Transportation Research Part C 86 (2018) 23–36

Contents lists available at ScienceDirect

Transportation Research Part C

journal homepage: www.elsevier.com/locate/trc

An online estimation of driving style using data-dependent pointer model

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ARTICLE INFO

Keywords:
Driving style
Fuel consumption
Mixture-based clustering
Data-dependent pointer
Recursive mixture estimation

ABSTRACT

The paper focuses on a task of stochastic modeling the driving style and its online estimation while driving. The driving style is modeled by means of a mixture model with normal and categorical components as well as a data-dependent pointer. The mixture parameters and the actual driving style are estimated with the help of a recursive algorithm under the Bayesian methodology. The main contributions of the presented approach are: (i) the online estimation of the driving style while driving, taking into account data up to the current time instant; (ii) the joint model for continuous and discrete data measured on a vehicle; (iii) the data-dependent model of the driving style conditioned by the values of fuel consumption; (iv) the use of the model both for detection of clusters according to the driving style and prediction of the fuel consumption along with other variables; and (v) the universal modeling with the help of mixtures, which allows us to use different combinations of components and pointer models as well as to specify the initialization approach suitable for the considered problem. Results of the driving style detection in real measurements and comparison with the theoretical counterparts are demonstrated.

1. Introduction

Modeling the driving style is important for many reasons. Timely recognition of the driving style in the online mode and its prediction can be beneficial in aspects of providing this information to a driver by means of driver assistance systems (Li et al., 2015).

Definitions of the driving style, which can be found in literature (Elander et al., 1993; Lajunen and Özkan, 2011; Sagberg et al., 2015) describe it as a way of driving (i.e., a set of individual driving habits), which is formed gradually with the accumulation of driving experience. The accumulated habits are reflected in a driver's activities while driving, which can be taken into account for performing the analysis of driving style (Cheng and and Fujioka, 1997; Toledo et al., 2007). The extensive multi-layer scheme of such driving activities is presented in Li et al. (2017), where they are generally divided among the primary driving tasks of route planning (Dia, 2002), maneuvering (Ehsani et al., 2015) as well as vehicle operating (Toledo et al., 2008) and the secondary tasks performed by the driver while driving (Ferdinand and Menachemi, 2014), e.g., phone using, talking, eating, smoking, etc. The mentioned scheme in Li et al. (2017) distinguishes the existing studies about modeling the driving style according to its definition.

Another way to categorize the studies on the driving style can be done in terms of the area where the driving style has a direct impact. As reported by a number of studies, driving style has a strong impact on driving safety (Evans, 1996), vehicle dynamics control (Plöchl and Edelmann, 2007) and the economic as well as ecologic efficiency of driving (Mensing et al., 2014).

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https://doi.org/10.1016/j.trc.2017.11.001
Received 29 December 2016; Received in revised form 22 August 2017; Accepted 1 November 2017

One of the study groups is related to the driving safety. In this area, the extensive overview of studies which deal with modeling the driving style is given by Sagberg et al. (2015). They introduced the systematic scheme for categorizing and operationalizing the driving styles in dependence on individual dispositions and sociocultural factors. Among recent related studies, Eboli et al. (2017) aimed at distinguishing cautious and aggressive driving styles. The relationship between the family driving-related atmosphere and young driver's driving styles was explored in Gil et al. (2016) and Taubman-Ben-Ari (2016). Taubman-Ben-Ari and Skvirsky (2016) investigated the young driver's driving style too and resulted in four driving styles with the suggestion of an insignificant effect of sociodemographic characteristics on the driving style.

Another group of studies with driving style modeling can be found in the area of applications concerned with the overall vehicle dynamics control. Specific control issues with the use of driving style models are discussed in Plöchl and Edelmann (2007), Zhang et al. (2010), Wang and Lukic (2011), Xu et al. (2015), and Bellem et al. (2016).

From an emissions' point of view, driving style modeling is discussed in the great number of publications, e.g., in Sentoff et al. (2015), Rangaraju et al. (2015), and Gallus et al. (2017), which deal with the ecological driving style. This area of the driving style analysis is closely related with its impact on fuel consumption as well (Mensing et al., 2014). This paper focuses on the modeling the driving style from the fuel economy point of view.

1.1. Related work

Studies, which are directed at exploring driving style in terms of reducing fuel consumption can be found in literature.

Murphey et al. (2009) in their study noticed that the information about a driver's driving style can be used for the aims of fuel economy. They took into account the driver's accelerating and decelerating, created jerk profiles for drivers and classified the driving style with the help of these profiles analysis. Manzoni et al. (2010) proposed a method to quantify the driving style from the fuel economy point of view using measurements of the longitudinal speed and the lateral acceleration. In Kamal et al. (2007), detection of abnormalities in driving style was solved by means of the adaptive fuzzy system. Malikopoulos and Aguilar (2012) investigated driving styles, which have a major effect on fuel consumption and optimized them via specific optimization framework with the use of polynomial metamodels. Categorization of the driving styles between normal and aggressive was considered by Vaitkus et al. (2014). They proposed using a pattern recognition approach to evaluate driving style automatically without expert intervention.

More recent studies are as follows: Ma et al. (2015) dealt with the effects of driving style on fuel consumption of city buses. Using a vehicle-engine combined model, they analyzed a great number of parameters related to fuel consumption found while accelerating, normal running and decelerating processes of vehicles. They reported that the influence of the driving style parameters on fuel consumption changed with road conditions and vehicle mass. A comparative study from two different countries was presented by Son et al. (2016), where they examined the relationship between driving style and real-world fuel consumption. Based on the analysis of data of fuel consumption, vehicle speed and acceleration pedal usage, they reported a high correlation of driving styles with the real-world fuel consumption and cultural factors. In a study by Ferreira et al. (2015), the driving styles, which are optimal from a fuel economy point of view were determined by means of data mining techniques. This study took data from public transportation buses and showed that the fuel consumption can be significantly reduced using the optimized driving practices. Mental models of three driving styles, which were defined as "normal","safe" and "fuel-efficient" were considered by Pampel et al. (2015). They conducted the experiment with a driving simulator, where participants had to drive according to instructions and then analyzed changes in their behavior. The used characteristics were accelerating, braking, coasting and car-following.

A question which factors have the greatest influence on driving style with respect to fuel economy was investigated by Akena et al. (2017). They identified and categorized such factors among driver factors, operating the vehicle, vehicle dynamics and driver awareness. Analysis of their impact on fuel economy was performed with the help of a multi-criterial hierarchical approach. According to the obtained results, driver awareness belongs to the most influential category. Factors related to vehicle control (primarily acceleration and speed) comprise the second most influential category and the driver-related factors have the least influence on fuel economy.

In addition, approaches to modeling the driving style can be also distinguished according to the formalisms they use. A variety of approaches are applied in all of the mentioned areas influenced by driving style, e.g., the correlation analysis (Eboli et al., 2017), fuzzy logic (Kamal et al., 2007; Dörr et al., 2014), *k*-means clustering (Guo and Fang, 2013), hierarchical clustering (Constantinescu et al., 2010), unsupervised learning (Nikulin, 2016), Bayesian networks (Amata et al., 2009), etc.

This paper considers the driving style estimation problem in the Bayesian context (Peterka, 1981; Kárný et al., 1998; Kárný et al., 2006; Nagy et al., 2011) and uses the mixture-based cluster analysis of data measured on a driven vehicle. The measurements are modeled by a mixture of normal and categorical components, where each of them describes variables within individual driving styles. A component, which is active at the current time instant, represents the actual driving style. To estimate which driving style is currently active, the recursive Bayesian mixture estimation algorithm is used. Bayesian methods were used for closely related problems in Mudgal et al. (2014) and Wang et al. (2016). However, the specific feature of the presented algorithm is its recursiveness, which (i) enables us to obtain a driving style estimate at each time instant and to update it online with the new data and (ii) guarantees the fixed computational time, which does not depend on algorithm convergence, which is characteristic for iterative techniques.

The presented paper continues the previous study (Suzdaleva and Nagy, 2014), where a stochastic data-based description of a driven vehicle was considered via the normal regression model within the context of the optimal (from the eco-driving viewpoint) control problem. Here, the focus is on the application of the recursive mixture estimation for the detection of the actual driving style and clustering the related measurements. The main contributions of the presented approach are:

- the online estimation of the driving style while driving;
- the joint model for continuous and discrete data measured on a driven vehicle;
- the data-dependent model of the driving style conditioned by the values of fuel consumption;
- the use of the model both for the detection of clusters according to the driving style and prediction of the fuel consumption along with other variables;
- and finally, universal modeling with the help of mixtures, which allows us to (i) use different combinations of component distributions and models of their switching as well as (ii) specify the initialization approach suitable for the application area.

The presented approach is explained in the remainder of the paper, which is organized as follows. Section 2 formulates a problem. Section 3 provides the theoretical background, introducing models, describes a clustering solution and summarizes it in the form of an algorithm with remarks on its practical aspects. Section 4 demonstrates the application of the mentioned algorithm to a problem of online detection of the driving style. It focuses on the data-based construction of the driving style model, initialization of the algorithm, results and a discussion. Conclusions are given in Section 5.

2. Problem formulation

A driven vehicle is considered as the observed system, which in discrete time instants $t \in \{1,\ldots,T\}$ (seconds in the presented paper) generates the data vector $y_t = [y_{1;t},y_{2;t},y_{3;t}]'$ of the continuous variables, where.

- $y_{1;t}$ is instantaneous fuel consumption [μl/s],
- $y_{2;t}$ is vehicle speed [km/h],
- $y_{3;t}$ is gas pedal position [%]

and the discrete variable z_t, which is the selection of the gear. The gear variable has the set of the possible values $\{-1,0,1,2,\ldots,6\}$, where value -1 corresponds to the reverse gear and value 0 denotes the neutral gear. The rest of the values are gear shifts from 1 to 6. The whole number of the possible gear values is denoted by m_z.

The behavior of the observed system changes according to a driving style, i.e., the system is multi-modal. A suitable tool for the description of such a system is a mixture model successfully applied in a variety of domains, e.g., (Park et al., 2010; Yu, 2012; Zou et al., 2014), etc. The mixture model consists of components describing data within the individual mode and a model of the pointer (Kárný et al., 1998), whose values indicate a currently active component. In this paper, the currently active component represents the driving style. It cannot be measured and should be estimated. With the use of the mixture model and the Bayesian methodology (Peterka, 1981; Kárný et al., 1998; Kárný et al., 2006; Nagy et al., 2011), a task solved in the paper is formulated as follows:

- specify the data-based description of the observed system;
- estimate recursively the mixture parameters based both on the available data collection and permanently arriving new measurements (which means the online estimation);
- estimate recursively the pointer, which points to the driving style;
- validate the model with the help of real data and a comparison with the theoretical counterparts.

The theoretical background necessary for a solution is given in the subsequent section.

3. Theoretical background

3.1. Models

The observed system is modeled by a mixture of m_c components in the form of probability density functions (pdf), where the i-th component, $\forall\ i \in \{1,\ldots,m_c\}$

$$f(\underbrace{y_t,z_t = l}_{\text{data}}|\Theta,\beta,\underbrace{c_t = i}_{\text{pointer}},\underbrace{y(t-1),z(t-1)}_{\text{past data}}) \tag{1}$$

describes the data y_t and z_t in dependence of the value i of the pointer c_t (here it points to the driving style), l is a value of the variable z_t (here the gear) and Θ and β are unknown parameters. A denotation of the type $y(t-1)$ means the collection of data $\{y_0,y_1,\ldots,y_{t-1}\}$ up to the time $t-1$ including the prior knowledge y_0.

According to the chain rule, e.g., (Peterka, 1981), in this paper the joint pdf (1) is decomposed in two conditional pdfs, where one of them describes the continuous variables dependent on the pointer and the discrete variable. The other describes the discrete variable z_t depending on the pointer. The decomposed pdf (1) takes the form

$$f(y_t|\Theta,c_t = i,\underbrace{z(t)}_{\psi_t})f(z_t = l|\beta,c_t = i,\underbrace{z(t-1)}_{z_{t-1}}), \tag{2}$$

where its left part is assumed to be the linear static regression model with normal noise for each $i \in \{1,\ldots,m_c\}$

$$f(y_t|\Theta, c_t = i, \psi_t) \sim \mathcal{N}_y(\underbrace{\psi_t \theta_i}_{\text{mean}}, \underbrace{r_i}_{\text{variance}}), \tag{3}$$

where $\mathcal{N}_y$ denotes the normal distribution of the i-th component and ψ_t is (here) the static regression vector. The parameter θ_i is the collection of regression coefficients of the i-th component, r_i is the constant covariance matrix of the normal noise and $\{\theta_i, r_i\}_{i=1}^{m_c} \equiv \Theta$. In the considered context, each normal component describes the data from the fuel consumption to the gas pedal position depending on the gear value and the driving style.

Under the assumption that the discrete variable does not depend on the continuous vector y_t (to avoid the use of logistic regression, which cannot be used for recursive estimation) and the parameter Θ, the second pdf in (2) is a transition table

$$f(z_t = l|\beta, c_t = i, z_{t-1} = q) \equiv \tag{4}$$

	$z_t = 1$	$z_t = 2$	$\cdots$	$z_t = m_z$
$z_{t-1} = 1$	$(\beta_{1\|1})_i$	$(\beta_{2\|1})_i$	$\cdots$	$(\beta_{m_z\|1})_i$
$z_{t-1} = 2$	$(\beta_{1\|2})_i$	$\cdots$	$\cdots$	$\cdots$
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$z_{t-1} = m_z$	$(\beta_{1\|m_z})_i$	$\cdots$	$\cdots$	$(\beta_{m_z\|m_z})_i$

where the parameter β_i belongs to the i-th component (for $c_t = i$) and $\{\beta_i\}_{i=1}^{m_c} \equiv \beta$. Here β_i is a matrix of non-negative probabilities $(\beta_{l|q})_i$ of the gear value $z_t = l$ conditioned by the last gear $z_{t-1} = q$ for the driving style $c_t = i$.

Component (2) describes data depending on the pointer variable c_t. The dynamic data-dependent pointer is modeled by the transition table, where it is conditioned by the previous pointer c_{t-1} and by the discrete or discretized data φ_{t-1}, i.e.,

$$f(c_t = i|\alpha, c_{t-1} = j, \varphi_{t-1} = k) \equiv \tag{5}$$

	$c_t = 1$	$c_t = 2$	$\cdots$	$c_t = m_c$
$c_{t-1} = 1$	$(\alpha_{1\|1})_k$	$(\alpha_{2\|1})_k$	$\cdots$	$(\alpha_{m_c\|1})_k$
$c_{t-1} = 2$	$(\alpha_{1\|2})_k$		$\cdots$	
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$c_{t-1} = m_c$	$(\alpha_{1\|m_c})_k$		$\cdots$	$(\alpha_{m_c\|m_c})_k$

The data items φ_{t-1} are selected from the data collection $y(t-1)$ or $z(t-1)$ for each $i,j \in \{1,\ldots,m_c\}$ and $k \in \{1,2,\ldots,m_\psi\}$. Similarly to (4), this transition table exists for each k-th value of φ_{t-1} with m_φ as the number of its values. The parameter $\alpha \equiv \{\alpha_k\}_{k=1}^{m_\varphi}$ contains non-negative probabilities $(\alpha_{i|j})_k$ of the pointer $c_t = i$ under the condition that the previous pointer $c_{t-1} = j$ and $\varphi_{t-1} = k$. Parameters Θ, β and α are assumed to be mutually independent.

For these models, the problem formulated in Section 2 is specified as the recursive estimation of the unknown parameters α,Θ,β and the value of the pointer c_t.

3.2. Recursive mixture-based clustering

The presented solution is based on the construction of the joint pdf of variables to be estimated (α,Θ,β and c_t) and the application of the Bayes rule. The approach uses recursive Bayesian algorithms avoiding numerical computations proposed for individual normal regression pdfs in Peterka (1981), categorical models (Kárný et al., 2006), mixtures with the static pointer in Kárný et al. (1998) and with the dynamic pointer in Nagy et al. (2011). Here an extension of the mentioned algorithms for the dynamic data-dependent pointer model and mixed-type components in the unified recursive form allowing a real-time performance is used.

For the sake of brevity, let the data pair $\{y_t, z_t\}$ measured at time t be denoted by D_t and then the data collection available up to the time instant t is $D(t)$. The estimation algorithm is derived with the help of the following scheme.

$$\begin{aligned}
&\underbrace{f(\overbrace{\Theta,\beta,c_t = i,\alpha,c_{t-1} = j}^{\text{unknown variables}}|\overbrace{D(t)}^{\text{data}})}_{\text{joint pdf}} \propto \underbrace{f(y_t, z_t = l, \Theta, \beta, c_t = i, \alpha, c_{t-1} = j|D(t-1))}_{\text{via Bayes and chain rule}} \\
&= \underbrace{f(y_t|\Theta, c_t = i, \psi_t)}_{\text{model (3)}} \underbrace{f(\Theta|D(t-1))}_{\text{prior GiW pdf}} \underbrace{f(z_t = l|\beta, c_t = i, z_{t-1} = q)}_{\text{model (4)}} \underbrace{f(\beta|D(t-1))}_{\text{prior Dir pdf for}\beta} \\
&\times \underbrace{f(c_t = i|\alpha, c_{t-1} = j, \varphi_{t-1} = k)}_{\text{model (5)}} \underbrace{f(\alpha|D(t-1))}_{\text{prior Dir pdf for}\alpha} \underbrace{f(c_{t-1} = j|D(t-1))}_{\text{prior pointer pdf}},
\end{aligned} \tag{6}$$

where GiW denotes the conjugate prior Gauss-inverse-Wishart pdf used for each normal component according to Peterka (1981) and

Kárný et al. (1998) and Dir denotes the conjugate prior Dirichlet pdf used for the categorical models (4) and (5) according to Kárný et al. (2006).

To obtain the posterior pdf of the pointer c_t, the relation (6) should be marginalized firstly over all parameters. It gives

$$\begin{aligned}\underbrace{\overbrace{f(c_t=i,c_{t-1}=j|D(t))}^{\text{denoted by} W_{i,j;t}}}_{\text{posterior pdf of } c_t \text{ and } c_{t-1}} &\propto \int_{\Theta^*}\int_{\beta^*}\int_{\alpha^*}\underbrace{f(y_t,z_t=l,\Theta,\beta,c_t=i,\alpha,c_{t-1}=j|D(t-1))}_{(6)}d\Theta d\beta d\alpha\\ &= \int_{\Theta^*}f(y_t|\Theta,c_t=i,\psi_t)f(\Theta|D(t-1))d\Theta\int_{\beta^*}f(z_t=l|\beta,c_t=i,z_{t-1}=q)f(\beta|D(t-1))d\beta\\ &\times\int_{\alpha^*}f(c_t=i|\alpha,c_{t-1}=j,\varphi_{t-1}=k)f(\alpha|D(t-1))d\alpha\, f(c_{t-1}=j|D(t-1)).\end{aligned} \tag{7}$$

Here the first integral is approximated by substituting the current measurement y_t and the previous point estimates of the parameters denoted by $\hat{\theta}_{i;t-1}$ and $\hat{r}_{i;t-1}$ to each i-th component, $\forall\, i \in \{1,\ldots,m_c\}$. It provides a proximity of the current data y_t to the i-th component. The point estimates of parameters are obtained using the conjugate prior GiW distribution for each normal component in the Bayes rule, which leads to recursive updating the initially chosen statistics denoted by $V_{i;t-1}$ and $k_{i;t-1}$ of appropriate dimensions according to Peterka (1981) and Kárný et al. (1998).

Similarly, the second integral represents the probability of the current measurement z_t conditioned by z_{t-1} taken from the table with the previous-time point estimate denoted here by $\hat{\beta}_{i;t-1}$, for each i-th component. The point estimates of the parameters of each i-th categorical component are obtained via the Bayes rule using the conjugate prior Dirichlet pdf according to Kárný et al. (2006) with the recomputable statistics denoted by $\vartheta_{i;t-1}$.

The third integral is a computation of the point estimate of the parameter α using the previous-time statistics (here denoted by $\gamma_{k;t-1}$) of the pointer model for the actual value $\varphi_{t-1} = k$.

The prior pointer pdf $f(c_{t-1} = j|D(t-1))$ expresses the probability of each component activity at the previous time instant $t-1$. Initially, it is chosen and then it is updated into the posterior pdf of the current pointer c_t by the marginalization of the result (7) over values of c_{t-1}

$$\underbrace{f(c_t=i|D(t))}_{\text{posterior pointer pdf}} \propto \overbrace{\sum_{j=1}^{m_c}\underbrace{f(c_t=i,c_{t-1}=j|D(t))}_{(7)}}^{\text{denoted by } w_{i;t}}, \tag{8}$$

which is the updated probability of the i-th component activity with respect to the current data y_t and z_t, and it is the i-th entry of the m_c-dimensional weighting vector w_t. The index of the maximum entry of the vector w_t denotes the point estimate of the pointer c_t, which indicates the component declared to be active at time t.

This scheme leads to the recursive update of statistics $V_{i;t-1}$ and $\kappa_{i;t-1}$ for the estimation of parameters Θ of normal components (Peterka, 1981; Kárný et al., 1998), i.e., $\forall\, i \in \{1,\ldots,m_c\}$

$$V_{i;t} = V_{i;t-1} + w_{i;t}\begin{bmatrix} y_t \\ \psi_t\end{bmatrix}\begin{bmatrix} y_t \\ \psi_t\end{bmatrix}', \tag{9}$$

$$\kappa_{i;t} = \kappa_{i;t-1} + w_{i;t}. \tag{10}$$

Using (9) and (10), the point estimates $\hat{\theta}_{i;t}$ and $\hat{r}_{i;t}$ are recomputed (Peterka, 1981) as follows:

$$\hat{\theta}_{i;t} = V_1^{-1}V_y, \quad \hat{r}_{i;t} = \frac{V_{yy}-V_y'V_1^{-1}V_y}{\kappa_{i;t}} \tag{11}$$

with the help of partition

$$V_{i;t} = \begin{bmatrix} V_{yy} & V_y' \\ V_y & V_1\end{bmatrix}, \tag{12}$$

where V_{yy}, V_y' and V_1 are submatrices of appropriate dimensions in dependence on the dimension of the vector y_t.

According to Kárný et al. (2006), the statistics $\vartheta_{i;t-1}$ of categorical components is updated $\forall\, i \in \{1,\ldots,m_c\}$ and $\forall\, l,q \in \{1,\ldots,m_z\}$

$$(\vartheta_{l|q})_{i;t} = (\vartheta_{l|q})_{i;t-1} + \delta(l,q;z_t,z_{t-1})w_{i;t}, \tag{13}$$

where the Kronecker delta function $\delta(l,q;z_t,z_{t-1}) = 1$, when $z_t = l$ and $z_{t-1} = q$ and it is equal to 0 otherwise. The point estimate $(\hat{\beta}_{l|q})_{i;t}$ is computed for each categorical part of the i-th component according to Kárný et al. (2006) as follows:

$$(\hat{\beta}_{l|q})_{i;t} = \frac{(\vartheta_{l|q})_{i;t}}{\sum_{s=1}^{m_c}(\vartheta_{s|q})_{i;t}}. \tag{14}$$

The update of the pointer statistics $\gamma_{k;t}$ is performed in the following way. In (7) the pdf $f(c_t = i,c_{t-1} = j|D(t))$ denoted by $W_{i,j;t}$ is joint for both pointers c_t and c_{t-1}. This joint pdf is used in the update of the pointer model statistics

$$(\gamma_{i|j})_{k;t} = (\gamma_{i|j})_{k;t-1} + \delta(k;\varphi_{t-1})W_{i,j;t}, \tag{15}$$

where the Kronecker delta function $\delta(k;\varphi_{t-1})$ is defined similarly as in (13). This form of the update was proposed in Kárný et al. (1998) for the static pointer model. In Nagy et al. (2011) the solution was proposed for the dynamic pointer but with the help of approximation based on the Kerridge inaccuracy (Kerridge, 1961). Here it is used with discrete (discretized) data in the condition, using for simplicity the approximation similarly to Kárný et al. (1998). The update is performed only for a currently measured data item.

The point estimate of the parameter α at the time instant t is obtained similarly to (14) with the help of the normalization for the actual value $k \in \{1,2,...,m_\psi\}$, i.e.,

$$(\hat{\alpha}_{i|j})_{k;t} = \frac{(\gamma_{i|j})_{k;t}}{(\sum_{s=1}^{m_c} \gamma_{s|j})_{k;t}}. \tag{16}$$

The derivations are now summarized in the form of the following algorithm.

3.3. Algorithm

Initialization (for t = 1)

- Specify the number of components m_c.
- For all components, set the initial statistics $V_{i;t}$,$\varkappa_{i;t}$ and $\vartheta_{i;t}$.
- For all values $k \in \{1,2,...,m_\psi\}$ set the initial statistics $\gamma_{k;t}$.
- Using these initial statistics, compute the initial point estimates $\hat{\theta}_{i;t}$,$\hat{r}_{i;t}$,$\hat{\beta}_{i;t}$ and $\hat{\alpha}_{k;t}$ of all parameters and for all components.
- Set the initial weighting vector w_t.

Online part (for t = 2,3,…)

1. Measure the new data y_t and z_t.
2. For each component, substitute y_t and the previous point estimates $\hat{\theta}_{i;t-1}$ and $\hat{r}_{i;t-1}$ in a corresponding pdf (3). It gives the proximity of a component to a data item. Construct the m_c-dimensional vector from the obtained proximities of all components.
3. Similarly, for each component, take the probability $(\hat{\beta}_{l|q})_{i;t-1}$ for the current values $z_t = l$ and $z_{t-1} = q$. Construct the m_c-dimensional vector from results from all components.
4. According to (7), multiply entry-wise the resulted vectors from the two previous steps, the prior weighting vector w_{t-1} and the point estimate matrix $\hat{\alpha}_{k;t-1}$ for the actual k.
5. The result of this entry-wise multiplication is the matrix with entries $W_{i,j;t}$. Normalize this matrix.
6. Perform the summation of the above normalized matrix over rows and obtain the vector with updated entries $w_{i;t}$ according to (8).
7. Classify the data according to the currently active component given by the index of the maximum entry of w_t.
8. Update all statistics according to (9)–(13) and (15).
9. Recompute the point estimates of all parameters according to (11), (14) and (16) and use them as the previous ones for Step 1 of the online part of the algorithm.

Remark 1. *Initialization is an important part of the algorithm and known as a critical task in the field of the mixture estimation. Firstly, it is concerned with the determination of the number of components. In the considered context, the initialization is solved under the assumption that some type of prior transportation data is available (previously measurements, data from realistic simulators, etc.).*

1. *In this case, one of the possibilities to determine the number of components is to apply the expert-based procedure of the visual analysis (*Suzdaleva et al., 2016*).*
2. *The application of one of the well-known clustering algorithms, e.g., k-means (*Jain, 2010*) for prior data can be also beneficial.*
3. *Expert knowledge about the expected number of components in the discussed field is suitable too.*
4. *A choice of the initial statistics of normal components influences the start of the estimation. Again, prior data can be utilized for this aim using procedures described by* Kárný et al. (2003), Suzdaleva et al. (2016)*.*
5. *The rest of the statistics and the initial weighting vector can be initialized either uniformly or randomly in combination with their updating by prior data.*

4. Online driving style estimation

In this section, the above general algorithm is applied to the problem formulated in Section 2. Here, the active component stands for the active driving style and it is searched online while driving.

The approach was validated with the help of experiments with the data introduced in Section 2. The aim of the experiments was to show the following key features of the approach:

- The driving style can be recognized from the measured data at each time instant while driving. It means that the data in the model accumulated before are updated by the actual measurements and the driving style estimate is re-computed online. This is done by

means of detection of clusters in the data space.
- Fuel consumption, vehicle speed, gas pedal position and gear can be modeled jointly despite their completely different nature (continuous and discrete).
- The model (5), which describes the switching of the driving styles, can benefit from using the measured data in the condition.
- With the recognized driving style, the values of fuel consumption, vehicle speed, gas pedal position and gear can be predicted.
- Due to generality of the mixture modeling, the initialization of the estimation can be tailored to the driving style analyzed from a fuel consumption point of view. It means that the prior data of fuel consumption can be used for this aim.

Implementation of the algorithm was prepared in Scilab 5.5.2 (www.scilab.org) which is known as a powerful programming free and open source environment for engineering applications. All codes are editable and adjustable for other applications of Algorithm 3.3 as well.

4.1. Data

The data was collected on a route which led through all important surroundings: a highway, outside the city and in the city. The way outside the city passed through several villages. Several drivers repeatedly drove along the route and each driver was instructed to drive some routes very carefully, then in a normal way and also in a sport manner.

For the experiments, 12 data sets were taken. Each data set contained 1500 measurements of fuel consumption, vehicle speed, gas pedal position and the gear. Their values were measured by seconds, which means that the average trip was 25 min.

4.2. Driving style model specification

The mixture model introduced in Section 3.1 was taken in the following form.

The pointer model (5) was specified as follows:

$$f(\textit{driving style}|\textit{previous driving style}, \textit{discretized instantaneous fuel consumption}), \tag{17}$$

where discretized values of the instantaneous fuel consumption were used as the past data in the condition. Three possible values were obtained via intervals 1, 500, 800, 1300 [μl/s] observed in the data.

Normal models (3) had the form

$$f(\textit{fuel consumption}, \textit{speed}, \textit{gas pedal position}|\textit{driving style}, \textit{gear}). \tag{18}$$

Categorical models (4) were taken in the form

$$f(\textit{gear}|\textit{driving style}, \textit{previous gear}). \tag{19}$$

4.3. Data preprocessing

The values of the gear variable have been preprocessed as follows. For programming reasons, the value 0, which corresponds to the neutral gear was denoted by 7. The reverse gear −1 was denoted by a value of 8.

4.4. Initialization

The data-dependent model (17) enables us to use data from the condition for the initialization purposes. 200 data items of the instantaneous fuel consumption discretized according to the intervals above were taken for this aim. Firstly, the initial statistics were set as small-valued diagonal matrices for normal models (18) and randomly for categorical models (19) as well as for (17). The initial weighting vector was set uniformly, which means that all of the driving styles had the same probability of activity in the beginning of the estimation. Then, the online part of Algorithm 3.3 was running for these 200 prior data items only, where three values of the discretized fuel consumption were applied as the known driving styles to update the initial statistics (Kárný et al., 2003; Kárný et al., 2006). This was done to imitate the parameter estimation with the known active driving style that allowed us to accumulate the statistics for the successful start of the online estimation. Finally, the statistics updated by prior data were taken as initial for the main online part of Algorithm 3.3.

To determine the number of driving styles more precisely, it was necessary to use a combination of the expert-based procedure of the visual analysis of prior data according to Suzdaleva et al. (2016) and *k*-means clustering (Jain, 2010). Visualization of the prior data is given in Fig. 1 (top), which is constructed of three plots, where the variables from the modeled data vector y_t are plotted against each other. The prior data set was measured on a highway that explains the speed from 110 to 155 [km/h]. The first two top figures show values of the fuel consumption plotted against the values of the speed and the gas pedal position. The top figure in third position plots the values of vehicle speed against the gas pedal position. The aim of this procedure is to distinguish visually the locations of the clusters, which are formed by prior data and determine the number of the driving styles.

The nature of the visualized data is complicated. Clusters can be guessed in the top three figures. However, for this prior data sample it is difficult to determine its number. That is why the *k*-means method known as a successful classifier was also applied for clustering the prior data. The results of the *k*-means application were 7 detected clusters created by the variables among each other,

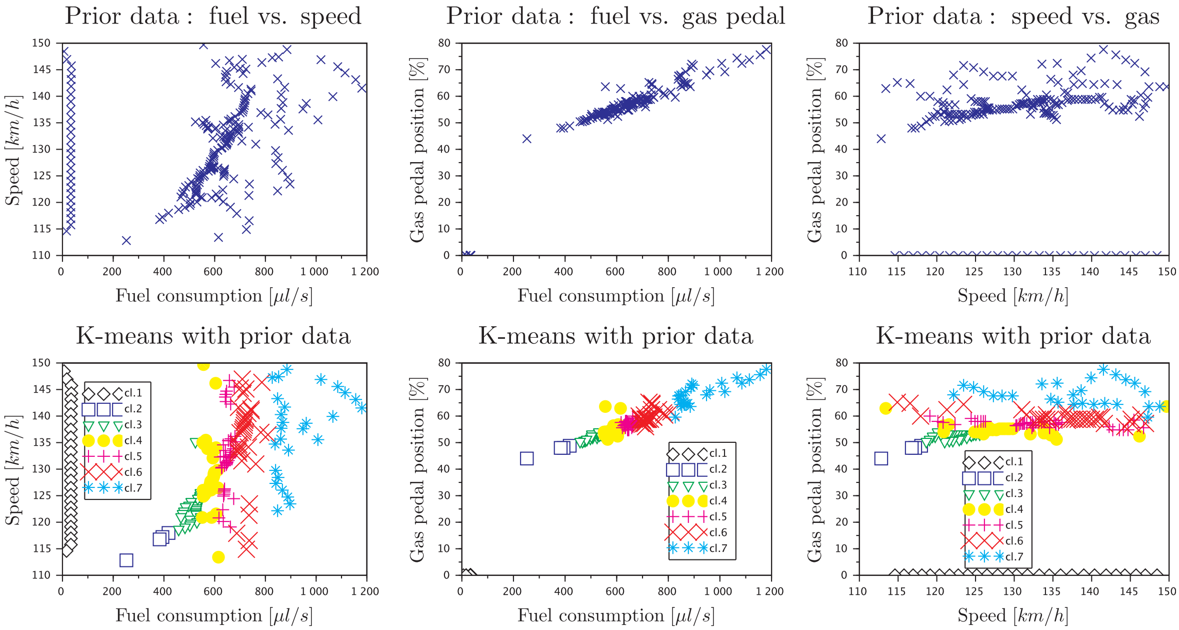

Fig. 1. 200 prior data items plotted against each other for initialization purposes (top) and 7 clusters detected by *k*-means in 200 prior data (bottom).

see Fig. 1 (bottom). Each plot in Fig. 1 (bottom) shows the clusters detected in the corresponding plot in Fig. 1 (top), i.e., right above it. Even in this small set of data, which was from the highway but from different drivers, 7 clusters were found among all of the data pairs. This means that the number of driving styles can be initialized as 7. The most illustrative plot in Fig. 1 (bottom) is that with the fuel consumption and the gas pedal position, where one of the driving styles corresponds to very low fuel consumption and almost zero gas pedal pressing (cluster 1 denoted by ◇). This driving style was also found in the first plot in Fig. 1 (bottom) with the very low fuel consumption and highway speed. Another driving style corresponds to the fuel consumption values around 300 [μl/s], a speed about 115 [km/h] and a gas pedal position from 40% to 50% (cluster 2 denoted by □). The rest of the initialized driving styles is shown as clusters 2–7 in the plots. In the third plot in Fig. 1 (bottom), the driving styles are partially overlapped, which is explained by their mixed nature in real data.

The initialization from this prior data set was used for the online estimation with all of the mentioned 12 tested data sets.

4.5. Results

The online part of Algorithm 3.3 implies that the driving style estimate is actualized each second all the time when the variables are measured. Results of the application of the online estimation were evaluated according to the following criteria:

- The driving style is not measurable and its estimates cannot be compared with its real values. However, the effectiveness of its estimation can be determined via the clustering of fuel consumption, speed and the gas pedal position according to the detected driving styles. To verify the reasonability of the obtained clustering, it can be compared with well-known successful clustering methods, which do not model the driving style but they look for the data groups in the measurements. A good choice in this case is the *k*-means (Jain, 2010) and the fuzzy *c*-means (Pal et al., 2005; Ghosh and Dubey, 2013) methods available in the Scilab toolboxes CLUSTER (http://atoms.scilab.org/toolboxes/CLUSTER/3.2) and NaN-toolbox (http://atoms.scilab.org/toolboxes/nan/1.3.4). These methods search for the clusters, each with the help of a different approach but both of them work with the data set at once. It means the data must be measured before the clustering starts, i.e., the clusters are obtained after driving. This is their main difference from Algorithm 3.3, which enables us to accumulate the data and actualize the clusters while driving. The aim of this part of the experiments was to compare whether the *k*-means and fuzzy *c*-means methods give clusters with a similar shape (Reilly et al., 2005) and location.
- The point estimates of parameters were substituted into normal models to obtain predictions of fuel consumption, speed as well as the gas pedal position and into categorical models to predict the gear shifts. Here, the aim was to compare the predicted values and the real measurements.
- The evolution of the activity of driving styles was monitored during the online part of the algorithm. If some driving style is active rarely or even not active at all, it means that the number of driving styles is unnecessarily high. The regular activity of all driving styles testifies to the correct choice of their number.

A series of experiments with the 12 data sets that were mentioned were performed. Graphical results were similar to each other which is why the results of one of the 12 tested data sets are presented. The quantitative evaluation of the estimation quality is given

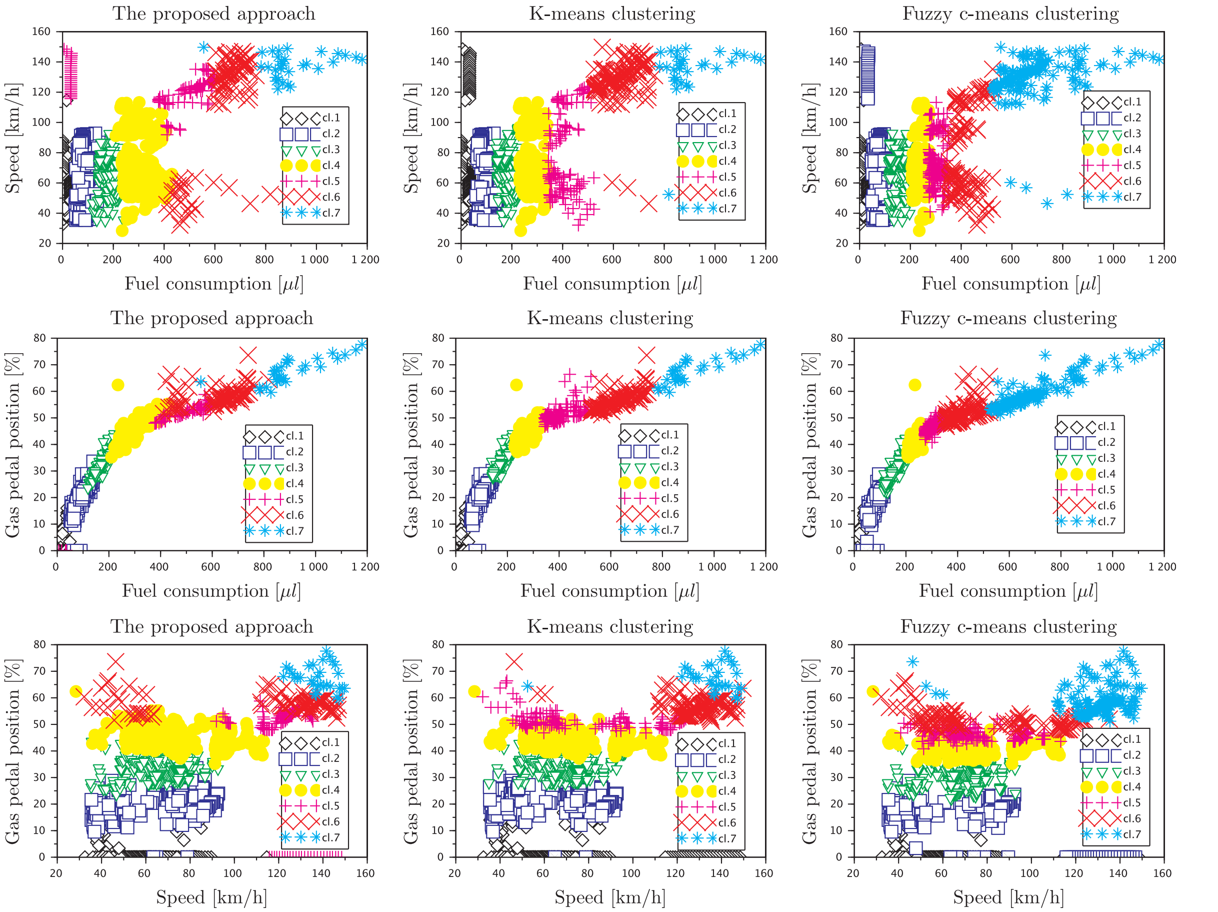

Fig. 2. The comparison of two-dimensional clusters of the fuel consumption, the speed and the gas pedal position obtained by the proposed algorithm, *k*-means and fuzzy *c*-means.

by means of the average result for all of the data sets.

4.5.1. Detected driving styles

Fig. 2 verifies 7 clusters obtained according to the driving style estimates (i.e., while driving) by their comparison with *k*-means and fuzzy *c*-means clustering (after driving). The upper row of plots in Fig. 2 presents the two-dimensional clusters created by the fuel consumption and the speed obtained by these three methods respectively. It can be seen that the shape and the locations of the clusters of all of the methods are very close to each other. The difference is in the detection of the cluster related to the driving style with low fuel consumption of about 40 [μl/s] and a highway speed from 115 to 150 [km/h]. In the online estimation (see the first plot), this driving style was detected as cluster 5 denoted by '+'. The *k*-means method detected it as cluster 1 denoted by '◇', which contains also the values of speed in the city and outside the city, i.e., from 30 to 90 [km/h], see the second plot. The fuzzy *c*-means method found this driving style as part of cluster 2 denoted by '□'. This cluster corresponds to a fuel consumption of around 100 [μl/s] but the fuzzy *c*-means method connected it with the highway speed as well.

The driving style recognized as cluster 6 denoted by '×' also differs among the compared methods. The online estimation related it mostly to highway speed and partially to the city speed. The fuzzy *c*-means detected it through all the speed values. The *k*-means found cluster 6 primarily as the highway speed driving style.

The identical driving styles detected in the two-dimensional space of the pair of fuel consumption and the gas pedal position are compared in the middle row of Fig. 2. These plots are probably the most illustrative ones. The shape and the locations of the clusters can be clearly seen and they are similar among all of the compared methods with the mentioned difference for clusters 1 and 6.

The bottom row of Fig. 2 presents the clusters detected for the speed and gas pedal position, where again the shape and the location of the clusters are similar for all of the methods with a difference in clusters 1 and 6 while there was an insignificant difference in data items assigned to cluster 7 denoted by '☆'.

The mentioned inconsistencies in the results can be explained by a different approach to finding clusters of each of the algorithms used. To evaluate whether the differences in the clusters are significant or not, the results were analyzed with the help of statistical tests. As noticed in Reilly et al. (2005), the clustering results can be compared using, e.g., the statistics based on the Cohen's kappa coefficient (Cohen, 1960). However, Reilly et al. (2005) found that this statistics was not advantageous for the model-based methods

Table 1
The *p*-values of the Kruskal-Wallis tests.

	The cluster centers	The numbers of data
The online estimation vs. *k*-means	0.715	0.798
The online estimation vs. fuzzy *c*-means	0.949	0.949
k-means vs. fuzzy *c*-means	0.772	0.949
The online estimation, *k*-means, fuzzy *c*-means	0.929	0.929

seeking for clusters of arbitrary shapes as Algorithm 3.3 does. The *k*-means method also shown the sensitivity to the initial values while testing (Reilly et al., 2005). That is why the centers of the detected clusters along with the numbers of data in the clusters were chosen for the comparison. The Kruskal-Wallis test (Kruskal and Wallis, 1952) was used to test whether the results of the three methods originate from the same distribution. For the 12 tested data sets with 1500 measurements, the *p*-values of the pairwise tests as well as comparisons of the three methods are given in Table 1. This table shows that the *p*-values in all of the tests are higher than the significance level of 0.05, which means that the differences among the results are not statistically significant.

In this way, it can be seen that the driving styles detected online while driving were verified by (i) two well-known methods performing the clustering after driving, which means that the evaluation of the driving styles was done after the vehicle had stopped, (ii) the shapes and the locations of clusters in all of the used data pairs and (iii) the quality of results among the 12 data sets.

The computational time of clustering was calculated by the Scilab functions `tic` and `toc` in seconds. For the 12 tested data sets with 1500 measurements, the average computational time (CT) with its standard deviation is given in Table 2 for all of the methods. This table indicates that the online estimation of the driving style while driving took on average 0.042 s with a very small standard deviation of 0.006, which confirms the fixed computational time. The *k*-means clustering, which evaluated data after driving, had a insignificantly longer computational time but with a greater standard deviation of 0.021. It means the time changed during the computations in dependence on convergence of the algorithm, which is typical for iterative methods. The fuzzy *c*-means methods searched for the clusters on average for the longest time and it was about 0.42 [s] for all of the data sets, which can be seen from the small standard deviation.

4.5.2. Data prediction quality

Fig. 3 compares the predicted expectations of fuel consumption, the speed as well as the gas pedal position and their real values. Here, again the results for one of the data sets are demonstrated. The expectations of the fuel consumption and gas pedal position are in good correspondence with real values. The results of the speed prediction are a bit worse. For each tested data set, a root-mean-square error (RMSE) was computed

$$\mathrm{RMSE} = \sqrt{\frac{1}{T}\sum_{t=1}^{T}(y_{n;t}-\hat{y}_{n;t})^2}, \tag{20}$$

where $n = 1,2,3$ denotes the index of the variable; $\hat{y}_{n;t}$ is the corresponding prediction; and $T = 1500$. The average RMSE among the 12 data sets is given in Table 3, which shows that the gas pedal position has the lowest average error and the fuel consumption – the highest one. Table 3 also presents the average percentage of incorrect predictions (IP) of the gear selection among the 12 data sets. Graphical results of the gear prediction are also demonstrated in Fig. 4. Gear shifts from the value 6 to 7 (which is neutral) and backwards are explained by the use of the automatic gearbox.

4.5.3. Switching the driving styles

The evolution of weights and switching the driving styles (while driving) is worth observing during the online estimation in order to see whether the pointer indicates the active driving style unambiguously. This is expressed in probabilities of the activity near the values 0 and 1. In the case of 7 driving styles, the graphical representation of the weights is poorly visible. However, the pointer values obtained as indices of maximum entries of the weighting vector are demonstrated in Fig. 5. They show switching the driving styles while driving. The regular activity of all of the detected driving styles is reported, which confirms that the considered number of driving styles is close to reality and the model was chosen correctly.

4.6. Discussion

The main aim of this study was to demonstrate the driving style estimation from a fuel economy point of view online while driving with the help of Bayesian recursive algorithms for mixture estimation (Kárný et al., 1998; Kárný et al., 2006; Nagy et al., 2011).

Table 2
A comparison of the average computational time and its standard deviation.

	The online estimation	*k*-means	Fuzzy *c*-means
Average CT, [s]	0.042	0.051	0.42
Standard deviation	0.006	0.021	0.005

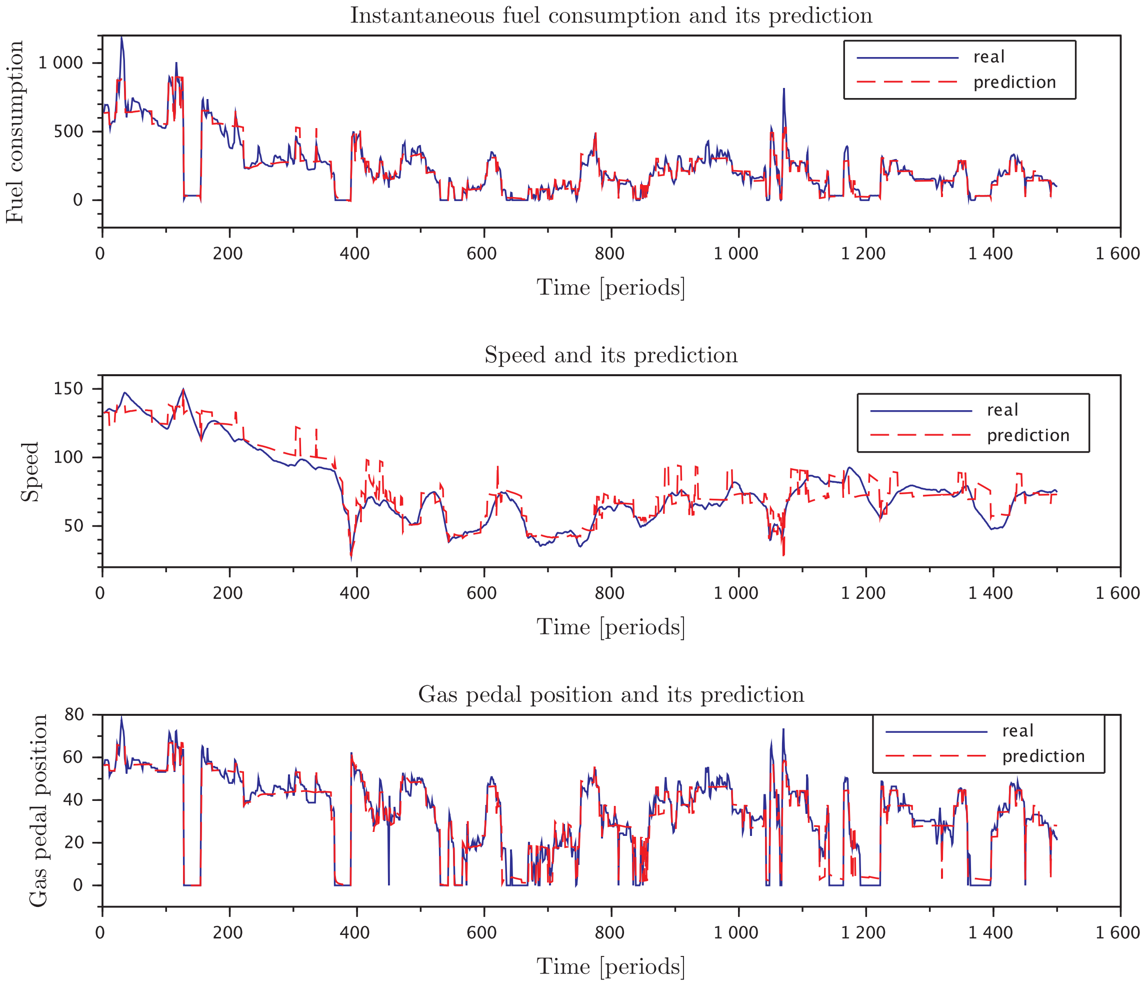

Fig. 3. The comparison of the prediction of fuel consumption, speed and gas pedal position with their real values.

Table 3
Average RMSE and IP.

	RMSE	IP, %
Fuel consumption	1.09	
Vehicle speed	0.24	
Gas pedal position	0.13	
Gear selection		2.71

Previous studies dealt with this problem via Bayesian methods (Mudgal et al., 2014; Wang et al., 2016) but the recursiveness was not used for the online estimation. Moreover, the majority of Bayesian algorithms for mixture estimation is based on the iterative expectation-maximization (EM) algorithm (Gupta and Chen, 2011), where the computational time depends on convergence of the algorithm.

As stated in Section 4.5, this aim has been accomplished and it is one of the main contributions of the study. Advantages of the online estimation enable us to recognize the driving style (i) while driving and (ii) with a fixed computational time. Driving style recognition is demonstrated by means of clustering the data. The cluster analysis was applied in Guo and Fang (2013) as well. However, the analysis was performed after driving. In contrast, here it was done while driving with regular updating from the new data. As it was noted by Li et al. (2017), data mining techniques could be utilized to examine existing databases of driving data. Here, they are used but the data sets serve to imitate the situation of real driving.

Seven driving styles were recognized during the online estimation and validated by the other classifiers. In the beginning of this research, a smaller number of driving styles were assumed to be in accordance with other studies (e.g., two driving styles in Vaitkus et al. (2014), three driving styles in Guo and Fang (2013), Pampel et al. (2015), etc.). However, experiments from Section 4.5 performed for a smaller number of driving styles produced the clusters of the shape and the location, which were not verified by other

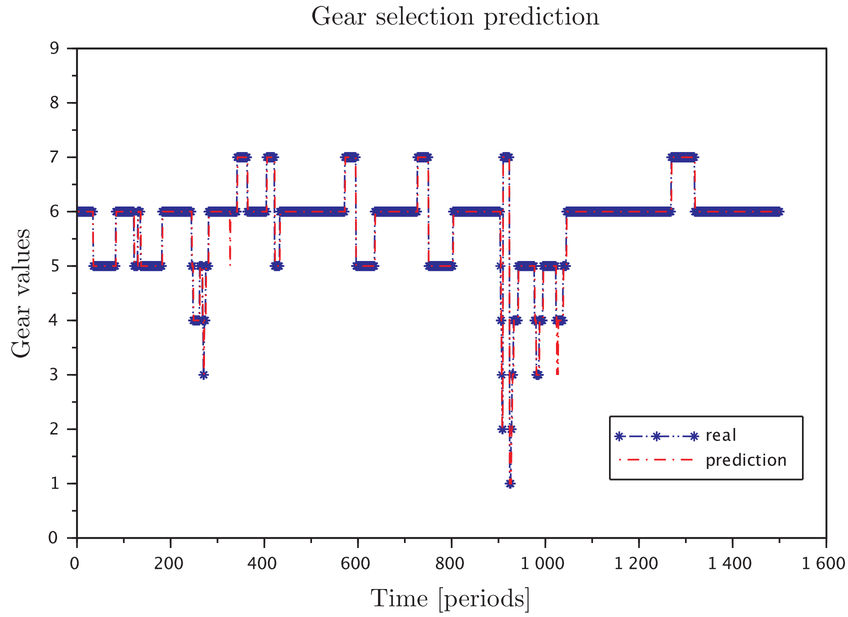

Fig. 4. The comparison of the gear shift prediction with real data.

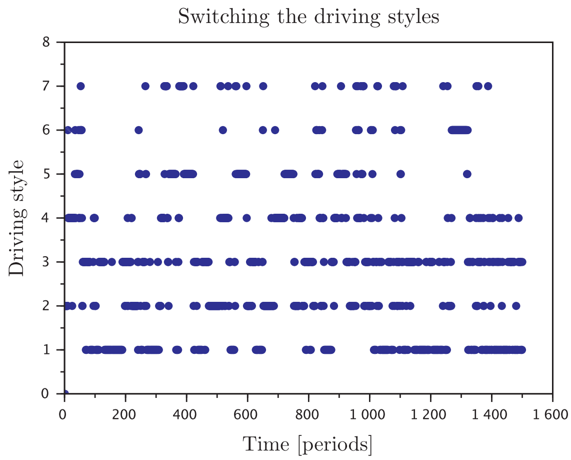

Fig. 5. Driving style switching as the pointer values during the online estimation.

classifiers and the data prediction quality was also bad (this is not shown here to simply save space). That is why the initialization from Section 4.4 was applied, which led to seven driving styles being initialized. It means that with the data collection used, the driving styles should be classified into seven groups. Such number of driving styles was also classified by Wang et al. (2016).

Fuel consumption, vehicle speed, gas pedal position and gear were modeled jointly in the mixture model. With the estimated driving styles, their prediction was successful. Comparing the obtained prediction with the previous study (Suzdaleva and Nagy, 2014), it should be noted that certain drawbacks in the prediction quality obviously caused by the nonlinearity in relationship of some variables here were compensated by the use of the mixture model. The nature of the data collected can be best seen in the plot with vehicle speed prediction, see Fig. 3, where the speed changes among the highway, outside the city and in the city. In general, the proposed approach can be used for the driving style estimation in all kinds of traffic situations, as it is the data-based method and depends on which data was measured. However, the driving style corresponding to the highway speed in the upper row of Fig. 2 (cluster 5 denoted by '+') was detected incorrectly, i.e., the online estimation was more successful for the traffic outside the city and in the city.

In addition, regarding the remaining contributions of the study, it should be stated that (i) the data-dependent pointer model enabled to facilitate the initialization of the algorithm and (ii) the mixture model, due to its generality, could have been comprised from the pdfs as it was most suitable for the considered practical problem.

A practical application of the online driving style estimation is seen primarily in driver assistance systems, starting with drivers being informed about the economic efficiency of their driving style and ending with in-vehicle information systems, depending on the degree of integration of the method into vehicle control systems.

The limitation of the approach is the complicated procedure of initialization. A number of driving styles should be initialized before the online estimation and its incorrect choice will lead to a failure of the estimation. However, the discussed initialization procedure should always be performed under the assumption of available prior data, which is not a problem in the transportation domain. Worst case scenario, an expert can advise how many driving styles would be expected. An advantage is that a small number of prior data is necessary for starting the algorithm.

5. Conclusion

The paper is devoted to the online detection of a driving style based on the recursive Bayesian estimation of a mixture of normal and categorical components. Seven driving styles related to fuel economy were recognized with help of the online estimation algorithm, which assumes that the driving style is being recognized while driving. The algorithm was also used for modeling and predicting of fuel consumption, speed, gas pedal position and gear selection. The generality of mixture modeling also enabled us to use the pointer conditioned by the fuel consumption and the specific initialization based on the preliminary analysis of prior data, which is crucial for the successful application of the online algorithm while driving.

A significant contribution of the discussed data-based algorithm is expected from the universal approach to modeled data. For instance, as reported in Mensing et al. (2014), a bound between an economic and ecological style of driving is not sufficiently discussed in the available literature. The presented data-based algorithm seems to be perspective in this sense, as it is limited by none of them and a success of the estimation depends on the data informativeness. It means that using other measurements, different model for the driving style detection can be constructed. The algorithm is limited neither by the presented specific choice of mixed components nor by the application domain. Other pdfs with reproducible statistics as well as another application area can be selected. Here, the normality was assumed for continuous modeled variables, which can also be one of the limitations of the study. A description of data by means of non-gaussian models is the task extensively solved within the presented research project. In addition, a potential extension can be seen in the multi-step prediction of the driving style, which could be based on the evolution of weights.

Driving style modeling related to the minimization of fuel consumption and compliance with emission standards is an issue which is important both for automobile manufactures and drivers. Research in this field is still highly desired and calls for novel solutions, despite the growing popularity and affordability of hybrid as well as electric vehicles. Investigations in this area often focus on optimal control design as one of the main problems. Modeling the driving style can significantly influence the construction of the optimal eco-driving strategy.

Acknowledgements

The research was supported by the project GAČR GA15-03564S.

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