference from goodness-of-fit hypothesis testing.

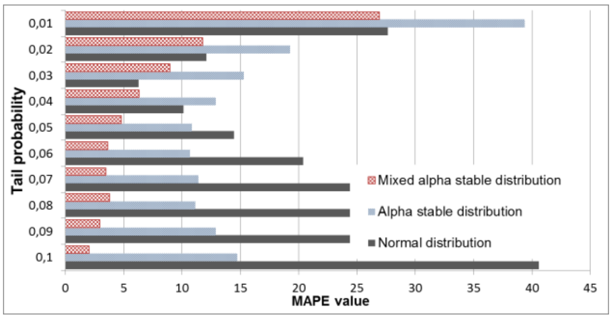

Figure 2: MAPE of VaR for different ε

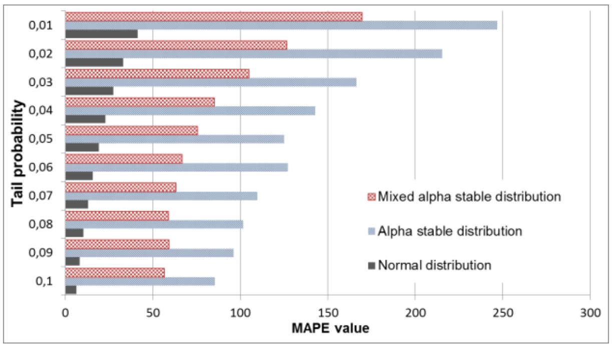

Figure 3: MAPE of CVaR for different ε

CONCLUDING REMARKS

The simulation experiment presented in this paper has shown that taking into account the goodness-of-fit testing results, simulated VaR and CVaR measures are always estimated with the smallest MAPE (comparing to empirical VaR and CVaR) if the underlying series are modelled by mixed α-stable distribution instead of standard α-stable distribution. It holds only for the case study performed in this research.

The future research will focus on the derivation of CVaR theoretical formula for mixed α-stable distribution. Next, Student's t-distribution will be also included in the research of modelling young financial markets that exhibit daily zero returns.

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