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RESEARCH ARTICLE

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Panel quantile regressions for estimating and predicting the value-at-risk of commodities

František Čech[1,2] | **Jozef Baruník**[1,2]

[1]Department of Macroeconomics and Econometrics, Institute of Economic Studies, Charles University, Prague, Czech Republic

[2]Department of Econometrics, Institute of Information Theory and Automation, Academy of Sciences of the Czech Republic, Prague, Czech Republic

Correspondence
Jozef Baruník, Institute of Economic Studies, Charles University, Opletalova 26, 110 00 Prague, Czech Republic.
Email: barunik@utia.cas.cz

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Abstract
Using a flexible panel quantile regression framework, we show how the future conditional quantiles of commodities returns depend on both ex post and ex ante uncertainty. Empirical analysis of the most liquid commodities covering main sectors, including energy, food, agriculture, and precious and industrial metals, reveal several important stylized facts. We document common patterns of the dependence between future quantile returns and ex post as well as ex ante volatilities. We further show that the conditional returns distribution is platykurtic. The approach can serve as a useful risk management tool for investors interested in commodity futures contracts.

KEYWORDS
implied volatility, panel quantile regression, realized volatility, value-at-risk

JEL CLASSIFICATION
C14, G17, G32, Q41

1 | INTRODUCTION

Commodities play an increasingly significant role in the asset allocations of institutional investors and, with the onset of exchange-traded funds, have become a regular asset class. Academic debate spurred by the developments has provided valuable insights into the economics of commodity markets, as well as several crucial aspects, such as price forecasting, risk measurement, or hedging. One of the main challenges faced by researchers is the fact that commodities are nonhomogeneous assets, and their risk, as well as return, may differ substantially, as each commodity is driven by specific supply and demand forces. A traditional economist's view on an asset price being a stream of future discounted expected cash flows is hence not directly applicable, and pricing of commodities is instead driven by short-term variations in the supply. In addition, exogenous factors, such as weather conditions, inventory levels, storage costs, production shocks, and even geopolitical events, play a crucial role, rendering risk measurement a difficult task. In this paper, we propose a simple, robust framework that can be used to model and forecast the value-at-risk (VaR) of commodities semiparametrically without the need for traditional assumptions. Empirical results support our approach, and we uncover stylized facts useful for investors and policy-makers.

The complex nature of commodity pricing results in risk characteristics that are different from those of financial assets such as stocks, bond, and currencies. Returns distributions—as measured by volatility, skewness, kurtosis, and empirical quantiles—are different from traditional asset classes; hence, we need more flexible techniques to measure risk. Many researchers have tried to model the VaR of commodities without reaching a consensus about the appropriate model. The three main approaches used in the literature are useful but lack the ability to handle the complexity of commodity data. First, RiskMetrics (Longerstaey & Spencer, 1996) does not necessarily capture the correct returns distribution conditional on the changing volatility. Second, the historical simulations used by, for example, Cabedo and Moya (2003) have the opposite problem: They capture the empirical returns distribution but do not make it conditional on volatility. Third, more advanced parametric models mostly built within the family of generalized autoregressive conditional heteroskedasticity (GARCH) models improve fits (Aloui & Mabrouk, 2010; Chiu, Chuang, & Lai, 2010; Giot & Laurent, 2004; Hung, Lee, & Liu, 2008; Lux, Segnon, & Gupta, 2016; Youssef, Belkacem, & Mokni, 2015); however, they require fat-tailed distributions, long memory, and other features that lead to heavy parameterization, making the approach less tractable.

Since the seminal work of Koenker and Bassett (1978), quantile regression models have been increasingly used in many disciplines. Notable contributions in finance include that by Engle and Manganelli (2004), who were among the first to use quantile regression and develop the conditional autoregressive VaR (CAViaR) model. Important for our work, Žikeš and Baruník (2016) show that various realized and implied volatility measures are useful in forecasting quantiles of future returns without making assumptions about the underlying conditional distributions. The resulting semiparametric model well captures conditional quantiles of financial returns in a flexible framework. Moving the research focus toward the multivariate framework and concentrating on interrelations between quantiles of more assets, White, Kim, and Manganelli (2015) pioneer the extension. A different stream of multivariate quantile regression-based literature concentrates on the analysis using factors (Ando & Bai, 2017; Chen, Dolado, & Gonzalo, 2016), but the research is recent and awaits further development. Although the application of quantile regression to forecasting quantiles of various economic variables is not new in finance, quantile regression has rarely been applied in the context of commodities. Among the few applications, Li, Hurn, and Clements (2017) adapts quantile regression for forecasting day-ahead electricity load quantiles, and Reboredo and Ugolini (2016) studies the quantile dependence of oil price movements and stock returns.

In this context, the work by Žikeš and Baruník (2016) is important because it provides a link between future quantiles of the returns distribution and its past variation. Despite the nonhomogeneous nature of commodities, Christoffersen, Lunde, and Olesen (2019) uncover several stylized facts pointing to factor structure in volatility. Being interested in future quantiles of commodity returns distributions, it is tempting to ask whether there is a common structure in the quantiles of commodity returns. Inspired by our previous findings regarding financial markets (Čech & Baruník, 2017), we hypothesize that there might be useful commonalities to be uncovered. In the quantile regression setup, no similar study uncovers information captured in the panels of volatility series of commodity markets. Hence, our work can possibly open new questions in modeling the VaR of commodity markets.

This paper contributes to the literature by identifying common patterns in the dependence between future quantiles of commodity returns and ex post/ex ante volatility measures using a flexible panel quantile regression (PQR) approach. Our simple yet robust modeling strategy utilizes all the advantages offered by PQR and commodity datasets. We document interesting empirical regularities by controlling for otherwise-unobserved heterogeneity among commodities. In particular, we reveal common factors in volatility that have direct influences on the future quantiles of their returns. Our research is important since the current literature contains little information about the potential of the uncertainty factors in the precise identification of extreme-tail events of the commodity returns distribution. More important, even less is known about commonalities between more commodities in this respect. Our research attempts to contribute in this direction.

In the first part of our empirical application, we study the behavior of energy (crude oil and natural gas), precious metal (gold and silver), industrial metal (copper), agricultural (cotton), and food (corn) commodities during a period of the global financial crisis. We document common effects of the ex post uncertainty measured by realized volatility on the estimation of the VaR of commodities. We demonstrate these effects to be time-varying; however, they do not dramatically change when we compare the results from the nonoverlapping crisis and after-crisis periods. In contrast to our expectations, we document homogeneous behavior across commodities. Moreover, the conditional distribution of the returns standardized by their realized volatility is *platykurtic*, in contrast to previous parametric studies, in which a variety of GARCH models were used. To match the empirical data, GARCH models required a fat-tailed distribution (Charles & Darné, 2017; Cheong, 2009; Giot & Laurent, 2003; Marimoutou, Raggad, & Trabelsi, 2009) or combination with extreme value theory (Youssef et al., 2015). Since commodities are considered to be relatively less risky compared to financial assets (Bodie & Rosansky, 1980; Conover, Jensen, Johnson, & Mercer, 2010; Gorton & Rouwenhorst, 2006), our findings are in line with those of Andersen, Bollerslev, Diebold, and Labys (2000), who document that returns of financial assets standardized by their realized volatility are almost Gaussian. Our findings can be attributed to the flexibility offered by the framework we use. Our model does not require an assumption about the distribution and estimates the volatility nonparametrically.

In the second part, we employ option implied volatility as an ex ante measure of uncertainty and relate it to future returns quantiles. Volatility implied by option prices reveals the market's expectations and is often used as an ex ante measure of investor sentiment. Relatively recently, new indexes measuring the market's expectation of the 30-day volatility of commodity prices by applying the well-known Volatility Index (VIX) methodology have been introduced for more commodities. Still, the availability of the commodity option implied VIXs is limited; therefore, we concentrate on the main ones: crude oil, gold, and silver. We document patterns driving the VaRs of the selected commodities that are similar for both ex post and ex ante volatility measures. Moreover, once we control for ex post uncertainty, the ex ante volatility exhibits the greatest importance for the VaR estimation.

2 | THEORETICAL BACKGROUND

2.1 | VaR and modeling quantiles of returns

VaR, introduced by J. P. Morgan in 1994, quickly became an industry standard for risk measurement in finance and still attracts great attention from researchers. The popularity of VaR stems from its simplicity, as it represents the maximum potential loss of a portfolio at a given probability as a single number. According to Longerstaey and Spencer (1996), VaR can be parametrically calculated as

$$VaR_\tau = \gamma_\tau \sigma, \tag{1}$$

where γ_τ is the τ quantile of the standard normal distribution and σ is the volatility of the asset.

The growing financialization of commodity markets[1] motivates researchers to apply standard time-series techniques, well established in the financial industry, to study the riskiness of commodities. Many researchers, therefore, study VaR using parametric approaches, in which the volatility in Equation (1) comes from a variety of GARCH models. Recent attempts include Youssef et al. (2015), who combine long-memory GARCH models with extreme value theory; Lux et al. (2016), in which Markov-switching multifractal and various GARCH models are applied to model and forecast oil price volatility; Chkili, Hammoudeh, and Nguyen (2014), in which a wide range of linear and nonlinear GARCH models are used to study the VaR of energy and precious metal commodities; and Giot and Laurent (2003), in which it is shown that skewed Student APARCH works best in forecasting VaR in commodity markets.

The more general definition of VaR presented in Jorion (2007) suggests thinking about VaR as being the quantile of the projected distribution of returns over a certain period of time. In this spirit, Žikeš and Baruník (2016) show that VaR estimation can be formulated as a quantile regression of returns on their ex post and ex ante volatility without making any distributional assumptions. Commonalities in the panels of realized volatilities (Bollerslev, Hood, Huss, & Pedersen, 2018) motivate Čech and Baruník (2017) to extend the previous work of Žikeš and Baruník (2016) into multivariate space and study common factors in volatility that have a direct influence on the future quantiles of returns. In particular, they propose to control for otherwise-unobserved heterogeneity among financial assets and measure common market risk factors using a PQR approach.

An important link between commodities and stocks is demonstrated in the work of Christoffersen et al. (2019), who find a strong relationship between commodity and stock market volatility. In our work, we hypothesize that commodities share similarities with the stock market in terms of the behavior of the conditional returns distributions. To identify common patterns driving the VaR of commodities, we adopt the PQR model for returns (Čech & Baruník, 2017). In particular, we study a quantile pricing equation of the following form:

$$Q_{r_{i,t+1}}(\tau | X_{i,t}) = \alpha_i(\tau) + X_{i,t}^{\top}\beta(\tau), \tag{2}$$

where $\tau \in (0, 1)$, $r_{i,t+1} = p_{i,t+1} - p_{i,t}$ are logarithmic daily returns, $X_{i,t}$ is a matrix of ex post and/or ex ante volatility measures, and α_i represents individual fixed effects.

The model defined in Equation (2) allows us to study the influence of the ex post/ex ante uncertainty on the specific quantiles of the commodity returns through the β coefficients, which are common to all commodities and account for unobserved heterogeneity among assets represented by individual fixed effects, α_i. To obtain parameter estimates, we apply the fixed-effect estimator of Koenker (2004)[2] and solve the following:

[1]For a detailed overview, see Cheng and Xiong (2014).

[2]We refrain from using penalization as originally suggested by Koenker (2004)—our cross-sectional dimension is much smaller than the time dimension, so the number of estimated parameters is small.

$$\min_{\alpha_i(\tau),\beta(\tau)} \sum_{t=1}^{n} \sum_{i=1}^{t_i} \rho_\tau\left(r_{i,t+1} - \alpha_i(\tau) - X_{i,t}^{\top}\beta(\tau)\right), \tag{3}$$

where $\rho_\tau(u) = u(\tau - I(u(<0)))$ is the quantile loss function defined in Koenker and Bassett (1978).

2.2 | Measures of uncertainty

We distinguish between ex post and ex ante uncertainty about the commodity price. The former measures variation in the historical data, whereas the latter describes the market's sentiment and expectation of future risk. Although underlying commodity is the same in both cases, the information content of each measure might differ (Giot & Laurent, 2007).

Ex post uncertainty is represented by the realized volatility estimated at a 5-min frequency. The choice of uncertainty measure is motivated by the results of Liu, Patton, and Sheppard (2015), who show that it is difficult to beat the performance of the 5-min RV using more sophisticated realized measures. We construct the estimator as the square-root of the sum of the squared intraday returns (Andersen, Bollerslev, Diebold, & Labys, 2003):

$$\widehat{RV}_{i,t}^{1/2} = \sqrt{\sum_{k=1}^{N} (\Delta_k p_{i,t})^2}, \tag{4}$$

where $\sum_{k=1}^{N} (\Delta_k p_{i,t})^2$ is the realized variance estimator, with $\Delta_k p_{i,t} = p_{i,t-1+\nu_k/N} - p_{i,t-1+\nu_{k-1}/N}$ being a discretely sampled vector of the kth intraday log-returns of the ith commodity in $[t-1, t]$, with N intraday observations.

We are interested in the realized volatilities of the following commodities: crude oil (CL), corn (CN), cotton (CT), gold (GC), copper (HG), natural gas (NG), and silver (SV). In the calculation, we consider trades from the period May 10, 2007 until December 31, 2015 during regular trading hours. To ensure sufficient liquidity, we explicitly exclude public holidays, such as Christmas, Thanksgiving, or Independence Day. From the raw tick data, we extract 5-min prices using the last-tick method, and we calculate open–close returns. Additionally, to study the impact of the global financial crisis on the commodity market, we divide our sample into two nonoverlapping subsamples: crisis (May 10, 2007–September 9, 2011) and after-crisis (September 11, 2011–Decmber 31, 2015). Figure 1 shows the daily returns and their realized volatility, whereas Table 1 presents descriptive statistics regarding the data used.

For the measurement of the ex ante uncertainty, we use the crude oil (OVX), gold (GVZ), and silver (VXSLV) VIXs, commodity counterparts of the CBOE VIX. Similar to VIX, commodity indexes measure the market's expectation of the 30-day volatility using the US Oil Fund, SPDR Gold Shares, and Silver ETF options and are calculated according to the VIX methodology,[3] that is, the CBOE implied volatility is calculated as

$$\sigma_{CBOE}^2 = \frac{2}{T} \sum_i \frac{\Delta K_i}{K_i^2} \mathrm{e}^{RT} Q(K_i) - \frac{1}{T}\left(\frac{F}{K_0} - 1\right)^2, \tag{5}$$

where T is time to expiration, F is the forward index level derived from index option prices, K_0 is the first strike below the forward index level F, K_i is the strike price of ith out-of-the-money option (call if $K_i > K_0$ and put if $K_i < K_0$), ΔK_i is the interval between strike prices, R is the risk-free interest rate to expiration, and $Q(K_i)$ is the midpoint of the bid–ask spread for each option with strike K_i. The value of the VIX that CBOE reports is

$$Index = 100 \times \sigma_{CBOE}. \tag{6}$$

Since the CBOE VIXs report the annual percentage volatility, we construct the daily implied volatility by dividing the index by $\sqrt{250}$ and 100 to scale it to units of realized volatility, for example,

[3]Full details of the VIX calculation can be found here: http://www.cboe.com/products/vix-index-volatility/vix-options-and-futures/vix-index/the-vix-index-calculation

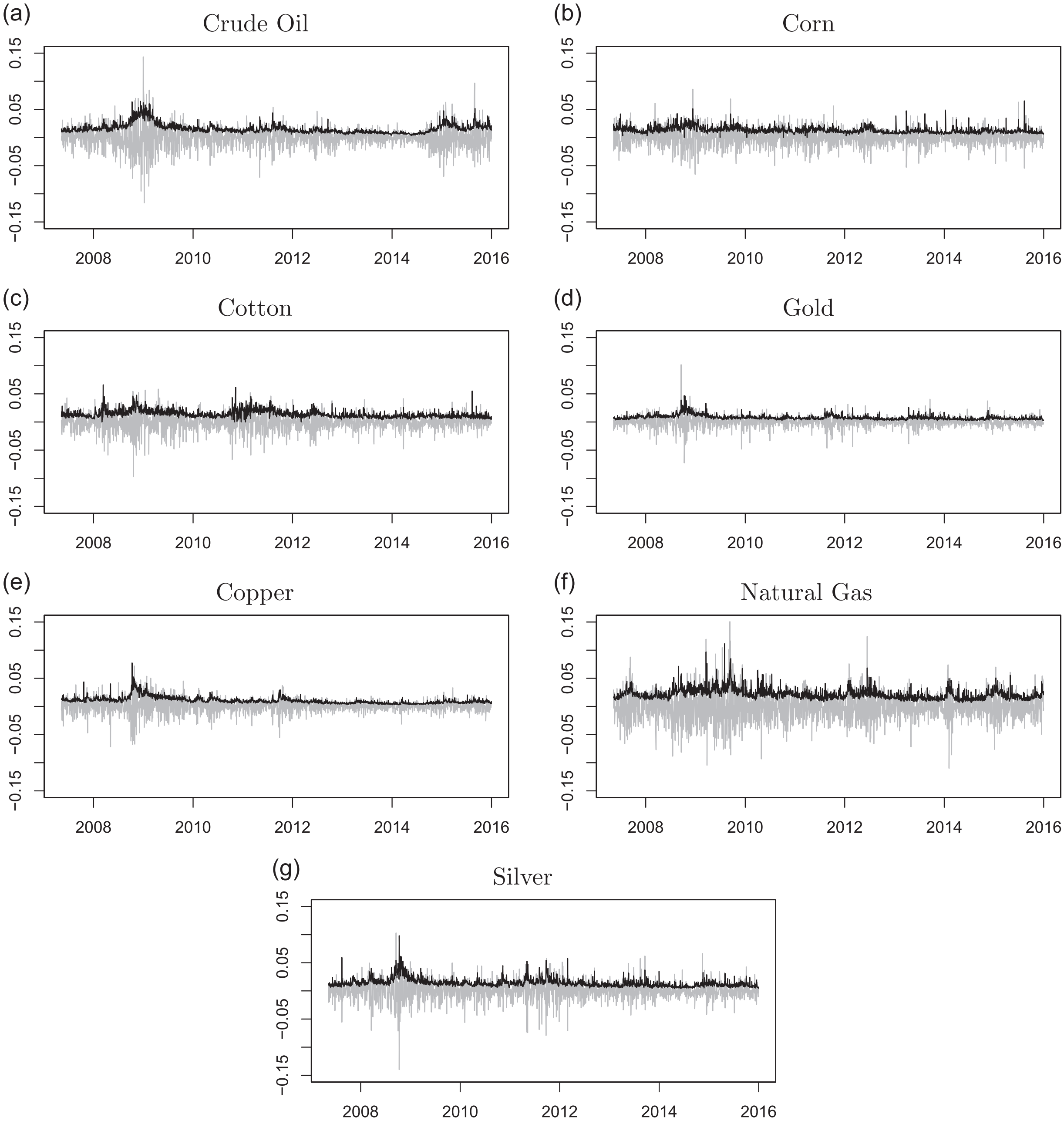

FIGURE 1 Daily returns and realized volatility. (a) Crude oil; (b) corn; (c) cotton; (d) gold; (e) copper; (f) natural gas; (g) silver. It is noted that plot displays open–close returns in gray and realized volatility in black during period May 10, 2007–December 31, 2015

$$OVX_{\text{daily}} = \frac{(1/100) \times OVX_{\text{annual}}}{\sqrt{250}}.$$

The crude oil, gold, and silver CBOE VIXs are obtained from the Federal Reserve Bank of Saint Louis.[4] The data availability differs across commodities, with crude oil having the longest history[5], since May 10, 2007, with gold being available from June 3, 2008, and silver from March 16, 2011. To have a balanced panel in our application, we set the starting date of all indexes to March 16, 2011. Figure 2 shows the plot of the VIXs, and Table 2 presents descriptive statistics.

[4]https://fred.stlouisfed.org/categories/32425

[5]The OVX was officially launched on July 15, 2008, but values were calculated back to May 10, 2007, when CBOE began trading the US Oil Fund options.

TABLE 1 Descriptive statistics—Returns and realized volatility

	Mean	*SD*	Skewness	Kurtosis	Median	Minimum	Maximum
Returns							
Crude oil	−0.000	0.018	0.103	4.936	0.001	−0.116	0.143
Corn	0.000	0.014	0.061	1.999	0.000	−0.065	0.086
Cotton	−0.001	0.014	−0.278	2.551	−0.000	−0.097	0.058
Gold	−0.000	0.009	0.066	12.269	0.000	−0.073	0.102
Copper	0.000	0.012	−0.309	4.955	0.000	−0.072	0.072
Natural gas	−0.001	0.024	0.233	2.537	−0.001	-0.110	0.151
Silver	−0.000	0.016	−0.510	6.034	0.000	-0.140	0.103
Realized volatility							
Crude oil	0.016	0.009	1.912	4.792	0.014	0.004	0.064
Corn	0.013	0.005	1.973	8.244	0.012	0.000	0.065
Cotton	0.014	0.007	1.688	5.639	0.012	0.000	0.066
Gold	0.008	0.004	2.585	12.069	0.007	0.002	0.047
Copper	0.011	0.006	2.667	13.228	0.009	0.003	0.077
Natural gas	0.022	0.009	2.156	9.794	0.020	0.007	0.112
Silver	0.014	0.008	2.615	12.860	0.013	0.004	0.098

Note: It displays descriptive statistics for returns and realized volatility estimates during the period from May 10, 2007 to December 31, 2015.

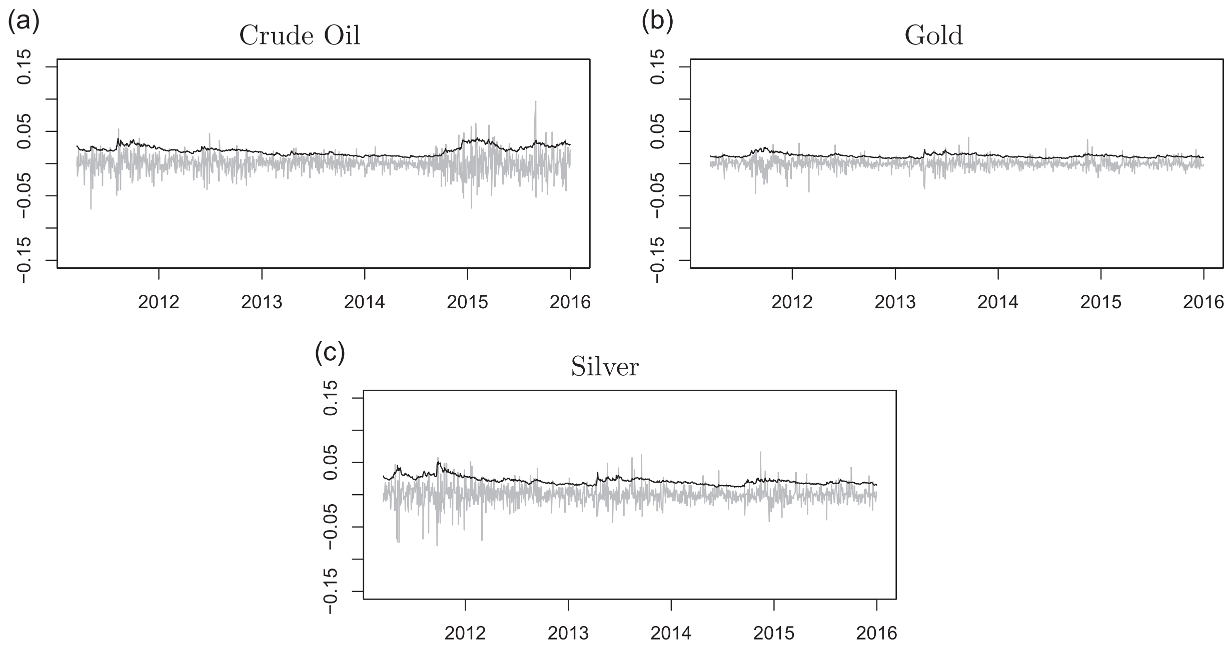

FIGURE 2 Daily returns and CBOE volatility indexes. (a) Crude oil; (b) gold; (c) silver. It is noted that plot displays open–close returns in gray and CBOE volatility indexes in black during the period from March 16, 2011 to December 31, 2015

TABLE 2 Descriptive statistics—CBOE volatility indexes

	Mean	*SD*	Skewness	Kurtosis	Median	Minimum	Maximum
Crude oil (OVX)	0.020	0.007	0.502	−0.608	0.020	0.009	0.040
Gold (GVZ)	0.012	0.003	1.372	2.304	0.011	0.008	0.025
Silver (VXSLV)	0.021	0.006	1.437	2.975	0.020	0.012	0.051

Note: It displays descriptive statistics of CBOE volatility indexes during the period from March 16, 2011 to December 31, 2015.

3 | EMPIRICAL APPLICATION

In the first part of the empirical analysis, we study the information content of the realized volatility for the commodity VaR estimation. Specifically, we concentrate on the commonalities in the dependence of the VaRs and corresponding realized volatilities during the period of the global financial crisis. We also conduct a small forecasting exercise and study the actual performance of the PQR estimator. The second part complements the realized volatility analysis and studies the role of the commodity CBOE VIXs in the VaR estimation.

3.1 | Influence of realized volatility

To study the VaR of commodities, we propose the following PQR model that links future return quantiles with the past realized volatility:

$$Q_{r_{i,t+1}}(\tau) = \alpha_i(\tau) + \beta_{RV^{1/2}}(\tau) \times RV_{i,t}^{1/2}, \tag{7}$$

where $i \in \{CL, CN, CT, GC, HG, NG, SV\}$. The appealing features of the model described by Equation (7) are the possibilities of identifying common patterns in realized volatilities by controlling for unobserved heterogeneity among commodities and directly relating the VaR and realized volatility as represented by the basic definition of the parametric VaR in Longerstaey and Spencer (1996). It is important to highlight that we do not need to assume a parametric distribution.

Table 3 and Figures 3 and 4 summarize the first part of our results. We identify strong common patterns of the dependence between quantiles of future commodity returns and ex post realized volatility and find this dependence to be highly statistically significant across the entire returns distribution. Specifically, the conditional returns distribution shares commonalities within the group of selected commodities, and these commonalities are stable over time.

For clarity and better readability, the results presented in Table 3 are divided into three groups according to the period included in the analysis. In the first part, we analyze the role of the global financial crisis in commodity markets; in the second part, we analyze the after-crisis period; and the last part provides estimates covering the full data set.

Throughout all of Table 3, we can observe the high statistical significance of the $\hat{\beta}_{RV^{1/2}}$ coefficient estimates for all but the median quantiles. The median performance, especially the lack of explanatory power to model median returns, constitutes a stylized fact of the unpredictability of expected returns and is in line with the efficient market hypothesis (Fama, 1970).

Turning our attention to the remaining quantiles, we can observe minor differences in the relative influence of the realized volatility on the VaR estimation within the studied periods. During the turbulent times of the financial crisis, the role of the realized volatility is slightly more important compared to the after-crisis period. We draw this conclusion since the absolute values of coefficient estimates are always higher during the crisis time, for example, the during-crisis 95% quantile coefficient estimate of 0.975 versus the after-crisis value of 0.878. Our results bring new insight to the stylized facts presented by Christoffersen et al. (2019) who documented increased uncertainty in the commodity

TABLE 3 Panel quantile regression parameter estimates: Realized volatility

τ	5%	10%	25%	50%	75%	90%	95%
Crisis: May 10, 2007–September 9, 2011							
$\hat{\beta}_{RV^{1/2}}$	−0.986	−0.831	−0.402	−0.000	0.339	0.800	0.975
	(−9.331)	(−7.956)	(−6.410)	(−0.000)	(6.004)	(9.798)	(6.678)
After crisis: September 11, 2011–December 31, 2015							
$\hat{\beta}_{RV^{1/2}}$	−0.949	−0.709	−0.304	0.036	0.292	0.613	0.878
	(−11.484)	(−9.887)	(−5.039)	(1.263)	(5.192)	(4.966)	(4.933)
Full sample: May 10, 2007–December 31, 2015							
$\hat{\beta}_{RV^{1/2}}$	−1.110	−0.879	−0.395	0.041	0.403	0.825	1.052
	(−14.471)	(−11.539)	(−7.959)	(1.728)	(7.124)	(11.605)	(8.269)

Note: It displays coefficient estimates with bootstrapped t-statistics in parentheses. We use the weighted x–y pair bootstrap of Bose and Chatterjee (2003). Full results with the individual fixed effects $\alpha_i(\tau)$ are presented in appendix.

markets during times of financial distress. We document that the conditional distributions of commodity returns standardized by their realized volatility have fatter tails in the crisis compared to the after-crisis period, that is, higher absolute values of the coefficient estimates in the crisis period. We thus perceive a more dispersed conditional returns distribution during times of financial crisis as a sign of the increased uncertainty. Moreover, the absolute values of the $\hat{\beta}_{RV^{1/2}}$ estimates are higher in lower than in upper quantiles. We document the relatively higher influence of the ex post uncertainty on the downside risk estimation. This asymmetry in the lower/upper quantiles of the conditional returns is similar to unconditional gain/loss asymmetry, *Stylized Fact 3* defined in Cont (2001).

An interesting finding of our analysis is the shape of the conditional returns distribution that can be seen from the graphical representation of the parameter estimates in Figures 3 and 4. In Figure 3, we can see similar shapes of the common conditional returns distribution represented by PQR coefficient estimates (solid black lines) for all studied commodities.[6]

These similarities are also visible in the level of individual commodities represented by boxplots. A closer look at the tails of the distributions reveals a slight asymmetric influence in the lower and upper tails. Importantly, Figure 4 contrasts the parameter estimates with the standard normal distribution and indicates a platykurtic conditional returns distribution since both the lower and upper tails are thinner than those implied by the standard normal distribution. Moreover, in the lower tail, the conditional returns distributions of all samples lie in the intersection of the corresponding 95% confidence intervals. Thus, we conclude that the properties and characteristics of the commodity market downside risk are stable in the crisis and after-crisis periods and did not change significantly during the global financial crisis. We have also looked at the dynamics of the parameters using rolling window estimation, as described in the next section. We have found parameters to vary slightly over time—The results of our analysis are summarized in Table A2 and Figure A1 in appendix.

These findings are in contrast to parametric studies, in which fat-tailed distributions were required to match empirical data; we document that returns standardized by the realized volatility have thin tails and are *platykurtic*. Although our results are in sharp contrast to the general perception of returns behavior, they are in line with works that consider commodities to be less risky than stocks (Bodie & Rosansky, 1980; Conover et al., 2010; Gorton & Rouwenhorst, 2006). The results can be useful in building a classical location-scale model where one needs to make an assumption about distribution of disturbances.

3.1.1 | VaR forecasting exercise

In this section, we study the accuracy of the one-step-ahead VaR forecasts. For estimation and forecasting purposes, we use a rolling window with a fixed length of 1,072 observations, that is, the length of the crisis and after-crisis subsamples from the previous analysis, and we backtest the forecasts using standard and well-established methods from the literature. The actual performance of the PQR method of obtaining VaR forecasts is tested against those of the UQR approach introduced in Žikeš and Baruník (2016) and the RiskMetrics (Longerstaey & Spencer, 1996) approach, which is the industry standard in the VaR literature.

From the numerous VaR backtesting procedures, we employ the proportion of failures test proposed by Kupiec (1995), the test for conditional coverage introduced in Christoffersen (1998) and the dynamic quantile test of Engle and Manganelli (2004) to study the absolute performance of the VaR forecasts. We also concentrate on the pairwise comparison of the PQR estimator, UQR, and RiskMetrics forecasting performances using the standard Diebold–Mariano test (Diebold & Mariano, 1995).

The general idea of the absolute performance tests is to study the relationship between the number of VaR exceedances, that is, the "*hit*" rate defined as

$$hit_t = \begin{cases} 1 & \text{if} \quad r_t \leq Q_{r_t}(\tau), \\ 0 & \text{otherwise}, \end{cases}$$

and the total numbers of VaR forecasts. Whereas the first test concentrates on the unconditional coverage only, that is, the *hit* rate should be close to nominal quantile level τ, the second one require *hits* to be also serially independent. Both tests comes from the family of likelihood-ratio tests, and they are related through identity

[6]Note that we document outliers in the univariate quantile regressions (UQRs) represented by the dots in Figure 3 that are estimates of copper (65% and 70% quantiles in the crisis period), crude oil (10–35% quantiles [minima] in the after-crisis period), and silver in the 25% quantile (maximum) in the after-crisis period.

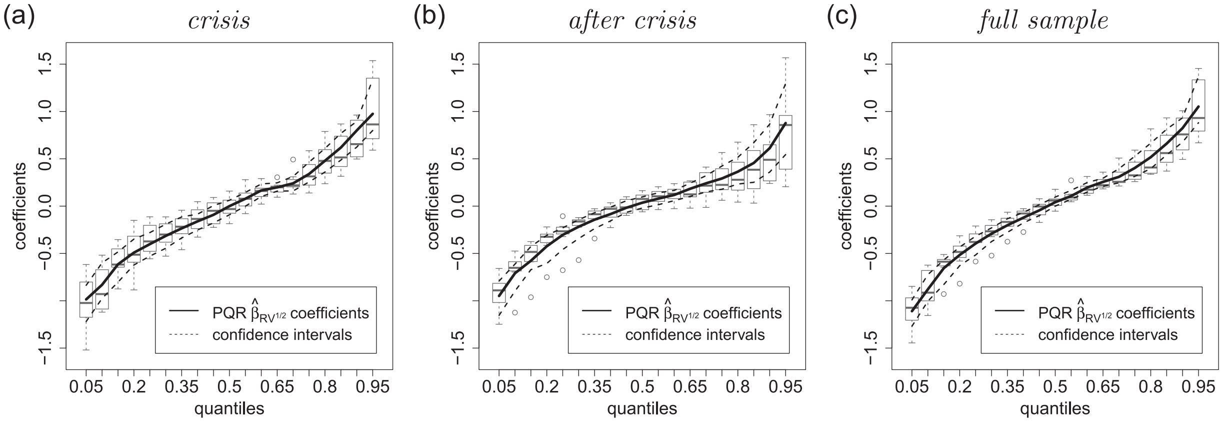

FIGURE 3 Panel quantile regression parameter estimates: Realized volatility. (a) Crisis; (b) after crisis; (c) full sample. It is noted that parameter estimates with corresponding 95% confidence intervals from the panel quantile regression are plotted by solid and dashed lines, respectively. Individual univariate quantile regression estimates are plotted in boxplots

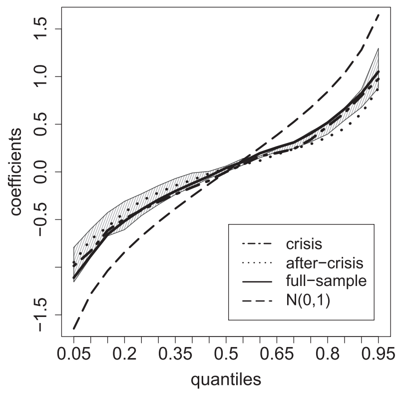

FIGURE 4 Conditional returns distributions versus standard normal distribution. It is noted that hatched area represents intersection of 95% confidence intervals of all studied samples estimates

$$LR_{\text{cc}} = LR_{\text{uc}} + LR_{\text{ind}},$$

where LR_{uc} stands for the unconditional coverage (proportion of failures) test, LR_{cc} for the conditional coverage test, and LR_{ind} for the independence of VaR violations. The third test (dynamic quantile test) can be observed as a general goodness-of-fit testing procedure for VaR forecast evaluation. It jointly examines the independence of the quantile estimates and the serial independence of the VaR violations.

In the pairwise comparison of the PQR approach with the competing models, we use the expected tick loss function as in Clements, Galvão, and Kim (2008) and Giacomini and Komunjer (2005), defined as

$$\mathscr{L}_{\tau,m} = E((\tau - I(e_{t+1}^m < 0)e_{t+1}^m),$$

where $I(\cdot)$ is the indicator function, $e_{t+1}^m = r_{t+1} - Q_{r_{t+1}}^m(\tau)$ and $Q_{r_{t+1}}^m(\tau)$ is the mth model quantile forecast. We use the tick loss functions as the input for the standard Diebold and Mariano (1995) test, where we test the equality of the predictive accuracy of the models, that is, H_0: $\mathscr{L}_{\tau,1} = \mathscr{L}_{\tau,2}$, against the general alternative.

The description of our findings starts with a graphical analysis. In Figure 5, the actual returns in the dark gray are plotted against the PQR (full black), UQR (dotted black), and RiskMetrics (dashed dim gray) 5% VaR forecasts.[7]In all commodities, the similarities of the PQR and UQR are clearly visible, as the forecasts almost overlap. They also closely mimic the behavior of returns, as is further documented in the formal statistical analysis in Table 4. In contrast, the

[7]The 10%, 90%, and 95% forecasts are presented in appendix.

TABLE 4 VaR forecasting performance: Absolute performance

		PQR					UQR					RiskMetrics				
Hits	τ	5%	10%	50%	90%	95%	5%	10%	50%	90%	95%	5%	10%	50%	90%	95%
Crude oil	$\hat{\tau}$	0.043	0.095	0.513	0.916	0.951	0.045	0.092	0.521	0.915	0.947	0.060	0.089	0.486	0.922	0.956
Corn	$\hat{\tau}$	0.031	0.008	0.487	0.921	0.959	0.032	0.077	0.480	0.923	0.962	0.041	0.077	0.487	0.923	0.960
Cotton	$\hat{\tau}$	0.037	0.075	0.500	0.918	0.963	0.034	0.068	0.494	0.924	0.965	0.042	0.089	0.520	0.920	0.951
Gold	$\hat{\tau}$	0.046	0.098	0.544	0.907	0.954	0.045	0.099	0.551	0.915	0.958	0.134	0.175	0.523	0.862	0.890
Copper	$\hat{\tau}$	0.027	0.071	0.521	0.915	0.959	0.041	0.083	0.511	0.914	0.947	0.119	0.166	0.492	0.842	0.888
Natural gas	$\hat{\tau}$	0.043	0.083	0.493	0.904	0.960	0.040	0.079	0.500	0.903	0.953	0.011	0.030	0.526	0.972	0.987
Silver	$\hat{\tau}$	0.031	0.082	0.541	0.922	0.955	0.031	0.090	0.544	0.927	0.957	0.046	0.079	0.522	0.928	0.955
Backtests																
Kupiec test																
Crude oil	LR_{uc}	1.174	0.274	0.637	**3.190***	0.048	0.626	0.882	1.930	**2.819***	0.229	2.026	1.573	0.637	**5.934****	0.878
Corn	LR_{uc}	**9.561*****	**4.919****	0.637	**5.414****	1.904	**8.584*****	**6.480****	1.622	**7.053*****	**3.357***	1.904	7.053	0.236	7.053	2.339
Cotton	LR_{uc}	**3.941****	**8.277*****	0.000	**4.005****	**3.941****	**6.810*****	**13.430*****	0.236	**7.652*****	**6.001****	1.516	1.573	**3.008***	**4.919****	0.048
Gold	LR_{uc}	0.418	0.046	**8.439*****	0.534	0.418	0.626	0.013	**11.113***	**2.819***	1.516	**110.123*****	**55.461*****	2.644	**15.702*****	**61.728*****
Copper	LR_{uc}	**14.115*****	**11.052*****	1.930	**2.819***	1.904	1.904	**3.585***	0.446	2.472	0.229	**77.882*****	**44.403*****	0.000	**34.437*****	**65.188*****
Natural gas	LR_{uc}	1.174	**3.585***	0.236	0.176	2.339	2.339	**5.414****	0.000	0.101	0.253	**48.877*****	**78.591*****	**4.449****	**83.855*****	**43.054*****
Silver	LR_{uc}	**9.561*****	**4.005****	**7.075*****	**5.934****	0.626	**9.561*****	1.320	**8.439*****	**9.609*****	1.174	0.418	**5.414****	**4.449****	**10.317*****	0.626
Christoffersen test																
Crude oil	LR_{cc}	1.810	0.341	3.054	3.228	0.162	1.458	2.207	3.753	3.274	0.582	2.035	3.495	2.669	**5.999****	1.608
Corn	LR_{cc}	**11.662*****	**6.511****	3.453	**5.424***	4.106	**10.816*****	**7.642****	3.782	**7.146****	**6.287****	**5.679***	**8.149****	1.869	**7.565****	2.384
Cotton	LR_{cc}	4.108	**8.471****	2.541	**6.062****	**5.230***	**7.275****	**13.658*****	3.177	**10.120*****	**7.854****	2.124	1.618	**4.907***	**7.413****	3.990
Gold	LR_{cc}	0.674	0.103	**8.439****	**5.087***	**5.123***	0.955	0.041	**11.135*****	**7.166****	**5.468***	**111.699*****	**57.769*****	3.443	**16.115*****	**63.439*****
Copper	LR_{cc}	**14.172*****	**12.491*****	2.563	2.906	2.368	1.925	**5.759***	1.410	2.602	2.237	**78.581*****	**44.676*****	0.971	**38.638*****	**67.253*****
Natural gas	LR_{cc}	1.810	**7.460****	4.593	2.321	2.384	2.727	**8.421****	3.838	2.432	0.309	**49.149*****	**78.593*****	**7.896****	**83.885*****	**43.425*****
Silver	LR_{cc}	**10.361*****	4.014	**7.579****	**8.740****	**5.137***	**10.361*****	1.705	**10.316*****	**13.186*****	**5.308***	1.357	**5.995***	**4.705***	**10.360*****	0.639
Dynamic quantile test																
Crude oil	DQ	**14.412***	**14.398***	5.153	**16.802****	**16.811****	2.587	11.357	6.923	12.584	**13.960***	**47.196*****	**44.281*****	3.927	**32.052*****	**27.148*****
Corn	DQ	11.525	9.690	9.244	**14.389***	8.168	11.641	10.790	9.035	**14.313***	9.594	**26.969*****	**37.071*****	5.346	**40.011*****	**26.788*****

(Continues)

TABLE 4 (Continued)

				PQR					UQR					RiskMetrics		
Cotton	*DQ*	11.077	8.964	**14.386***	**14.980***	10.049	12.462	**15.761****	**16.096****	**17.918****	12.483	**22.787*****	**21.431*****	11.579	**42.092*****	**30.322*****
Gold	*DQ*	7.589	6.687	12.251	12.960	12.535	8.160	6.878	14.553	11.391	**14.692***	**255.936*****	**131.871*****	9.922	**79.130*****	**185.416*****
Copper	*DQ*	**20.098*****	**14.009***	8.160	3.900	4.917	6.814	9.276	6.862	5.628	8.934	**171.078*****	**101.308*****	5.630	**83.492*****	**161.928*****
Natural gas	*DQ*	8.640	12.537	13.216	5.358	9.759	12.921	**14.055***	**18.071****	7.780	8.909	**35.443*****	**65.421*****	10.429	**66.716*****	**32.731*****
Silver	*DQ*	9.205	6.316	**22.742*****	11.859	3.164	9.444	5.288	**24.516*****	**15.143***	10.463	**18.087****	**29.487*****	**14.627***	**25.604*****	**16.629****

Abbreviation: PQR, panel quantile regression; UQR, univariate quantile regression; VaR, value-at-risk.

Note: It reports absolute forecasting performance of the PQR, UQR, and RiskMetrics VaR estimators. For each model and quantile τ, we report the unconditional coverage ($\hat{\tau}$) and the values of the unconditional coverage Kupiec test (LR_{uc}), conditional coverage Christoffersen test (LR_{cc}), and dynamic quantile test for correct dynamic specification (DQ).

Not correctly specified models at the 1% (***), 5% (**), and 10% (*) significance levels are given in bold.

TABLE 5 VaR forecasting performance: Pairwise comparison

		Benchmark									
		UQR					RiskMetrics				
PQR	τ	5%	10%	50%	90%	95%	5%	10%	50%	90%	95%
Crude oil	$\widehat{DM}$	0.343	−1.265	0.795	*−2.117***	*−1.809***	**8.890*****	**7.889*****	−0.410	**5.271*****	**3.940*****
Corn	$\widehat{DM}$	−0.150	0.337	−0.14	0.054	0.571	**4.713*****	**6.773*****	−0.862	**5.687*****	**4.072*****
Cotton	$\widehat{DM}$	0.288	0.689	0.824	−0.473	0.054	**1.741****	**1.909****	−0.141	**5.437*****	**4.136*****
Gold	$\widehat{DM}$	**3.041*****	**1.619****	**1.451*****	−1.192	−0.913	**3.078*****	**3.123*****	0.087	**4.698*****	**4.941*****
Copper	$\widehat{DM}$	*−2.316***	−1.190	−0.993	−0.240	0.327	**3.393*****	**3.521*****	−0.702	**2.998*****	**3.848*****
Natural gas	$\widehat{DM}$	1.274	**1.842*****	0.363	0.601	−0.539	**12.657*****	**13.435*****	0.441	**12.877*****	**11.630*****
Silver	$\widehat{DM}$	1.092	0.864	1.028	−0.058	0.256	**4.294*****	**6.653*****	−1.070	**6.625*****	**4.737*****

Abbreviation: PQR, panel quantile regression; UQR, univariate quantile regression; VaR, value-at-risk.
Note: It reports the relative performance of the panel quantile regression estimator. For each commodity and quantile τ, we report the Diebold–Mariano test statistics ($\widehat{DM}$). Statistically more/less accurate forecasts with respect to the benchmark are given in bold/italics.
*Significant at 10% level.
**Significant at 5% level.
***Significant at 1% level.

forecasts of RiskMetrics are far from the returns in certain periods and highly overestimate the risk, for example, crude oil in 2015 or natural gas and silver in 2012. In Figure 5, we can also observe fairly stable behavior of the PQR/UQR forecasts, whereas RiskMetrics tends to be rather volatile. We now turn to a statistical comparison of the out-of-sample VaR forecasts. In the *hits* part of Table 4, the unconditional coverage, $\hat{\tau}$, is presented. In the below median quantiles, the PQR VaR forecasts overestimate the potential loss of an investor with the long position in the commodities since the unconditional coverage is below the nominal level. The same applies to the above median quantiles and the investor holding a short position. We can thus think of the PQR means of VaR modeling as of a conservative risk strategy.

In the *Backtest* section of Table 4, we can see that the results of both PQR and UQR exhibit similar patterns. For the majority of commodities, the Kupiec and Christoffersen tests suggest that the models are not correctly specified in almost all quantiles, whereas the dynamic quantile test identified incorrect dynamic specification only for crude oil and the lower quantiles of copper. The differences in the results stem from the construction of the tests: The Kupiec/Christoffersen approaches are likelihood-ratio-based, whereas the dynamic quantile test is regression-based. The performance of the RiskMetrics is the worst, similar to graphical analysis, with all but median quantiles being incorrectly dynamically specified according to the *DQ* test (Table 5).

In Table 5, we can see a comparison of PQR against UQR and RiskMetrics. Overall, our results indicate good forecasting performance of the PQR approach. Specifically, PQR performs significantly better than RiskMetrics in all studied quantiles but the median. It also performs better than UQR in the 5%, 10%, and 50% quantiles for gold and 10% quantile for natural gas. In contrast, UQR outperforms PQR in the upper quantiles for crude oil and in the 5% quantile for copper, that is, in the commodity/quantile pairs, where PQR was not dynamically correctly specified but UQR was Figure 6.

3.2 | Role of ex ante uncertainty

Having revealed the importance of ex post uncertainty for future commodity VaR and identifying common patterns in the panel of commodities, it is tempting to ask what the role of ex ante information is. In this section, we extend our analysis and study the role of the ex ante implied volatility measure. Since the availability of the commodity CBOE VIXs is limited, the period used for the study spans from March 16, 2011, to December 31, 2015 and almost corresponds to the after-crisis period from the previous section.

Similar to the previous section, we concentrate on the role of the ex post or ex ante volatility measure for the VaR estimation in line with the parametric definition of VaR. In addition to the realized volatility, we also use implied VIX

TABLE 6 Panel quantile regression parameter estimates: RV and CBOE index

τ	5%	10%	25%	50%	75%	90%	95%
Panel A							
	$Q_{r_{i,t+1}}(\tau) = \alpha_i(\tau) + \beta_{RV^{1/2}}(\tau) \times RV_{i,t}^{1/2}$						
$\hat{\beta}_{RV^{1/2}}$	− 1.167	− 0.884	− 0.392	0.098	0.438	0.786	0.994
	(− 69.007)	(− 8.292)	(− 5.751)	(4.411)	(5.977)	(5.080)	(3.326)
$\hat{\alpha}_{CL}$	− 0.009	− 0.005	− 0.003	− 0.000	0.002	0.006	0.007
	(− 29.258)	(− 3.536)	(− 3.662)	(− 2.008)	(2.385)	(3.086)	(2.071)
$\hat{\alpha}_{GC}$	− 0.004	− 0.002	− 0.001	− 0.001	0.001	0.003	0.005
	(− 31.619)	(− 3.622)	(− 3.076)	(− 5.226)	(1.243)	(3.332)	(2.654)
$\hat{\alpha}_{SV}$	− 0.007	− 0.004	− 0.002	− 0.001	0.001	0.005	0.009
	(− 42.111)	(− 3.699)	(− 2.739)	(− 6.360)	(1.340)	(2.777)	(2.922)
	$Q_{r_{i,t+1}}(\tau) = \alpha_i(\tau) + \beta_{INDEX^{1/2}}(\tau) \times INDEX_{i,t}^{1/2}$						
$\hat{\beta}_{INDEX^{1/2}}$	− 1.229	− 0.859	− 0.404	0.098	0.449	0.821	1.153
	(− 18.269)	(− 15.299)	(− 23.766)	(3.750)	(27.785)	(13.448)	(16.901)
$\hat{\alpha}_{CL}$	0.001	0.000	− 0.000	− 0.001	− 0.001	− 0.001	− 0.002
	(0.412)	(0.506)	(− 0.568)	(− 2.003)	(− 4.774)	(− 1.043)	(− 1.744)
$\hat{\alpha}_{GC}$	0.003	0.002	0.001	− 0.001	− 0.002	− 0.001	− 0.003
	(4.263)	(2.784)	(4.822)	(− 4.401)	(− 10.799)	(− 2.304)	(− 4.098)
$\hat{\alpha}_{SV}$	0.004	0.003	0.002	− 0.002	− 0.003	− 0.003	− 0.003
	(3.661)	(2.815)	(4.285)	(− 4.703)	(− 8.572)	(− 2.310)	(− 2.411)
Panel B							
	$Q_{r_{i,t+1}}(\tau) = \alpha_i(\tau) + \beta_{RV^{1/2}}(\tau) \times RV_{i,t}^{1/2} + \beta_{INDEX^{1/2}}(\tau) \times INDEX_{i,t}^{1/2}$						
$\hat{\beta}_{RV^{1/2}}$	− 0.465	− 0.299	− 0.159	0.033	0.148	0.190	0.020
	(− 3.652)	(− 6.916)	(− 2.003)	(0.934)	(3.393)	(1.558)	(0.083)
$\hat{\beta}_{INDEX^{1/2}}$	− 0.838	− 0.654	− 0.281	0.083	0.355	0.678	1.131
	(− 5.115)	(− 8.429)	(− 6.527)	(3.213)	(19.644)	(15.871)	(6.692)
$\hat{\alpha}_{CL}$	− 0.001	0.001	− 0.001	− 0.001	− 0.001	− 0.001	− 0.002
	(− 0.538)	(0.747)	(− 1.520)	(− 2.425)	(− 7.344)	(− 0.914)	(− 1.656)
$\hat{\alpha}_{GC}$	0.001	0.001	0.001	− 0.001	− 0.002	− 0.001	− 0.003
	(1.063)	(1.862)	(3.464)	(− 4.980)	(− 18.255)	(− 2.649)	(− 3.424)
$\hat{\alpha}_{SV}$	0.002	0.002	0.001	− 0.002	− 0.003	− 0.002	− 0.003
	(1.362)	(2.302)	(3.047)	(− 4.689)	(− 11.949)	(− 2.797)	(− 2.222)

Note: It displays coefficient estimates with bootstrapped *t*-statistics in parentheses. We use the weighted *x-y* pair bootstrap of Bose and Chatterjee (2003).

to explain the future conditional returns; hence, we estimate the following two equations and contrast the parameters first:

$$Q_{r_{i,t+1}}(\tau) = \alpha_i(\tau) + \beta_{RV^{1/2}}(\tau) \times RV_{i,t}^{1/2}, \tag{8}$$

$$Q_{r_{i,t+1}}(\tau) = \alpha_i(\tau) + \beta_{INDEX^{1/2}}(\tau) \times INDEX_{i,t}^{1/2}, \tag{9}$$

whereas both approaches result in semiparametric VaR and give us conditional returns distribution and the later one stresses the importance of the anticipated risk level of the market participants.

In the last part of our analysis, we examine the information content of ex ante uncertainty after controlling for ex post uncertainty, and we formulate the problem as

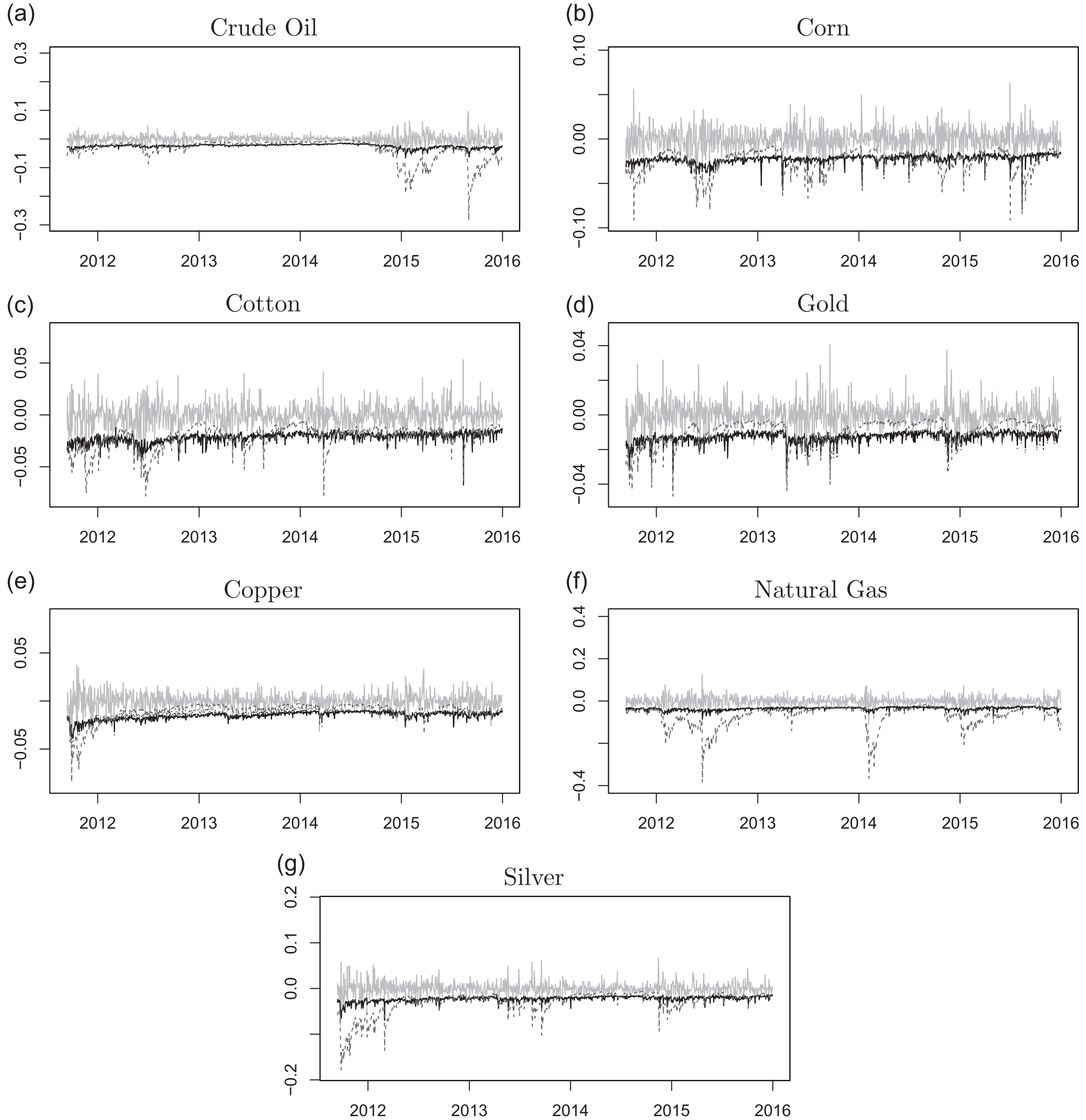

FIGURE 5 5% VaRs. (a) Crude oil; (b) corn; (c) cotton; (d) gold; (e) copper; (f) natural gas; and (g) silver. It is noted that plot displays actual returns (dark gray) during the period September 13, 2011–December 31, 2015, plotted against the PQR (full black), UQR (dotted black), and RiskMetrics (dashed dim gray) 10% VaR forecasts. PQR, panel quantile regression; UQR, univariate quantile regression; VaR, value-at-risk

$$Q_{r_{i,t+1}}(\tau) = \alpha_i(\tau) + \beta_{RV^{1/2}}(\tau) \times RV_{i,t}^{1/2} + \beta_{INDEX^{1/2}}(\tau) \times INDEX_{i,t}^{1/2}. \tag{10}$$

This specification allows us to directly compare the role and importance of the ex post and ex ante risk measures in the determination of quantiles of future returns. Here we follow previous literature that uses both implied and realized volatilities in a single regression despite possible collinearity (Busch, Christensen, & Nielsen, 2011; Jiang & Tian, 2005). The literature commonly finds the coefficients of implied volatility as well as explanatory power of the models unchanged when realized volatility is added to the regression pointing to the result that implied volatility is more informative. Similarly, in case the implied volatility is informative for future quantiles of returns, we expect it to drive the quantiles even when controlled for realized volatility. Equations (8)–(10) are estimated on the panel of $i \in \{CL, GC, SV\}$ commodities.

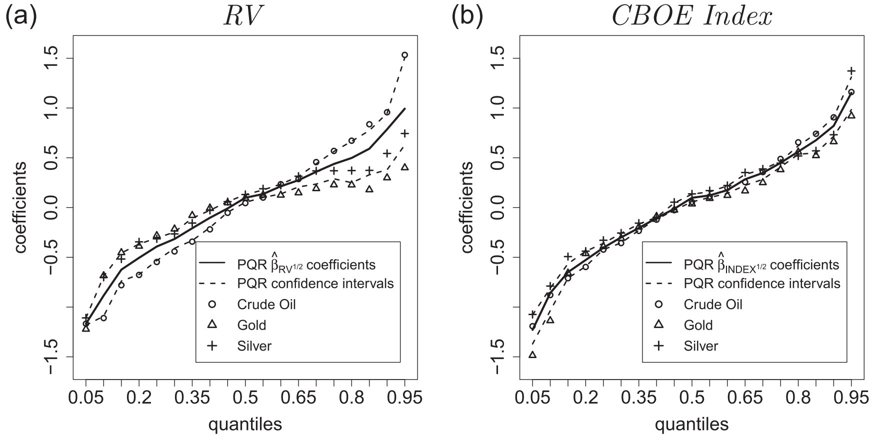

FIGURE 6 Panel quantile regression parameter estimates: Individual RV and CBOE Index. (a) RV; (b) CBOE index. It is noted that both realized volatility and volatility index parameter estimates and the corresponding 95% confidence intervals from the panel quantile regression are plotted by solid and dashed lines, respectively. Individual univariate quantile regression estimates are indicated by circles (crude oil), triangles (gold), and plus signs (silver)

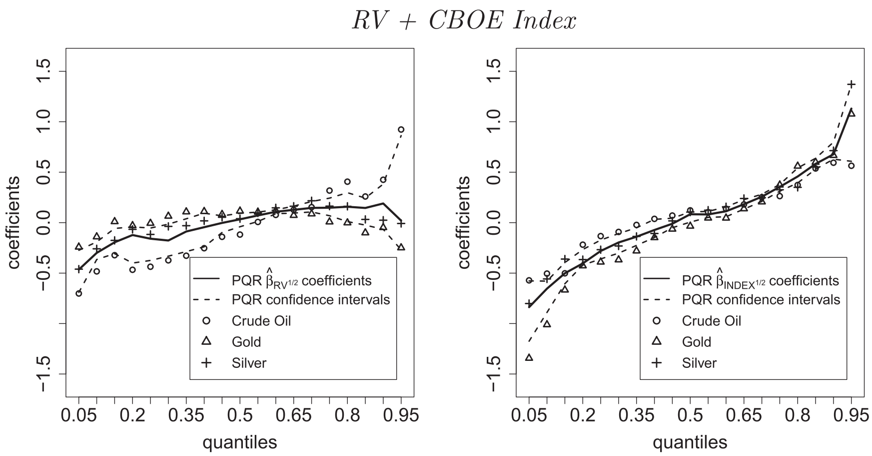

FIGURE 7 Conditional returns distributions versus standard normal distribution. It is noted that hatched area represents the intersection of the 95% confidence intervals of separate estimation of ex post and ex ante volatility based models

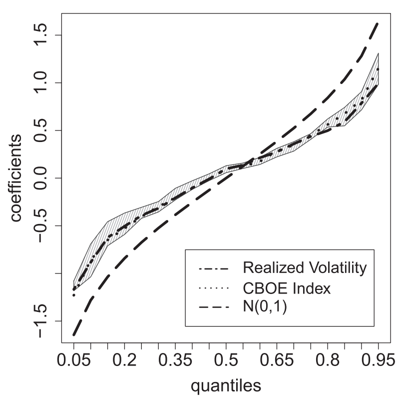

FIGURE 8 Panel quantile regression parameter estimates: RV + CBOE index. (a) RV + CBOE index. It is noted that both realized volatility and volatility index parameter estimates and the corresponding 95% confidence intervals from the panel quantile regression are plotted by solid and dashed lines, respectively. Individual univariate quantile regression estimates are indicated by circles (crude oil), triangles (gold), and plus signs (silver)

Panel A of Table 6 highlights the importance of both individual realized volatility and CBOE VIXs for future commodity returns quantile modeling. According to the proximity of the conditional returns distributions represented by the coefficient estimates $\hat{\beta}_{RV^{1/2}}$ and $\hat{\beta}_{INDEX^{1/2}}$, the impacts of both ex post and ex ante volatility on future return quantiles are of similar extent. We support this finding by visual inspection of Figure 7—the conditional returns distributions from the ex post and ex ante volatility model specifications are close to each other, and importantly, they both lie in the intersection of the 95% confidence intervals of these estimates.

In *Panel A*, we further see the high statistical significance of all $\hat{\beta}_{RV^{1/2}}$ and $\hat{\beta}_{INDEX^{1/2}}$ coefficients, including the median. In contrast to our previous analysis, the median individual fixed-effect estimates α_i are also statistically significant. Since the signs of α and β are opposite, they offset the influence of each other and make median returns difficult to predict. Therefore, returns distributions conditional on either of the volatility measures qualitatively match the results of our previous analysis. There is an asymmetric influence of the volatility measures in the lower and upper quantiles. Greater asymmetry is present in the $\hat{\beta}_{RV^{1/2}}$ coefficients, where the difference between the 5% and 95% quantile values is $1.167 - 0.994 = 0.173$, compared to the $\hat{\beta}_{INDEX^{1/2}}$ difference of $1.229 - 1.153 = 0.076$. We also document that conditional returns distributions have thinner tails than the standard normal distribution and therefore are *platykurtic*. This characteristic is visible in Figure 7, where the 5% (95%) conditional returns distribution quantile of the realized volatility specification is −1.167 (0.994), as opposed to −1.645 (1.645) for the standard normal distribution. The results are similar for the CBOE index, for which the 5% (95%) conditional returns distribution quantile is −1.229 (1.153).

Panel B of Table 6 emphasizes the role of ex ante volatility in the VaR estimation. The CBOE VIXs exhibit great importance once we control for realized volatility. The dominance of the CBOE VIXs is well documented in the above median quantiles—The $\hat{\beta}_{INDEX^{1/2}}$ estimates are always statistically significant, whereas this is not generally true for the $\hat{\beta}^{RV^{1/2}}$ coefficients. In the far upper quantiles, the $\hat{\beta}_{RV^{1/2}}$ coefficient estimates are even not statistically different from zero. Figure 8 confirms our findings visually. The confidence bands of the RV coefficients are much wider than the confidence intervals of the CBOE VIXs coefficients and often contain zero. We conclude that implied volatility subsumes most of the information carried by realized volatility, although realized volatility still plays role in forecasts.

4 | CONCLUSION

In this paper, we propose to use realized volatility and CBOE VIXs together with PQR to model the VaR of the representatives of the commodity market. The choice of volatility measures allows us to directly compare differences in the impacts of ex post and ex ante uncertainty on the conditional quantiles of commodity returns. Using PQR, we can control for unobserved heterogeneity among commodities and study the common influence of the uncertainty measures on the VaR estimation. The flexibility offered by the PQR approach, moreover, does not require us to make any distributional assumption about the distribution of commodity returns. The advantage of our approach is revealed in the empirical application.

In the first part of our empirical application, we study the role of the ex post uncertainty proxied by the RV and document common effects in the VaR estimation of seven representatives of the commodity market. These effects hold within all studied samples, and they did not change during the global financial crisis. We further show that the conditional distribution of returns standardized by the lagged RV has thinner tails than the standard normal distribution, which is commonly used in VaR applications. We also confirm our previous findings in a forecasting exercise, in which the PQR approach exhibits good performance.

In the second part, we complement ex post uncertainty analysis by the option implied volatility, an ex ante uncertainty measure. We concentrate on volatility implied by option prices since it reveals the market's sentiment and expectations about future riskiness. Due to the limited availability of CBOE VIXs, we concentrate on the crude oil, gold, and silver commodity alternatives to the VIX. Analogous to the results from the first part, our analysis reveals similarities in the patterns driving commodity VaRs. These patterns are almost identical for both ex post and ex ante volatility measures. In the model specification in which we control for ex post volatility, we also highlight the importance of the ex ante uncertainty for commodity VaR estimation.

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ORCID

František Čech http://orcid.org/0000-0003-3514-0735
Jozef Baruník http://orcid.org/0000-0001-5097-2607

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APPENDIX A

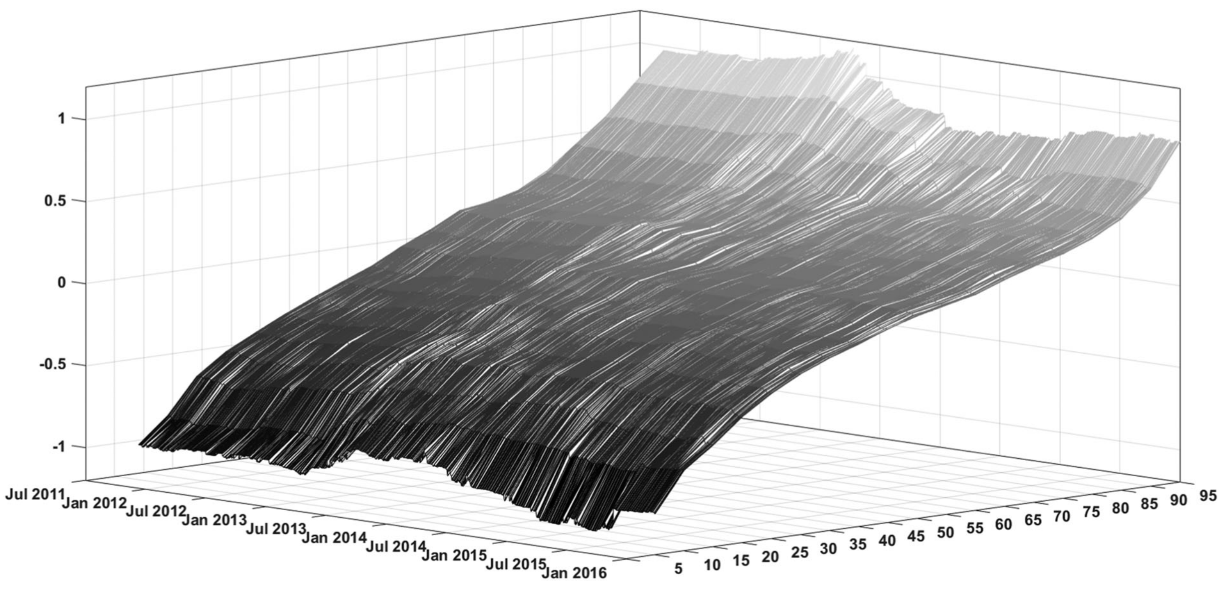

FIGURE A1 Time-varying coefficients. It is noted that plot displays time-varying coefficients for given quantiles during the period September 12, 2011–December 31, 2015

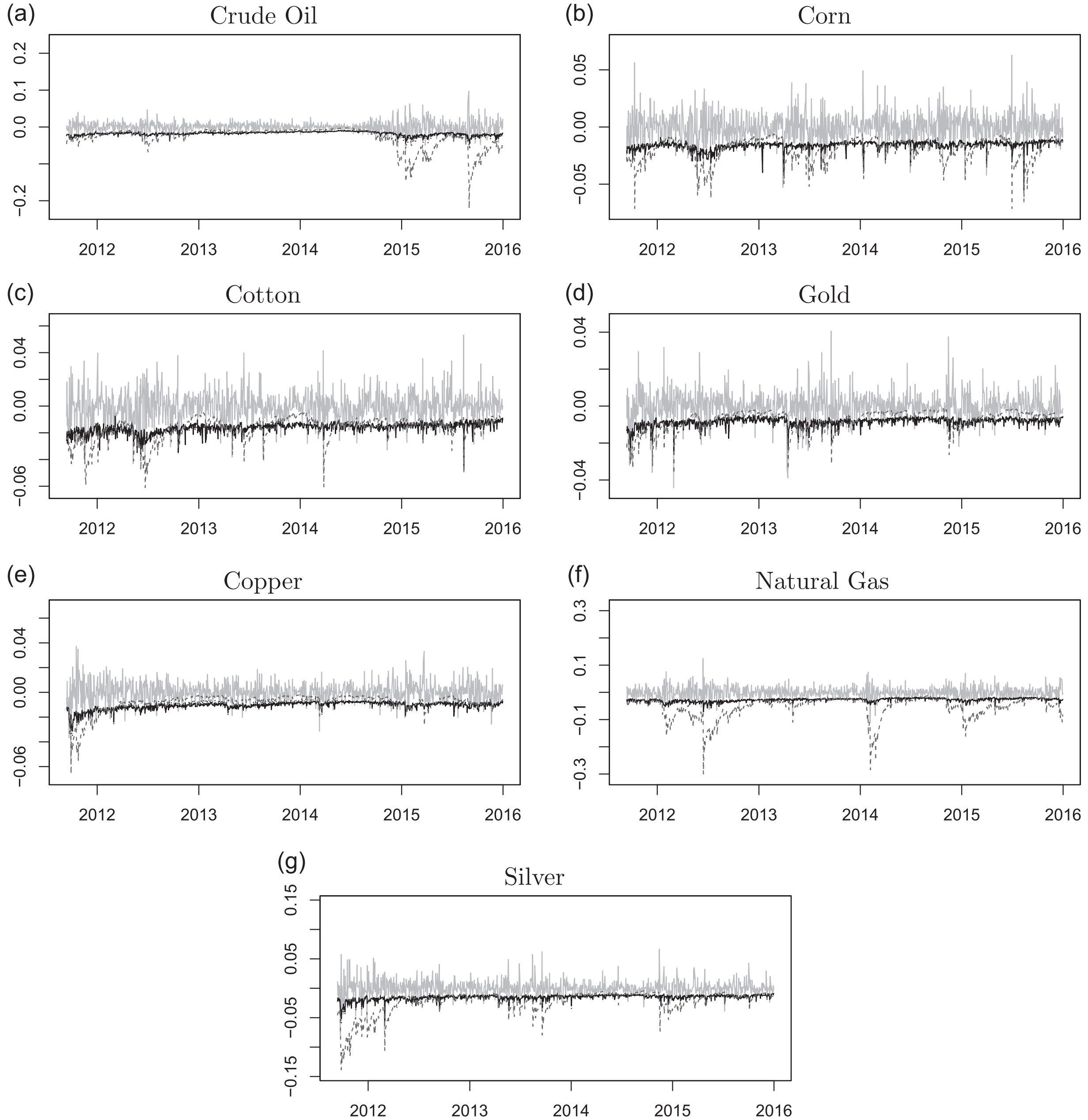

FIGURE A2 10% VaRs. (a) Crude oil; (b) corn; (c) cotton; (d) gold; (e) copper; (f) natural gas; and (g) silver. It is noted that plot displays actual returns (dark gray) during the period September 13, 2011–December 31, 2015, plotted against the PQR (full black), UQR (dotted black), and RiskMetrics (dashed dim gray) 10% VaR forecasts. PQR, panel quantile regression; UQR, univariate quantile regression; VaR, value-at-risk

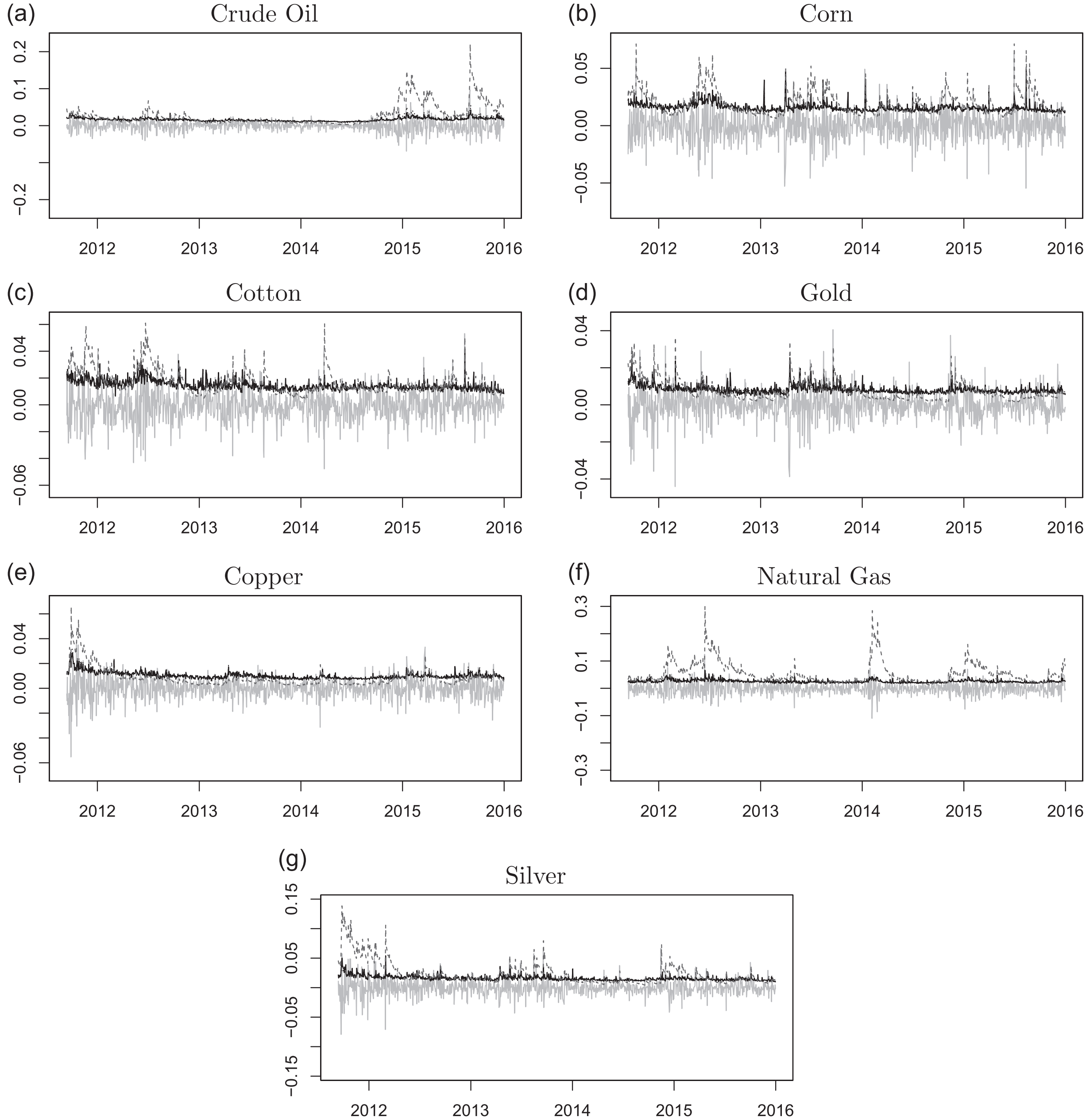

FIGURE A3 90% VaRs. (a) Crude oil; (b) corn; (c) cotton; (d) gold; (e) copper; (f) natural gas; and (g) silver. It is noted that plot displays actual returns (dark gray) during the period September 13, 2011–December 31, 2015, plotted against the PQR (full black), UQR (dotted black), and RiskMetrics (dashed dim gray) 10% VaR forecasts. PQR, panel quantile regression; UQR, univariate quantile regression; VaR, value-at-risk

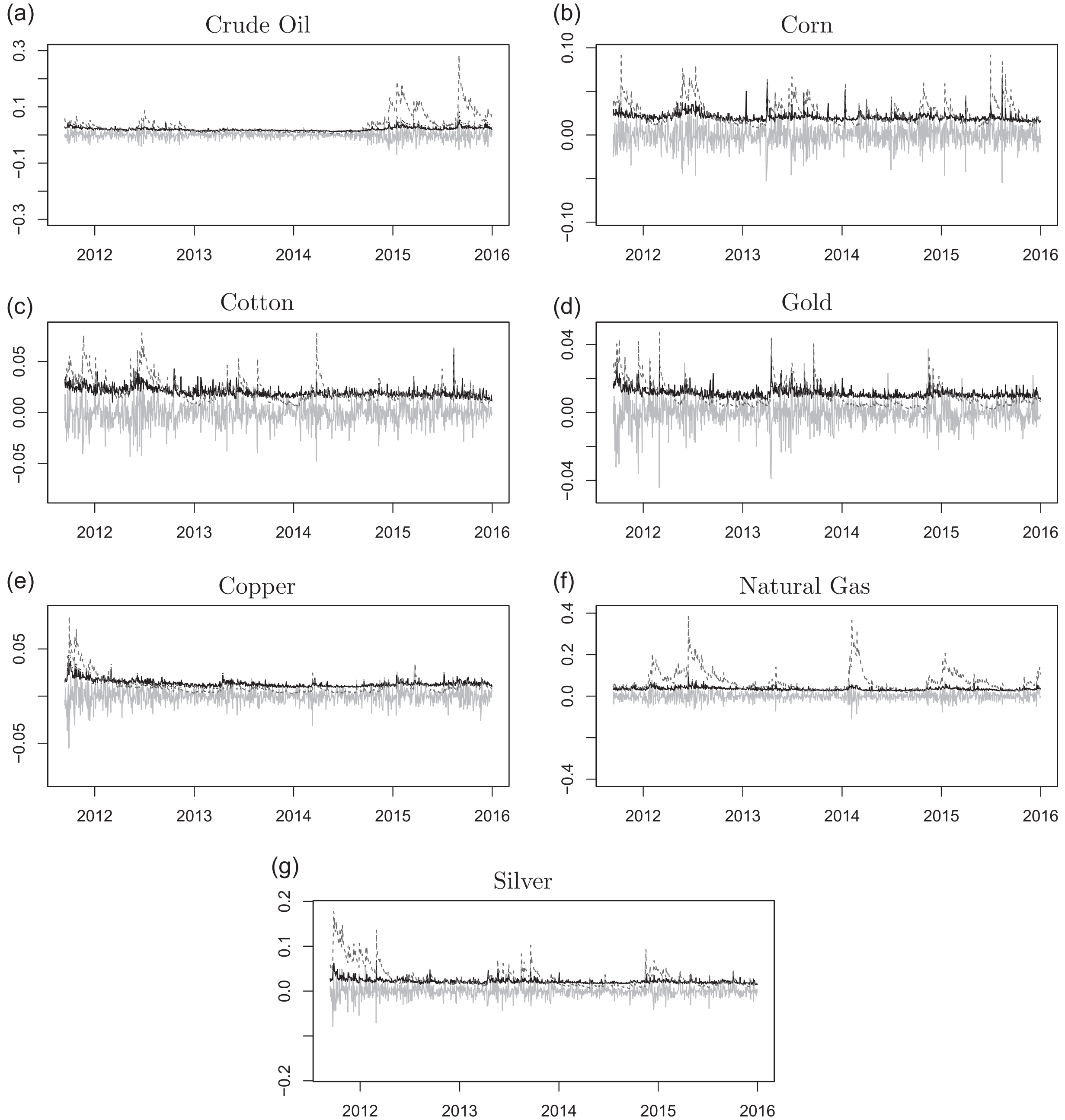

FIGURE A4 95% VaRs. (a) Crude oil; (b) corn; (c) cotton; (d) gold; (e) copper; (f) natural gas; and (g) silver. It is noted that plot displays actual returns (dark gray) during the period September 13, 2011–December 31, 2015, plotted against the PQR (full black), UQR (dotted black), and RiskMetrics (dashed dim gray) 10% VaR forecasts. PQR, panel quantile regression; UQR, univariate quantile regression; VaR, value-at-risk

TABLE A1 Panel quantile regression parameter estimates: Realized volatility

τ	5%	10%	25%	50%	75%	90%	95%
	Crisis: 2006–2010						
$\hat{\beta}_{RV^{1/2}}$	−0.986	−0.831	−0.402	−0.000	0.339	0.800	0.975
	(−9.331)	(−7.956)	(6.410)	(−0.000)	(6.004)	(9.798)	(6.678)
$\hat{\alpha}_{CL}$	−0.012	−0.009	−0.004	0.001	0.006	0.009	0.011
	(−5.956)	(−4.236)	−3.387	(1.450)	(6.736)	(6.112)	(4.609)
$\hat{\alpha}_{CN}$	−0.013	−0.007	−0.003	0.000	0.005	0.009	0.012
	(−9.718)	(−5.518)	−2.948	(0.000)	(5.918)	(7.445)	(5.416)
$\hat{\alpha}_{CT}$	−0.012	−0.007	−0.004	−0.000	0.004	0.007	0.011
	(−9.287)	(−4.798)	(−3.912)	(−0.000)	(5.148)	(5.525)	(4.492)
$\hat{\alpha}_{GC}$	−0.006	−0.003	−0.001	0.001	0.002	0.003	0.005
	(−6.419)	(−3.633)	(−2.504)	(1.524)	(5.697)	(5.073)	(4.263)
$\hat{\alpha}_{HG}$	−0.010	−0.006	−0.002	0.001	0.004	0.005	0.008
	(−6.901)	(−4.991)	(−2.075)	(2.436)	(5.499)	(5.119)	(4.162)
$\hat{\alpha}_{NG}$	−0.020	−0.015	−0.009	− 0.001	0.008	0.011	0.018
	(−7.720)	(−6.633)	(−6.854)	(− 1.267)	(5.630)	(5.393)	(5.300)
$\hat{\alpha}_{SV}$	−0.012	−0.006	−0.003	0.001	0.005	0.007	0.010
	(−6.620)	(−3.757)	(−2.615)	(2.149)	(6.519)	(4.953)	(5.281)
	After crisis: 2011–2015						
$\hat{\beta}_{RV^{1/2}}$	− 0.949	− 0.709	− 0.304	0.036	0.292	0.613	0.878
	(− 11.484)	(− 9.887)	(− 5.039)	(1.263)	(5.192)	(4.966)	(4.933)
$\hat{\alpha}_{CL}$	− 0.011	− 0.008	− 0.004	0.000	0.003	0.007	0.009
	(− 11.489)	(− 7.243)	(− 6.042)	(0.115)	(5.533)	(4.968)	(4.005)
$\hat{\alpha}_{CN}$	− 0.009	− 0.006	− 0.004	− 0.000	0.004	0.007	0.009
	(− 11.866)	(− 7.672)	(− 6.588)	(− 1.223)	(6.497)	(5.717)	(5.049)
$\hat{\alpha}_{CT}$	− 0.009	− 0.006	− 0.003	− 0.001	0.002	0.005	0.007
	(− 10.857)	(− 8.411)	(− 4.777)	(− 3.413)	(3.675)	(4.380)	(4.068)
$\hat{\alpha}_{GC}$	− 0.006	− 0.004	− 0.002	− 0.001	0.001	0.004	0.005
	(− 11.974)	(− 8.032)	(− 4.613)	(− 3.825)	(3.856)	(5.408)	(4.988)
$\hat{\alpha}_{HG}$	− 0.005	− 0.004	− 0.002	− 0.000	0.002	0.005	0.006
	(− 10.306)	(− 7.928)	(− 4.924)	(− 1.088)	(5.133)	(4.701)	(4.819)
$\hat{\alpha}_{NG}$	− 0.015	− 0.011	− 0.007	− 0.002	0.006	0.012	0.016
	(− 8.631)	(− 10.323)	(− 7.703)	(− 3.795)	(6.548)	(6.051)	(5.103)
$\hat{\alpha}_{SV}$	− 0.009	− 0.006	− 0.003	− 0.001	0.002	0.007	0.010
	(− 10.504)	(− 7.585)	(− 4.693)	(− 3.433)	(3.868)	(4.938)	(5.685)
	Full sample: 2006–2015						
$\hat{\beta}_{RV^{1/2}}$	− 1.110	− 0.879	− 0.395	0.041	0.403	0.825	1.052
	(− 14.471)	(− 11.539)	(− 7.959)	(1.728)	(7.124)	(11.605)	(8.269)
$\hat{\alpha}_{CL}$	− 0.010	− 0.006	− 0.003	0.000	0.003	0.006	0.008
	(− 9.765)	(− 5.062)	(− 4.789)	(0.443)	(4.473)	(5.910)	(4.383)
$\hat{\alpha}_{CN}$	− 0.008	− 0.005	− 0.003	− 0.000	0.003	0.006	0.009
	(− 10.764)	(− 5.976)	(− 5.248)	(− 1.532)	(4.909)	(7.877)	(5.623)
$\hat{\alpha}_{CT}$	− 0.008	− 0.005	− 0.003	− 0.001	0.002	0.005	0.008
	(− 9.764)	(− 5.260)	(− 4.304)	(− 2.580)	(2.598)	(4.952)	(4.961)

(Continues)

TABLE A1 (Continued)

τ	5%	10%	25%	50%	75%	90%	95%
$\hat{\alpha}_{GC}$	− 0.005	− 0.003	− 0.001	− 0.000	0.001	0.003	0.004
	(− 9.180)	(− 4.990)	(− 3.770)	(− 1.319)	(2.697)	(5.983)	(4.912)
$\hat{\alpha}_{HG}$	− 0.006	− 0.003	− 0.001	0.000	0.002	0.004	0.005
	(− 7.640)	(− 5.161)	(− 3.489)	(0.632)	(3.747)	(5.497)	(5.006)
$\hat{\alpha}_{NG}$	− 0.014	− 0.011	− 0.007	− 0.002	0.005	0.010	0.014
	(− 9.555)	(− 6.983)	(− 7.204)	(− 4.309)	(4.492)	(6.948)	(5.798)
$\hat{\alpha}_{SV}$	− 0.008	− 0.005	− 0.002	− 0.000	0.003	0.005	0.009
	(− 8.422)	(− 4.846)	(− 3.656)	(− 1.511)	(3.231)	(6.195)	(6.049)

Note: It displays coefficient estimates with bootstrapped *t*-statistics in parentheses. We use the weighted *x*–*y* pair bootstrap of Bose and Chatterjee (2003).

TABLE A2 Time-varying coefficients: Summary statistics

τ	Mean	Median	*SD*	Minimum	Maximum
5%	−0.961	−0.967	0.077	−1.109	−0.761
10%	−0.741	−0.722	0.075	−0.869	−0.617
25%	−0.318	−0.289	0.055	−0.433	−0.240
50%	0.056	0.070	0.034	−0.018	0.113
75%	0.350	0.353	0.022	0.292	0.409
90%	0.718	0.724	0.106	0.543	0.898
95%	0.908	0.878	0.113	0.731	1.158

Note: It displays summary statistics of coefficient estimates for given quantiles during the period September 12, 2011–December 31, 2015.