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# Multi-stage emissions management of a steel company

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## Abstract

We present a multi-stage model for determining the optimal production and emissions coverage for an industrial company participating in the European Emissions Trading System. This model is adapted for a real-life European steel company. A mean-multiperiod CVaR is used as a decision criterion. There are two stochastic parameters—market demand for products and emissions allowance price. The aim of this paper is to explore the costs and risk of a company caused by emissions trading. The presented model is solved for various values of the risk aversion parameters and initial price of the allowance. As a result, it is found that the production is little influenced by the price of allowances and it nearly does not depend on risk-aversion. The probability of the company's default, on the other hand, is significantly influenced by the emission prices. Futures on allowances as well as banking (i.e., transferring allowances between periods) are used to reduce the risks of the emissions trading. We further exploit the same situation under different settings, namely, given random price margins, and time-dependent, deterministic and positively contaminated distributions of demand. In all these cases, the results follow patterns similar to those given the original setting.


**Keywords** Multiperiod CVaR · Multi-stage model · Stochastic programming · Emission allowance · Steel company

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## 1 Introduction

The European Emissions Trading System (EU ETS) has become a subject of discussion among both politicians and researchers since immediately after its launch in 2005. This system exerts pressure on cleaner production and thus on protection of the environment on one side. But, on the other side, it has also a negative effect on competitiveness of European companies as it brings along additional costs and financial risks.

Under the EU ETS, each tonne of $CO_2$ released to the atmosphere during a single year must be covered by one emission allowance (EUA—European Union Allowance) by the end of March in the following year (see Council of European Union 2003). In this paper, we explore a quantitative impact of the ETS on a $CO_2$ emitting company, namely on its profitability and default probability. Moreover, we explore to what extent the risks associated with emission trading can be reduced.

The EUAs can be purchased either in auctions or on the secondary market. To compensate the burden imposed by the system, a certain amount of allowances is given to the company for free from the state, which is called *grandfathering*.

The main goal of this paper is to assess the impact of the EU ETS on the economic situation of a participating company and study ways of reducing the risk brought by emissions trading. In principle, these risks can be reduced either by using derivatives, most often by futures, or by so called *banking*, i.e. transferring unused allowances to the future trading periods.

In particular, the following questions are to be explored:

- How does risk aversion, allowance price and existence of derivatives influence production?
- Do these factors influence the chance of the company's survival?
- Which combination of spots (the EUAs themselves) and their futures is optimal in reducing the financial risk?
- To what extent can banking be used to reduce the risk?

Answers to these questions are sought by means of a case study of an anonymous Czech steel company, production of which is modelled by a multi-stage stochastic model, calibrated by means of the data provided by the company. Two parameters of the model are taken as stochastic—demand for products and price of emission allowances. The decision variables of the model include the production, the amount of EUAs purchased, and the amount of futures purchased. As a decision criterion, multi-period mean-CVaR criterion is used. The decision period is equal to 1 year.

As the true parameter of risk aversion can hardly be known, we solve the problem for several levels of risk aversion. Further, to study the impact of the allowance price on the company, the problem is solved at several levels of allowance prices as well as for the hypothetical case when the futures cannot be used.

In addition, four alternative settings are studied. First, to reflect the fact that our data come from times of recession, we study the possibility of a positive deman shock, modelled by means of contamination. Second, the restricting assumption of deterministic price margins is relaxed and the margins are taken as the third random parameter. Third, the demand is taken as time dependent rather than i.i.d. Fourth, to isolate the effect of emission trading, the original analysis is repeated while assuming the demand to be deterministic.

Several studies exist that have analysed optimal decisions of a single company under the EU ETS. However, the assumptions of these studies are much more restrictive than ours. Moreover, the other studies were built under different (past) rules and conditions of the emission trading system, and most of them have been designed for only one trading period, see Šmíd et al. (2017), Tang and Song (2013) and Zapletal and Šmíd (2016). Only two multi-stage stochastic models have been published so far: the one by Rong and Landhelma (see Rong and Lahdelma 2007) and the one by Gong and Zhou (2013), the latter even involving futures (excluding spots, on the other hand); however, neither of them involves any risk measures, using only expectation. An optimization model considering derivatives and the mean-CVaR risk measure has been published (Šmíd et al. 2017); however, this model only covers a single period.

Obviously, using dynamic CVaR as a risk measure in multi-stage optimization of industrial companies' policy is not a new idea. Pisciella et al. (2016), for instance, developed the three-stage model for power generation capacity expansion planning and used the nested-CVaR risk measure there. Moreover, several papers exist using the CVaR criterion in multi-stage asset allocation (see. e.g., Kopa et al. 2018) and in asset liability management (see. e.g., Moriggia et al. 2019), which is, interestingly, similar to the problem solved in this paper (emission covering can be seen as a liability). Nobody has, however, applied this or a similar risk measure to the optimal decision of a company trading with emissions.

Below we list the basic assumptions of our model together with their justifications.

- *The company decides on its production and allowance portfolios.*
- *Four periods are considered* ($T = 4$) *and, therefore, four decisions are made at the beginning of each period.* The time period has been chosen as 2017–2020, reflecting the fact that the current trading phase of the EU ETS ends just in 2020. After this time, some modifications of the trading rules can be expected.
- *Banking of allowances is enabled.* In this way, the company can hedge against the risks caused by price fluctuation. Because all the models published so far except of Rong and Lahdelma (2007) have only been single-period, it is very desirable to explore this option.
- *Certain amounts of EUAs are given to the company for free (grandfathered).* The actual numbers of the allowances grandfathered are fixed, given by the rules of the system.
- *Additional/excess EUA spots may be bought/sold at a secondary market for exogeneous price.*
- *At each time* $t = 0, 1, 2, 3$, *futures with maturities* $\tau = t + 1, t + 2, \ldots, T$ *may be purchased.* Recall that a future is a standardised financial instrument entitling its holder, at its maturity time, to buy the corresponding spot for the current (future) price, payable at the maturity date.
- *Mean-risk with multi-period CVaR serves as the decision criterion.* Use of CVaR for single-stage models is very frequent and reasonable because this risk measure is coherent by Artzner et al. (1999) and can also be easily linearised (cf. Rockafellar and Uryasev 2002). The (single period) CVaR has already been used even for modelling of companies' behaviour under the EU ETS, see Šmíd et al. (2017) or Luo and Desheng (2016). The multi-period CVaR is used in this paper because it is coherent (see Shapiro and Ugurlu 2016) and time consistent (see Kovacevic and Pflug 2009).
- *The company may fund its emission trading by loans with the interest rate of* $\varrho = 4\%$. *A prohibitive interest rate of* $\iota = 15\%$ *is paid in case of lack of money at the end of the last period.* These values have been determined by an anonymous expert on economics.
- *Demands and allowance prices are stochastic.* All the remaining factors are supposed to be deterministic.

This paper is organized as follows. After this Introduction, the structure of the model is presented in Sect. 2. Section 3 is devoted to the description and analysis of input data. Section 4 describes our analysis and studies its results. The next Sect. 5 describes various perturbations of our model: its stress-testing by means of contaminating the demand distribution, a model with stochastic profit margins, and a model with time-dependent demand. Finally, the paper is concluded (Sect. 6).

## 2 The model

At each time $t = 0, \dots, T-1$, the company decides on its final production $x_t \in \mathbb{R}^n_+$, $n \in \mathbb{N}$, which has to be no greater than the current demand $d_t \in \mathbb{R}^n_+$:

$$x_t \le d_t. \quad (D_t)$$

The raw production, needed for the final production $x$, is given by

$$y_t = R x_t$$

where $R \in \mathbb{R}^{n\times n}$, $R = (I - A)^{-1}$ is a matrix derived using the Leontief's input/output model of the company's production process (where $A \in \mathbb{R}^{n\times n}$ is a matrix of production coefficients, and $I$ is an identity matrix of size $n$), cf. Miller and Blair (2009). The raw production is subject to production limits:

$$y_t \le w \quad (P_t),$$

where $w \in \mathbb{R}^n_+$ is a constant.

The profit resulting from production $x_t$ is $m x_t$ where $m \in \mathbb{R}^n_+$ is a deterministic vector of margins corresponding to individual final products.

The $CO_2$ emissions released by production $y_{t-1}$ are given by

$$h' y_{t-1}$$

where $h$ is a deterministic vector, constant over time, which determines the amounts of $CO_2$ stemming from the individual raw products.

At each $t = 1, \dots, T$, the company is given $r_t$ (grandfathered) allowances. It is assumed that $r_t$ is deterministic for each $t$. In addition, at each $t = 0, \dots T$, they buy $s_t \in \mathbb{R}$ of spots and $f_t^{t+1}, \dots, f_t^T \in \mathbb{R}_+$ of futures with maturities $t+1, \dots, T$, respectively. Consequently, the total number of spots $e_t$ held at time (immediately after) $t$ is given by

$$\begin{aligned} e_0 &= s_0, \quad (E_0) \\ e_t &= e_{t-1} + s_t + \sum_{0 \le \tau < t} f_\tau^t + r_t - h' y_{t-1}, \qquad (E_t) \end{aligned}$$

$1 \le t \le T$. (In words, the increment in the spots quantity held is given by the sum of the spots bought at $t$, the number of the spots obtained for free, and the total number of futures maturing at $t$ minus the number needed to cover the emissions from the previous period.)

The company does note speculate, which we express by the following restrictions:

$$\begin{aligned} &s_0 \ge 0, \quad (S_0) \\ &s_t \ge -r_t, \quad f_t^{t+1}, \dots, f_t^T \ge 0, \quad e_t \ge 0, \quad (S_t) \end{aligned}$$

$t = 1, \dots T$, i.e., no short selling is allowed, the futures cannot be sold, and no more than the grandfathered number of spots may be sold.

The spots are paid immediately, the futures are paid at their maturity. Purchasing of the allowances may be funded by loans with the interest rate of $\varrho$. The cash missing at the last period is penalized by a prohibitive interest rate of $\iota$. Thus, the cash-flows values at individual times are given by the profit from production minus the sum of costs on allowances and the debt maintenance:

$$z_0 = -p_0 s_0, \quad (Z_0)$$
$$z_t = m' x_{t-1} - p_t s_t - \sum_{\tau=0}^{t-1} q_\tau^t f_\tau^t - \varrho c_{t-1}, \quad (Z_t)$$

for $1 \leq t \leq T-1$, and

$$z_T = m' x_{T-1} - p_T s_T - \sum_{\tau=0}^{T-1} q_\tau^T f_\tau^T - \varrho c_{T-1} - \iota c_T. \quad (Z_T)$$

Here, $c_\tau = \left[\sum_{s=0}^{\tau} z_s\right]_-$ is the amount owed at time $\tau$, $p_\tau, q_\tau^t$ are the prices of spots, futures with maturity $t$, respectively, at $\tau$.

Naturally, we require the decision processes to be non-anticipative

$$x_t, s_t, f_t^{t+1}, \ldots, f_t^T \in \mathcal{F}_t, \quad (M_t)$$
$$\mathcal{F}_t = \sigma((\pi_\tau)_{\tau \leq t}), \quad \pi_t = (p_t, q_t, d_t), \ t = 0, \ldots, T.$$

The optimisation model itself is expressed as follows:

$$\begin{aligned}
&\min \rho_\lambda(-z_0, \ldots, -z_T) \\
&\text{s.t. } x_t \leq d_t, && 0 \leq t \leq T-1, (D_t) \\
&\quad y_t \leq w, && 0 \leq t \leq T-1, (P_t) \\
&\quad x_t, s_t, f_t^{t+1}, \ldots, f_t^T \in \mathcal{F}_t, && 0 \leq t \leq T-1, (M_t) \\
&\quad e_0 = s_0, && (E_0) \\
&\quad e_t = e_{t-1} + s_t + \textstyle\sum_{0 \leq \tau < t} f_\tau^t + r_t - h' y_{t-1}, && 1 \leq t \leq T, \quad (E_t) \\
&\quad s_0 \geq 0, && (S_0) \\
&\quad s_t \geq -r_t, \quad f_t^{t+1}, \ldots, f_t^T \geq 0, \quad e_t \geq 0, && 1 \leq t \leq T, \quad (S_t) \\
&\quad z_0 = -p_0 s_0, && (Z_0) \\
&\quad z_t = m' x_{t-1} - p_t s_t - \textstyle\sum_{\tau=0}^{t-1} q_\tau^t f_\tau^t - \varrho c_{t-1}, && 1 \leq t \leq T-1, (Z_t) \\
&\quad z_T = m' x_{T-1} - p_T s_T - \textstyle\sum_{\tau=0}^{T-1} q_\tau^T f_\tau^T - \varrho c_{T-1} - \iota c_T, && (Z_T)
\end{aligned}$$

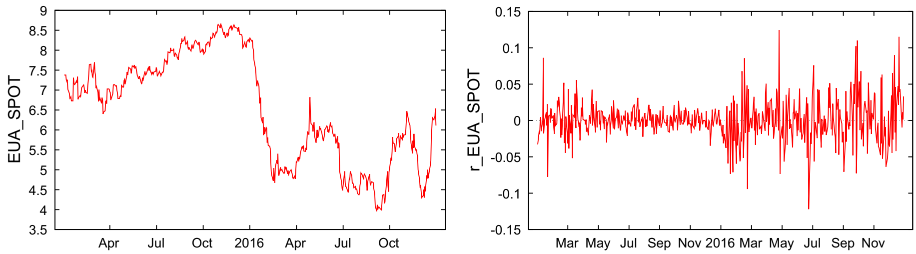
where the decision criterion is mean-multiperiod CVaR:

$$\rho_\lambda(u_0, \ldots, u_T) = (1-\lambda)\mathbb{E}\left(\sum_{\tau=0}^{T} u_\tau\right) - \lambda \mathcal{R}(u_0, \ldots, u_T),$$
$$\mathcal{R}(u_0, \ldots, u_T) = \sum_{t=0}^{T} \mathbb{E}\left[\text{CVaR}_{\alpha_t}[u_t | \mathcal{F}_{t-1}]\right].$$

## 3 Random parameters

As already mentioned, the random parameters considered in the model include the prices of EUA spots, and consequently their futures, and the demands for the products.

Historical prices of the EUA spots, which we obtained from the ICE trading platform www.theice.com, and their returns are depicted in Fig. 1. Even though the graphs suggest




**(a)** Prices **(b)** Returns

**Fig. 1** EUA spot prices and returns (2015–2016)

some heterogeneity, we decided to assume the price returns to be mutually independent; as only yearly returns enter our model, the assumption of independence would not much harm reality. In particular we assume that

$$p_t = p_0 \exp\left\{\sum_{\tau=1}^{t} u_\tau\right\}$$

where $u_1, \ldots, u_t$ are i.i.d. normal with standard deviation $\sigma = 0.439$, where the latter value was estimated from the price time series. In order to preclude speculation, we assumed $p_t$ to be a martingale (i.e. $\mathbb{E}u_\tau = -\frac{\operatorname{var} u_\tau}{2}$). The initial price $p_0$ was set to the value from the first trading day in 2017 on the ICE trading platform ($p_0 = 6.54$ EUR).

The historical future prices and their differences from the spot prices are depicted in Figs. 2 and 3, respectively. It is clearly seen that the differences are positive at the majority of times, decreasing with the time approaching maturity. It follows from the arbitrage arguments that the differences have to converge to zero at the maturity. Thus, to fit the dynamics of the price $q_t^s$ of a future with the maturity $s$, we consider a noised cost-of-carry model

$$q_t^s = \exp\{a(s-t) + \epsilon_t\} p_t, \quad \operatorname{var}(\epsilon_t) = (s-t)^2 \sigma^2, \quad t \le s,$$

where $a$ is a real parameter and $\epsilon_1, \epsilon_2, \ldots$ are centred mutually independent random variables. Note that, in this model, the future prices are on average higher than the spot ones and their difference are decreasing in time, converging to zero at maturity.

As for the estimation of the parameters, it follows that, for any $s$, the log-future-spot spreads $y_{t,s} = \log q_t^s - \log p_t$, $t \le s$, satisfy a simple linear regression model

$$\frac{y_{t,s}}{s-t} = a + \eta_t$$

where $\eta_t$ is a white noise. By its estimation, we obtain $a \doteq 0.00974$, $\operatorname{stdev}(\eta_t) = 0.010$.

For computational simplicity, we further assumed $\eta_t \equiv 0$. Consequently, our model for the future prices becomes

$$q_t^s = \exp\{0.00974(s-t)\} p_t$$

Unfortunately, we do not have historical records of the demand $d_t$ at our disposal. The only information we have is an estimate of the demands' means; therefore, we based our estimate of the demand distribution on the data of the Czech nationwide demands for the long, flat and semi-finished steel products, coming from the Steel Union of the Czech Republic (Fig. 4). After a basic analysis of these series, we have decided to take these series as three-dimensional i.i.d. Gaussian. Consequently, we take the distribution of $d_t$ as the one of the nationwide demands, rescaled so as to have expectations $\mathbb{E}d_t$. The correlation coefficient

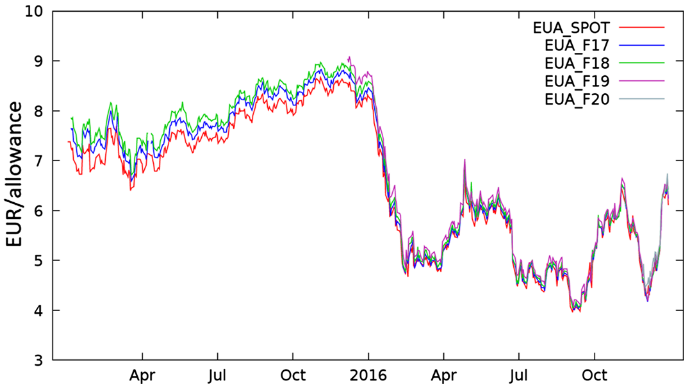



**Fig. 2** Prices of EUA futures (2015–2016)

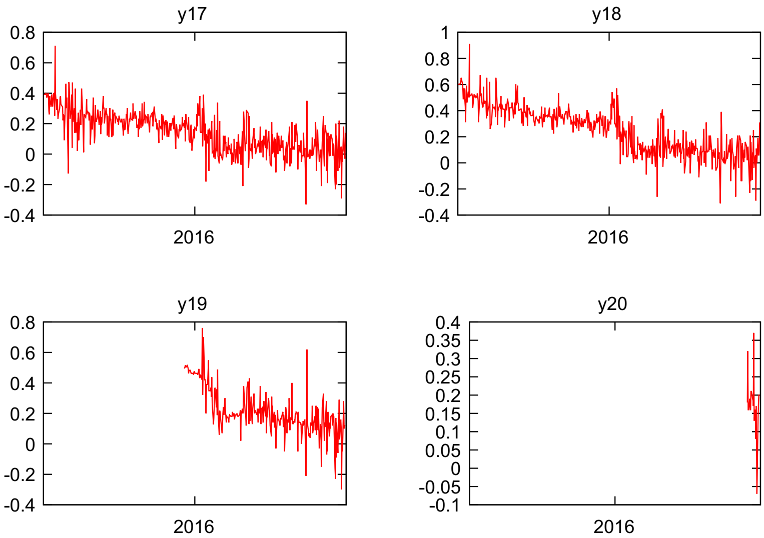



**Fig. 3** Future-spot spreads for futures with maturity 2017–2020

between the demands for plates and cuts, which both belong to the same category (flat), is assumed to equal one. As a result, $d_t$ came out as a series of i.i.d. normal variables with

$$\mathbb{E}d_t = \begin{bmatrix} 510 \\ 28 \\ 90 \\ 20 \end{bmatrix}, \qquad \operatorname{var}(d_t) = \begin{bmatrix} 36219.4 & 603.2 & 145.2 & 0.0 \\ & 10.0 & 2.9 & 0.0 \\ & & 14.2 & 0.0 \\ & & & 5.8 \end{bmatrix}.$$

Finally, as the correlations between the spot prices and the demmand proxies has come out insignificant, we assumed $u_t$ and $d_t$ to be independent.

For the optimization problem to be tractable, the (Gaussian) distribution of $(u_t, d_t)$ is approximated by a product of distributions $\tilde{u}_t$ and $\tilde{d}_t$, where $\tilde{u}_t$ is a discrete variable having three-atom symmetric distribution matching the mean and variance of $u_t$ and $\tilde{d}_t = \mathbb{E}d_t + W\upsilon_t$ where $W \in \mathbb{R}^{4\times 3}$ is a matrix fulfilling $WW' = \operatorname{var}(d_t)$ and $\upsilon_t \in \mathbb{R}^3$ is a random vector with

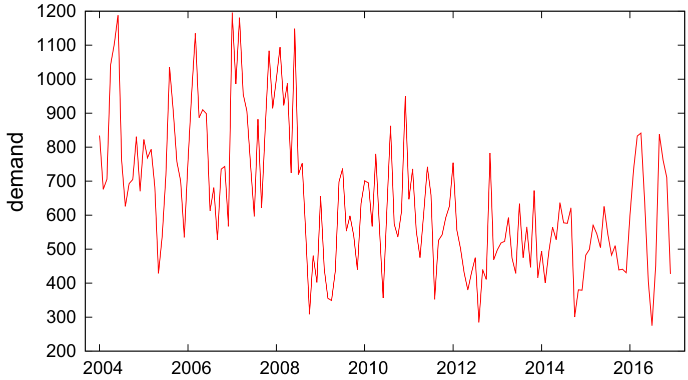



**Fig. 4** Time series of monthly (approximated) overall demand (in thousands of tons)

the uniform distribution with atoms $\{-1, 1\} \times \{-1, 1\} \times \{-1, 1\}$. Note that $(\tilde{u}_t, \tilde{d}_t)$ matches the first two moments of $(u_t, d_t)$.

Consequently, the process of the random parameters was approximated by a scenario tree, each of its nodes at stage $0, 1, \ldots, T-2$ has $3 \times 8 = 24$ branches, and each node at stage $T-1$ has three branches, i.e. the tree has a total of $24^3 \times 3 = 41{,}472$ scenarios. For the discussion on the approximation error risen by replacing a continuous distribution by a discrete one, see e.g., Šmíd (2009).

## 4 Results

The problem has been solved for values $0, 0.1, \ldots, 1$ of the risk-aversion coefficient $\lambda$ and different levels of the initial price of allowance $p_0$—the multiples of $0, 1, 2, 4$ and 8 of the actual price $p_0 = 6.54$ EUR, the zero value corresponding to the case without the covering obligation. For each combination of the parameters, two cases are tested—the first one allowing for trading futures and the second one without the futures. A total of $11 \times 5 \times 2 = 110$ instances of the problem has thus been solved.

Each instance is solved via its deterministic equivalent. The whole procedure is implemented in `C++` program using `CPlex` to solve those deterministic equivalents. The total running time of all the instances was about 24 h using PC with Intel Core I7, 3.40 GHz and 16 GB RAM.

Next, we discuss how our results answered our research questions.

The first question concerns the influence of an allowance price, existence of derivatives and risk aversion on production.

Table 1 shows a comparison between the amount of the expected total production for the state without emissions trading ($p_0 = 0$) and situations under various multiples of the current value of $p_0$, in particular, with $p_0 = 6.54, 13.08, 26.16$ and $52.32$. The values in this table are percentages of the maximal possible production (i.e. maximum of the demand and the production capacity), averaged over all the scenarios. In case the percentage is the same for all $\lambda$'s, the result is a singleton. Otherwise, it is given by an interval.

For the values of $p_0$ lower than or equal to 13.08, the production is the maximum one. But, the fourfold increase ($p_0 = 26.16$) begins to influence the production for the last 2 years (but the drop is less than 1 pp of the total produced volume). In case of the eightfold increase

**Table 1** Total amount of production for various $p_0$ (100% = amount without the emissions trading)

| $(p_0)$ | 2017 (%) | 2018 (%) | 2019 (%) | 2020 (%) |
|---|---|---|---|---|
| 0 | 100 | 100 | 100 | 100 |
| 6.54 | 100 | 100 | 100 | 100 |
| 13.08 | 100 | 100 | 100 | 100 |
| 26.16 | 100 | 100 | 99.65 | 99.53–99.65 |
| 52.32 | 100 | 99.04 | 99.04 | 95.91–97.78 |

in the initial spot price, a restricting effect occurs already in 2018. However, the decrease is still not significant.

The two remaining factors, i.e., existence of futures and risk aversion, do not influence the production at all under the current emission price.

Hence, the answer to the first question is that neither allowance price, nor existence of derivatives can influence the production of our company significantly. The reason is that the production is profitable even under relatively high allowance prices; the restriction occurs only when the price is very high, which, in our setting, corresponds to only a small percentage of the scenarios.

The second question to explore was the probability of the company's default. As the default, we consider the state when the total loss accumulated at up to $T$ is greater than a certain threshold, set by an expert at economics. In particular, the default happens if

$$z_0 + \sum_{t=1}^{T}(z_i - \phi_i) < -u$$

where $u$ is the threshold and $\phi_t$ are fixed costs at $t$. The actual values of $u$ and $\phi_t$ are kept confidential according to our agreement with the company.

The results of the analysis for $\lambda = 0.2$ are displayed in Table 2 together with the corresponding values of the expected loss. Different values of the initial allowance price $p_0$ are taken. If the EU ETS had not existed (i.e. if $p_0 = 0$), the probability of a default would be equal to 73.4%. The current EUA price increases this probability by 0.066. Naturally, the probability further grows with increasing $p_0$ (emissions trading brings an additional risk). The high values of this probability even for the situation without the impact of the system stems from poor economic situation of the modelled company (and also of the steel sector itself). An effect of risk aversion and existence of futures is negligible; the differences in percentages values are below 0.2%.

Thus, the answer to the second question is that the emissions trading, namely the allowance price, affects the probability of a default, whereas the influence of risk aversion and existence of futures is negligible.

The results answering the third question (the optimal portfolio of allowances) are shown in Figs. 5 and 6. Two facts are apparently common for all these stages—futures are always used for hedging when considering a risk-averse decision maker ($\lambda > 0$) and the traded amount of spots decreases with a rising value of $\lambda$. Figure 5 presents the structure of the purchased/sold allowances for all years averaged by scenarios.

In 2017 (Fig. 5a), the same amount of futures are purchased for all positive $\lambda$'s, whereas the spots are also traded in all the following stages. In particular, spots are purchased (the volume decreases with rising risk-aversion) in 2018 (Fig. 5b). In the last two stages (2019

**Table 2** Results for various initial prices of permits ($\lambda = 0.2$)

| ($p_0$) (EUR) | Probability of default (%) | Expected loss (M EUR) |
|---|---|---|
| 0 | 73.4 | 23.35 |
| 6.54 | 80.0 | 24.94 |
| 13.08 | 83.5 | 26.54 |
| 26.16 | 87.9 | 29.74 |
| 52.32 | 95.6 | 35.88 |

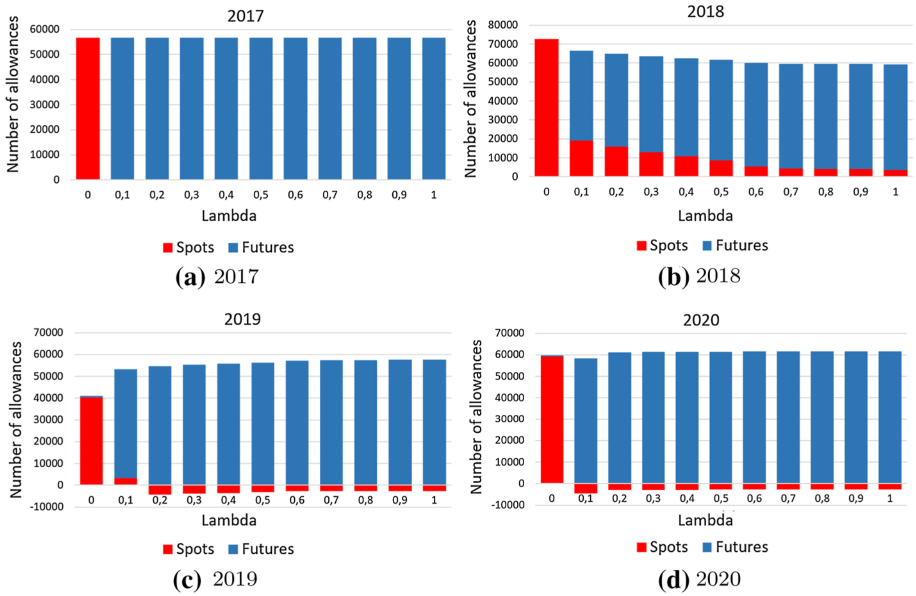



**Fig. 5** Optimal structure of purchased/sold allowances for various $\lambda$'s (average values for all scenarios)

and 2020), which are displayed in Fig. 5c, d, respectively, spots are only sold in relatively small amounts (except for 2019 when $\lambda = 0.1$) .

The differences between the volume of allowances needed and purchased within all the periods fall with increasing $\lambda$, see Fig. 6. A more detailed structure of the allowances needed and bought is provided by Fig. 7.

To summarize these results, it appears that the EUA futures represent a desirable way to hedge against the risks caused by the emissions trading. It is worth noting that, in this instance, only the futures maturing at $t + 1$ are purchased at any $t$. This is given by the nature of the multi-period CVaR criterion, which assigns zero risk to all the futures, regardless of their maturity; so it is always optimal to buy those with the nearest maturity, which are always the cheapest in our setting.

The last question asked in the introduction concerns banking of allowances between time periods. As the prices of the allowances are martingales in our setting, there is no sense in banking the futures given risk neutrality; however, given risk aversion, banking can be used for hedging against the risks. Two cases are studied: the situation when futures are allowed (see Fig. 8) and the case with the futures excluded (see Fig. 9).

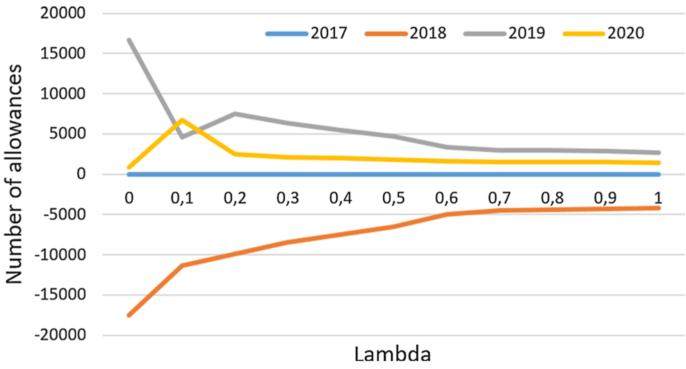
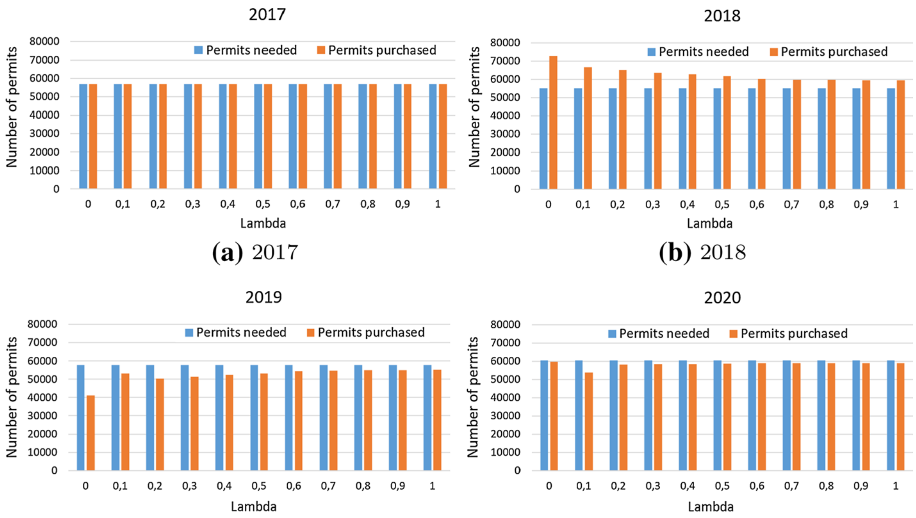



**Fig. 6** Differences between needed and purchased volume of allowances for various λ's (average values for all scenarios)




**(a)** 2017 **(b)** 2018 **(c)** 2019 **(d)** 2020

**Fig. 7** Amounts of allowances needed/purchased for various λ's (average values for all scenarios)

With the futures allowed (Fig. 8), banking is only used for 2018 → 2019 and for 2019 → 2020, with the volumes of the banked allowances falling with the increasing λ. This means, that the futures appear to be more suitable for risk hedging than banking.

The situation is almost opposite when the use of futures is forbidden, see Fig. 9. Here, banking is the only way to decrease risk and it is thus used for all the time periods and for all positive values of λ. In general, the banked volumes increase together with a degree of risk aversion.

Based on the information written above, banking can help in emission trading both with and without the futures. However, quite naturally, banking is used more and the banked amounts are greater when the futures are excluded.

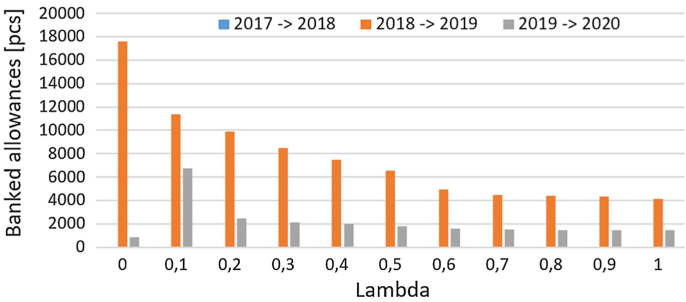



**Fig. 8** Banking of allowances when futures are allowed (average values for all scenarios)

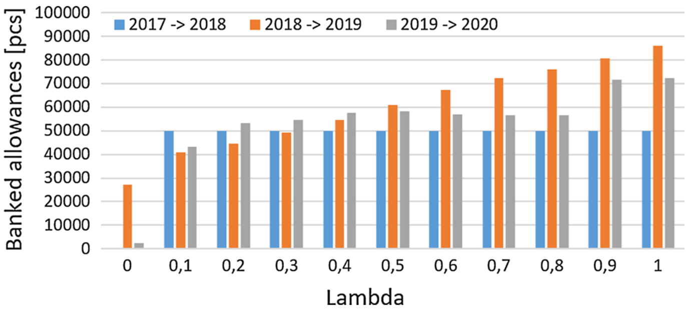



**Fig. 9** Banking of allowances when futures are excluded (average values for all scenarios)

## 5 Alternative models

Due to the fact that our model is based on many simplifying assumptions, we provide a further what-if analysis. In this section, we test how different settings of the model influence the results. Namely, we run the model with contaminated probability distribution of the market demand, non-stationary demand and stochastic profit margins.

### 5.1 Positive demand shocks

As the data used are highly influenced by the recession in the steel industry and a bad financial situation of the modelled company, a question arises, whether the results would be different if the situation got better. To answer it, a stress testing analysis is performed. In particular, it is explored how the optimal policy of the company as well as the probability of a default responds to positive shocks in demand.

For the stress testing analysis, we use contamination, which consists of perturbing the underlying probability distribution using an additional information about the possible changes in the distribution. In this section, we analyze the effect of the contamination on the production and the probability of default. Similarly to Dupačová and Kopa (2012) and Dupačová and Kopa (2014), both the objective and the constraints of the problem are contaminated by an alternative probability distribution. In our model, contrary to Dupačová and Kopa (2012), or Dupačová and Kopa (2014) we contaminate the conditional distribution $P_{d_t|d_{t-1}}$ by an alternative conditional distribution $Q_{d_t|d_{t-1}}$ for $t = 1, 2, 3$. The contaminated conditional distribution with parameter $\nu_t$ is defined as: $P_{d_t|d_{t-1}}(\nu_t) = (1 - \nu_t)P_{d_t|d_{t-1}} + \nu_t Q_{d_t|d_{t-1}}$,

**Table 3** The results after contamination

| | Without contamination | $\nu = 8\%$ | $\nu = 16\%$ | $\nu = 24\%$ |
|---|---|---|---|---|
| Amount of production (%) | 100.00 | 101.46 | 102.93 | 104.39 |
| Probability of default (%) | 80.00 | 75.64 | 71.01 | 66.12 |

$\nu_t \in (0, 1)$. This construction guarantees that the cumulative distribution function of the contaminated distribution is a linear convex combination (with parameter $\nu_t$) of the cumulative distribution functions of the original and alternative conditional distributions.

In stress testing, the alternative (stress) distribution is typically given exogenously, often given by just one (very unfavorable) scenario. However, in our model, the alternative conditional distributions are derived from the original ones, because we assume that a shock (negative or positive) in the demand will affect all the scenarios. Since the original scenario tree is inter-stage independent, $P_{d_t|d_{t-1}}$ is the same for all $t$ and all realizations $d_{t-1}(\omega^s)$ : $\mathtt{P}(d_t = v_i|d_{t-1}) = \frac{1}{8}$ for all $i = 1, ..., 8$, $t = 1, ..., 3$. A shock in demand will cause a shift in this distribution, i.e., the alternative conditional distribution is: $\mathtt{P}(d_t = v_i + c_t|d_{t-1}) = \frac{1}{8}$ for all $i = 1, ..., 8$, $t = 1, ..., 4$ where $c_t$ is a demand shock which may differ in time however; it is the same for all the scenarios. Then the contaminated conditional distribution with parameter $\nu_t$ may be described as follows:

$$\mathtt{P}(d_t = v_i|d_{t-1}) = (1 - \nu_t)\frac{1}{8}, \quad \mathtt{P}(d_t = v_i + c_t|d_{t-1}) = \nu_t \frac{1}{8}, \quad i = 1, ..., 8, \quad t = 1, ..., 3.$$

If moreover $c_t$ and $\nu_t$ is constant in time, i.e. $c_t = c$, $\nu_t = \nu$ then the contaminated scenario tree again be inter-stage independent. Anyway, the contaminated scenario tree is regular, but every node now has 16 children instead of 8 in the original tree. This, of course, increases the computational demand of the proble but, at the same time, it allows us to analyse the results with respect to both the value of shocks $c_t$ and the contamination parameters $\nu_t$, which express the chances of shocks.

The values of shocks for all products are set in line with the most optimistic scenario ($S5$) in Zapletal and Šmíd (2016). In particular, 1.33 multiples for flat products and 2.03 multiples for long products and semiproducs are considered. The resulting vector of changes in demand is

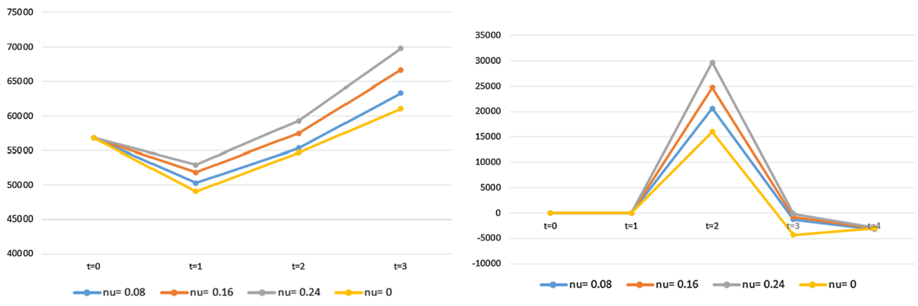
$$\boldsymbol{c}^{\mathrm{T}} = [168.3, 9.24, 92.7, 20.6].$$

It would be reasonable to use the continuum of values of contamination parameters $\nu$ and risk aversion coefficients $\lambda$. However, as the contaminated scenario tree is eight times larger than the original one ($48 \times 48 \times 48 \times 3 = 331{,}776$ scenarios), the computational time increases (8 h for a single instance), the computations have been run only for a single risk aversion $\lambda = 0.2$ and three values of contamination parameters $\nu$: $\nu^1 = 0.08$, $\nu^2 = 0.16$ and $\nu^3 = 0.24$ (these values have been chosen to reasonably cover the upper quartile) which means that the probability that the demand will jump by $\boldsymbol{c}$ at the stage $t$ is 8%, 16%, and 24%, respectively.

We have decided to use the same contamination parameter as well as values of shocks for all the stages. But, if some evidence of irregular chance, or impact of shock exists, different values for each stage can be chosen.

The results of the stress testing are shown in Table 3 and Fig. 10.

It follows from Table 3 that, even after the shock, the production still follows the demand, and the average values of the production as well as $CO_2$ emissions increase.




**(a)** Futures **(b)** Spots

**Fig. 10** Impact of contamination on traded allowances (average values for all scenarios)

The impact of the changes on the optimal number of allowances to trade is provided by Fig. 10. It can be seen that the optimal policies are very similar for all four considered situations. The transactions of allowances only differ in volume. Naturally, the higher value of $\nu$, meaning a higher average demand, the larger amount of allowances (regardless of their type) to purchase, or vice versa, the smaller amount of allowances to sell.

Table 3 also shows the changes of a default probabilities depending on the contaminating parameter. As it could have been expected, the positive change in demand decreases the probability of the company's default. The value of 80%, achieved without the contamination (see Table 2) is decreased to 75.6% (for $\nu^1 = 0.08$), 71% (for $\nu^2 = 0.16$) and 66.1% (for $\nu^3 = 0.24$), respectively. Note that the probability of a default for the non-contaminated scenario without emissions trading (i.e., 73.4%, see Table 2) is between the values for contaminated scenarios $\nu^1 = 0.08$ and $\nu^2 = 0.16$.

Summarised, the effect of positive shocks in demand is not surprising. In particular, the production as well as the way of the risk hedging, albeit changing the expected way, follow the same pattern as those given by the low demand. The default probabilities are naturally descreased; however, they still remain large. Maybe it is worth noting here, that the modelled company has indeed been closed down before these results are published.

## 5.2 Stochastic margins

A question may arise to what extent our results are influenced by our simplifying assumption of constant product margins $m$ while these numbers vary in practice. To examine this question, we ran our computations with stochastic margin $\tilde{m}$ taking two possible equiprobable values $(1-\alpha)m$, $(1+\alpha)m$, where $0 < \alpha < 1$. Due to the lack of empirical knowledge, we assume $\tilde{m}$ to be independent of the demand and the spot prices. Similarly to the case of contamination, the size of the scenario tree increased eight times by adding random parameter $\tilde{m}$; hence we run the computation only for a single $\lambda = 0.2$ and three values of $\alpha = 0.05, 0.15, 0.25$.

The results, namely the default probabilities and the differences in the average amounts of the spots/futures purchased, for the three values of parameter $\alpha$, are shown in Tables 4 and 5, respectively.

The decrease in the default probability, seemingly counter-intuitive (one would expect this risk to increase with additional randomness), can easily be explained: as the default probability is always above 50% and as the increasing variability widens the income distribution symmetrically, the default probability decreases because the lower half of the distribution is "lost anyway" and widening of the distribution increases its mass above the probability threshold level.

**Table 4** Probability of default in case of stochastic demands (and their comparison with the results of the original model), ($\lambda = 0.2$, $p_0 = 6.54$)

| $\alpha$ | 0.05 | 0.15 | 0.25 |
|---|---|---|---|
| Probability of default (%) | 78.23 | 75.65 | 72.55 |
| Comparison with the original results (p.p.) | – 1.77 | – 4.65 | –7.45 |

**Table 5** Traded allowances in case of stochastic demands compared to the original model ($\lambda = 0.2$, $p_0 = 6.54$)

| $\alpha$ | Deviation from the original model | 2017 | 2018 | 2019 | 2020 |
|---|---|---|---|---|---|
| 0.05 | Futures | 0 | 2738 | 849 | –264 |
| | Spots | 0 | – 5146 | 1893 | – 70 |
| 0.15 | Futures | 0 | – 4118 | – 2784 | – 961 |
| | Spots | 0 | 7740 | 517 | – 395 |
| 0.25 | Futures | 0 | – 5338 | – 3918 | – 1652 |
| | Spots | 0 | 10034 | 1544 | – 670 |

As for the emissions trading, only slight changes in the optimal portfolio have occurred without any clear pattern. For $\alpha = 0.05$, the company buys more futures in comparison with the original model by 1.5%. However, for $\alpha = 0.15$ and $\alpha = 0.25$, the original amounts of the traded futures are reduced by 3.5% and 4.9%, respectively.

### 5.3 Dependent demands

Another simplification we have made is that the distribution of demands $d_t$ is i.i.d. In practice, however, the demands are likely to be autocorrelated. To examine the effect of autocorrelation, we rerun our analysis with demands following a simple AR(1) process $d_t = 0.5d_{t-1} + 0.5(\mathbb{E}d_1 + \varepsilon_t)$ with $\varepsilon_t$ binomial symmetric with its atoms set so that the average of unconditional standard deviations of $d_1, d_2$ and $d_3$ is the same as the standard deviation of the original $d_i$ (see Sect. 3). Again, we run our analysis for a single value $\lambda = 0.2$ and the actual price level.

As a result of the change in demand distribution, only small changes in the production occur. In particular, the total production increases by roughly 1% in the second period and decreases in the remaining two periods (about 1% each year). It is worth noting that despite the overall production (of all the products and all the periods together) is slightly decreased, the company needs more allowances. This is given by the fact that the company produces less plates, but more cuts, which are less carbon-friendly.

The structure of the traded allowances has also been changed; the results can be found in Table 6. It can be seen that the company decreases the number of the traded futures (by almost 8%), and more spots are purchased instead (by almost 59%). Obviously, these changes are caused by the changes in the risk associated with demand at different times (originally, the risk was constant but now it increases in time).

Figure 11 shows that the probability of default has decreased in comparison with the original model with the stationary demand, namely, from 0.0344 (for $p_0 = 52.32$) to 0.0985 (for $p_0 = 13.08$) .

**Table 6** Traded allowances in case of dependent demand compared to the original model ($\lambda = 0.2,\ p_0 = 6.54$)

| Deviation from the original model | 2017 | 2018 | 2019 | 2020 |
|---|---|---|---|---|
| Number of futures | 0 | – 3796 | – 5876 | – 2624 |
| Number of spots | 0 | 9325 | 6799 | – 2422 |

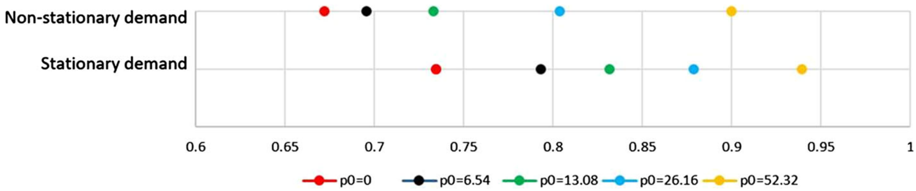



**Fig. 11** Default probability in case of dependent demands compared to the original model ($\lambda = 0.2$)

## 6 Conclusions

The impact of the emissions trading on a real-life risk-averse European steel company is studied in this paper.

Our results are unique in several ways. No such complex model focused on the relationships between the European Emissions Trading System and an industrial company has ever been published before. No analysis focused on a single particular steel company has been performed so far. Moreover, the way of applying the contamination to the post-optimal analysis is also innovative.

The most significant result is that the production of the company, hence the amount of emissions, is little influenced by the emission prices. The main reason for this is that the company typically produces the maximum of the demand and its production limits, and stops doing so only if much higher emission prices are given than those of the 2017 level.[1] What the emission prices do influence, however, are the economic results of the company, and consequently the probability of its survival.

Naturally, such a result raises questions about effectiveness of the emission trading system—once the company is closed, economic and social harms arise, and the steel will be produced abroad, outside of the European regulation framework. Finding a more exhaustive answer to the question of the system's effectiveness would, however, require much broader research than the present case study.

Similarly to the price level, the production and the default probabilities depend little on the risk aversion by the producer. This, however, could have been expected as the production, contrary to the emission trading, is risk-less in our model.

Another question we studied has been to what extent the risk stemming from the emission trading can be reduced by using derivatives, in particular futures, and by banking. Here, apart from zero risk aversion, we obtained non-trivial results showing that it is optimal to use a combination of both with a higher weight of the futures in this mix.

Further we studied the same situation under various perturbations of our model, namely, given positive demands shocks, stochastic margins, and a time dependent demand distribution. Under these perturbations, the results, namely the production amounts, default

[1] Even if the prices started at the prices from the time of submission of this paper, i.e., 20 EUR, the production still would not be affected.

probabilities and optimal emission trading, changed only slightly, which suggests that our results are quite robust.

Although this analysis is based on the data of only one company and despite several simplifications (especially, the approximations of the random parameters) having been assumed, the results of this paper can shed at least some light on the way companies are affected by emission reducing policies.

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