Credal Networks and Compositional Models: Preliminary Considerations

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Abstract

In the paper we present basic concepts concerning credal networks and compositional models for credal sets and describe the problem of imprecision increase in the first type of these models.

Keywords: Credal sets, credal networks, compositional models, strong independence.

1 Introduction

The most widely used models managing uncertainty and multidimensionality are, at present, so-called *probabilistic graphical Markov models*. The problem of multidimensionality is solved in these models with the help of the concept of conditional independence, which enables factorization of a multidimensional probability distribution into small parts (marginals, conditionals or just factors). Among them, the most popular are Bayesian networks. Therefore, it is not very surprising, that analogous models have been studied also in several theories of imprecise probability [1, 2, 3].

Credal networks are a generalization of Bayesian networks, able to deal with imprecision. Compositional models for credal sets, on the other hand, are intended to be generalization of compositional models for precise probabilities [6, 7, 8]. As the equivalence between Bayesian networks and precise compositional models is well known, it seems quite natural to ask a similar question also in this, more general case.

Compositional models were introduced also in possibility theory [12, 13] (here the models are parameterized by a continuous t-norm) and a few years ago also in evidence theory [9, 10]. In all these frameworks the original idea is kept, but there exist some slight differences among them.

Although Bayesian networks and compositional models represent the same class of distributions, they do not make it in the same way. Bayesian networks use *conditional distributions* whereas compositional models consist of *unconditional distributions*. Naturally, both types of models contain the same information but while some marginal distributions are explicitly expressed in compositional models, it may happen that their computation from a corresponding Bayesian network is rather computationally expensive. Therefore it appears that some of computational procedures designed for compositional models are (algorithmically) simpler than their Bayesian network counterparts.

Furthermore, the research concerning relationship between compositional models in evidence theory and evidential networks [14] revealed probably a more important thing. Even though any evidential network (with proper conditioning rule and conditional independence concept) can be expressed as a compositional model, if we do it in the opposite way and transform a compositional model into an evidential network, we may realize, that the model is more imprecise than the original one. It is caused by the fact that conditioning increases imprecision.

The goal of this paper is twofold. First, we want to show that the operator of composition can also be defined for credal sets (at least under specific conditions). Second, we want to argue that it is worth-developing, as conditioning in the framework of credal sets also increases imprecision.

The contribution is organized as follows. In Section 2 we summarize basic concepts and notation. Definition of the operator of composition is introduced in Section 3, where also its basic properties can be found. Finally, in Section 4 we recall the concept of credal networks and demonstrate how conditioning increases imprecision of the resulting model.

2 Basic Concepts and Notation

In this section we will recall basic concepts and notation necessary for understanding the contribution.

2.1 Variables and Distributions

For an index set $N = \{1, 2, \ldots, n\}$ let $\{X_i\}_{i \in N}$ be a system of variables, each X_i having its values in a finite set $\mathbf{X}_i$ and $\mathbf{X}_N = \mathbf{X}_1 \times \mathbf{X}_2 \times \ldots \times \mathbf{X}_n$ be the Cartesian product of these sets.

In this paper we will deal with groups of variables on its subspaces. Let us note that X_K will denote a group of variables $\{X_i\}_{i \in K}$ with values in

$$\mathbf{X}_K = \times_{i \in K} \mathbf{X}_i$$

throughout the paper.

Having two probability distributions P_1 and P_2 of X_K we say that P_1 is *absolutely continuous* with respect to P_2 (and denote $P_1 \ll P_2$) if for any $x_K \in \mathbf{X}_K$

$$P_2(x_K) = 0 \Longrightarrow P_1(x_K) = 0.$$

This concept plays an important role in the definition of the operator of composition.

2.2 Credal Sets

A *credal set* $\mathcal{M}(X_K)$ about a group of variables X_K is defined as a closed convex set of probability measures about the values of this variable.

In order to simplify the expression of operations with credal sets, it is often considered [11] that a credal set is the set of probability distributions associated to the probability measures in it. Under such consideration a credal set can be expressed as a *convex hull* of its extreme distributions

$$\mathcal{M}(X_K) = \mathrm{CH}\{\mathrm{ext}(\mathcal{M}(X_K))\}.$$

Consider a credal set about X_K, i.e. $\mathcal{M}(X_K)$. For each $L \subset K$ its *marginal credal set* $\mathcal{M}(X_L)$ is obtained by element-wise marginalization, i.e.

$$\mathcal{M}(X_L) = \mathrm{CH}\{P^{\downarrow L} : P \in \mathrm{ext}(\mathcal{M}(X_K))\}, \tag{1}$$

where $P^{\downarrow L}$ denotes the marginal distribution of P on $\mathbf{X}_L$.

Having two credal sets $\mathcal{M}_1$ and $\mathcal{M}_2$ about X_K and X_L, respectively (assuming that $K, L \subseteq N$), we say that these credal sets are *projective* if their marginals about common variables coincide, i.e. if

$$\mathcal{M}_1(X_{K\cap L}) = \mathcal{M}_2(X_{K\cap L}).$$

Let us note that if K and L are disjoint, then $\mathcal{M}_1$ and $\mathcal{M}_2$ are projective, as $\mathcal{M}(X_\emptyset) = 1$.

Conditional credal sets are obtained from the joint ones by point-wise conditioning of the extreme points and subsequent linear combination of the resulting conditional distributions. More formally: Let $\mathcal{M}(X_I X_J)$ be a credal set about (groups of) variables $X_I X_J$. Then for any $x_J \in \mathbf{X}_J$

$$\mathcal{M}(X_I|x_J) = \mathrm{CH}\{P(X_I|x_J) : P \in \mathrm{ext}(\mathcal{M}(X_I X_J))\}, \tag{2}$$

is a *conditional credal set* about X_I given $X_J = x_J$.

2.3 Strong Independence

Among numerous definitions of independence for credal sets [4] we have chosen strong independence, as it seems to be the most appropriate for multidimensional models.

We say that (groups of) variables X_K and X_L (K and L disjoint) are *strongly independent* with respect to $\mathcal{M}(X_{K\cup L})$ iff (in terms of probability distributions)

$$\mathcal{M}(X_{K\cup L}) \quad = \quad \mathrm{CH}\{P_1 \cdot P_2 : P_1 \in ext(\mathcal{M}(X_K)), P_2 \in ext(\mathcal{M}(X_L))\}. \tag{3}$$

Again, there exist several generalizations of this notion to conditional independence, see e.g. [11], but since the following definition is suggested by the authors as the most appropriate for the marginal problem, it seems to be a suitable concept also in our case, since the operator of composition can also be used as a tool for solution of a marginal problem, as shown (in the framework of possibility theory) e.g. in [13].

Given three groups of variables X_K, X_L and X_M (K, L, M be mutually disjoint subsets of N, such that K and L are nonempty), we say analogous[1] to [11] that X_K and X_L are *conditionally independent on the distribution* given X_M under global set $\mathcal{M}(X_{K\cup L\cup M})$ (in symbols $K \perp\!\!\!\perp L|M$) iff

$$\begin{aligned}\mathcal{M}(X_{K\cup L\cup M}) \; &= \; \mathrm{CH}\{(P_1 \cdot P_2)/P_1^{\downarrow M} : P_1 \in ext(\mathcal{M}(X_{K\cup M})), \\ &\qquad P_2 \in ext(\mathcal{M}(X_{L\cup M})), P_1^{\downarrow M} = P_2^{\downarrow M}\}\,. \end{aligned} \tag{4}$$

This definition is a generalization of stochastic conditional independence: if $\mathcal{M}(X_{K\cup L\cup M})$ is a singleton, then also $\mathcal{M}(X_{K\cup M})$ and $\mathcal{M}(X_{L\cup M})$ are (projective) singletons and the definition collapses into definition of stochastic conditional independence.

3 Operator of Composition and Its Properties

Now, let us start considering how to define composition of two credal sets. Consider two index sets $K, L \subset N$. At this moment we do not put any restrictions on K and L; they may be but need not be disjoint, one may be subset of the other.

In order to enable the reader the understanding of this concept, let us first present the definition of composition for precise probabilities [6]. Let P_1 and P_2 be two probability distributions of (groups of) variables X_K and X_L. Then

$$(P_1 \triangleright P_2)(X_{K\cup L}) = \frac{P_1(X_K) \cdot P_2(X_L)}{P_2(X_{K\cap L})}, \tag{5}$$

whenever $P_1(X_{K\cap L}) \ll P_2((X_{K\cap L})$. Otherwise, it remains undefined.

Let $\mathcal{M}_1$ and $\mathcal{M}_2$ be credal sets about X_K and X_L, respectively. Our goal is to define a new credal set, denoted by $\mathcal{M}_1 \triangleright \mathcal{M}_2$, which will be about $X_{K\cup L}$ and will contain all of the information contained in $\mathcal{M}_1$ and as much as possible of information of $\mathcal{M}_2$.

The required properties are met by Definition 1[2] in [15]. However, the definition exhibits a kind of discontinuity and should be reconsidered. In this paper we will deal only with the composition of projective credal sets.

Definition 1 For two projective credal sets $\mathcal{M}_1$ and $\mathcal{M}_2$ about X_K and X_L, a *composition* $\mathcal{M}_1 \triangleright \mathcal{M}_2$ is defined by the following expression:

$$\begin{aligned}(\mathcal{M}_1 \triangleright \mathcal{M}_2)(X_{K\cup L}) \; &= \; \mathrm{CH}\{(P_1 \cdot P_2)/P_2^{\downarrow K\cap L} : P_1 \in ext(\mathcal{M}_1(X_K)), \\ &\qquad P_2 \in ext(\mathcal{M}_2(X_L)), P_1^{\downarrow K\cap L} = P_2^{\downarrow K\cap L}\}. \end{aligned}$$

[1] Let us note that our definition differs somehow from that presented in [11]: the authors require point-wise satisfaction in (3) and (4), which leads to non-convexity. In [5] this type of independence is called *complete*.

[2] Let us note that the definition was based on Moral's concept of conditional independence with relaxing convexity.

The following lemma presents basic properties possessed by this operator of composition.

Lemma 1 *For two projective credal sets $\mathcal{M}_1$ and $\mathcal{M}_2$ about X_K and X_L, respectively, the following properties hold true:*

(i) $\mathcal{M}_1 \triangleright \mathcal{M}_2$ *is a credal set about* $X_{K\cup L}$.

(ii) $\mathcal{M}_1 \triangleright \mathcal{M}_2 = \mathcal{M}_2 \triangleright \mathcal{M}_1$.

Proof.

(i) To prove that $\mathcal{M}_1 \triangleright \mathcal{M}_2$ is a credal set about $X_{K\cup L}$ it is enough to show that any $P \in \{ext(\mathcal{M}_1 \triangleright \mathcal{M}_2)\}$ is a probability distribution on $\mathbf{X}_{K\cup L}$, as the convexity of $\mathcal{M}_1 \triangleright \mathcal{M}_2$ is evident. But it is obvious, as any $P \in \{ext(\mathcal{M}_1 \triangleright \mathcal{M}_2)\}$ is obtained via formula for composition of probability distributions (5).

(ii) For any distribution P of $\{ext(\mathcal{M}_1 \triangleright \mathcal{M}_2)(X_{K\cup L})\}$ there exist $P_1 \in \{ext(\mathcal{M}_1(X_K))\}$ and $P_2 \in \{ext(\mathcal{M}_2(X_L))\}$ such that $P_1^{\downarrow K\cap L} = P_2^{\downarrow K\cap L}$ and $P = (P_1 \cdot P_2)/P_2^{\downarrow K\cap L}$. But simultaneously (due to projectivity) $P = (P_1 \cdot P_2)/P_1^{\downarrow K\cap L}$, which is an element of $(\mathcal{M}_2 \triangleright \mathcal{M}_1)(X_{K\cup L})$. Hence
$$(\mathcal{M}_1 \triangleright \mathcal{M}_2)(X_{K\cup L}) = (\mathcal{M}_2 \triangleright \mathcal{M}_1)(X_{K\cup L}),$$
as desired. □

Let us now illustrate the application of the operator of composition and its properties by two examples. The first shows what happens when $K \cap L = \emptyset$.

Let use note the all variables in the examples in this paper are binary.

Example 1 Let
$$\mathcal{M}_1(X_1) = \mathrm{CH}\{[0.2, 0.8], [0.7, 0.3]\}$$
and
$$\mathcal{M}_2(X_2) = \mathrm{CH}\{[0.6, 0.4], [1, 0]\}$$
be two credal sets about X_1 and X_2, respectively. Then, as mentioned above, $\mathcal{M}_1(X_1)$ and $\mathcal{M}_2(X_2)$ are projective, and therefore $\mathcal{M}_1 \triangleright \mathcal{M}_2$ is obtained via Definition 1:
$$\begin{aligned}(\mathcal{M}_1 \triangleright \mathcal{M}_2)(X_1X_2) &= \mathrm{CH}\{[0.12, 0.08, 0.48, 0.32], [0.2, 0, 0.8, 0], \\ &\qquad [0.42, 0.28, 0.18, 0.12], [0.7, 0, 0.3, 0]\}.\end{aligned}$$
It is evident, that not every element of $\mathcal{M}_1 \triangleright \mathcal{M}_2$ can be expressed as a product of its marginals, as e.g.
$$\begin{aligned}[0.41, 0.04, 0.39, 0.16] &\in \mathrm{CH}\{[0.12, 0.08, 0.48, 0.32], [0.2, 0, 0.8, 0], \\ &\qquad [0.42, 0.28, 0.18, 0.12], [0.7, 0, 0.3, 0]\},\end{aligned}$$
but $[0.41, 0.04, 0.39, 0.16] \notin \{P_1 \cdot P_2 : P_1 \in \mathcal{M}(X_K), P_2 \in \mathcal{M}(X_L)\}$. ♢

The following example is devoted to the case, when $K \cap L \neq \emptyset$.

Example 2 Let

$$\begin{aligned}\mathcal{M}_1(X_1X_2) \quad = \quad & \mathrm{CH}\{[0.2, 0.2, 0, 0.6], [0.1, 0.4, 0.1, 0.4], \\ & [0.25, 0.25, 0.25, 0.25], [0.2, 0.3, 0.3, 0.2]\}.\end{aligned}$$

be a credal set about variables X_1X_2 and

$$\begin{aligned}\mathcal{M}_2(X_2X_3) = \mathrm{CH}\{&[0.2, 0, 0.3, 0.5], [0, 0.2, 0, 0.8], \\ &[0.5, 0, 0.5, 0], [0.2, 0.3, 0.2, 0.3]\},\end{aligned}$$

be a credal set about variables X_2X_3. These two credal sets are projective, as

$$\mathcal{M}_1(X_2) = \mathrm{CH}\{[0.2, 0.8], [0.5, 0.5]\} = \mathcal{M}_2(\mathrm{X}_2),$$

therefore Definition 1 can be applied:

$$\begin{aligned}&(\mathcal{M}_1 \triangleright \mathcal{M}_2)(X_1X_2X_3) \\ &\quad = \mathrm{CH}\{[0.2, 0, 0.075, 0.125, 0, 0, 0.275, 0.375], [0, 0.2, 0, 0.2, 0, 0, 0, 0.6], \\ &\qquad [0.1, 0, 0.15, 0.25, 0.1, 0, 0.15, 0.25], [0, 0.1, 0, 0.4, 0, 0.1, 0, 0.4], \\ &\qquad [0.25, 0, 0.25, 0, 0.25, 0, 0.25, 0], [0.1, 0.15, 0.1, 0.15, 0.1, 0.15, 0.1, 0.15], \\ &\qquad [0.2, 0, 0.2, 0, 0.3, 0, 0.3, 0], [0.08, 0.12, 0.08, 0.12, 0.12, 0.18, 0.12, 0.18]\}.\end{aligned}$$

It can easily be checked that both $(\mathcal{M}_1 \triangleright \mathcal{M}_2)(X_1X_2) = \mathcal{M}_1(X_1X_2)$ and $(\mathcal{M}_1 \triangleright \mathcal{M}_2)(X_2X_3) = \mathcal{M}_2(X_2X_3)$. ♢

The following theorem reveals the relationship between strong independence and the operator of composition. It is, together with Lemma 1, the most important assertion enabling us to introduce multidimensional models.

Theorem 1 *Let $\mathcal{M}$ be a credal set about $X_{K\cup L}$ with marginals $\mathcal{M}(X_K)$ and $\mathcal{M}(X_L)$. Then*

$$\mathcal{M}(X_{K\cup L}) = (\mathcal{M}^{\downarrow K} \triangleright \mathcal{M}^{\downarrow L})(X_{K\cup L}) \tag{6}$$

iff

$$(K \setminus L) \perp\!\!\!\perp (L \setminus K)|(K \cap L). \tag{7}$$

Proof. Let us suppose that (6) holds. Since $\mathcal{M}_1(X_K)$ and $\mathcal{M}_2(X_L)$ are projective, Definition 1 can be applied and therefore

$$\begin{aligned}\mathcal{M}(X_{K\cup L}) \quad = \quad & \mathrm{CH}\{(P_1 \cdot P_2)/P_2^{\downarrow K\cap L} : P_1 \in \mathcal{M}(X_K), \\ & P_2 \in \mathcal{M}(X_L), P_1^{\downarrow K\cap L} = P_2^{\downarrow K\cap L}\}).\end{aligned}$$

To prove (7) means to find for any P from $\mathcal{M}(X_{K\cup L})$ a pair of projective distributions P_1 and P_2 from $\mathcal{M}(X_K)$ and $\mathcal{M}(X_L)$, respectively, such that $P = (P_1 \cdot P_2)/{P_1}^{\downarrow K\cap L}$. But due to condition of projectivity, $\mathcal{M}(X_{K\cup L})$ consists of exactly this type of distributions.

Let on the other hand (7) be satisfied. Then any P from $\mathcal{M}(X_{K\cup L})$ can be expressed as conditional product of its marginals, namely

$$P = (P^{\downarrow K} \cdot P^{\downarrow K})/P^{\downarrow K\cap L},$$

$P^{\downarrow K} \in \mathcal{M}(X_K)$ and $P^{\downarrow L} \in \mathcal{M}(X_L)$. Therefore,

$$\mathcal{M}(X_{K\cup L}) \quad = \quad \{(P^{\downarrow K} \cdot P^{\downarrow K})/P^{\downarrow K\cap L} : P^{\downarrow K} \in \mathcal{M}_1(X_K), P^{\downarrow L} \in \mathcal{M}_2(X_L))\},$$

which concludes the proof. □

4 Credal Networks

A *credal network* [1] over X_N is (analogous to Bayesian networks) a pair $(\mathcal{G}, \{\mathbf{P}_1, \ldots, \mathbf{P}_k\})$ such that for any $i = 1, \ldots, k$ $(\mathcal{G}, \mathbf{P}_i)$ is a Bayesian network over X_N.

The resulting model is a credal set, which is the convex hull of the Bayesian networks, i.e.

$$\text{CH}\{P_1(X_N), \ldots P_k(X_N)\}.$$

It is evident, that this definition looses the attractiveness of Bayesian networks, where the overall information is computed from the local pieces of information.

The most popular (and also effective) type of credal networks are those called separately specified. A *separately specified credal networks* over X_N is a pair $(\mathcal{G}, \mathbf{M})$, where $\mathbf{M}$) is a set of conditional credal sets $\mathcal{M}(X_i|pa(X_i))$ for each $X_i \in X_N$.

Here the overall model is obtained analogous to Bayesian networks as the strong extension of the $\mathcal{M}(X_i|pa(X_i)), i \in N$.

Nevertheless, the reverse side of this nice property is the imprecision increase of this type models, as can be seen even from the following simple example.

Example 3 Let $\mathcal{M}(X_1X_2)$ be a credal set about variables X_1 and X_2 with values in $\mathbf{X}_1$ and $\mathbf{X}_2$ ($\mathbf{X}_i = \{x_i, \bar{x}_i\}$), respectively, defined as in Example 2.

From its extreme points we obtain the following distributions:

$P_1(x_2) = 0.2$	$P_1(x_1\|x_2) = 1$	$P_1(x_1\|\bar{x}_2) = 0.25$
$P_2(x_2) = 0.2$	$P_2(x_1\|x_2) = 0.5$	$P_2(x_1\|\bar{x}_2) = 0.5$
$P_3(x_2) = 0.5$	$P_3(x_1\|x_2) = 0.5$	$P_3(x_1\|\bar{x}_2) = 0.5$
$P_4(x_2) = 0.5$	$P_4(x_1\|x_2) = 0.4$	$P_2(x_1\|\bar{x}_2) = 0.6,$

These are, together with the graph $X_2 \longrightarrow X_1$ four Bayesian networks. Their convex hull is exactly the set $\mathcal{M}_1(X_1X_2)$. Nevertheless, it is not separably specified credal network. To obtain it we need the credal sets $\mathcal{M}(X_2)$, $\mathcal{M}(X_1|x_2)$ and $\mathcal{M}(X_1|\bar{x}_2)$

From the above values one will get the "extreme" points of $\mathcal{M}(X_1|x_2)$ and $\mathcal{M}(X_1|\bar{x}_2)$:

$$[1, 0], [0.5, 0.5], [0.5, 0.5], [0.4, 0.6],$$

and

$$[0.25, 0.75], [0.5, 0.5], [0.5, 0.5], [0.6, 0.4],$$

respectively.

As $[0.5, 0.5]$ is a linear combination of both [1,0] and [0.4,0.6], and [0.25,0.75] and [0.6,0.4], the resulting (conditional) credal sets are

$$\begin{aligned}\mathcal{M}(X_2) &= \text{CH}\{[0.2, 0.8], [0.5, 0.5]\},\\ \mathcal{M}(X_1|x_2) &= \text{CH}\{[1, 0], [0.4, 0.6]\},\\ \mathcal{M}(X_1|\bar{x}_2) &= \text{CH}\{[0.25, 0.75], [0.6, 0.4]\}.\end{aligned}$$

The strong extension of these credal sets is

$$\begin{aligned}\mathcal{M}_1(\tilde{X}_1X_2) = \text{CH}\{&[0.2, 0.2, 0, 0.6], [0.2, 0.48, 0, 0.32], [0.08, 0.2, 0.12, 0.6],\\ &[0.08, 0.48, 0.12, 0.32], [0.5, 0.125, 0, 0.375],\\ &[0.5, 0.3, 0, 0.2], [0.2, 0.125, 0.3, 0.375], [0.2, 0.3, 0.3, 0.2]\}.\end{aligned}$$

which is less precise than the original model. ♢

5 Conclusions

We introduced an operator of composition of projective credal sets — a generalization of that introduced about 15 years ago in (precise) probability framework. The operator satisfies the basic properties necessary for the introduction of compositional models of credal sets. Nevertheless, the definition must be extended to non-projective credal sets, which seems to be the most important problem to be solved in the near future.

We also recalled the concept of credal networks and we suggested that compositional models of credal sets are potentially good counterpart of these models, which are either not separately specified (contrary to our expectation concerning compositional models), or more imprecise.

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