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A novel approach to handle intransitive judgements in industrial control problems

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Abstract

As top managers have to lead decision-making processes every day, they use a various palette of supporting tools, often without the knowledge of their theoretical backgrounds. Nevertheless, especially when dealing with judgements provided by experts, some theoretical assumptions have to be fulfilled. One of them is transitivity, an intuitively appealing property that is usually taken for granted. When incorporated in the decision-making process, intransitivity of judgements or preferences can lead to unwanted consequences and the results of decision-supporting tools can be misleading. It seems that such an issue is not often addressed in industrial control problems involving manufacturing industries. To this end, we offer a simple and efficient method to deal with intransitive preferences in this research domain. To illustrate the applicability of this general mathematical method, we use it to support the risk management process in the food industry. The obtained results provide meaningful managerial implications.

Keywords: Decision Support Systems; Analytic Hierarchy Process; Intransitive Preferences; Risk Management and Control

1. Introduction and objectives

Given the complexity and unpredictability of most business environments, managers generally prefer to deal with core decision-making problems by relying on the experience of decision-maker(s). To such an aim, establishing a link between the possibly non-transitive nature of human preferences and the consistency of mathematical models is often required by the traditional decision-making practice. However, literature does not show compact opinions on such an issue (Amenta et al. (2020)). Indeed, such traditional *multi-criteria decision-making* (MCDM) models as, for instance, the *analytic hierarchy process* (AHP) somehow tends to get a "synthetic" consistency of experts' opinions by manipulating original assessments.

The present paper proposes a decision-making approach making use of spontaneous judgements attributed by selected stakeholders when pairwise comparing decision-making elements. The originally expressed opinions may be fundamental to represent practical business problems despite being (in some cases) potentially non-transitive. In such a direction, their manipulation will be avoided. As a representative industrial field, we present a real case study on the topic of risk management and manufacturing system control in the food industry. We specify that the proposed method can be extended to any sector of activity.

For the sake of brevity, we postponed the detailed theoretical analysis of the proposed method to another, preferably journal, article.

The present paper is organized as follows. The next Section 2 reports the literature review, while section 3 describes the methodological approach proposed to handle the inconsistency of judgements. Section 4 shows a real-world case study of industrial reality. In particular, a real problem on risk management in the food industrial sector is dealt with to test the validity of our method and its applicability to practical problems.

2. State of art

MCDM methods enable decision-makers to establish which solution (or which set of alternatives) represents the best trade-off according to differently weighted evaluation criteria referring to such practical aspects as, for instance, safety & security, cost, productivity, and so on. Among the plethora of existing methods, the literature agrees on considering the AHP as one of the most popular. The AHP is based on the concept of pairwise comparisons between pairs of elements expressed by a decision-making team (Dong and Cooper, 2016), or maybe, by a single expert in the field of interest, in the form of linguistic variables (Franek and Kresta, 2014). judgements of pairwise comparisons have to be collected and aggregated into input matrices, called *pairwise comparison matrices* (PCMs) - see Grzybowski and Starczewski (2020). PCMs will be mathematically manipulated to obtain the vector of weights reflecing the degrees of importance or priorities of the involved elements, the last ones being eventually ranked based on the calculated weights (Liu et al. (2020)). The final ranking well represents evaluations of the expert(s) given an additional assumption of consistency, that may be easily verified mathematically. The key aspect of the AHP is indeed consistency of pairwise comparisons attributed by experts, what directly influence the quality of final decisions (Hsieh et al. (2018)).

However, this normative position of the AHP may, sometimes, be a limiting factor. Indeed, one may easily design decision-making problems leading directly to inconsistent PCMs. This corresponds well to current discussion in the domain of the *expected utility theory* (EUT), introduced in Von Neumann and Morgenstern (1953), where many systematic violations of the basic assumptions, see e.g. Tversky (1969), motivated the development of alternative decision-making theories, such as Fishburn (1988); Starmer (2000); Machina (2004). In particular, the axiom of *transitivity* of preferences is not always supported by empirical evidence, see, e.g., Bar-Hillel and Margalit (1988); Butler et al. (2016). A concise mathematical model of non-transitive decision-making has been proposed in Kreweras (1961); Fishburn (1982), representing preferences with a skew-symmetric bilinear (SSB) functional. Such theory may deal with inconsistent PCMs seamlessly, therefore it will be used to transform the above-elicited PCMs into weight vectors. One thus obtains a method converging to a shared choice among various decision-makers that may express their preferences with no additional limitations on their judgements.

Effective risk management and systems control in the industry have to be undertaken and tailored to the specific sector of activity to maximize safety and security for human resources, as a primary objective, but also to optimize activity outcomes in terms of productivity, quality, and cost-effectiveness. The utmost importance of risk-oriented management is then clear Wu et al. (2014). However, especially under the current circumstances, having the COVID-19 pandemic strongly impacted industrial business worldwide, enterprises are experiencing many practical difficulties when it comes to the real implementation of risk management plans Hubbard (2020). Therefore, offering a structured mathematical approach based on expert experience is important to support companies throughout the optimization of their results.

3. Notation and methodology

Let k be a positive integer, we denote $\mathcal{P}(k) = \left\{p \in \mathbb{R}^k : p \geq 0, \sum_{i=1}^{k} p_i = 1\right\}$ the set of all probability distributions having a finite support of cardinality k, i.e. all convex combinations of k elements. Given a square matrix $\boldsymbol{X}$, we denote its transpose by $\boldsymbol{X}^\intercal$. Matrix $\boldsymbol{X}$ is *skew-symmetric* if $\boldsymbol{X}^\intercal = -\boldsymbol{X}$. A square matrix with positive entries obtained from comparisons between certain attributes following a predefined scale is a *pairwise comparison matrix* (PCM). In detail, the entries of a generic PCM are numerical values translating linguistic judgments of preference between pairs of decision-making elements, i.e. criteria, sub-criteria or alternatives. A PCM matrix $\boldsymbol{X}$ of order k is *reciprocal*, $\boldsymbol{X} \in \mathcal{R}(k)$, if $x_{ji} = 1/x_{ij}$ for all $i, j = 1, \ldots, k$, and *homogeneous* if $x_{ii} = 1$ for all $i = 1, \ldots, k$. A reciprocal matrix $\boldsymbol{X} \in \mathcal{R}(k)$ is *consistent* with a weight vector $w \in \mathcal{P}(k)$ if w reflects the priorities expressed by elements of $\boldsymbol{X}$ in such a way that $x_{ij} = w_i/w_j$ for all $i, j = 1, \ldots, k$.

For a (binary) relation $\succ$ defined on a set S, an element $s \in S$ is a *maximal element* of S with respect to $\succ$ if set $\{q \in S : q \succ s\}$ is empty. We say that $\succ$ is *asymmetric* if $p \succ q$ implies $q \not\succ p$ for all $p, q \in S$, and $\succ$ is *transitive* if $p \succ q$ and $q \succ r$ implies $p \succ r$ for all $p, q, r \in S$. Further, we assume that S is $\mathcal{P}(k)$. If there is a skew-symmetric matrix $\boldsymbol{X}$ that represents relation $\succ$ on set $\mathcal{P}(k)$ as follows:

$$p \succ q \quad \Longleftrightarrow \quad p^\intercal \boldsymbol{X} q > 0 \quad \text{for all} \quad p, q \in \mathcal{P}(k),$$

we say that matrix $\boldsymbol{X}$ is *skew-symmetric bi-linear* (SSB) *representation* of $\succ$, see e.g. Fishburn (1988); Pištěk (2018, 2019). A relation admitting a SSB representation will further be called *SSB preference relation*. Note that such a (preference) relation $\succ$ on $\mathcal{P}(k)$ is asymmetric, but not necessarily transitive.

Finally, for a decision-making problem with m criteria $\{c_1, c_2, \ldots, c_m\}$ and n options $\{o_1, o_2, \ldots, o_n\}$, let us consider pairwise comparison matrices $\boldsymbol{A}, \boldsymbol{B}^{(l)}, l = 1, \ldots, m$, where $\boldsymbol{A} \in \mathcal{R}(m)$ with $a_{ij} > 0$ is a PCM representing the importance of different criteria. $\boldsymbol{B}^{(l)} \in \mathcal{R}(n)$ with $b_{ij}^{(l)} > 0$ is expressing the degree of preference of option o_i over option o_j with respect to l-th criterion, $l \in \{1, \ldots, m\}$.

Example (Leader example). *Let us consider a decision-making problem with $n = 3$ alternatives and $m = 4$ evaluation criteria. We will use the "Tom, Dick, and Harry" example that is described by Wikipedia contributors (2020). This example introduces the real situation in choosing a leader for a company whose founder is about to retire. There are three competing candidates (Tom, Dick, and Harry) and four criteria (Age, Charisma, Education, Experience) for choosing the most suitable candidate. Criteria are pairwise compared and the related judgements of preference a_{ij} are collected in the following input matrix $\boldsymbol{A}$:*

Table 1. $\boldsymbol{A}$: criteria pairwise comparison. Note that the vector of criteria weights for matrix $\boldsymbol{A}$ calculated using the standard AHP-based way reads $\mathbf{w} = [0.0559,\ 0.2699,\ 0.1266,\ 0.5476]^{\mathrm{T}}$.

	Age	Charisma	Education	Experience	CR
Age	1	1/5	1/3	1/7	0.04435
Charisma	5	1	3	1/3	
Education	3	1/3	1	1/4	
Experience	7	3	4	1	

Element a_{ij} of matrix $\boldsymbol{A}$ is the measure of preference of the item in row i when compared to the item in column j. Note that just $\frac{m(m-1)}{2}$ judgements need to be elicited (recall $m = 4$ in this case).

Similarly, we can evaluate the preference of each candidate with respect to a given criterion. We will denote respective preference matrices as $\boldsymbol{B}^{(l)}$ where $l \in \{1, 2, 3, 4\}$.

Table 2. $\boldsymbol{B}^{(l)}$: alternative pairwise comparison in each criterion

(a) $\boldsymbol{B}^{(1)}$: Age

	Tom	Dick	Harry	CR
Tom	1	1/3	5	0.02795
Dick	3	1	9	
Harry	1/5	1/9	1	

(b) $\boldsymbol{B}^{(2)}$: Charisma

	Tom	Dick	Harry	CR
Tom	1	5	9	0.06852
Dick	1/5	1	4	
Harry	1/9	1/4	1	

(c) $\boldsymbol{B}^{(3)}$: Education

	Tom	Dick	Harry	CR
Tom	1	3	1/5	0.06239
Dick	1/3	1	1/7	
Harry	5	7	1	

(d) $\boldsymbol{B}^{(4)}$: Experience

	Tom	Dick	Harry	CR
Tom	1	1/4	4	0.03548
Dick	4	1	9	
Harry	1/4	1/9	1	

The AHP provides a measure of consistency using the so-called *consistency ratio CR*:

$$CR = \frac{CI}{RI}, \tag{1}$$

with CI being the *consistency index*, and RI being the *random index*. RI values correspond to average consistencies of randomly generated matrices, provided by Saaty (2000). For a matrix of order k, CI is defined as:

$$CI = \frac{\lambda_{max} - k}{k - 1}, \tag{2}$$

λ_{max} being the unique largest eigenvalue of the matrix that gives the Perron eigenvector as an estimate of the weight (or priority) vector. Average consistencies (RI values) of randomly generated matrices are provided by Saaty (2000). The CR value has to be lower than a defined threshold for assuring the quality of pairwise comparisons. A $CR \leq 0.1$ generally implies acceptable consistency, being the threshold value even lower for smaller size matrices. One may verify that matrices $\boldsymbol{A}$ and $\boldsymbol{B}^{(l)}$ in the Leader example are consistent enough to be treated by the standard AHP approach. However, consistency conditions are not met for the vast majority of practical problems. This will be the case of the real-world problem in the case study, see Section 4.

3.1. Aggregated preference matrix

We assume that the pairwise comparison of criteria represented by matrix $\boldsymbol{A}$ is consistent (to a high-enough degree), thus we may compute the vector of evaluation criteria weights, $\mathbf{w} \in \mathcal{P}(m)$, in the standard AHP-way. Alternatively, vector $\mathbf{w}$ may be also given directly by experts, cf. the case study in Section 4. However, in our problem setting, this is not assumed for matrices $\boldsymbol{B}^{(l)}$, $l = 1, \ldots, k$, representing the evaluation of options by individual criteria. Trying to represent $\boldsymbol{B}^{(l)} \in \mathcal{P}(n)$ by a vector of weights may thus lead to high information loss. To avoid this issue, we combine matrices $\boldsymbol{B}^{(l)}$ into a PCM matrix called *aggregated preference matrix* $\boldsymbol{P} \in \mathcal{R}(n)$ employing also the weight vector of evaluation criteria $\boldsymbol{w}$.

As underlined by Blagojevic et al. (2016), there are various possible procedures for aggregating judgements of pairwise comparisons and obtaining matrix $\boldsymbol{P}$. The most common is the aggregation of individual judgements and the aggregation of individual weights Abel et al. (2015); Ramanathan and Ganesh (1994), but also models based on consensus convergence Lehrer and Wagner (2012) and 'soft' consensus computations Wu and Xu (2012) have been applied. We use an element-wise (weighted) geometric mean of matrices $\boldsymbol{B}^{(l)}$, i.e.,

$\boldsymbol{P} = (p_{ij})$ is composed of:

$$p_{ij} = \prod_{l} b_{ij}^{(l)^{w_l}}, \; i, j \in \{1, 2, \dots, k\}, \tag{3}$$

where $w_l \geq 0$, $l = 1, 2, \dots, m$ such that $\sum_l w_l = 1$ are criteria weights obtained from matrix A. Let us highlight that both product $\prod$ and power $(B)^w$ in (3) are performed element wise. Note that aggregated preference matrix $\boldsymbol{P}$ is reciprocal.

Example (Continuation of the Leader example). *Note that in case of matrices $\boldsymbol{A}, \boldsymbol{B}^{(l)}$ from the Leader Example, as defined in Tables 1 and 2, respectively, matrix $\boldsymbol{P}$ can be found in Table 3.*

Table 3. Aggregated preference matrix $\boldsymbol{P}$ for the Leader example (with the numbers rounded to three decimal places), computed from (3) using vector $\mathbf{w}$ derived from matrix $\boldsymbol{A}$ in the standard AHP-way, see Table 1, and matrices $\boldsymbol{B}^{(l)}$ from Table 2.

	Tom	Dick	Harry
Tom	1	0.836	2.231
Dick	1.196	1	2.654
Harry	0.448	0.377	1

3.2. Skew-symmetric bi-linear representation of preferences

Let k be a positive integer and $\succ$ be a (preference) relation on $\mathcal{P}(k)$. From the perspective of the EUT, preference relation $\succ$ is rational if and only if it may be represented by a linear functional on $\mathcal{P}(k)$, i.e. there has to exist $\boldsymbol{x} \in \mathcal{P}(k)$ such that

$$\boldsymbol{p} \succ \boldsymbol{q} \quad \Longleftrightarrow \quad \boldsymbol{x}^\mathsf{T}\boldsymbol{p} > \boldsymbol{x}^\mathsf{T}\boldsymbol{q} \quad \text{for all} \quad \boldsymbol{p}, \boldsymbol{q} \in \mathcal{P}(k).$$

Such a representation may not account for possible intransitives of individual preferences. To this end the theory of the SSB representation of preferences has been proposed Fishburn (1982). Let an asymmetric matrix $\boldsymbol{X}$ be a SSB representation of $\succ$, that is

$$\boldsymbol{p} \succ \boldsymbol{q} \quad \Longleftrightarrow \quad \boldsymbol{p}^\mathsf{T}\boldsymbol{X}\boldsymbol{q} > 0 \quad \text{for all} \quad \boldsymbol{p}, \boldsymbol{q} \in \mathcal{P}(k).$$

For such a general $\boldsymbol{X}$ one may easily find examples of $\boldsymbol{p}, \boldsymbol{q}, \boldsymbol{r} \in \mathcal{P}(k)$ such that $\boldsymbol{p} \succ \boldsymbol{q}$, $\boldsymbol{q} \succ \boldsymbol{r}$, and $\boldsymbol{r} \succ \boldsymbol{p}$. This seems to be an insurmountable obstacle for decision-making. However, the well-known Minimax Theorem, see, e.g., Von Neumann and Morgenstern (1953), implies that there is a maximal element $\boldsymbol{s} \in \mathcal{P}(k)$ such that $\boldsymbol{s} \succ \boldsymbol{q}$, $\boldsymbol{s} \succ \boldsymbol{p}$, and $\boldsymbol{s} \succ \boldsymbol{r}$.

Theorem 3.1. *Let $\succ$ be a preference relation on $\mathcal{P}(k)$ that has a SSB representation, then there exists a maximal element of $\mathcal{P}(k)$ w.r.t. $\succ$.*

Recall that elements x_{ij} of $\boldsymbol{X}$ are proportional to the scale of preference of alternative i over j. Thus, for any $\boldsymbol{p}, \boldsymbol{q} \in \mathcal{P}(k)$, one may evaluate the probability vector of $\boldsymbol{p}$ yielding a more preferred outcome than $\boldsymbol{q}$ by $\boldsymbol{p}^\mathsf{T}\boldsymbol{X}\boldsymbol{q}$. This gives a clear interpretation to the maximal element $\boldsymbol{s} \in \mathcal{P}(k)$: satisfying $\boldsymbol{s}^\mathsf{T}\boldsymbol{X}\boldsymbol{q} \geq 0$ for all $\boldsymbol{q} \in \mathcal{P}(k)$, element $\boldsymbol{s}$ yields a more preferred outcome more (or equally) likely than any other probability vector in $\mathcal{P}(k)$. This condition can be equivalently stated as

$$\boldsymbol{X}\boldsymbol{s} \leq 0 \tag{4}$$

using skew-symmetry of $\boldsymbol{X}$. To get the solution of (4), one can easily employ the methods of polyhedral geometry.

3.3. New approach to treat inconsistency in pairwise comparisons

By applying the tools introduced above, we will obtain a new method that can handle the possible inconsistency of experts' judgements well (note, however, that from the perspective of the SSB representation, the AHP-inconsistency is, actually, not an inconsistency).

Let $\boldsymbol{P} \in \mathcal{R}(n)$ be the aggregated preference matrix $\boldsymbol{P}$ given by (3) . To apply the theory of the SSB representation, one needs a skew-symmetric matrix $\boldsymbol{X}$ such that x_{ij}, if positive, represents the scale of preference of i over j. One may come with many ways how to transform aggregated preference matrix P into an SSB matrix; we propose to use an element-wise logarithm $\boldsymbol{X} = \log \boldsymbol{P}$ to this end. Indeed, such a matrix is skew-symmetric using reciprocity of $\boldsymbol{P}$, the sign of x_{ij} indicates if i is preferred to j or vice versa, and the absolute value of x_{ij} corresponds to the scale of such a preference.

The maximal preferred element with respect to such a matrix will be called *the final distribution of preference*, and denoted by $\zeta \in \mathcal{P}(n)$. By using (4), vector ζ satisfies $(\log \boldsymbol{P})\zeta \leq 0$, and so all final distributions of preference form a non-empty polyhedron determined by

$$(\log \boldsymbol{P})\, \zeta \leq 0, \zeta \geq 0, \sum_{i=1}^{n} \zeta_i = 1. \tag{5}$$

Let us recall that such $\zeta \in \mathcal{P}(n)$ leads to a more preferred outcome with higher (or equal) probability than any other probability distribution in $\mathcal{P}(n)$. The whole procedure has been summarised through the flowchart of Figure 1.

Example (Continuation of the Leader example). *To illustrate the fact that $\log \boldsymbol{P}$ is an SSB matrix, let us apply the element-wise $\log$ transformation to matrix $\boldsymbol{P}$ of Table 3. We obtain a matrix $\log \boldsymbol{P}$ shown in Table 4.*

The final distribution of preference ζ calculated by solving the problem from polyhedral geometry (5) is shown in Table 5; note that for the given matrix $\boldsymbol{P}$ it is unique.

The final distribution of preference $\zeta \in \mathcal{P}(n)$ deter-

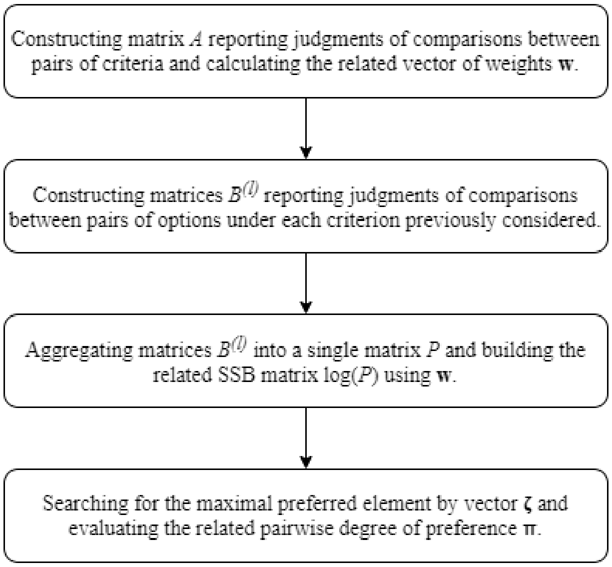

Figure 1. Flowchart summarising the proposed procedure

Table 4. $\log \boldsymbol{P}$ – an SSB matrix

	Tom	Dick	Harry
Tom	0	-0.179	0.802
Dick	0.179	0	0.976
Harry	-0.802	-0.976	0

mined by (5) often has many zero elements, see, e.g. the solution of the Leader example in Table 5. Thus, ζ indicates well which element of $\mathcal{P}(n)$ leads to the best choice more (or equally) likely than any other, but it may not be reasonably used to rank all alternatives. To this end one may evaluate *pair-wise degree of preference* of ζ, defined by

$$\pi = \boldsymbol{P}^{\intercal}\, \zeta. \tag{6}$$

For any $i = 1, \dots, n$ value π_i corresponds to the (expected) degree of preference of ζ over an option i that is measured on the Saaty's scale. By using (5) and skew-symmetry of $\log P$ we have $(\log P)^{\intercal}\zeta \geq 0$, leading to $\pi_i \geq 1$ due to Jensen iequality. This corresponds well to the fact that ζ is optimal.

Example (Continuation of the Leader example)**.** *Having matrix* **P***, the pair-wise degree of preference* π *is shown in Table 6.*

The values 1.196 and 2.654 in Table 6 mean that Dick is preferred 1.196 times more than Tom and 2.654 times than Harry in the holistic evaluation.

4. Case study

The present case study deals with the topic of risk management and system control in the food industrial sector. In detail, we propose a structured way to prioritize interventions of risk management aimed at

Table 5. The final distribution of preference ζ

Tom	Dick	Harry
0	1	0

Table 6. The pair-wise degree of preference π.

Tom	Dick	Harry
1.196	1	2.654

guaranteeing the safety of operators working with a core subsystem belonging to the packaging plant of a food company. The company is located in Italy and is deputed to the production, packaging, and commercialization of marine salt for alimentary use. The core subsystem of the mentioned industrial plant represents one of the packaging lines to where marine salt is routed upon the production stage, to be packed as a finite product and got ready for being dispatched and/or stored in the industrial warehouse. The subsystem is made of five main machines that are: cartoning machine, bundler, shrink-wrapped, palletizer, wrapping machine. Six interventions represent the set of options $(o_n,\ n = 1, \dots, 6)$ of the decision-making problem. They aim to deal with three relevant types of risks connected with the use of the machines belonging to the analyzed subsystem. Main risks had been identified during the previous risk assessment stage. They are: 1. physical and mechanical risks; 2. tripping, entanglement, and falling risks; 3. postural and ergonomic risks. Related interventions aim to potentiate system control and are summarized in Table 7. Their ranking will support to prioritize their implementation. This procedure will constitute an important part of the risk management plan incorporating procedures and best practices aiming at facing uncertainty characterizing current times, strongly affected by the COVID-19 pandemic.

Interventions are going to be evaluated under four criteria, that are: c_1 safety and security, c_2 cost, c_3 productivity, and c_4 hygiene. Criterion c_1 refers to safety of human resources as wells as to the plant's adherence to the regulations in force. Criterion c_2 refers to the implementation of interventions as well as to the potential occurrence of plant shutdown. Criterion c_3 is related to the fulfillment of production standards and system availability. Criterion c_4 lastly evaluates the respect of hygiene conditions for personnel and plant sanitation according to the HACCP manual and to the COVID-19 protocol in force (Iavicoli et al. (2020)). The described criteria have been attributed diverse importance to eventually rank the six interventions. In such a direction, the following numerical weights have been established by collecting judgments of pairwise comparisons within the company and, in particular, from stakeholders familiar with the process under analysis: $\mathbf{w} = [0.399, 0.116, 0.070, 0.415]$. This indicates the maximum prominence of such aspects as hygiene and security. It is clear indeed as these aspects are considered as being much more important than cost and

Table 7. Risk management interventions to be prioritised

ID code	*Intervention description*
o_1	Implementing a semi-automatic lubrication system for those equipment requiring grease on a periodic basis by permanent installation of electro-mechanical grease dispensers with possibility for manual pumping.
o_2	Contracting a specialized external company for regular preventive maintenance of electrical equipment as well as periodic arrangements for necessary settings and normalization operations on the operating machines.
o_3	Improving aspects related to emergency management by simulating several types of industrial plant accidents and sharing information with human resources at any level of the hierarchy structure of the company.
o_4	Implementing, where possible, a program of equipment lockout aimed at minimizing the contact with operators during periodic activities of power supplies control on each work section and machine shutdown components.
o_5	Optimising the control process of power supply by reviewing the number of compressors in operation and by managing their control settings through personnel adequately trained on the compressed air supply side.
o_6	Increasing the frequency of sanitizing interventions on the machines operating in the core subsystem of interest by respecting measures of social distance and adhering to the COVID-19 regulation in force.

Table 8. Evaluation of options with respect to criteria and *CR* values

(a) $B^{(1)}$

	o_1	o_2	o_3	o_4	o_5	o_6	*CR*
o_1	1	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	0.008
o_2	3	1	3	1	1	1	
o_3	2	$\frac{1}{3}$	1	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{2}$	
o_4	3	1	3	1	1	1	
o_5	3	1	2	1	1	1	
o_6	3	1	2	1	1	1	

(b) $B^{(2)}$

	o_1	o_2	o_3	o_4	o_5	o_6	*CR*
o_1	1	2	3	$\frac{1}{3}$	$\frac{1}{3}$	1	0.081
o_2	$\frac{1}{2}$	1	3	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{4}$	
o_3	$\frac{1}{3}$	$\frac{1}{3}$	1	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{4}$	
o_4	3	2	3	1	1	$\frac{1}{2}$	
o_5	3	2	3	1	1	$\frac{1}{3}$	
o_6	1	4	4	2	3	1	

(c) $B^{(3)}$

	o_1	o_2	o_3	o_4	o_5	o_6	*CR*
o_1	1	1	5	1	1	4	0.178
o_2	1	1	4	$\frac{1}{2}$	$\frac{1}{2}$	4	
o_3	$\frac{1}{5}$	$\frac{1}{4}$	1	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	
o_4	1	2	3	1	3	$\frac{1}{2}$	
o_5	1	2	3	$\frac{1}{3}$	1	4	
o_6	$\frac{1}{4}$	$\frac{1}{4}$	3	2	$\frac{1}{4}$	1	

(d) $B^{(4)}$

	o_1	o_2	o_3	o_4	o_5	o_6	*CR*
o_1	1	3	2	1	1	$\frac{1}{7}$	0.103
o_2	$\frac{1}{3}$	1	3	1	1	$\frac{1}{3}$	
o_3	$\frac{1}{2}$	$\frac{1}{3}$	1	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	
o_4	1	1	3	1	1	$\frac{1}{7}$	
o_5	1	1	3	1	1	$\frac{1}{7}$	
o_6	7	3	3	7	7	1	

productivity for risk management.

The general manager of the company has been asked to provide judgements of comparisons between pairs of options under the four previously weighted criteria. The evaluations are reported in Table 8, the last columns giving the values of consistency ratios of comparisons, calculated according to the AHP (Saaty (1977)). We can observe that judgements' consistency is verified via AHP just in the first and in the second cases since the remaining *CR* values surpass the threshold of 0.1. This evidence is particularly strong under the third criterion of productivity while judgements are only slightly inconsistent when comparing options under the fourth criterion of hygiene. The list of vectors $\mathbf{v}^{(l)}$ derived from matrices in Table 8 is in Table 9. The matrix of aggregated preferences P, calculated by substituting vector $\mathbf{w}$ above and matrices from Table 8 into formula (3), is shown in Table 10.

From the optimal distribution of preference ζ in Table 11 it is clear that the execution of intervention o_6 should be scheduled, see also the respective degree of preference π in Table 12. This intervention aims to optimize all the considered criteria with a particular focus on hygiene, safety, and security aspects, that have been associated with higher weights by means of the AHP application. By exchanging feedback with the management of the company, it emerges indeed the primary importance of minimizing potential infection phenomena that may arise within the workplace deputed to the packaging activity. This would have the twofold objective of protecting people and the business activity on the whole. Other actions playing an important part in significantly reducing the main risks for workers will consist in upgrading the lubrication system and the emergency management plan. Such interventions as equipment lockout, systems of power supply control and dedicated maintenance of electrical equipment, despite having certainly a positive impact on people safety and company results, will be postponed on time.

5. Conclusion

We introduced a new approach to deal with problems where transitivity of judgements is not granted. Frequently, experts involved in decision-making are not keen to modify their elicited assessment for mathematical reasons. This can be likely the case of complex industrial control problems. By extending the AHP method with the so-called SSB representation of (possibly) intransitive preferences, we considerably extended its applicability, keeping, however, the well-known problem structure of the AHP. The solution uses proven

Table 9. Matrix V of local weights $\mathbf{v}^{(l)}$

	c1	c2	c3	c4
o1	0.065	0.147	0.215	0.132
o2	0.217	0.096	0.176	0.112
o3	0.095	0.053	0.047	0.061
o4	0.217	0.200	0.236	0.107
o5	0.203	0.191	0.201	0.107
o6	0.203	0.313	0.126	0.480

Table 10. P: Case study

	o1	o2	o3	o4	o5	o6
o1	1	1.108	1.280	0.571	0.571	0.312
o2	0.903	1	3.059	0.883	0.883	0.592
o3	0.781	0.327	1	0.333	0.392	0.380
o4	1.752	1.132	3	1	1.077	0.389
o5	1.752	1.132	2.552	0.928	1	0.428
o6	3.208	1.688	2.635	2.573	2.338	1

Table 11. The optimal distribution of preference ζ.

o1	o2	o3	o4	o5	o6
0	0	0	0	0	1

Table 12. The pair-wise degree of preference π.

o1	o2	o3	o4	o5	o6
3.208	1.688	2.635	2.573	2.338	1

mathematical concepts as polyhedral geometry and linear programming. The proposed approach has been applied to a real-world case study on risk management in the food industry. The goal is to potentiate system control and human safety by relying on original experts' judgments. The following tree actions of risk management have been scheduled as priority interventions: increasing the frequency of sanitizing interventions, implementing a semi-automatic lubrication system and improving aspects related to emergency management. This will have positive influence on enhancing the global level of hygiene, safety and security. A detailed theoretical analysis of the proposed method has been postponed for future research.

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