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Consensus of a nonlinear multi-agent system with output measurements ⋆

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Abstract: The consensus problem of a multi-agent system with nonlinear agents is solved. It is assumed only the outputs of the agents are measurable, in order to obtain the estimate of the state, the nonlinear Luenberger observer is applied. The control of the agents is based on the exact feedback linearization. The method is illustrated by an example.

Keywords: Nonlinear discrete-time system, nonlinear observer, functional equation.

1. INTRODUCTION

Synchronization problem, also known as consensus problem in the recent terminology of control theory, see e.g. Olfati-Saber and Murray (2004); Wen et al. (2013); Li et al. (2010) is a problem that gains a strong attention nowadays. Even if the pioneering works were written for synchronization of linear multi-agent systems, the mathematical models of the complex networks often demonstrate the nonlinear behaviour. Several ways to handle this problem were derived, e.g. by estimating the nonlinearity via the Young inequality, as in Cao et al. (1998). Application of the exact feedback linearization, see Khalil (2001), allows to match the nonlinearities more precisely. Cases of implementation of this method in control of complex systems are numerous. Consensus of nonlinear agents using static output feedback is solved in Wu et al. (2017). Synchronization of complex networks with nontrivial zero dynamics, but, without taking uncertainties into account, was derived in Rehák et al. (2018). Later on, this method was applied in Rehak and Lynnyk (2019) to synchronization of multi-agent systems with nonlinear agents that exhibit time delays.

Let us mention that theory of synchronization of large-scale systems composed of identical subsystems is closely related to the problem of large-scale systems stabilization. Control algorithms for linear systems with interconnections "every subsystem with every other" can be found e.g. in Bakule et al. (2016), which are applicable to systems with delays in the control loop as well. Control of more generally interconnected identical systems is described in Demir and Lunze (2011). Similarly to the aforementioned application of the exact feedback linearization, control of nonlinear large-scale systems based on this approach was presented in Rehák and Lynnyk (2019). Several ideas of these paper are adopted here for the synchronization problem of multi-agent system with a dynamical feedback.

As shown in Kazantzis and Kravaris (1998), one can find a nonlinear counterpart of the Luenberger observer derived in the linear systems theory. The crucial part of the observer design is to derive an equation corresponding to the Sylvester equation that arises in the case of linear Luenberger observers. This equation is a linear first-order partial differential equation (PDE) with non-constant coefficients. An approximation of its solution based on the Taylor polynomials is presented in the original paper Kazantzis and Kravaris (1998).

To show existence of these approximations, the so-called Lyapunov auxiliary theorem is applied. The drawback is that this theorem has too restrictive assumptions to be practically usable: all eigenvalues of the linearization of the original observed system around the origin must have the same sign of the real parts. In particular, systems with purely imaginary eigenvalues are not admitted. This assumption was relaxed in Sakamoto et al. (2014) by proposing an iterative method to solve the PDE. This method is based on an iterative method originally developed for computation of the stable, center-stable etc. manifolds, see Sakamoto and Rehák (2011). This approach was successfully applied in Tran et al. (2017).

The method for center-manifold computation exhibits also analogies to the regulator equation known in the nonlinear output regulation problem. This equation was be numerically approximated by the finite-element method (FEM) in Rehák and Čelikovský (2008); Rehák et al. (2009), the FEM was used for the observer design as well. An advantage of the FEM is possibility to prove existence of an L^2 solution of this PDE on a pre-defined domain, see Rehák (2011). This approach was successfully extended to the observer problem described in Kazantzis and Kravaris (1998) in the paper Rehák (2019). This encouraged us to apply this approach the presented paper as well.

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Purpose of this paper

The purpose of this paper is to combine the exact feedback linearization-based method for the solution of the synchronization problem of nonlinear multi-agent systems with the nonlinear observer developed by Kazantsis and Kravaris. This yields an algorithm for synchronization of a nonlinear multi-agent system with output measurements fully employing the nonlinear structure. We believe this problem has not been studied before.

2. GRAPH THEORY

In this section, the basic notions of the graph theory necessary for developing the multi-agent control are repeated. For a most thorough description, the reader is advised to check e.g. Li et al. (2010). The network describing the interconnected agents is composed of N nodes.

The nodes are denoted by integers, the set of all nodes is $\mathbf{N} = \{1, \dots, N\}$. Assume the set $\mathbf{E} \subset \mathbf{N} \times \mathbf{N}$ be defined as follows: $(i, j) \in \mathbf{E}$ if and only if the node i sends information to the node j.

Assumption 1. It is supposed $(i, i) \notin \mathbf{E}$.

The graph describing the topology of the entire network is given as $\mathbf{G} = (\mathbf{N}, \mathbf{E})$. As evident from Assumption 1, it contains no loops.

The $N \times N$-dimensional *adjacency matrix* $J = (e_{ij})$ is defined as $J_{ij} = 1$ if and only if $(i, j) \in \mathbb{E}$, otherwise $J_{ij} = 0$. Define also the Laplacian matrix L by $L = \mathrm{diag}(\sum_{j=1}^{N} J_{1j}, \dots, \sum_{j=1}^{N} J_{Nj}) - J$.

The graph $\mathbf{G}$ is said to *contain a spanning tree* if, for every $i, j \in \mathbf{N}$, there exists a directed path from the node i to j.

Assumption 2. The graph $\mathbf{G}$ is undirected: if $(i, j) \in \mathbf{E}$ then also $(j, i) \in \mathbf{E}$.

This assumption says that, if the control of the ith agent uses information from the jth agent then also the information about the state of the ith agent is required to compute the control of the jth agent. This assumption is just to simplify the computations; extension of the results to the directed graphs will be a subject of a future publication.

For the proof of the following result see Chen et al. (2010):

Lemma 1. If the undirected graph $\mathbf{G}$ contains a spanning tree then 0 is a simple eigenvalue of the Laplacian matrix L corresponding to the eigenvector $\mathbf{e} = (1, \dots, 1)^T \in \mathbb{R}^N$. Moreover, there exist an orthogonal matrix T and a diagonal matrix Δ such that

$$T^T L T = \Delta. \tag{1}$$

Corollary 1. Under the assumptions of Lemma 1, one has

$$L\mathbf{e} = 0. \tag{2}$$

Without loss of generality, it is possible to assume that $\Delta = \mathrm{diag}(0, d_1, \dots, d_{N-1})$ where d_i are constants satisfying $0 < d_1 \leq \dots \leq d_{N-1}$.

3. PROBLEM SETTING

Let $f, g : \mathbb{R}^n \to \mathbb{R}^n$, $h : \mathbb{R}^n \to \mathbb{R}$ be smooth functions, $f(0) = 0$, $g(0) \neq 0$.

Let $\mathcal{A}$ and $\mathcal{C}$ be Jacobi matrices of f and h, respectively, evaluated at the origin.

Assumption 3. The pair $(\mathcal{C}, \mathcal{A})$ is observable.

The multi-agent system is composed of N identical agents

$$\dot{x}_i = f(x_i) + g(x_i)u_i, \tag{3}$$

$$y_i = h(x_i). \tag{4}$$

The goal is to find a synchronizing control u_i for every agent so that, with $\bar{x} = \frac{1}{N}\sum_{i=1}^{N} x_i$, the following holds

$$\lim_{t\to\infty} \|x_i(t) - \bar{x}(t)\| = 0. \tag{5}$$

It is assumed the only measurable quantity is the output y. Since the controller requires the state knowledge, an estimate of the state must be provided by an observer.

Due to this, the control law of the ith agent is

$$u_i = K \sum_{j\in\mathcal{N}_i} (\mathcal{T}(\hat{x}_j) - \mathcal{T}(\hat{x}_i)) \tag{6}$$

where:

- $\hat{x}_i$ is the state estimate of the ith agent provided by a state observer (to be proposed).
- $\mathcal{T} : \mathbb{R}^n \to \mathbb{R}^n$ is a diffeomorphism to be determined later.
- K is the control gain to be also determined later.

The state observer is given by

$$\dot{\hat{x}}_i = f(\hat{x}_i) + g(\hat{x}_i)u_i + L(\hat{x}_i)(y_{-}h(\hat{x}_i)). \tag{7}$$

where the observer gain is proposed in the following section.

4. LUENBERGER OBSERVER

The nonlinear Luenberger observer was introduced in Kazantzis and Kravaris (1998), further generalized to systems with delays in Kazantzis and Wright (2005). In this paper, this observer is used.

At the beginning, matrix $\tilde{A} \in \mathbb{R}^{n\times n}$ is chosen so that

$$\max\ \mathrm{Re}\ \mathrm{eig}(\tilde{A}) < \min\Big(\min\ \mathrm{Re}\ \mathrm{eig}(\frac{\partial f}{\partial x}(0)), 0\Big). \tag{8}$$

Moreover, choose a vector $b \in \mathbb{R}^n$ so that the pair $(\tilde{A}, b)$ is controllable.

For each agent define function $\Phi : \mathbb{R}^n \to \mathbb{R}^n$ that satisfies equation

$$\frac{\partial \Phi}{\partial x} f(x) = \tilde{A}x + bh(x),\ \Phi(0) = 0. \tag{9}$$

Then, the observer gain is defined for $x' \in \mathbb{R}^n$ as

$$L(x') = \Big(\frac{\partial \Phi}{\partial x}(x')\Big)^{-1} b. \tag{10}$$

With this definition, let the observer of the ith agent be given as

$$\hat{x}_i = f(\hat{x}_i) + g(\hat{x}_i)u_i + L(\hat{x}_i)\Big(h(x_i) - h(\hat{x}_i)\Big). \tag{11}$$

As shown in Kazantzis and Kravaris (1998), if $u = 0$, then $\lim_{t\to\infty} \|x(t) - \hat{x}(t)\| = 0$. As will be evident from the considerations below, this holds true even in presence of the control but under the condition that function g is constant.

Note also that there is a neighborhood of the origin where function Φ has a non-singular Jacobi matrix, hence the observer gain is correctly defined there. This fact also implies that the mapping Φ is a diffeomorphism.

If this condition is violated, some more thorough analysis is necessary. It is conducted along the lines of the proof given in Kazantzis and Kravaris (1998). Let $z_i = \Phi(x_i)$, $\hat{z}_i = \Phi(\hat{x}_i)$. Then

$$\begin{aligned}&\dot{z}_i - \dot{\hat{z}}_i\\ =&\frac{\partial \Phi}{\partial x}(x_i)(f(x_i) + g(x_i)u_i)\\ &-\frac{\partial \Phi}{\partial x}(\hat{x}_i)\Big(f(\hat{x}_i) + g(\hat{x}_i)u + L(\hat{x}_i)(h(x_i) - h(\hat{x}_i))\Big)\\ =&\tilde{A}(\Phi(x_i) - \Phi(\hat{x}_i)) + \Big(\frac{\partial \Phi}{\partial x}(x_i)g(x_i) - \frac{\partial \Phi}{\partial x}(\hat{x}_i)g(\hat{x}_i)\Big)u_i.\end{aligned}$$

Assumption 4. There exists a positive constant $M > 0$ so that for every $x', x'' \in \mathbb{R}^n$ holds

$$\|\frac{\partial \Phi}{\partial x}(x')g(x') - \frac{\partial \Phi}{\partial x}(x'')g(x''\| \leq M\|x' - x''\|. \tag{12}$$

Eq. 12 together with the fact that function Φ is a diffeomorphism imply existence of a constant $M' > 0$ so that

$$\|\frac{\partial \Phi}{\partial x}(x_i)g(x_i) - \frac{\partial \Phi}{\partial x}(\hat{x}_i)g(\hat{x}_i)\| \leq M'\|z_i - \hat{z}_i\|. \tag{13}$$

Moreover, matrix $\tilde{A}$ was chosen to be Hurwitz. Therefore, for every $c > 0$ there exists matrix $P \in \mathbb{R}^{n\times n}$, $P > 0$ satisfying

$$\tilde{A}^T P + P\tilde{A} = -cI_n. \tag{14}$$

Define for $i = 1, \ldots, n$ Lyapunov functions $V_i = (z_i - \hat{z}_i)^T P(z_i - \hat{z}_i)$. Then

$$\dot{V}_i \leq -c(z_i - \hat{z}_i)^2 + M'(z_i - \hat{z}_i)^2\|u_i\|. \tag{15}$$

Hence $\dot{V}_i < 0$ if $c > M'\|u_i\|$ for all $t \geq 0$.

Remark 1. The pioneering work Kazantzis and Kravaris (1998) proves existence of a solution of (9) under rather restrictive assumptions. To be specific, it was required that eigenvalues of the Jacobi matrix of f lie all in the left complex half-plane or all lie in the right complex half-plane. This was required as the original proof relied on the Lyapunov auxiliary theorem. An alternative proof was given in Sakamoto et al. (2014) where the requirement (8) was introduced. A proof of existence of a solution of (9) based on the finite element method was presented in Rehák (2019), again under milder assumptions that the original proof.

5. SYNCHRONIZATION OF THE MA SYSTEM

The exact feedback linerization is applied to every agent.

Assumption 5. The relative degree of system (3,4) is n.

Denote

$$\begin{aligned}\xi_{1,i} =& h(x_i),\\ \xi_{2,i} =& (L_F h)(x_i),\\ &\vdots\\ \xi_{n,i} =& (L_f^{n-1}h)(x_i).\end{aligned}$$

Let also $\xi_i = (\xi_{i,1}, \ldots, \xi_{i,n})^T$. Then there exists a pair of functions $\varphi, \psi : \mathbb{R}^n \to R$ such that

$$\dot{\xi}_{n,i} = \varphi(\xi_i) + \psi(\xi)u_i.$$

Let $\overline{\varphi} = \frac{\partial \varphi}{\partial \xi_i}(0)$, $\widetilde{\varphi}(\xi_i) = \varphi(\xi_i) - \overline{\varphi}\xi_i$ and $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^n$ be defined as

$$A = \begin{pmatrix} 0 & 1 & 0 & \ldots & 0\\ 0 & 0 & 1 & \ldots & 0\\ \vdots & \vdots & \vdots & & \vdots\\ 0 & 0 & 0 & \ldots & 1\\ \hline & & \overline{\varphi} & & \end{pmatrix},\ B = \begin{pmatrix}0\\0\\ \vdots\\0\\1\end{pmatrix}. \tag{16}$$

Moreover, let the transformation of the control be defined as $v_i = \widetilde{\varphi}(\xi_i) + \psi(\xi_i)u_i$. Then

$$\dot{\xi}_i = A\xi_i + Bv_i. \tag{17}$$

Let also $\xi = (\xi_1^T, \ldots, \xi_N^T)^T$ and $v = (v_1, \ldots, v_N)^T$. Moreover, define the mapping $\mathcal{T} : \mathbb{R}^n \to \mathbb{R}^n$ as $\mathcal{T}(x') = (h(x'), (L_f h)(x'), \ldots, (L_f^{n-1}h)(x'))^T$. Due to Assumption 5, this mapping is well defined and is a diffeomorphism on a neighborhood of the origin.

The goal is to synchronize the set of systems (17). As the synchronized systems are linear, this does not pose any particular problems.

Assume that there is a synchronizing control $v = (I \otimes K)\xi$. Then, the controlled system obeys the relation

$$\dot{\xi} = (I_N \otimes A)\xi + (L \otimes BK)\xi. \tag{18}$$

This translates into the (x, u) coordinates as

$$u_i = \frac{1}{\psi(\mathcal{T}(x_i))}\Big(\sum_{j\in\mathcal{N}_i} K(\mathcal{T}(x_j) - \mathcal{T}(x_i)) - \widetilde{\varphi}(\mathcal{T}(x_i))\Big). \tag{19}$$

This control synchronizes the MA system composed of agents (3), however, this control is not realizable as it requires knowledge of the states. Hence the states x_i and x_j are replaced by their estimates provided by the observers (it is assumed the ith agent has access to these estimates of its neighbors). Thus (19) is replaced by

$$u_i = \frac{1}{\psi(\mathcal{T}(\hat{x}_i))}\Big(\sum_{j\in\mathcal{N}_i} K(\mathcal{T}(\hat{x}_j) - \mathcal{T}(\hat{x}_i)) - \widetilde{\varphi}(\mathcal{T}(\hat{x}_i))\Big). \tag{20}$$

The goal now is to find the control gain K. With the transformed system (17), define the disagreement vector $\varepsilon = \xi - \mathbf{1} \otimes \bar{\xi}$.

Let us assume that the state ξ is available to the controller and, consequently, the state feedback can be implemented. The disagreement dynamics obeys the following equation in the case of the state feedback:

$$\dot{\varepsilon} = (I_N \otimes A)\varepsilon + (L \otimes BK)\varepsilon. \tag{21}$$

Using the well-known procedure (see e.g. Li et al. (2010)), one can design the control gain K so that $\lim_{t\to\infty} \|\varepsilon\| = 0$. Therefore there exists a matrix $Q \in \mathbb{R}^{n\times n}$, $Q > 0$ so that there exists a constant $c_W > 0$ so that

$$A^T Q + QA = -c_W I_N. \tag{22}$$

Hence, for function $W = \varepsilon^T(I_N \otimes Q)\varepsilon$ holds

$$\dot{W} \leq -c_W\|\varepsilon\|^2. \tag{23}$$

However, the state x_i is not available to the controller. Hence also the vectors ξ_i and ξ are not available as well, hence the vector ξ must be replaced by some estimate $\hat{\xi} = \mathcal{T}(\hat{x})$. Equation (18) is replaced by

$$\dot{\xi} = (I_N \otimes A)\xi + (L \otimes BK)\hat{\xi}. \tag{24}$$

This implies that (21) is replaced by

$$\dot{\varepsilon} = (I_N \otimes A)\varepsilon + (L \otimes BK)\varepsilon + (L \otimes BK)(\hat{\xi} - \xi). \tag{25}$$

From this, one has using (23)

$$\dot{W} \le -c_W\|\varepsilon\|^2 + \varepsilon^T(L\otimes QBK)(\hat{\xi}-\xi). \tag{26}$$

Define now the Lyapunov function $\mathcal{V}(\xi)$ by

$$\mathcal{V} = \sum_{i=1}^{N} V(z_i - \hat{z}_i) + W(\varepsilon). \tag{27}$$

Then its derivative obeys

$$\begin{aligned}\dot{\mathcal{V}} \le& \sum_{i=1}^{n} \dot{V}(z_i-\hat{z}_i) + \dot{W}(\varepsilon)\\ \le& \sum_{i=1}^{N} -c(z_i-\hat{z}_i)^2 + M'(z_i-\hat{z}_i)^2\|u_i\|\\ &- c_W\|\varepsilon\|^2 + \varepsilon^T(L\otimes QBK)(\hat{\xi}-\xi).\end{aligned}$$

First, note that, since $\xi = \mathcal{T}(x)$, $z_i = \Phi(x_i)$ and both mappings Φ and $\mathcal{T}$ are diffeomorphisms, there exists a constant $\varkappa > 0$ so that, with $z = (z_1^T,\ldots,z_N^T)^T$ and $\hat{z} = (\hat{z}_1^T,\ldots,\hat{z}_N^T)^T$

$$\varkappa\|\xi-\hat{\xi}\| \ge \|z-\hat{z}\|. \tag{28}$$

Then

$$\begin{aligned}\dot{\mathcal{V}} \le& \sum_{i=1}^{N} -c\varkappa^2\|\xi-\hat{\xi}\|^2 + M'\varkappa^2\|\xi-\hat{\xi}\|^2\|u_i\|\\ &- c_W\|\varepsilon\|^2 + \varepsilon^T(L\otimes QBK)(\hat{\xi}-\xi).\end{aligned}$$

Moreover, there exists a constant $\bar{c} > 0$ so that

$$\varepsilon^T(L\otimes QBK)(\hat{\xi}-\xi) \le \bar{c}\|\varepsilon\|\|\xi-\hat{\xi}\|.$$

Hence, for every $\alpha > 0$ holds

$$\varepsilon^T(L\otimes QBK)(\hat{\xi}-\xi) \le \frac{\bar{c}\alpha}{2}\|\varepsilon\| + \frac{\bar{c}}{2\alpha}\|\xi-\hat{\xi}\|.$$

Hence, with $C = \sup_{t\ge 0}\|u_i(t)\|$

$$\begin{aligned}\dot{\mathcal{V}} \le& \sum_{i=1}^{N} -\Big(c\varkappa^2 - M'\varkappa^2 C - \frac{\bar{c}}{2\alpha}\Big)\|\xi-\hat{\xi}\|^2\\ &- (c_W - \frac{\bar{c}\alpha}{2})\|\varepsilon\|^2.\end{aligned}$$

Theorem 5.1. The synchronization is achieved with control (20) if conditions

$$0 < c_W - \frac{\bar{c}\alpha}{2}, \tag{29}$$

$$0 < c\varkappa^2 - M'\varkappa^2 C - \frac{\bar{c}}{2\alpha}, \tag{30}$$

$$C > \sup_{t\ge 0}\|u_i(t)\| \tag{31}$$

hold.

Remark 2. From the above considerations follows that one has to assume boundedness of the control signals in all agents. This is due to the presence of the term $\frac{\partial\Phi}{\partial x}(x)g(x) - \frac{\partial\Phi}{\partial x}(\hat{x})g(\hat{x})$ in the dynamics of the difference $z - \hat{z}$.

Remark 3. The specific procedure how to design the synchronizing the control for a nonlinear system admitting the exact feedback linearization is presented e.g. in Rehak and Lynnyk (2019) or Rehák and Lynnyk (2020), hence it is not presented here in detail. The procedure presented in Rehák and Lynnyk (2021) can be used to synchronize the linearized multi-agent system in presence of time delays and disturbances.

6. EXAMPLE

As an example system, five interconnected systems (agents) interconnected into a ring topology are chosen. Each agent system is governed by the following equations (the index denoting the agent's number is omitted in this introductory part):

$$\begin{aligned}\dot{x}_1 =& x_2,\\ \dot{x}_2 =& -(x_1+x_1^3)e^{x_1} - 0.1x_2,\\ y =& x_1\end{aligned}$$

It is chosen

$$\tilde{A} = \begin{pmatrix}-1 & 0\\ 0 & -2\end{pmatrix},\quad b = \begin{pmatrix}1\\1\end{pmatrix}.$$

From these definitions follows that function Φ is determined as

$$\Phi(x_1,x_2) = \begin{pmatrix} 0.2368x_1 - 0.2632x_2 - 0.1657x_1^2\\ +0.0975x_1x_2 - 0.1218x_2^2 - 0.1590x_1^3\\ +0.1383x_1^2x_2 - 0.0544x_1x_2^2 + 0.0777x_2^3\\ \\ 0.1979x_1 - 0.1042x_2 - 0.0391x_1^2\\ +0.0260x_1x_2 - 0.0144x_2^2 - 0.0467x_1^3\\ +0.0368x_1^2x_2 - 0.0207x_1x_2^2 + 0.0122x_2^3 \end{pmatrix}$$

and finally, with $x' = (x_1', x_2')^T$,

$$L(x') = \begin{pmatrix} 0.0264 + 0.2339x_1' - 0.1461x_2' - 0.3387x_1'^2\\ +0.1678x_1'x_2' + 0.1787x_2'^2\\ \\ 0.0937 - 0.0522x_1' + 0.0548x_2' - 0.1033x_1'^2\\ +0.0322x_1'x_2' + 0.0159x_2'^2 \end{pmatrix}.$$

The exact feedback linearization of an agent yields a linear system

$$\dot{\xi} = \begin{pmatrix}0 & 1\\ -1 & -0.1\end{pmatrix}\xi + \begin{pmatrix}0\\1\end{pmatrix}v \tag{32}$$

with $\xi_1 = x_1, \xi_2 = x_2$ and $v = u + (-x_1 - x_1^3)e^{x_1} + x_1$.

Then, the LQ control design yields $K = (0.414,\ -1.256)$.

Let us turn attention to the implementation the aforementioned observer design to the multi-agent synchronization. The ith agent (whose state is denoted $x_i = (x_{i,1}, x_{i,2})^T$) is endowed with an observer

$$\begin{aligned}\dot{\hat{x}}_{i,1} =& \hat{x}_{i,2} + L_1(\hat{x}_{i,1},\hat{x}_{i,2})(x_{i,1}-\hat{x}_{i,1}),\\ \dot{\hat{x}}_{i,2} =& -(\hat{x}_{i,1}+\hat{x}_{i,1}^3)e^{\hat{x}_{i,1}} - 0.1\hat{x}_{i,2}\\ &+ L_2(\hat{x}_{i,1},\hat{x}_{i,2})(x_{i,1}-\hat{x}_{i,1}) + u_i.\end{aligned}$$

The control v_i designed as the synchronizing control for a linearized agents is (since $\xi_i = x_i$):

$$\begin{aligned}v_i =& K(\hat{\xi}_{i+1} + \hat{\xi}_{i-1} - 2\hat{\xi}_i) = K(\hat{x}_{i+1} + \hat{x}_{i-1} - 2\hat{x}_i)\\ &\text{if } i = 2,3,4\\ v_1 =& K(\hat{\xi}_2 + \hat{\xi}_5 - 2\hat{\xi}_1) = K(\hat{x}_2 + \hat{x}_5 - 2\hat{x}_1),\\ v_5 =& K(\hat{\xi}_1 + \hat{\xi}_4 - 2\hat{\xi}_5) = K(\hat{x}_1 + \hat{x}_4 - 2\hat{x}_5),\end{aligned}$$

Then u_i is given as

$$u_i = v_i - (-\hat{x}_{i,1} - \hat{x}_{i,1}^3)e^{\hat{x}_{i,1}} - \hat{x}_{i,1} \tag{33}$$

The initial conditions of the states $x_{i,1}$ were chosen as $(0.1, 0.2, -0.1, 0.01, 0.15)$, initial conditions in the states $x_{i,2}$ as well as all initial conditions of the observers were set to zero.

The results can be seen in the following figures. In Fig. 1, the synchronization of the states $x_{i,1}$ is depicted for all agents. The analogous phenomenon is shown in Fig. 2, in this case, for the second states. One can see that the synchronization is achieved.

The state estimate of the second part of the first and third agents (the state $x_{2,i}$ and its estimate) can be seen in Fig. 3. The blue line stands for the first agent: solid line: state $x_{1,2}$, dashed line: $\hat{x}_{1,2}$. The red lines represent the analogous states for the third agent: $x_{3,2}$, dashed line: $\hat{x}_{3,2}$. One can see that the estimate converges to the state of the system faster than the synchronization is achieved.

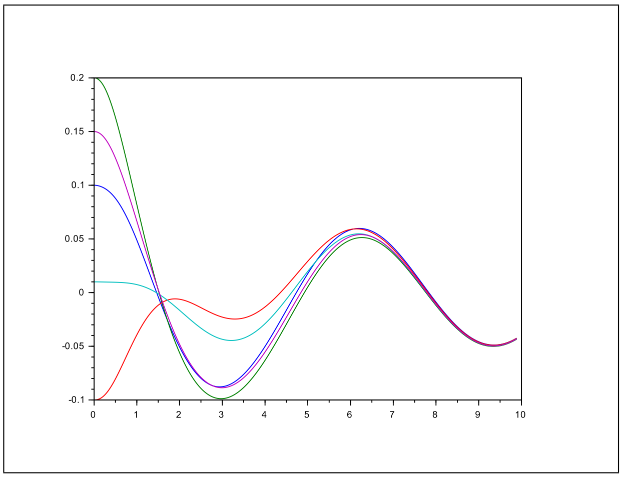

Fig. 1. Synchronization of the first state of the agents

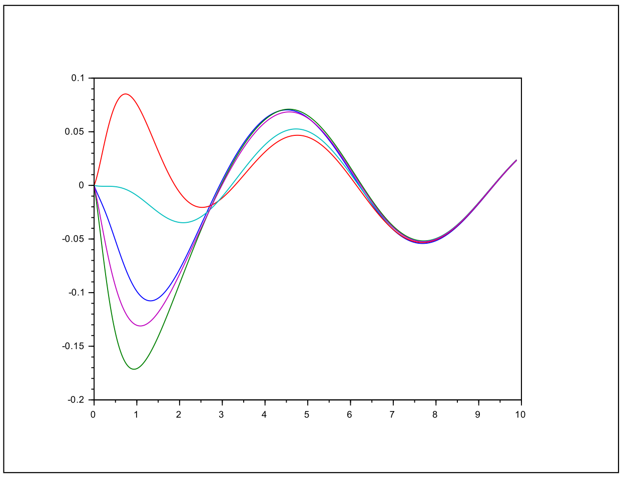

Fig. 2. Synchronization of the second state of the agents

7. CONCLUSIONS

An algorithm for synchronization of a multi-agent system with nonlinear agents is presented. The dynamic output feedback is used, the observer is designed using the adaptation of the Luenberger approach. It was shown that the synchronization is achieved if the synchronizing control as well as the observer satisfy certain conditions that are derived in the paper. The results are illustrated by an example. In future, the case of delayed measurements of the agent's states will be investigated.

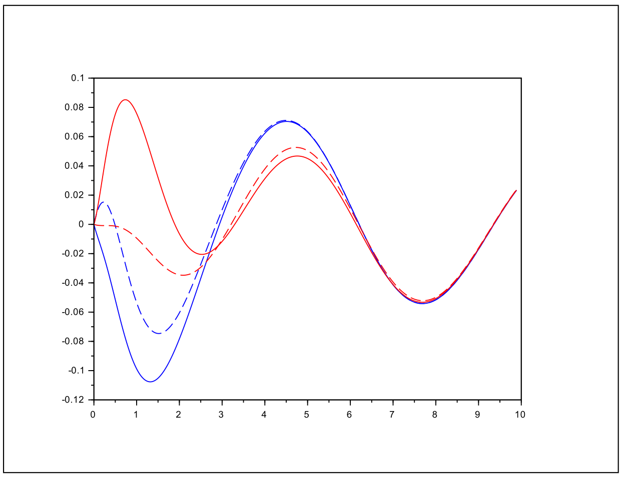

Fig. 3. Selected states and their estimates

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