A Note on Optimal Value of Loans

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Abstract. People try to gain (in the last decades) own residence (a flat or a little house). Since young people do not posses necessary financial resources, the bank sector offers them a mortgage. Of course, the aim of any bank is to profit from such a transaction. Therefore, according to their possibilities, the banks employ excellent experts to analyze the financial situation of potential clients. Consequently, the banks know what could be a maximal size of the loan (in dependence on the debtor's position, salary and age) and what is a reasonable size of the installments. The aim of this contribution is to analyze the situation from the second side. In particular, the aim is to investigate the possibilities of the debtors not only in dependence on their present–day situation, but also on their future private and subjective decisions and on possible "unpleasant" events. Moreover, consequently according to these indexes, the aim of this contribution is to suggest a method for a recognition of a "safe" loan and simultaneously to offer tactics to state a suitable environment for future time. The stochastic programming theory will be employed to it.

Keywords: Loan, debtor, installments, multistage stochastic programming.

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1 Introduction

Decisions of households, one of which taking of a mortgage is, are usually studied by microeconomics [2]. A standard approach to the analysis of such a decision would be to quantify subjective gains from the living in own estate and compare them with a discomfort incurred by the repayment of the mortgage. The analysis of risks associated with a such a decision, is less common, both in theory and, unfortunately, in practice. Neglecting the risks, nevertheless, can easily lead to situations, considered as catastrophic by the decision makers. The aim of the present paper is to outline a methodology, which could be used to a responsible analysis of risks associated with mortgage taking from the debtor's point of view.

In the paper, first, an example of a "classical" situation will be explained (Section 2), followed by stochastic programming models (Section 3). A simple stochastic programming problems will be constructed employing the original example (Section 4). Conclusion can be found in Section 5.

2 Problem Analysis – Example

Let us start with simple standard situation. A young married couple wants to gain own flat. Evidently, these young people have first to decide if they prefer flat or a little house. This decision depends on their nature, financial possibilities and conditions about loans (in this time). Young people are responsible and so they will try to analyze their possibilities. To this end let us assume that (in the start time) their a monthly income is

$$Z_0 = U_0 + V_0, \quad \text{where} \quad U_0 \quad \text{is an income of husband and} \quad V_0 \quad \text{is an income of wife.}$$

Evidently, this income can be divided into three parts Z_0^1, Z_0^2, Z_0^3, where Z_0^1 denotes means for a basic consumption, Z_0^2 denotes means that can be employed for a repayment of installments and Z_0^3 can be

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considered as an allocation to saving. Consequently

$$Z_0 = Z_0^1 + Z_0^2 + Z_0^3, \quad Z_0^1,\, Z_0^2 > 0,\, Z_0^3 \ge 0. \tag{1}$$

Given the annuity repayments, which is the most standard way of repaying the loan and if we denote by a symbol M the value of the loan, by m number of identical installments and by ζ the loan interest rate, then the identical installments $b(M) := b(\zeta)$ in time points $t = 1, 2, \ldots, m$ (see, e.g., [7] or [9]) are given by

$$\begin{aligned} b(M) \;:=\; b(\zeta) \;=\; & \tfrac{M\zeta}{1-v^m}, \quad \zeta \neq 0, \quad v = v(\zeta) = (1+\zeta)^{-1}, \\ & \tfrac{1}{m}, \qquad \zeta = 0. \end{aligned} \tag{2}$$

It follows from the relations (1), (2) that (in the case when $\zeta \neq 0$) it is desirable (in "static" approach) the following inequality

$$\frac{M\zeta(1+\zeta)^m}{(1+\zeta)^m - 1} \le Z_0^2 \tag{3}$$

to be fulfilled. Of course, this condition (in the extreme case) can be replaced by the inequality

$$\frac{M\zeta(1+\zeta)^m}{(1+\zeta)^m - 1} \le Z_0^2 + Z_0^3. \tag{4}$$

If it is possible to assume that the relations (1), (2) will be fulfilled also in future, then the young people can take the loan equal to the maximal value M for which the inequality (3) (respective (4)) is fulfilled. However mostly it is necessary to assume that the financial situation of young married couple can change. For example: it is reasonable to assume that in some time period, say (m_1, m_2), $0 < m_1 < m_2 \le m$ the married couple plan to have a baby. According to this fact and to the social politics of a state the young people can assume the less income in this time, approximately equal to

$$Z_1 = U_0 + V_1 = Z_0^1 + Z_1^2 + Z_1^3, \quad Z_1^2,\, Z_1^3 \ge 0,$$

where V_1 is the supposed income of wife in the time interval (m_1, m_2); Z_1^2 denotes the means, that can be employed for a repayment of installments (of course $Z_1^2 \le Z_0^2$) and Z_1^3 saved amount in every year of this time interval (of course mostly $0 \le Z_1^3 \le Z_0^3$). Evidently without financial reserve the inequalities

$$Z_0^1 + Z_0^2 \le U_0 + V_1$$

need to be fulfilled. Consequently, if

$$U_0 + V_1 < Z_0^1 + Z_0^2,$$

then a very serious trouble could arise. However, if the young couple saved every time point $t \in (1, \ldots, m_1 - 1)$ the amount Z_0^3 and if the inequality

$$\frac{(m_2 - m_1)M[\zeta(1+\zeta)^m]}{(1+\zeta)^m - 1} \le (m_2 - m_1)[Z_0^2 - Z_1^2] + (m_1 - 1)Z_0^3 \tag{5}$$

is fulfilled, then they endure the time period (m_1, m_2) without financial troubles.

To construct the relation (5), it has been assumed that the amount Z_0^3 is deterministic, the same in every time point $t \in (1, \ldots, m_1 - 1)$ and that this amount can not be changed. However this situation can be a little different. To explain a new approach we suppose $m_1 = 3, \quad m_2 - m_1 = 2$ and one of the situations:

A 1. The deterministic value Z_0^3 (in the relation (1)) can be replaced by random values $Z_0^3(t)$; $Z_0^3(t)$, $t \in (1, m_1 - 1)$ with probability one positive. Consequently the deterministic income $Z_0 = Z_0^1 + Z_0^2 + Z_0^3$ is replaced by random $Z_{0,0} = Z_0^1 + Z_0^2 + Z_0^3(1)$ in the start point $t = 1$ and by $Z_{0,1} = Z_0^1 + Z_0^2 + Z_0^3(2)$ in the time point $t = 2$. Furthermore it is reasonable to assume that young people can these random amount invest (for example) into two assets to obtain:

a

in the first year the value	$\xi_{0,1}x_{0,1} + \xi_{0,2}x_{0,2}$
under the assumptions	$x_{0,1} + x_{0,2} \le Z_0^3(1), \quad x_{0,1}, x_{0,2} \ge 0,$

- in the second year the value $\xi_{1,1}x_{1,1} + \xi_{1,2}x_{1,2}$

  under the assumptions $x_{1,1} + x_{1,2} \le Z_0^3(2), \quad x_{1,1}, x_{1,2} \ge 0$

  (under the assumptions that the profit in the time $t = 1$ can not influence the invested amount in the time $t = 2$). Evidently, it is desirable (for young people) the fulfilling of the relation

$$\frac{(m_2 - m_1)M[\zeta(1+\zeta)^m]}{(1+\zeta)^m - 1} \le (m_2 - m_1)[Z_0^2 - Z_1^2] + \sum_{i=0}^{1}[\xi_{i,1}x_{i,1} + \xi_{i,2}x_{i,2}], \tag{6}$$

  and of course the maximization of a possible profit, or

b. in the first year the value $\xi_{0,1}x_{0,1} + \xi_{0,2}x_{0,2}$

   under the assumptions $x_{0,1} + x_{0,2} \le Z_0^3(1), \quad x_{0,1}, x_{0,2} \ge 0,$

- in the second year the value $\xi_{1,1}x_{1,1} + \xi_{1,2}x_{1,2}$

  under the assumptions $x_{1,1} + x_{1,2} \le Z_0^3(2) + \xi_{0,1}x_{0,1} + \xi_{0,2}x_{0,2}, \quad x_{1,1}, x_{1,2} \ge 0.$

  (The profit obtained in the time $t = 1$ can be invested in the time moment $t = 2$).

Evidently, it is desirable (for young people) that the following relation holds:

$$\frac{(m_2 - m_1)M[\zeta(1+\zeta)^m]}{(1+\zeta)^m - 1} \le (m_2 - m_1)[Z_0^2 - Z_1^2] + \xi_{1,1}x_{1,1} + \xi_{1,2}x_{1,2} \tag{7}$$

and of course the maximization of a total profit.

Remark. $Z_0^3(1)$, $Z_0^3(2)$, $\xi_{0,1}$, $\xi_{0,2}$, $\xi_{1,2}$, $\xi_{1,2}$ are generally supposed to be random variables with "positive support". Consequently, it is necessary to "specify" the sense of relations in A.1. In details, it is necessary to "specify" when the operator of mathematical expectation, probability constraints, risk constraints or stochastic dominance constraints are employed in the optimization problems.

A.2 $Z_0^3(1)$, $Z_0^3(2)$ have a deterministic character. Let us assume that these amounts can be investigated into two assets (portfolio) with returns $\bar{\xi}_{0,1}$, $\bar{\xi}_{0,2}$, $\bar{\xi}_{1,1}$, $\bar{\xi}_{1,2}$. Mathematically saying, it is possible to determine $x_{0,1}$, $x_{0,2}$, $x_{1,1}$, $x_{1,2}$ fulfilling the relations

$$x_{0,1} + x_{0,2} \le Z_0^3(1), \quad x_{0,1},\, x_{0,2} \ge 0,$$
$$x_{1,1} + x_{1,2} \le Z_0^3(2), \quad x_{1,1},\, x_{1,2} \ge 0$$

to obtain random values

$$g_0 = \bar{\xi}_{0,1}x_{0,1} + \bar{\xi}_{0,2}x_{0,2},$$
$$g_1 = \bar{\xi}_{1,1}x_{1,1} + \bar{\xi}_{1,2}x_{1,2}.$$

Evidently, it is possible also to define random values Y_0, Y_1 by the following relation

$$\begin{aligned} Y_0 &= \tfrac{1}{2}\bar{\xi}_{0,1} + \tfrac{1}{2}\bar{\xi}_{0,2}, \\ Y_1 &= \tfrac{1}{2}\bar{\xi}_{1,1} + \tfrac{1}{2}\bar{\xi}_{1,2}. \end{aligned} \tag{8}$$

g_1, Y_1 are random values "depending" on $Z_0^3(1)$, and g_2, Y_2 "depending" on $Z_0^3(2)$. Employing the theory of a stochastic dominance [8] it is "reasonable" to determine $x_{0,1}$, $x_{0,2}$, $x_{1,1}$, $x_{1,2}$ such that

$$\begin{aligned} & F_{g_0} \succeq_1 F_{Y_0}, \qquad F_{g_1} \succeq_1 F_{Y_1}, \\ \text{or} \quad & F_{g_0} \succeq_2 F_{Y_0}, \qquad F_{g_1} \succeq_2 F_{Y_1}. \end{aligned} \tag{9}$$

(Symbols $\succeq_1, \succeq_2$ denote first and second order stochastic dominance; F_{g_0}, F_{g_1}, F_{Y_0}, F_{Y_1} distribution functions of g_0, g_1, Y_0, Y_1.) The definition of the stochastic dominance will be given in the next section. Moreover, we can evaluate the decision of $x_{0,1}, x_{0,2}, x_{1,1}, x_{1,2}$ for example by linear forms

$$c_{1,1}x_{1,1} + c_{12}x_{1,2}, \qquad c_{2,1}x_{2,1} + c_{2,2}x_{2,2}, \tag{10}$$

with $c_{0,1}$, $c_{0,2}$, $c_{1,1}$, $c_{1,2}$ considered generally to be random.

In the introduction we have tried to give a simple analysis of debtor's situation (for a time interval $(0,\, m_2)$) under very simple conditions. We have neglected many troubles and situations that can happen (e.g. illness, a loss of employment). We also omitted a possibility to gain "better" career or only increasing salary. In the next section we shall try to recall a survey of suitable mathematical models corresponding to introduced situations.

3 Stochastic Programming Problems

In this section we try to recall suitable types of the stochastic programming problems in static setting. To this end let $(\Omega, \mathcal{S}, P)$ be a probability space; $\xi\, (:= \xi(\omega) = (\xi_1(\omega), \ldots, \xi_s(\omega),\, \omega \in \Omega)$ an s–dimensional random vector defined on $(\Omega, \mathcal{S}, P)$; $F (:= F_\xi(z), z \in R^s)$ the distribution function of ξ; P_F the probability measure corresponding to F. Let, moreover, $g_0(:= g_0(x, z))$ be a real-valued function defined on $R^n \times R^s$; $X_F \subset X \subset R^n$ a nonempty set generally depending on F, $X \subset R^n$ a nonempty "deterministic" set. If E_F denotes the operator of mathematical expectation corresponding to F and if for $x \in X$ there exists $\mathsf{E}_F g_0(x,\, \xi)$, then one-stage (static) "classical" stochastic optimization problem can be introduced ([6], [8]) in the form:

$$\text{Find} \qquad \varphi(F,\, X_F) = \inf\{\mathsf{E}_F g_0(x, \xi) | x \in X_F\}. \tag{11}$$

To our purpose we recall only special cases of X_F. We consider the case $X_F = X$ "deterministic" constraints; the case when there exist functions $\bar{g}_i(:= \bar{g}_i(x),\, x \in R^n)$, $i = 1, \ldots, s$ such that

- either

$$\begin{aligned} X_F(:= X_F(\alpha)) \;&=\; \bigcap_{i=1}^{s} \{x \in X : P_F[\omega : \bar{g}_i(x) \le \xi_i] \ge \alpha_i\}, \\ &\quad\; \alpha_i \in (0,\, 1),\, i = 1, \ldots, s, \quad \alpha = (\alpha_1, \ldots, \alpha_s), \end{aligned} \tag{12}$$

- or

$$\begin{aligned} X_F(:= X_F(u_0,\, \alpha)) \;&=\; \bigcap_{i=1}^{s} \{x \in X : \min_{u^i}\{P_F[\omega : L_i(x,\, \xi) \le u^i] \ge \alpha_i\} \le u_0^i\}, \\ &\quad\; u_0^i > 0,\, \alpha_i \in (0,\, 1),\, i = 1, \ldots, s, \\ &\quad\; u_0 = (u_0^1, \ldots, u_0^s),\; \alpha = (\alpha_1, \ldots, \alpha_s), \\ L_i(x,\, z) \;&=\; \bar{g}_i(x) - z_i,\, i = 1, \ldots, s,\; z = (z_1, \ldots, z_s). \end{aligned} \tag{13}$$

Evidently, the case (12) corresponds to a special class of individual probability constraints and $L_i(x,\, z)$, $i = 1, \ldots, s$, in the case (13), can be considered as loss functions (for more details see, e.g., [5]).

Second order stochastic dominance constraints are the last considered type of constraints. To recall them let $g(:= g(x,\, z)) := g(x,\, \xi)$ be a function defined on $R^n \times R^s$, $Y(:= Y(z)) := Y(\xi)$ a random value with the distribution function F_Y. We can define first and second stochastic dominance constraints by:

- first order

$$X_F \;=\; \{x \in X : F_{g(x,\, \xi)}(u) \le F_Y(u) \quad \text{for every}\, u \in R^1\}, \tag{14}$$

- second order

$$X_F \;=\; \{x \in X : F^2_{g(x,\, \xi)}(u) \le F^2_Y(u) \quad \text{for every} \quad u \in R^1\}, \tag{15}$$

where $\quad F^2_{g(x,\, \xi)}(u) = \int\limits_{-\infty}^{u} F_{g(x,\, \xi)}(y)dy, \quad F^2_Y(u) = \int\limits_{-\infty}^{u} F_Y(y)dy, \quad u \in R^1.$

(For more information about stochastic dominance see, e.g., [8]).

4 Simple Mathematical Models

In this section we try to introduce simple optimization models in multiobjective (and multiperiod) setting corresponding to A.1. To this end we have to recall and generalize the notions mentioned firstly in the Introduction:

- M .... value of loan,
- m .... number of identical installments,
- ζ .... interest rate corresponding to loan,
- Z_t .... income of young married couple in time point $t \in \{0, 1, \ldots, m\}$,
- U_t .... income of husband in time point $t \in \{0, 1, \ldots, m\}$,
- V_t .... income of wife in time point $t \in \{0, 1, \ldots, m\}$,
- Z_t^1 .... means determined for basic consumption in time point $t \in \{0, 1, \ldots, m\}$,
- Z_t^2 .... means determined for repayment of installment in time point $t \in \{0, 1, \ldots, m\}$,
- Z_t^3 .... allocation (maybe random) for saving in time point $t \in \{0, 1, \ldots, m\}$,
- (m_1, m_2) ...... time interval in which income of wife is supposed to be smaller,
- $\xi_{t,j},\ t = 0, 1, \ldots, m,\ j = 1, 2$ random returns in time t and asset j in the approach A.1a,
- $\bar{\xi}_{t,j},\ t = 0, 1, \ldots, m,\ j = 1, 2$ random returns in time t and asset j in the approach A.1b,
- $x_{t,j},\ \bar{x}_{t,j},\quad t = 0, \ldots,\quad m,\quad j = 1, 2,$ ...... decision variables,
- F ...... a distribution function covering all random values occur that in the corresponding model.

First we generalize the approach of the situation A. 1a: Z_t^3 are for $t \in (1, m_1-1) \bigcup (m_2+1, m)$ supposed to be with probability one positive. Moreover, we assume that the corresponding amount can be investigated (of course in the case of positive value) in two assets with random returns $\xi_{t,1}, \xi_{t,2}$. If moreover we can assume that the profit obtained in the time point $t \in \{1, \ldots, m\}$ can not be investigated in the time $t+1, \ldots, m$, then evidently one of the possible corresponding stochastic optimization problem can be constructed as following:

$$\text{Find} \qquad \max M \tag{16}$$

under the system of constraints

$$\frac{M\zeta(1+\zeta)^m}{(1+\zeta)^m - 1} \le Z_t^2, \quad t = 0, 1, \ldots, m_1 - 1, \tag{17}$$

$$P_F\{x_{t,1} + x_{t,2} \le Z_t^3\} \quad \ge \quad 1 - \varepsilon_t, \quad \varepsilon_t \in (0,1),\ x_{t,1}, x_{t,2} \ge 0, \quad t = 0, 1, \ldots m_1 - 1, \tag{18}$$

$$P_F\{\frac{(m_2 - m_1)M[\zeta(1+\zeta)^m]}{(1+\zeta)^m - 1} \le \sum_{i=m_1}^{m_2} [Z_i^2 - Z_0^2] + \sum_{i=0}^{m_1-1})[\xi_{i,1}x_{i,1} + \xi_{i,2}x_{i,2}]\} \ge 1 - \varepsilon_0, \quad \varepsilon_0 \in (0, 1). \tag{19}$$

Evidently, in this case it is reasonable to add to an objective function (16) the second one

$$\mathsf{E}_F \sum_{i=0}^{m} [\xi_{i,1}x_{i,1} + \xi_{i,2}x_{i,2}] \tag{20}$$

with the corresponding constraints

$$\begin{aligned} P_F\{x_{t,1} + x_{t,2} \le \max(0, Z_t^3)\} \quad &\ge \quad 1 - \varepsilon_t,\ \varepsilon_t \in (0, 1),\ x_{t,1}, x_{t,2} \ge 0, \quad t = m_1, \ldots m_2 - 1, \\ P_F\{x_{t,1} + x_{t,2} \le Z_t^3\} \quad &\ge \quad 1 - \varepsilon_t,\ \varepsilon_t \in (0, 1),\ x_{t,1}, x_{t,2} \ge 0, \quad t = m_2 + 1, \ldots, m. \end{aligned} \tag{21}$$

Consequently, we have constructed two objective stochastic programming problem with objective (16) and (20) and constraints (17), (18), (19) and (21).

Starting with the situation A.1b we can obtain the problem:

$$\text{Find} \qquad \max M \tag{22}$$

under the system of constraints

$$\frac{M\zeta(1+\zeta)^m}{(1+\zeta)^m - 1} \le Z_t^2, \qquad t = 0, \ldots, m_1 - 1, \tag{23}$$

$$\begin{aligned} &P_F\{\bar{x}_{0,1} + \bar{x}_{0,2} \le Z_0^3\} \ge 1 - \varepsilon_0, \ \varepsilon_0 \in (0, 1), \ \bar{x}_{0,1}, \bar{x}_{0,2} \ge 0, \\ &P_F\{\bar{x}_{t,1} + \bar{x}_{t,2} \le \bar{Z}_t^3\} \ge 1 - \varepsilon_t, \ \varepsilon_t \in (0, 1) \quad \bar{x}_{t,1}, \bar{x}_{t,2} \ge 0, \quad t = 1, \ldots, m_1 - 1, \\ &\bar{Z}_t^3 = \max(0, Z_t^3) + \bar{\xi}_{t-1,1}\bar{x}_{t-1,1} + \bar{\xi}_{t-1,2}\bar{x}_{t-1,2}, \quad t = 1, \ldots, m, \end{aligned} \tag{24}$$

$$P_F\{\frac{(m_2 - m_1)M[\zeta(1+\zeta)^m]}{(1+\zeta)^m - 1} \le \sum_{i=m_1}^{m_2} [Z_i^2 - Z_0^2] + [\bar{\xi}_{m_2,1}\bar{x}_{m_2,1} + \bar{\xi}_{m_2,2}\bar{x}_{m_2,2}]\} \ge 1 - \varepsilon_0, \tag{25}$$

Evidently, in this case it is also reasonable to add to the objective function (22) the second one

$$\mathsf{E}_F[\bar{\xi}_{m,1}\bar{x}_{m,1} + \bar{\xi}_{m,2}\bar{x}_{m,2}] \tag{26}$$

and the corresponding constraints

$$P_F\{\bar{x}_{t,1} + \bar{x}_{t,2} \le Z_t^3 + \bar{\xi}_{t-1,1}\bar{x}_{t-1,1} + \bar{\xi}_{t-1,2}\bar{x}_{t-1,2},\} \quad \ge \quad 1 - \varepsilon_t, \ t = m_2 + 1, \ldots, m, \tag{27}$$

Remark. We have supposed (for simplicity) that a profit from the investigation in the time interval $(0, m_2)$ is included in the condition (25) and can not be employed in the time $t = m_2 + 1, \ldots, m$

5 Conclusion

In the last decades many people try to gain their own residence. Since they do not posses sufficient means, the bank sector offer them the loan. The aim of this contribution is to give a preliminary analysis of their situations and possible responsible behaviour. Three approaches have been analyzed in a very simple examples, two of them have been employed for a construction of stochastic optimization models. The results of [1], [3], [4] can be employed to investigate properties of these models. Employing these methodology a risk for young people can happen only with very small prescribed probability. However to deal with this new problem is over the possibilities of this contribution.

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